An Efficient Tabu Search Algorithm for Robust Solutions of Electromagnetic Design Problems

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A tabu search algorithm having robust solution searching power is proposed. To avoid excessive computation burdens which are needed for the expected fitness evaluations in tabu searches, it is proposed that the Gaussian distributed neighborhood vector, the strategy for assigning expected fitness only to promising solutions, and the utilization of neighborhood solutions to determine the fitness values of the potential solutions are to be used. A new distance-weighted formula for the expected fitness assignments is also proposed. Two case studies are reported to validate and demonstrate the feasibility and advantages of using the proposed algorithm in finding the robust solution of an optimal design problem.

***Index Terms*—Expected fitness, optimal design, robust solution, tabu search method.**

I. Introduction

TRADITIONALLY, the focus and indeed the ultimate goal of most optimization study is to find the global optimal designs. However, in practical engineering design problems, there are inevitably uncertainties or perturbations which are unavoidable. In such cases, if the global solution is very sensitive to perturbations in the design variables, a small variation of the optimized parameters could give rise to significant performance degradation. Therefore, the preferable design solution is not the globally optimal solution, but the one that is close to the optimal solution while having a high tolerance or robustness to small variations in the decision parameters. In this regard, there are increasing endeavors devoted to the study of robust design techniques [1], [2]. Nevertheless, studies on robust design techniques have a short history of less than ten years, and are still in their infancy. This paper proposes a tabu-search-based methodology for finding the robust design of an inverse problem while preserving the high computational efficiency of available tabu search algorithms.

Generally speaking, a robust solution means the design is insensitive to small variations of the design variables. To search for the robust solution of a design problem, an expected fitness function, as defined below, rather than the original objective function, is commonly used [1], [3]

$$f_{\exp}(x) = \int_{-\infty}^{\infty} f(x+\delta)p(\delta)d\delta \tag{1}$$

where δ is a disturbance on the design variable x that is distributed based on a probability function $p(\delta)$; $f(x)$ is the objective function of the optimal problem.

Since there is no closed-form objective function $f(x)$ for an inverse problem, the expected fitness function of a solution x is generally determined from

$$f_{\exp}(x) = \frac{1}{n}\sum_{i=1}^{n} f(x_i) \tag{2}$$

where n is the number of sample points generated in the neighborhood of the specific point x.

From (2) it is obvious that to determine the expected fitness value of a specific solution, a set of additional function evaluations are required. Considering the fact that thousands of iterations are generally required for a global optimizer to converge to the final solution and fitness evaluations are expensive because of the common usage of finite-element analysis, the determination of the expected fitness using (2) is clearly not viable for many engineering design problems. Consequently, the evaluation of the expected fitness function constitutes the most important issue in robust optimal studies.

II. Tabu Algorithm With Robust Solution Searching Methodologies

To develop an efficient and vigorous optimizer for the robust design of an engineering electromagnetic design problem, some novel robust solution searching methodologies are proposed and integrated into an universal tabu method [4]. To facilitate the description of the proposed algorithm, its iterative procedures are first summarized as follows.

Initialize: Define the neighborhood vector H; Generate the initial solution and evaluate its expected fitness.

stopcriterion := false

While stopcriterion := false do

> Generate a number of feasible solutions in the neighborhood of the current solution x using neighborhood vector H;
>
> Let x^* be the best one among these neighborhood solutions;
>
> Sort the neighborhood solutions whose distance to x^* is less than a threshold value $D_{\exp}$, and denote them by $x_1^*, x_2^*, \cdots, x_l^*$;
>
> If l is less than $l_{\exp}$, generate randomly $l_{\exp} - l$ new neighborhood solutions within the neighborhood size $D_{\exp}$ in accordance to a Gaussian distribution;
>
> Determine the expected fitness value of x^* using (4);
>
> $x = x^*$

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A. Determination of the Neighborhood Vector H

As similar to a general-purpose tabu search method, a set of new feasible solutions are generated in the neighborhood of each current point x according to a predefined neighborhood vector H in every iterative cycle of the proposed algorithm. However, to make full use of the neighborhood solutions so as to obtain an accurate estimate of the expected fitness with minimal number of additional samples, the components of the neighborhood vector H are determined in accordance to a Gaussian distribution, i.e., in accordance to the following probability density:

$$H(h) = \frac{\exp\left(-\frac{(h-x)^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma}. \tag{3}$$

Moreover, as described previously, when the number of the total neighborhood solutions, whose distances to the best one of all neighborhood solutions are less than a threshold value $D_{\rm exp}$, is smaller than a predefined integer $l_{\rm exp}$, some new samples will be generated around this best solution in accordance to the same Gaussian distribution rule for the same reason as explained previously.

B. Strategy for Expected Fitness Evaluation

For a design problem, only promising solutions, or more concretely the local optimals, have the potential to be the robust solutions. Therefore, it is unnecessary to evaluate the expected fitness values of all intermediate solutions. However, it is difficult to determine if an intermediate solution is a local optimal during the optimizing process. To avoid the difficulty in checking whether an intermediate solution is a local optimal and eliminate the unnecessary assignment of the expected fitness values to intermediate solutions, a novel strategy for the expected fitness computations is introduced, i.e., in every iterative cycle, the expected fitness assignment procedure is activated only for the best neighborhood solution. Obviously, the application of such a strategy will save a huge amount of computation costs, which are otherwise required by common robust optimal methods, in which the expected fitness values for all the solutions are uniformly assigned.

C. Estimation of Expected Fitness

To reduce the excessive computational burden for the expected fitness evaluations, the information gathered from the neighborhood solutions of the current state is used in the proposed algorithm in the determination of the expected fitness value of a promising solution x^*. Since a Gaussian distribution neighborhood vector H is used, additional sample points are required for some very rare cycles in which the number of the neighborhood solutions, whose distances to the promising solution are less than a predefined value $D_{\rm exp}$, is smaller than a threshold integer $l_{\rm exp}$. Consequently, the expected fitness values of promising solutions can be determined with marginal increases in the number of function evaluations in the proposed algorithm. Moreover, the expected fitness value of a potential solution x^* is computed in the proposed algorithm using

$$f(x^*) = \sum_{i=1}^{l_{\rm exp}} w_i f(x_i^*) \Bigg/ \sum_{i=1}^{l_{\rm exp}} w_i \tag{4}$$

where $w_i = 1/\|x^* - x_i^*\|$ is the distance-weighted contribution of the ith nearest neighbors of x^*, as defined previously.

TABLE I
DETAILS OF THE 2-D TEST FUNCTION

Centre (Peak Location)	Width (σ)	Amplitude (β)
(1,1) (Local optimal)	0.3	0.7
(1,3) (Local optimal)	0.4	0.75
(3,1) (Robust optimal)	1.0	1.0
(3,4) (Global optimal)	0.4	1.2
(5,2) (Local optimal)	0.6	1.0

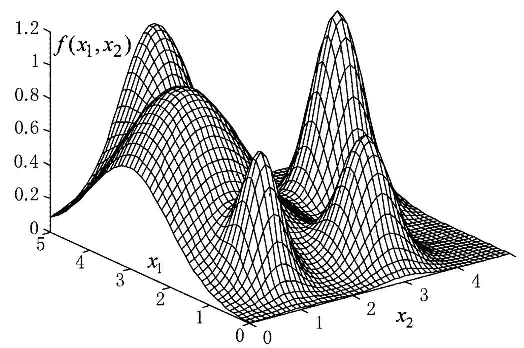

Fig. 1. Distribution of the peaks of the 2-D test mathematical function.

III. NUMERICAL EXAMPLES

A. Mathematical Validation

To validate the proposed algorithm, some well-designed mathematical functions, which are created by using an expansion in terms of the Gaussian basis function [5], are selected and extensively studied. The general definition of this series of functions is given by

$$f(x) = \sum_{i=1}^{m} \beta_i e^{-\sum_{j=1}^{d} \frac{(x_j - c_{ij})^2}{2\sigma_i^2}} \tag{5}$$

where β_i, c_{ij}, σ_i are, respectively, the amplitude, center, and width of the basic function; m is the total number of basis functions.

Because of space limitations, only the experimental results on a specific 2-D test function are reported. This 2-D test function is created by randomly generating the parameters $\beta_i, c_{ij}, \sigma_i$, and m of (5), and the details of the parameters are given in Table I. The mathematical expression of the function is formulated as

$$\begin{aligned} f(x) = &\, 0.7e^{-\frac{(x_1-1)^2+(x_2-1)^2}{0.18}} + 0.75e^{-\frac{(x_1-1)^2+(x_2-3)^2}{0.32}} \\ &+ e^{-\frac{(x_1-3)^2+(x_2-1)^2}{2}} + 1.2e^{-\frac{(x_1-3)^2+(x_2-4)^2}{0.32}} \\ &+ e^{-\frac{(x_1-5)^2+(x_2-2)^2}{0.72}}. \end{aligned} \tag{6}$$

Fig. 1 gives the peak distributions of this 2-D test functions. Obviously, this test function has five maximum values (local optimals) at points (1,1), (1,3), (3,1), (3,4), and (5,2). Moreover, its global and robust solutions lie, respectively, in point (3,4) and point (3,1).

TABLE II
PERFORMANCES OF DIFFERENT ALGORITHMS ON THE TEST FUNCTION

Algorithm	Averaged Iterative No.	No of solutions to be assigned $f_{\exp}$	x	$f(x)$	$f_{\exp}$
General Tabu	3546	/	(3.0,4.0)	1.211	0.520
Proposed	3595	205	(3.0,1.0)	1.001	0.836

To demonstrate and compare performances of the proposed algorithm with traditionally optimal approaches as well as with analytical solutions, the maximization of this test function is solved, respectively, by using a general-purpose tabu search and the proposed algorithms. In the numerical experiment, the intervals for all dimensions of the decision variables, say x_1 and x_2 in this case study, are normalized to [0 1]. Under this normalized condition and to empower the algorithm to have global search ability, the parameter σ of (3) is set to 0.1 so that 70% of the total neighborhood solutions are centered in a small neighborhood size of 0.1 of the current solution, and the neighborhood solutions are also used fully in order to obtain a more accurate estimate of the expected fitness with fewer samples without sacrificing the diversity of the neighborhood solutions. Also, the threshold value, $D_{\exp}$, is set to 0.1; and $l_{\exp}$ is set to $0.5h_{\text{number}}$ (h_{number} is the number of neighborhood solutions generated in each current solution). The other parameters for both the general-purpose and the proposed tabu search algorithms are the same as those used in [4]. Table II tabulates the average performances of the proposed and the general-purpose tabu search algorithms over ten independent runs. The convergence trajectories of a typical run for the two algorithms are shown in Figs. 2 and 3. From these numerical results the following can be seen.

1) Compared with other robust optimizers, only 205 promising solutions among the 3595 intermediate ones are required to be assigned the fitness values for the proposed algorithm; Consequently, a large amount of unnecessary computational burdens for the expected fitness evaluations are avoided.
2) Compared with a general-purpose global optimizer, the proposed algorithm can find the exact robust solution with negligible increase in computational burden for the expected fitness evaluations.
3) Actually, as seen from Figs. 2 and 3, the proposed algorithm has converged to the analytical robust solution in about 88 iterations while the general-purpose tabu search method needs 481 iterations in order to find the theoretical global optimal point. In other words, the proposed algorithm has a faster convergence speed compared to the general-purpose tabu search method.
4) Since the performance parameters given in Table II are the averaged ones of ten independent runs, the robustness of the proposed algorithm is positively confirmed.

In addition, one has experimented the proposed algorithm for problems of high dimension up to 10, and found that the increase in iterative number is marginal.

B. Application

The geometry optimization of the multisectional arcs of the pole shoes of a large salient pole hydrogenerator [4] is solved by using the proposed algorithm to elucidate its viability in finding

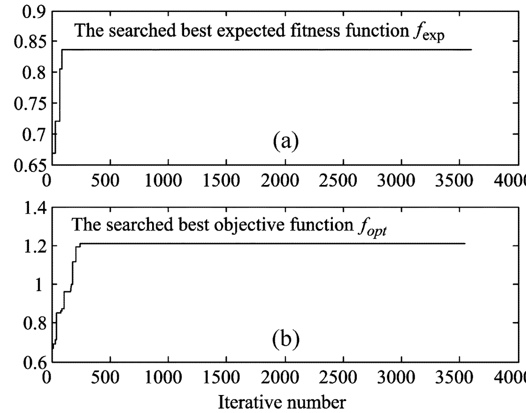

Fig. 2. Converge trajectories of the searched best expected fitness ($f_{\exp}$) or objective function (f_{opt}) values of a typical run. (a) For the proposed algorithm and (b) for the general-purpose tabu search algorithm.

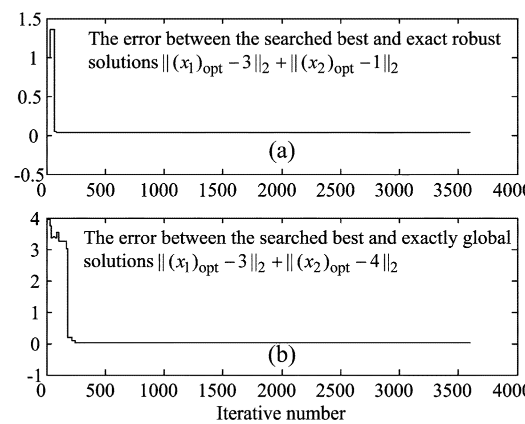

Fig. 3. Converge trajectories of the searched best optimal solutions of a typical run. (a) For the proposed algorithm and (b) for the general-purpose tabu search algorithm.

the robust solution of a practical engineering design problem. This optimal problem is formulated as

$$\begin{aligned} \max \quad & B_{f1}(X) \\ \text{s.t} \quad & e_v - e_{v0} \le 0 \\ & \text{THF} - \text{THF}_0 \le 0 \\ & \text{SCR} - \text{SCR}_0 \ge 0 \\ & X'_d - X'_{d0} \le 0 \end{aligned} \tag{7}$$

where B_{f1} is the amplitude of the fundamental component of the flux density in the air gap at no-load condition; e_v is the distortion factor of a sinusoidal voltage of the machine on no-load; THF is the abbreviations of the Telephone Harmonic Factor; X'_d is the direct axis transient reactance of the generator; SCR is the abbreviations of the short circuit ratio.

The decision parameters of this case study are the center positions and radii of the multisectional arcs of the pole shoes as shown in [4, Fig. 1]. In the numerical experiment of this case study, B_{f1} is directly determined from the finite-element solution of the no-load electromagnetic field of the machine, and the other performances of (7) are derived based on these finite-element solutions. Also, the intervals for the six decision variables are all transformed to [0 1] for the convenience and easiness of code developments.

TABLE III
FINAL OPTIMAL RESULTS AND PERFORMANCES OF DIFFERENT ALGORITHMS ON THE OPTIMAL DESIGN OF A 300-MW HYDROGENERATOR

Algorithm	No. FE analysis	No of solutions to be assigned $f_{\exp}$	B_{fl}(Tesla)	$(B_{fl})_{\exp}$(Tesla)
Gtabu	2278	/	1.086	1.001
Proposed	2365	165	1.055	1.042
Ctabu	3792	2290	1.055	1.042

For performance comparison purpose, a general-purpose tabu search method (Gtabu), the proposed algorithm (Proposed), and a conventional tabu search method (Ctabu) for robust design optimization in which the expected fitness values are assigned to all of the intermediate solutions, are used to solve this case study. In the numerical implementation of the proposed algorithm, the values of the parameter σ of (3) and the threshold parameter $D_{\exp}$ are set to be the same as those in the previous section. Table III lists the final optimal results and the performances of the aforementioned three optimal algorithms for the optimal designs of a 300-MW hydrogenerator. It should be noted that the computational efficiency of an optimal algorithm is measured by the number of the finite-element analysis since the CPU times required for finite-element analysis constitutes most of the running times of the corresponding algorithm for an inverse problem. The following observations can be made on these numerical results.

1) The proposed algorithm and a conventional tabu search method for robust design optimization both converge to the same robust solution, while the number of the function evaluations used by the former is only about 60% of that used by the later.
2) Compared with a general-purpose tabu search algorithm, the number of the total function evaluation for the proposed algorithm is increased from 2278 to 2365. In other words, the increase in computational burden required for the expected fitness evaluations for the proposed algorithm is relatively small compared to those required by a general-purpose tabu search method.
3) For the global optimal solution searched by the general-purpose tabu search method, a small variation (1%) of the decision parameter r_2 will degrade the objective function from 1.086 to 0.998 while the magnitude of the objective function for the robust solution searched by the proposed algorithm under the same perturbation condition is almost unchanged.

Nevertheless, the proposed algorithm could be made even more effective if a robust response surface had been used in the evaluations of function values for the additional neighborhood points.

IV. CONCLUSION

To search for robust solutions of electromagnetic design problems, this paper proposes a tabu-search-based robust optimizer with emphasis on preserving the high computational efficiency and conceptual simplicity of the available general-purposed tabu methods. The primary numerical results as reported on two different case studies have demonstrated that: 1) compared with a general-purpose global optimizer, the merit of the proposed algorithm is its ability to find a robust solution of an optimal design and 2) compared with a conventional optimal method for robust design optimization, the unique merit of the proposed algorithm is its high computation efficiency which is comparable to that of a general-purpose optimal method, notwithstanding the fact that additional function evaluations are required for the expected fitness assignments in the proposed algorithm. To enable the proposed algorithm to become a powerful and widely recognized robust optimizer in the study of optimization problems in different engineering disciplines, the future work of the authors will focus on the development of an adaptive neighborhood generating scheme which will automatically adjust the neighborhood sizes of the method according to the characteristics of the objective functions.

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