Production Scheduling with Supply and Delivery Considerations to Minimize the Makespan

Xiuli Wang[1, 2] T. C. E. Cheng[2, #]

[1]School of Economics and Management,
Nanjing University of Science and Technology,
Nanjing, China

[2]Department of Logistics,
The Hong Kong Polytechnic University,
Hung Hom, Kowloon, Hong Kong

Abstract

In this paper we study a scheduling model that simultaneously considers production scheduling, material supply, and product delivery. One vehicle with limited loading capacity transports unprocessed jobs from the supplier's warehouse to the factory in a fixed travelling time. Another capacitated vehicle travels between the factory and the customer to deliver finished jobs to the customer. The objective is to minimize the arrival time of the last delivered job to the customer. We show that the problem is NP-hard in the strong sense, and propose an $O(n)$ time heuristic with a tight performance bound of 2. We identify some polynomially solvable cases of the problem, and develop heuristics with better performance bounds for some special cases of the problem. Computational results show that all the heuristics are effective in producing optimal or near-optimal solutions quickly.

Keywords: scheduling; supply and delivery; heuristics; worst-case error bounds

---

[#]Corresponding author.

1. Introduction

Supply chain scheduling has in recent years gained new importance with the development of supply chain management. Supply chain scheduling research integrates the three stages of material supply, production arrangement, and product delivery into one model that seeks to achieve optimal overall system performance through proper coordination of these stages. Thomas and Griffin (1996), and Erengüc et al. (1999) emphasized the need for studying supply chain issues at the operational level. Hall and Potts (2003) showed that if decision makers at different stages of a supply chain make poorly coordinated decisions at the operational level, substantial inefficiencies may result. The supply chain scheduling problem is different from the traditional batch scheduling problem, which mainly uses batching as a means to reduce machine setup times and costs incurred from switching production between different job families. For example, the reader is referred to the papers on this area by Cheng et al. (1996, 1997), Potts and Van Wassenhove (1992), Potts and Kovalyov (2000), Quadt and Kuhn (2007), and Schaller (2007).

Research on supply chain scheduling mainly focuses on models that describe the coordination between production and delivery stages. For example, see Lee and Chen (2001), Chang and Lee (2004), Chen and Vairaktarakis (2005), Pundoor and Chen (2005), Wang and Lee (2005), Li et al. (2005), and Wang and Cheng (2007). To the best of our knowledge, for integrated three-stage supply chain scheduling models, Hall and Potts (2003) considered scheduling models integrating material supply, production scheduling, and product delivery with an arborescent supply chain structure. Li and Ou (2005) studied a single-machine scheduling model with material pickup and job delivery under the assumption that the material supplier and the customer are at located the same location, while the manufacturer resides at another location.

Many firms in Hong Kong are engaged in global supply chain business activities. For example, a clothing manufacturer in Hong Kong has received orders for fashion apparels from European customers. Taking into consideration such factors as availability of materials, material quality and price, manpower cost, and availability of workers, the manufacturer purchases raw materials such as cotton from South Korea and arranges production at its factories in mainland China. For such season-sensitive products, in order to reduce the high risk from market uncertainty, the firm finds it advantageous to consider the planning decisions on material supply, production and product delivery simultaneously. Since transportation spans long distances in this situation, both transport time and transport capacity constraints need to be considered in the planning decisions.

Motivated by the above example, we consider in this paper a scheduling model that integrates material supply, production scheduling, and product delivery. In our model the material warehouse, the factory, and the customer are located at three different places. There are two vehicles each with a limited loading capacity; one vehicle travels between the warehouse and the factory for material transportation, and the other travels between the factory and the customer for product delivery. The whole logistics activity embracing both production and transportation requires proper coordination in order to achieve low costs and a high level of customer service. Our model differs from that of Hall and Potts (2003) in that we incorporate the important factor of transport time in the model. Our model also differs from that of Li and Ou (2005) for we assume that the material supplier and the customer are at different locations.

The remainder of the paper is organized as follows. In Section 2 we formally describe the problem and introduce some notation. In Section 3 we first show that the problem is NP-hard in the strong sense, then we establish some optimal properties for the assignment of jobs to supply batches and delivery batches, and finally derive some lower bounds for the optimal solution of the problem. In Section 4 we identify special cases of the problem that can be solved in polynomial time. In Section 5 we devise several heuristics for the general problem and for some special cases, and analyze their worst-case performance bounds. In Section 6 we evaluate the performance of the heuristics computationally and present the experimental results. In the last section we make concluding remarks and suggest directions for future research.

2. Description of the problem

We formally describe our problem as follows: the material supplier, the factory, and the customer are located at different locations. There is a set of orders (jobs) $N = \{1, 2, \cdots, n\}$ from the customer. Each job requires to be processed by a single machine at the factory, where job preemption is not allowed. We assume that job $i\ (i = 1, 2, \cdots, n)$ has a processing time $p_i > 0$. Initially, the unprocessed jobs as materials are located at the supplier's warehouse, which need to be transported to the factory for processing. The finished jobs need to be delivered to the customer, too. A vehicle initially stays at the supplier's warehouse and is available for transporting unprocessed jobs from the supplier's warehouse to the factory. Each trip may load at most $K_1 > 0$ jobs due to the restriction of the vehicle capacity. It takes $t_1 > 0$ units of time for the vehicle to travel from the supplier's warehouse to the factory, and t_1 units of time to travel from the factory back to the supplier's warehouse. There is another vehicle that initially stays at the factory and transports finished jobs from the factory to the customer. It can load no more than $K_2 > 0$ jobs in each trip. The vehicle takes $t_2 > 0$ units of time to deliver processed jobs from the factory to the customer, and t_2 units of time to go back the factory from the customer. We assume that both t_1 and t_2 include the time of loading and unloading jobs. A job is available for processing once being delivered to the factory, and is available for delivery to the customer once its processing is finished. The logistical issues of our problem are concerned with determining the departure times of both of the supply trips and delivery trips, the jobs to be transported in each trip, and the starting time of processing each job. The objective is to minimize the makespan of the whole logistics activity, i.e., the arrival time of the last delivered job to the customer.

In a trip from the supplier's warehouse to the factory, all the unprocessed jobs loaded by the vehicle are denoted as a supply batch. For nonnegative integers q_1 and u_1 satisfying $n = q_1K_1 + u_1$ and $0 < u_1 \le K_1$, $q_1 + 1$ is the minimum number of supply batches the vehicle has to take in order to transport all the unprocessed jobs from the supplier's warehouse to the factory. For any solution for the problem, all the supply batches constitute a supply scheme $\varphi = (B_1^s, B_2^s, \cdots, B_w^s)$, where B_k^s denotes the kth supply batch, and $q_1 + 1 \le w \le n$. In a trip from the factory to the customer, all the finished jobs loaded by the vehicle are denoted as a delivery batch. For nonnegative integers q_2 and u_2 satisfying $n = q_2K_2 + u_2$ and $0 < u_2 \le K_2$, $q_2 + 1$ is the minimum number of delivery batches the vehicle has to take in order to deliver all the processed jobs from the factory to the customer. All the delivery batches constitute a delivery scheme $\psi = (B_1^d, B_2^d, \cdots, B_v^d)$, where B_k^d denotes the kth delivery batch, and $q_2 + 1 \le v \le n$. To minimize the makespan, both batch transporting and job processing must be carried out as early as possible. Thus, once a supply scheme φ, a schedule π, and a delivery scheme ψ are determined, we obtain a solution (φ, π, ψ) for the problem.

We define the following notation:

P: the sum of the processing times of all the jobs, i.e., $P = p_1 + p_2 + \cdots + p_n$;

$\pi(i)$: the ith processed job in schedule π;

$C_{\max}(\varphi, \pi, \psi)$: the makespan of the solution (φ, π, ψ);

$C_{\max}^{Hx}$: the makespan of Heuristic Hx;

$C_{\max}^{*}$: the optimal makespan of the problem.

3. Properties of the problem

In this section we first establish the computational complexity of our problem. We then discuss some properties for assigning jobs to supply or delivery batches in order to obtain an optimal solution. Finally, we establish some lower bounds for the optimal solution of the problem.

The following theorem states the computational complexity of the problem.

Theorem 1. The recognition version of the problem is strongly NP-complete even if $K_1 = K_2$ and $t_1 = t_2$.

We can prove the theorem using similar arguments in Li and Ou (2005), so we omit the proof. In view of Theorem 1, it is unlikely that the problem can be solved in polynomial time. The following optimal properties of the problem are obvious.

Lemma 1. There exists an optimal solution that satisfies the following conditions:

(i) $\varphi = (B_1^s, B_2^s, \cdots, B_{q_1+1}^s)$, where $|B_1^s| = |B_2^s| = \cdots = |B_{q_1}^s| = K_1$ and $|B_{q_1+1}^s| = u_1$. The supply batch B_k^s, $k = 1, 2, \cdots, q_1 + 1$, should be transported to the factory once the vehicle is idle at the supplier's warehouse. The vehicle must return from the factory immediately after unloading any supply batch.

(ii) The jobs of B_i^s, $i = 1, 2, \cdots, q_1$, should be processed before the jobs of B_{i+1}^s; the jobs of B_i^d, $i = 1, 2, \cdots, q_2$, should be processed before the jobs of B_{i+1}^d. There is no idle time on the machine in the factory if there are unprocessed jobs that are available for the machine to process.

(iii) $\psi = (B_1^d, B_2^d, \cdots, B_{q_2+1}^d)$, where $|B_1^d| = u_2$ and $|B_2^d| = |B_3^d| = \cdots = |B_{q_2+1}^d| = K_2$. The delivery batch B_k^d, $k = 1, 2, \cdots, q_2 + 1$, should be delivered from the factory to the customer once all the jobs in B_k^d are finished processing and the vehicle is idle at the factory. The vehicle must return from the customer immediately after unloading any delivery batch.

In order to search for an optimal solution for our problem, we may confine our attention to solutions that satisfy the conditions of Lemma 1. Once a schedule for processing the jobs is determined, we may generate a solution for the problem that complies with Lemma 1. The process is formally stated as a procedure as follows.

Procedure SD

Step 1. For a given schedule $\pi = (\pi(1), \pi(2), \cdots, \pi(n))$, assign jobs $\pi((i-1)K_1 + 1)$, $\cdots, \pi(iK_1)$ to supply batch B_i^s for $i = 1, 2, \cdots, q_1$, and $\pi(q_1K_1 + 1), \cdots,$ $\pi(q_1K_1 + u_1)$ to supply batch $B_{q_1+1}^s$.

Step 2. Assign jobs $\pi(1), \cdots$, $\pi(u_2)$ to delivery batch B_1^d and $\pi(u_2 + (i-1)K_2 + 1)$, $\cdots, \pi(u_2 + iK_2)$ to delivery batch B_{i+1}^d for $i = 1, 2, \cdots, q_2$.

Step 3. Departing at time 0, the vehicle carries $B_1^s, B_2^s, \cdots, B_{q_1+1}^s$ in turn from the supplier's warehouse to the factory with no idle time between any two consecutive trips. Another vehicle delivers $B_1^d, B_2^d, \cdots, B_{q_2+1}^d$ in turn from the factory to the customer once all the jobs in the delivery batch are finished processing and the vehicle returns to the factory. Stop.

Obviously, Procedure SD can be performed in $O(n)$ time. Having once determined a schedule π, we will use Procedure SD to produce a solution (φ, π, ψ) for the problem. Sometimes we denote $C_{\max}(\varphi, \pi, \psi)$ as $C_{\max}(\pi)$ for notational convenience.

For a given schedule $\pi = (\pi(1), \pi(2), \cdots, \pi(n))$, according to Lemma 1, there exists an integer pair (ξ_π, η_π) such that $C_{\max}(\pi) = t_1 + 2\xi_\pi t_1 + P_x + 2\eta_\pi t_2 + t_2$, where $0 \le \xi_\pi \le q_1$, $0 \le \eta_\pi \le q_2$, $\xi_\pi K_1 + \eta_\pi K_2 < n$, and $P_x = \sum_{i=\xi_\pi K_1+1}^{n-\eta_\pi K_2} p_{\pi(i)}$. For any integer pair (ξ, η) satisfying $0 \le \xi \le q_1$, $0 \le \eta \le q_2$, $\xi K_1 + \eta K_2 < n$, and $P_x = \sum_{i=\xi K_1+1}^{n-\eta K_2} p_{\pi(i)}$, let $L(\xi, \eta) = t_1 + 2\xi t_1 + P_x + 2\eta t_2 + t_2$, and we have $L(\xi, \eta) \le C_{\max}(\pi)$. Thus, we obtain the following lower bounds for the optimal solution for the problem.

Lemma 2. The optimal solution of the problem has the following lower bounds: $LB1 = t_1 + P + t_2$, $LB2 = t_1 + 2q_1 t_1 + t_2$, $LB3 = t_1 + 2q_2 t_2 + t_2$, and $LB4 = t_1 + 2\xi t_1 + 2\eta t_2 + t_2$ for all ξ and η satisfying $\xi K_1 + \eta K_2 < n$.

For the special case where $K_1 = K_2$, we have $q_1 = q_2$ and $u_1 = u_2$. Set $K_1 = K$, $q_1 = q$, $u_1 = u$, and $\max\{p_1, p_2, \cdots, p_n\} = p_{\max}$. We consider an instance of the case with n job processing times $\{\varepsilon, \varepsilon, \cdots, \varepsilon, p_{\max}\}$, where ε is a very tiny positive number and $\varepsilon < \min\{p_1, p_2, \cdots, p_n\}$. For this instance, there are n different schedules when the job with $p_{\max}$ is sequenced in n different positions. We denote the schedule in which the job with $p_{\max}$ is sequenced in the kth position as π_k. For schedule π_k, $k = 1, 2, \cdots, n$, its lower bound $L(\pi_k)$ is

$$L(\pi_k) = \begin{cases} t_1 + 2mt_1 + p_{\max} + (u-1)\varepsilon + 2(q-m)t_2 + t_2 & \text{if } k \in \{mK+1, mK+2, \\ & \qquad \cdots, mK+u\} \\ t_1 + 2mt_1 + p_{\max} + (K-1)\varepsilon + 2(q-m-1)t_2 + t_2 & \text{otherwise} \end{cases},$$

where $m = 0, 1, 2, \cdots, q$. When $t_1 \geq t_2$, we have

$$t_1 + 2qt_2 + p_{\max} - 2t_2 + t_2 \leq \min\{L(\pi_1), L(\pi_2), \cdots, L(\pi_n)\}.$$

When $t_1 < t_2$, we have

$$t_1 + 2qt_1 + p_{\max} - 2t_1 + t_2 \leq \min\{L(\pi_1), L(\pi_2), \cdots, L(\pi_n)\}.$$

Hence,

$$LB5 = \begin{cases} t_1 + 2qt_2 + p_{\max} - 2t_2 + t_2 & \text{if } t_1 \geq t_2 \\ t_1 + 2qt_1 + p_{\max} - 2t_1 + t_2 & \text{otherwise} \end{cases}$$

is a lower bound for the instance with processing times $\{\varepsilon, \varepsilon, \cdots, \varepsilon, p_{\max}\}$.

It is obvious that the optimal objective value of the problem with processing times $\{p_1, p_2, \cdots, p_n\}$ is no less than that of the instance with processing times $\{\varepsilon, \varepsilon, \cdots, \varepsilon, p_{\max}\}$. Therefore, we have the following lemma.

Lemma 3. When $K_1 = K_2 = K$, the optimal solution for the problem has a lower bound

$$LB5 = \begin{cases} t_1 + 2qt_2 + p_{\max} - 2t_2 + t_2 & \text{if } t_1 \geq t_2 \\ t_1 + 2qt_1 + p_{\max} - 2t_1 + t_2 & \text{otherwise} \end{cases}.$$

4. Polynomial solvable cases

Although the general problem is NP-hard in the strong sense, there are some special cases that are solvable in polynomial time. In this section we identify such solvable cases and give the respective algorithms to solve these cases in polynomial time.

Case $q_1 = 0$

We consider the case where $q_1 = 0$, i.e., all the unprocessed jobs can be carried by the vehicle in a single trip from the supplier's warehouse to the factory. In this case, supply transportation is not a bottleneck constraint on the entire three-stage logistics activity and only contributes a constant time t_1 to the optimal objective of the problem. So the case is essentially equivalent to the problem with only production and delivery coordination. For such a situation, the optimal solution can be obtained by the following procedure: Generate a schedule $\pi = (\pi(1), \pi(2), \cdots, \pi(n))$ such that $p_{\pi(1)} \le p_{\pi(2)} \le \cdots \le p_{\pi(n)}$. Then perform Procedure SD to produce a solution (φ, π, ψ). Obviously, the solution (φ, π, ψ) is optimal and its makespan is $\max\{t_1 + P + t_2, \quad t_1 + \sum_{j \in B_1^d} p_j + 2q_2 t_2 + t_2\}$.

Case q_2 = 0

We now consider the case where $q_2 = 0$, i.e., all the finished jobs can be transported by the vehicle in one trip from the factory to the customer. In this case, delivery transportation is not a bottleneck constraint on the entire three-stage logistics activity and only contributes a constant time t_2 to the optimal objective of the problem. So the case is essentially equivalent to the problem with only supply and production coordination. For this case, an optimal solution can be obtained by the following procedure: Generate a schedule $\pi = (\pi(1), \pi(2), \cdots, \pi(n))$ such that $p_{\pi(1)} \ge p_{\pi(2)} \ge \cdots \ge p_{\pi(n)}$. Then perform Procedure SD to produce a solution (φ, π, ψ). Obviously, the solution (φ, π, ψ) is optimal and its makespan is $\max\{t_1 + P + t_2,\ t_1 + 2q_1 t_1 + \sum_{j \in B_{q_1+1}^s} p_j + t_2\}$.

Case $P_1 \ge 2t_1$

Re-index the jobs such that $p_1 \le p_2 \le \cdots \le p_n$, and let $P_i = \sum_{j=(i-1)K_1+1}^{iK_1} p_j$ for $i = 1, 2, \cdots, q_1$, and $P_{q_1+1} = \sum_{j=q_1K_1+1}^{q_1K_1+u_1} p_j$. We consider the special case where $P_1 \ge 2t_1$. In this case, the ability to transport material supply is so high that any supply planning decision has the same effect on the planning decisions for the subsequent two stages. In such a situation, we can develop a polynomial time algorithm to solve the case optimally. The algorithm is performed as follows: Generate a schedule $\pi = (\pi(1), \pi(2), \cdots, \pi(n))$ such that $p_{\pi(1)} \le p_{\pi(2)} \le \cdots p_{\pi(n)}$. Then perform Procedure SD to produce a solution (φ, π, ψ). The optimal makespan is $C_{\max}(\pi) = \max\{t_1 + P + t_2,\ t_1 + D_0 + 2q_2t_2 + t_2\}$.

5. Heuristics

Since there are polynomial time algorithms to solve the above special cases optimally, we assume that the general problem studied in this section does not include the above special cases. In other words, we assume that $q_1 \ge 1$, $q_2 \ge 1$, and $P_1 < 2t_1$ hold for the general problem. We first provide a heuristic for the general problem. Then with some restrictions imposed on the parameters K_1 and K_2, we develop some better heuristics.

Heuristic H1

Step 1. For an arbitrary schedule $\pi = (\pi(1), \pi(2), \cdots, \pi(n))$, use Procedure SD to produce a solution (φ, π, ψ). Stop.

Heuristic H1 runs in $O(n)$ time. The following theorem provides a performance bound of Heuristic H1.

Theorem 2. $C_{\max}^{H1} / C_{\max}^{*} \le 2$ and the bound is tight.

The proof of Theorem 2 is trivial and is omitted. Let $\overline{P}_{q_1+1}$ denote the sum of the K_1 largest processing times of the jobs in *N*. For the special case where $\overline{P}_{q_1+1} \le 2t_1$, we can easily derive the following theorem.

Theorem 3. If $\overline{P}_{q_1+1} \le 2t_1$, then $C_{\max}^{H1} / C_{\max}^{*} \le 1 + 1/q_1$.

Since there are many parameters, namely K_1, K_2, t_1, and t_2, in the model of the general problem, developing a good heuristic for the general problem is very difficult. A better approach to study the problem is to consider some special cases. In the following, we study the special cases where $q_1 = 1$, $q_2 = 1$, and $K_1 = K_2$. In fact, even for these special cases, we can show in a manner similar to Theorem 1 that both of the cases where $q_1 = 1$ and $q_2 = 1$ are at least NP-hard in the ordinary sense. By Theorem 1, the case where $K_1 = K_2$ is NP-hard in the strong sense. We develop better heuristics for these special cases.

5.1 Case $q_1 = 1$

When $q_1 = 1$, i.e., all the unprocessed jobs can be transported to the factory in two supply batches. We provide the following heuristic.

Heuristic H2

Step 1. Re-index the jobs of *N* such that $p_1 \le p_2 \le \cdots \le p_n$.

Step 2. Generate a schedule $\pi_1 = (u_1 + 1, u_1 + 2, \cdots, u_1 + K_1, 1, 2, \cdots, u_1)$, and use Procedure SD to produce a solution $(\varphi_1, \pi_1, \psi_1)$.

Step 3. For the solution $(\varphi_1, \pi_1, \psi_1)$, where $\psi_1 = (B_1^d, B_2^d \cdots, B_{q_2+1}^d)$, when $q_2 = 2$, $\sum_{i \in B_1^d} p_i > P/2$, and $C_{\max}(\pi_1) = t_1 + \sum_{i \in B_1^d} p_i + 4t_2 + t_2$ hold, proceed with the following procedure:

1) Swap the job with the largest processing time in B_1^d and the job with the smallest processing time in B_3^d.

2) After swapping the two jobs, check the objective function: If $\xi = \eta = 1$ or $\sum_{i \in B_1^d} p_i < P/2$, then go to the next step. Otherwise, go to Step 1.

3) Generate a schedule $\pi_2 = (\hat{B}_1^d, \hat{B}_2^d, \hat{B}_3^d)$, where the jobs in $\hat{B}_i^d$ follow the shortest processing time (SPT) rule for $i = 1, 2, 3$. For schedule π_2, use Procedure SD to produce a solution $(\varphi_2, \pi_2, \psi_2)$.

Step 4. Generate a schedule $\pi_3 = (1, 2, \cdots, n-1, n)$, and use Procedure SD to produce a solution $(\varphi_3, \pi_3, \psi_3)$.

Step 5. Let $C_{\max}^{H2} = \min\{C_{\max}(\pi_1), C_{\max}(\pi_2), C_{\max}(\pi_3)\}$. Stop.

Clearly, Heuristic H2 runs in $O(n\log n)$ time. The following theorem provides a performance bound of Heuristic H2.

Theorem 4. If $q_1 = 1$, then $C_{\max}^{H2} / C_{\max}^* \le 3/2$.

The proof of Theorem 4 can be obtained from the authors.

Although the performance bound of Heuristic H2 is not tight, the following instance shows that the bound is no less than 7/5: $n = 4m$ $(m > 1)$ jobs with processing times $p_1 = p_2 = \cdots = p_{2m} = \varepsilon$, $p_{2m+1} = p_{2m+2} = \cdots = p_{4m} = 2 - \varepsilon$, $K_1 = 2m$, $K_2 = 2$, $t_1 = m$, and $t_2 = 1$. Applying Heuristic H2, for the solution $(\varphi_1, \pi_1, \psi_1)$, we have $\pi_1 = (2m+1, 2m+2, \cdots, 4m, 1, 2, \cdots, 2m)$ and $C_{\max}(\pi_1) = 7m - 2m\varepsilon + 1$. For the solution $(\varphi_3, \pi_3, \psi_3)$, we have $\pi_3 = (1, 2, \cdots, 4m)$ and $C_{\max}(\pi_3) = 7m - 2m\varepsilon + 1$. Since $q_2 > 2$, we need not consider the solution $(\varphi_2, \pi_2, \psi_2)$. In fact, the optimal solution is $(\varphi*, \pi*, \psi*)$, where $\pi* = (1, 2m+1, 2, 2m+2, \cdots, 2m, 4m)$, and $C_{\max}^* = 5m + 1$. So $C_{\max}^{H2} / C_{\max}^* = (7m - 2m\varepsilon + 1)/(5m+1)$. Hence, $C_{\max}^{H2} / C_{\max}^*$ approaches 7/5 as m approaches infinity and ε approaches zero.

We notice that an instance of Case $q_2 = 1$ can be easily transformed into an equivalent instance of Case $q_1 = 1$. Let an entry $(K_1, t_1; K_2, t_2)$ denote a kind of instances where the vehicle capacity and travelling time for supplies are K_1 and t_1, respectively; and the vehicle capacity and travelling time for deliveries are K_2 and t_2, respectively. Consider instances $(K_1', t_1'; K_2', t_2')$ of Case $q_1 = 1$ and $(K_1'', t_1''; K_2'', t_2'')$ of Case $q_2 = 1$ with the same parameters, except that $K_1'' = K_2', t_1'' = t_2', K_2'' = K_1'$ and $t_2'' = t_1'$. It is easy to show that the reversed schedule of any feasible schedule for the first instance is a feasible schedule for the second instance, and that these two schedules have the same makespan. From this property, similar to the analysis of the case where $q_1 = 1$, we can develop a heuristic with a performance bound of 3/2 for the case where $q_2 = 1$.

5.2 Case $K_1 = K_2$

We now consider the special case of the problem where $K_1 = K_2 = K$, i.e., $q_1 = q_2 = q$. For the special case where $q_1 = 1$, we have provided Heuristic H2 with a worst-case bound of 3/2. In the following heuristic, we suppose that $q \geq 2$ holds.

Heuristic H3

Step 1. Re-index the jobs of *N* such that $p_1 \leq p_2 \leq \cdots \leq p_n$.

Step 2. Generate a schedule $\pi_1 = (1, 2, \cdots, n-1, n)$, and use Procedure SD to produce a solution $(\varphi_1, \pi_1, \psi_1)$.

Step 3. Generate a schedule $\pi_2 = (n, 2, 3, \cdots, n-1, 1)$, and use Procedure SD to produce a solution $(\varphi_2, \pi_2, \psi_2)$.

Step 4. Let λ be an integer satisfying $p_{(\lambda-1)K+1} + \cdots + p_{\lambda K} < 2t_1 \leq p_{\lambda K+1} + \cdots + p_{(\lambda+1)K}$ and $\lambda \in \{1, 2, \cdots, q-1\}$, or satisfying $p_{(q-1)K+1} + \cdots + p_{qK} \leq 2t_1$

$\leq p_{(q-1)K+u+1} + \cdots + p_{qK+u}$ and $\lambda = q$. If such a λ exists and $2\lambda > q$, then produce a schedule π_3 as follows; otherwise, let $C_{\max}^{H3} = \min\{C_{\max}(\pi_1), C_{\max}(\pi_2)\}$. Stop.

When $(q-\lambda)K + u$ is even,

$$\pi_3 = (1,\ 3, \cdots, (q-\lambda)K + u - 1; \lambda K + 1, \lambda K + 3, \cdots, qK + u - 1;$$
$$(q-\lambda)K + u + 1, (q-\lambda)K + u + 2, \cdots, \lambda K;$$
$$2,\ 4, \cdots, (q-\lambda)K + u; \lambda K + 2, \lambda K + 4, \cdots, qK + u)\ .$$

When $(q-\lambda)K + u$ is odd,

$$\pi_3 = (1, 2, 4, 6, \cdots, (q-\lambda)K + u - 1; \lambda K + 2, \lambda K + 4, \cdots, qK + u - 1;$$
$$(q-\lambda)K + u + 1, (q-\lambda)K + u + 2, \cdots, \lambda K;$$
$$3,\ 5, \cdots, (q-\lambda)K + u; \lambda K + 1, \lambda K + 3, \cdots, qK + u)\ .$$

Use Procedure SD to produce a solution $(\varphi_3, \pi_3, \psi_3)$.

Step 5. Let $C_{\max}^{H3} = \min\{C_{\max}(\pi_1), C_{\max}(\pi_2), C_{\max}(\pi_3)\}$. Stop.

Clearly, Heuristic H3 runs in $O(n\log n)$ time. The following theorem provides a performance bound of Heuristic H3.

Theorem 5. If $K_1 = K_2 = K$ and $q \geq 2$, then $C_{\max}^{H3} / C_{\max}^{*} \leq 7/4$.

Proof. Let $P_i = p_{(i-1)K+1} + \cdots + p_{iK}$ for $i = 1, 2, \cdots q$ and $\overline{P}_{q+1} = p_{n-K+1} + \cdots + p_n$. We consider the cases: (1) $\overline{P}_{q+1} \leq 2t_1$, and (2) $2t_1 < \overline{P}_{q+1}$, respectively.

(1) $\overline{P}_{q+1} \leq 2t_1$

We consider the solution $(\varphi_1, \pi_1, \psi_1)$. $C_{\max}(\pi_1) = t_1 + 2\xi t_1 + P_x + 2\eta t_2 + t_2$, where $n - (\xi + \eta)K < K$ and $P_x = \sum_{i=\xi K+1}^{n-\eta K} p_i$, so $C_{\max}(\pi_1) - LB4 = P_x \leq 2t_1$ $\leq LB2/2$, thus $C_{\max}(\pi_1) \leq 3C_{\max}^{*}/2$.

(2) $2t_1 < \overline{P}_{q+1}$

In this situation, when $P_1 \geq 2t_1$, $C_{\max}(\pi_1)$ is an optimal solution; when $P_1 < 2t_1$, there exists an integer λ defined in Step 4 with $\lambda \in \{1,2,\cdots,q-1\}$ such that $P_\lambda \leq 2t_1 \leq P_{\lambda+1}$, or $\lambda = q$ such that $P_q \leq 2t_1 < \overline{P}_{q+1}$.

For $C_{\max}(\pi_1) = t_1 + 2\xi t_1 + P_x + 2\eta t_2 + t_2$, if $\xi \leq \lambda - 1$, we have $P_x \leq 2t_1$, then $C_{\max}(\pi_1) \leq 3C^*_{\max}/2$; otherwise, $C_{\max}(\pi_1) = t_1 + 2\lambda t_1 + P_x + 2(q-\lambda)t_2 + t_2$ or $C_{\max}(\pi_1) = t_1 + 2\lambda t_1 + (p_{\lambda K+1} + \cdots + p_n) + t_2$. When $C_{\max}(\pi_1) = t_1 + 2\lambda t_1 + P_x + 2(q-\lambda)t_2 + t_2$, $C_{\max}(\pi_1) - (t_1 + 2\lambda t_1 + 2(q-\lambda)t_2 + t_2) = P_x < 2t_2 \leq LB3/2$, so $C_{\max}(\pi_1) \leq 3C^*_{\max}/2$. When $C_{\max}(\pi_1) = t_1 + 2\lambda t_1 + (p_{\lambda K+1} + \cdots + p_n) + t_2$, if $p_{\lambda K+1} + \cdots + p_n \leq P/2$, then clearly $C_{\max}(\pi_1) \leq 3C^*_{\max}/2$. On the other hand, when $C_{\max}(\pi_1) = t_1 + 2\lambda t_1 + (p_{\lambda K+1} + \cdots + p_n) + t_2$ and $\lambda K < (q-\lambda)K + u$ hold, $\lambda \leq q - \lambda$, i.e., $2\lambda \leq q$, we also have $C_{\max}(\pi_1) \leq 3C^*_{\max}/2$. Thus, in the following discussion, we only consider the situation where $p_{\lambda K+1} + \cdots + p_n > P/2$ and $2\lambda > q$ hold for $C_{\max}(\pi_1) = t_1 + 2\lambda t_1 + (p_{\lambda K+1} + \cdots + p_n) + t_2$. We divide the case into two situations: 1) $p_n \geq P/2$, and 2) $p_n < P/2$.

1) $p_n \geq P/2$

If $t_1 \leq t_2$, by Lemma 3, $C_{\max}(\pi_1) - LB5 \leq (t_1 + p_{\lambda K+1} + \cdots + p_{n-1}) + t_1 \leq LB1/2 + LB2/6$, then $C_{\max}(\pi_1) \leq 5C^*_{\max}/3$.

If $t_1 > t_2$, we consider the solution $(\varphi_2, \pi_2, \psi_2)$. When $C_{\max}(\pi_2) = t_1 + P + t_2$ or $C_{\max}(\pi_2) = t_1 + 2\lambda t_1 + (p_{\lambda K+1} + \cdots + p_{n-1} + p_1) + t_2$, obviously $C_{\max}(\pi_2) \leq 3C^*_{\max}/2$. When $C_{\max}(\pi_2) = t_1 + (p_2 + \cdots + p_u + p_n) + 2qt_2 + t_2$, by Lemma 3, $C_{\max}(\pi_2) - LB5 \leq (p_2 + \cdots + p_u) + 2t_2 \leq LB1/2 + LB3/6 \leq 2C^*_{\max}/3$, i.e., $C_{\max}(\pi_2) \leq 5C^*_{\max}/3$.

2) $p_n < P/2$

We focus on the solution $(\varphi_3, \pi_3, \psi_3)$. For $C_{\max}(\pi_3) = t_1 + 2\xi t_1 + P_x + 2\eta t_2 + t_2$, when $q - \lambda + 1 \le \xi \le \lambda - 1$, since $\sum_{i \in B_l^s} p_i \le 2t_2$ for $l = q - \lambda + 2, \cdots, \lambda$, we have $C_{\max}(\pi_3) - (t_1 + 2\xi t_1 + 2\eta t_2 + t_2) = P_x < 2t_1 < LB1/2$, so $C_{\max}(\pi_3) \le 3C^*_{\max}/2$. Hence, in the following, we only consider the situations i) $\xi \le q - \lambda$, and ii) $\xi \ge \lambda$.

i) When $\xi \le q - \lambda$

If $\eta = 0$, since $\xi \le q - \lambda$ and $q - \lambda < \lambda$, we have $2\xi < q$, then $C_{\max}(\pi_3) - LB1 = 2\xi t_1 < LB2/2$, thus $C_{\max}(\pi_3) \le 3C^*_{\max}/2$.

If $\eta \ne 0$, let

$$U_{32} = \begin{cases} \{\lambda K + 1, \lambda K + 3, \cdots, qK + u\} & \text{if } (q-\lambda)K + u \text{ is odd} \\ \{\lambda K + 2, \lambda K + 4, \cdots, qK + u\} & \text{if } (q-\lambda)K + u \text{ is even} \end{cases}.$$

Clearly, $\eta \ge \lceil |U_{32}| / K \rceil$. In this situation, $\sum_{i \in U_{32}} p_i \ge (p_{\lambda K+1} + \cdots + p_n)/2 > P/4$. Therefore, $C_{\max}(\pi_3) - LB4 = P_x < 3P/4 < 3C^*_{\max}/4$. That is, $C_{\max}(\pi_3) \le 7C^*_{\max}/4$.

ii) When $\xi \ge \lambda$

Let

$$U_{11} = \begin{cases} \{1, 2, 4, 6, \cdots, (q-\lambda)K + u - 1\} & \text{if } (q-\lambda)K + u \text{ is odd} \\ \{1, 3, 5, \cdots, (q-\lambda)K + u - 1\} & \text{if } (q-\lambda)K + u \text{ is even} \end{cases},$$

$$U_{12} = \begin{cases} \{\lambda K + 2, \lambda K + 4, \cdots, qK + u - 1\} & \text{if } (q-\lambda)K + u \text{ is odd} \\ \{\lambda K + 1, \lambda K + 3, \cdots, qK + u - 1\} & \text{if } (q-\lambda)K + u \text{ is even} \end{cases},$$

$$U_2 = \{(q-\lambda)K + u + 1, (q-\lambda)K + u + 2, \cdots, \lambda K\},$$

$$U_{31} = \begin{cases} \{3, 5, \cdots, (q-\lambda)K + u\} & \text{if } (q-\lambda)K + u \text{ is odd} \\ \{2, 4, \cdots, (q-\lambda)K + u\} & \text{if } (q-\lambda)K + u \text{ is even} \end{cases}.$$

If $\eta \ne 0$, since the jobs in $U_{31} \cup U_{32}$ are sequenced in nondecreasing order of their processing times in π_3, then $P_x \le 2t_2$, so $C_{\max}(\pi_3) \le 3C^*_{\max}/2$.

If $\eta = 0$ and when $(q-\lambda)K + u \ne \lambda K$, we have $\sum_{i \in U_{12}} p_i \ge \sum_{i \in U_{32}} p_i - p_n$ and $\sum_{i \in U_{11}} p_i \ge \sum_{i \in U_{31}} p_i - p_{(q-\lambda)K+u}$, and U_2 is not an empty set. So $\sum_{i \in U_{12} \cup U_{11} \cup U_2} p_i$

$\geq \sum_{i \in U_{31} \cup U_{32}} p_i - p_n > \sum_{i \in U_{31} \cup U_{32}} p_i - P/2$, $2\sum_{i \in U_{12} \cup U_{11} \cup U_2} p_i > P - P/2 = P/2$, i.e., $\sum_{i \in U_{12} \cup U_{11} \cup U_2} p_i > P/4$, so $\sum_{i \in U_{31} \cup U_{32}} p_i < 3P/4$. We have $C_{\max}(\pi_3) - LB2 < P_x \leq \sum_{i \in U_{31} \cup U_{32}} p_i < 3P/4$. Thus, $C_{\max}(\pi_3) \leq 7C^*_{\max}/4$.

If $\eta = 0$ and when $(q - \lambda)K + u = \lambda K$, we have $u = K$, and U_2 is an empty set. In this situation, $P_x = \sum_{i \in B_{32}} p_i$. Since $\sum_{i \in U_{11} \cup U_{31}} p_i + 2\sum_{i \in U_{12}} p_i \geq P - p_n > P/2$, $\sum_{i \in U_{11} \cup U_{31}} p_i + \sum_{i \in U_{12}} p_i > P/4$, thus $\sum_{i \in U_{32}} p_i < 3P/4$. We have $C_{\max}(\pi_3) - LB2 < P_x \leq \sum_{i \in U_{32}} p_i < 3P/4$. Thus, $C_{\max}(\pi_3) \leq 7C^*_{\max}/4$. □

Although the performance bound of Heuristic H3 is not tight, the following instance shows that the bound is no less than 3/2: $n = 4m + 2$ jobs with processing times $p_1 = p_2 = \cdots = p_{2m+1} = \varepsilon$, $p_{2m+2} = p_{2m+3} = \cdots = p_{4m+2} = 2 - \varepsilon$, $K_1 = K_2 = 2$, $t_1 = 1 - \varepsilon$ and $t_2 = 1$. Applying Heuristic H3, for the solution $(\varphi_1, \pi_1, \psi_1)$, we have $\pi_1 = (1, 2, \cdots, 4m + 2)$ and $C_{\max}(\pi_1) = 6m - (4m + 1)\varepsilon + 4$. For the solution $(\varphi_2, \pi_2, \psi_2)$, we have $\pi_2 = (4m + 2, 2, 3, \cdots, 4m + 1, 1)$ and $C_{\max}(\pi_2) = 6m - (4m - 1)\varepsilon + 2$. Since $q = 2m$ and $\lambda = m$, $2\lambda > q$ does not hold. So we need not consider the solution $(\varphi_3, \pi_3, \psi_3)$. In fact, the optimal solution is $(\varphi*, \pi*, \psi*)$, where $\pi* = (1, 2m + 2, 2, 2m + 3, \cdots, 2m + 1, 4m + 2)$, and $C^*_{\max} = 4m - \varepsilon + 4$. So, $C^{H3}_{\max} = 6m - (4m - 1)\varepsilon + 2$. From $C^{H3}_{\max} / C^*_{\max} = (6m - (4m - 1)\varepsilon + 2)/(4m - \varepsilon + 4)$, $C^{H3}_{\max} / C^*_{\max}$ approaches 3/2 as m approaches infinity and ε approaches zero.

6. Computational experiments

In this section we report the results of computational experiments conducted to test the performance of the above heuristics.

For the problem under study, our heuristics are based on the idea of matching or balancing the abilities of the three logistical stages in a series of time periods. We use the notion of "logistics ability" to uniformly describe the transportation ability of the delivery vehicles, and the production ability of the processing machine. For the supply and delivery stages, we measure their logistics abilities by $\alpha = N/((2q_1+1)t_1)$ and $\alpha = N/((2q_2+1)t_2)$, respectively. For the processing machine, its logistics ability is measured by $\alpha = N/P$. In essence, the parameter α quantitatively scales the largest ability to pass the number of jobs per time unit at each logistical stage. Since the value of α is only taken as a comparative scale of the logistics abilities of three different stages, for the convenience of experimental computation, we first set an appropriate value of α at 0.04. The experimental scheme was designed to test the heuristics operating in situations characterized by different combinations of logistics abilities of the three logistical stages. Specifically, when $\alpha = 0.02$, 0.04 and 0.08 for the transportation or production stage, we consider that this stage has small, middle and big logistics ability, respectively. In the following tables, we use letters "S", "M" and "B" to denote small, middle and big logistics ability, respectively. For example, the symbol "BMS" represents the case where the supply, production and delivery stage have big, middle and small logistics ability, respectively. The total number of all possible combinations of the logistics abilities of the three stages is $3^3 = 27$. We did not distinguish the cases BBB, MMM and SSS in the experimental scheme since the logistics abilities of all of the three stages are equivalent. So we only consider 25 cases of the problem in the following experiments.

The heuristics were tested over problem sizes of $N = 25, 50, 100, 150, 200$ jobs. For any instance, the job processing times were independently and randomly generated from a discrete uniform distribution in the interval [1, U], where U = 100, 50, 25 when the production stage has a small, middle and big logistics ability, respectively. We also randomly generated parameter K_1 or K_2 from a discrete uniform distribution under the constraints of the problem. Furthermore, according to the specified logistics ability α of the supply or delivery stage, we calculated parameter t_1 or t_2. Considering the different number of jobs and the different combinations of the logistics abilities of the three stages, we examined the performance of each heuristic operating in 125 situations, and randomly generated 100 instances for each situation.

We evaluated the performance of the heuristics by the average relative error and the maximum relative error of each situation of the problem. For each instance, we computed $C_{\max}^{Hx}$ and the lower bound *LB*, where $LB = \max\{LB1, LB2, LB3, LB4\}$ for Heuristics H1 and H2, and $LB = \max\{LB1, LB2, LB3, LB4, LB5\}$ for Heuristic H3. The relative error of a solution is defined as $Error = C_{\max}^{Hx} / LB$, the average relative error of a situation as $avgE = (\sum Error) / InstanceNumber$, and the maximum relative error of a situation as $maxE = \max\{Error \mid all\ instances\ \ tested\ \ for\ a\ situation\}$.

Tables 1 to 3 exhibit the experimental results for Heuristics H1, H2 and H3, respectively. From Table 1, the average relative errors of all the 25 logistics ability combinations were no more than 10%, and the maximum relative errors no larger than 30%, which indicate that the performance of Heuristic H1 is good for the general problem. From Tables 2 and 3, we see that the performance of Heuristics H2 and H3, especially in terms of maximum relative errors, is almost always better than that of Heuristic H1. They are capable of generating near-optimal solutions or optimal solutions.

In our experimental scheme, since the average and maximum relative errors were evaluated with respect to the lower bounds of the test instances, this fact should be taken into account in interpreting the above insights about the performance of the heuristics. However, on further examining the experimental results in Tables 1 to 3, we notice that the performance of the heuristics is clearly related to different cases of logistics ability. This phenomenon indicates that our experimental scheme to distinguish the different logistics ability of the three stages is reasonable. On the other hand, this observation highlights that problems with different logistics ability characteristics require different scheduling strategies to deal with in order to achieve good results. We also observe that the effectiveness of Heuristics H2 and H3 increases as the number of the jobs increases, suggesting that they can be put to practice to effectively cope with real-life problems.

7. Conclusions

In this paper we studied the problem of production scheduling with supply and delivery considerations, where the material warehouse, the factory, and the customer are at different locations. Through the coordination of transportation and production, the objective is to minimize the makespan. We showed that the problem is NP-hard in the strong sense, and developed several heuristics for the general problem and for some special cases. The worst-case error bounds of the heuristics were analyzed. Computational results showed that all the heuristics are effective in producing optimal or near-optimal solutions quickly.

There are many interesting topics worthy of studying for models integrating material supply, production scheduling, and product delivery at the operational level. Within the framework of this paper, the actual transportation and production environments or characteristics may be taken into consideration. Another interesting research direction is to extend our model to consider the optimization of other objective functions such as minimizing the total flow time or minimizing the maximum lateness.

Acknowledgements

We are grateful to two anonymous referees for their constructive comments on an earlier version of this paper. This research was supported in part by The Hong Kong Polytechnic University under a grant from the *Area of Strategic Development in China Business Services*. The research was also supported in part by the School of Economics and Management, Nanjing University of Science and Technology under grant number EM-200604.

References

Chang Y.-C., Lee C.-Y., 2004. Machine scheduling with job delivery coordination. European Journal of Operational Research 158, 470-487.

Chen Z.-L., Vairaktarakis G.L., 2005. Integrated scheduling of production and distribution operations. Management Science 51, 614-628.

Cheng T.C.E., Gordon V.S., Kovalyov, M.Y., 1996. Single machine scheduling with batch deliveries. European Journal of Operational Research 94, 277-283.

Cheng, T.C.E., Kovalyov, M.Y., Lin, B.M.T., 1997. Single machine scheduling to minimize batch delivery and job earliness penalties. SIAM Journal on Optimization 7, 547-559.

Erengüc S.S., Simpson N.C., Vakharia A.J., 1999. Integrated production/distribution planning in supply chains. European Journal of Operational Research 115, 219-236.

Garey M.R., Johnson D.S., 1979. Computers and Intractability: A Guide to the Theory of NP-completeness, Freeman, New York.

Hall N.G., Potts C.N., 2003. Supply chain scheduling: batch and delivery. Operations

Research 51, 566-584.

Lee C.-Y., Chen Z.-L., 2001. Machine scheduling with transportation considerations. Journal of Scheduling 4, 3-24.

Li C.-L., Vairaktarakis G., Lee C.-Y., 2005. Machine scheduling with deliveries to multiple customer locations. European Journal of Operational Research 164, 39-51.

Li C.-L., Ou J.-W., 2005. Machine scheduling with pickup and delivery. Naval Research Logistics 52, 1-14.

Potts C.N., Van Wassenhove L.N., 1992. Integrating scheduling with batching and lot-sizing: a review of algorithms and complexity. Journal of the Operational Research Society 43, 395-406.

Potts C.N., Kovalyov M.Y., 2000. Scheduling with batching: a review. European Journal of Operational Research 120, 228-249.

Pundoor G., Chen Z.-L., 2005. Scheduling a production-distribution system to optimize the tradeoff between delivery tardiness and distribution cost. Naval Research Logistics 52, 571-589.

Quadt D., Kuhn H., 2007. Batch scheduling of jobs with identical process times on flexible flow lines. International Journal of Production Economics 105, 385-401.

Schaller J., 2007. Scheduling on a single machine with family setups to minimize total tardiness. International Journal of Production Economics 105, 329-344.

Thomas D.J., Griffin P.M., 1996. Coordinated supply chain management. European Journal of Operational Research 94, 1-15.

Wang H., Lee C.-Y., 2005. Production and transport logistics scheduling with two transport mode choices. Naval Research Logistics 52, 796-809.

Wang X-.L., Cheng T.C.E., 2007. Machine scheduling with an availability constraint and job delivery coordination. Naval Research Logistics 54, 11-20.

Table 1. Experimental results for Heuristic H1

Job No.	Error values	BBM	BBS	BMB	BMM	BMS	BSB	BSM	BSS	MBB	MBM	MBS	MMB
25	avgE	1.061	1.032	1.043	1.018	1.061	1.000	1.021	1.032	1.061	1.054	1.033	1.017
	maxE	1.201	1.113	1.122	1.197	1.223	1.008	1.139	1.189	1.242	1.225	1.111	1.178
50	avgE	1.064	1.037	1.042	1.017	1.077	1.000	1.018	1.032	1.070	1.058	1.032	1.016
	maxE	1.218	1.126	1.119	1.144	1.269	1.000	1.178	1.209	1.238	1.203	1.112	1.188
100	avgE	1.061	1.036	1.038	1.011	1.081	1.000	1.015	1.022	1.059	1.058	1.029	1.013
	maxE	1.221	1.120	1.108	1.109	1.262	1.000	1.142	1.168	1.245	1.198	1.108	1.122
150	avgE	1.069	1.035	1.041	1.022	1.077	1.000	1.017	1.024	1.067	1.057	1.032	1.014
	maxE	1.236	1.126	1.131	1.188	1.218	1.003	1.133	1.199	1.256	1.197	1.111	1.132
200	avgE	1.068	1.030	1.037	1.019	1.082	1.000	1.016	1.030	1.061	1.055	1.030	1.020
	maxE	1.249	1.108	1.123	1.129	1.236	1.001	1.161	1.147	1.239	1.196	1.117	1.105

To continue

MMM	MMS	MSB	MSM	MSS	SBB	SBM	SBS	SMB	SMM	SMS	SSB	SSM
1.040	1.071	1.026	1.036	1.063	1.039	1.035	1.032	1.080	1.073	1.062	1.027	1.043
1.162	1.237	1.195	1.142	1.254	1.121	1.134	1.094	1.284	1.256	1.233	1.190	1.224
1.046	1.066	1.023	1.038	1.070	1.040	1.033	1.033	1.081	1.063	1.068	1.020	1.019
1.177	1.219	1. 182	1.214	1.296	1.113	1.110	1.111	1.274	1.248	1.221	1.133	1.134
1.023	1.057	1.029	1.035	1.069	1.034	1.032	1.029	1.080	1.060	1.054	1.029	1.026
1.102	1.201	1.151	1.187	1.252	1.121	1.117	1.102	1.264	1.211	1.213	1.153	1.185
1.028	1.059	1.025	1.029	1.071	1.036	1.032	1.033	1.076	1.061	1.066	1.024	1.031
1.099	1.218	1.163	1.195	1.282	1.107	1.113	1.106	1.216	1.199	1.193	1.125	1.151
1.017	1.063	1.026	1.026	1.065	1.032	1.032	1.023	1.072	1.065	1.058	1.026	1.012
1.082	1.207	1.179	1.181	1.287	1.118	1.105	1.094	1.246	1.221	1.211	1.145	1.130

Table 2. Experimental results for Heuristic H2

Job No.	Error values	BBM	BBS	BMB	BMM	BMS	BSB	BSM	BSS	MBB	MBM	MBS	MMB
25	avgE	1.016	1.008	1.015	1.038	1.009	1.005	1.016	1.008	1.013	1.019	1.004	1.018
	maxE	1.080	1.043	1.141	1.114	1.058	1.044	1.066	1.048	1.059	1.086	1.028	1.122
50	avgE	1.014	1.007	1.011	1.034	1.011	1.004	1.012	1.007	1.008	1.009	1.003	1.015
	maxE	1.076	1.040	1.067	1.101	1.054	1.028	1.044	1.031	1.047	1.084	1.016	1.067
100	avgE	1.011	1.006	1.007	1.028	1.009	1.000	1.009	1.006	1.008	1.010	1.003	1.015
	maxE	1.058	1.031	1.075	1.100	1.045	1.027	1.035	1.031	1.051	1.058	1.020	1.063
150	avgE	1.009	1.005	1.006	1.031	1.009	1.000	1.007	1.005	1.008	1.007	1.002	1.014
	maxE	1.054	1.029	1.048	1.087	1.056	1.007	1.031	1.029	1.067	1.061	1.013	1.062
200	avgE	1.009	1.005	1.003	1.027	1.010	1.000	1.006	1.004	1.007	1.008	1.003	1.013
	maxE	1.058	1.031	1.054	1.077	1.055	1.012	1.026	1.024	1.057	1.060	1.014	1.043

To continue

MMM	MMS	MSB	MSM	MSS	SBB	SBM	SBS	SMB	SMM	SMS	SSB	SSM
1.097	1.007	1.003	1.034	1.045	1.006	1.006	1.008	1.019	1.011	1.018	1.003	1.035
1.224	1.053	1.049	1.152	1.151	1.030	1.029	1.042	1.098	1.058	1.086	1.042	1.131
1.087	1.006	1.001	1.039	1.059	1.004	1.004	1.003	1.012	1.007	1.009	1.003	1.035
1.174	1.064	1.035	1.110	1.156	1.024	1.023	1.042	1.067	1.043	1.084	1.032	1.084
1.093	1.006	1.001	1.034	1.057	1.004	1.004	1.004	1.010	1.007	1.009	1.003	1.037
1.177	1.039	1.025	1.115	1.132	1.026	1.025	1.029	1.050	1.048	1.057	1.021	1.087
1.092	1.004	1.000	1.042	1.072	1.004	1.004	1.002	1.011	1.007	1.006	1.002	1.038
1.191	1.022	1.021	1.094	1.150	1.035	1.033	1.030	1.067	1.065	1.060	1.018	1.083
1.094	1.006	1.000	1.040	1.063	1.004	1.003	1.003	1.009	1.006	1.007	1.001	1.037
1.199	1.031	1.010	1.088	1.134	1.029	1.028	1.030	1.056	1.071	1.058	1.017	1.077

Table 3. Experimental results for Heuristic H3

Job No.	Error values	BBM	BBS	BMB	BMM	BMS	BSB	BSM	BSS	MBB	MBM	MBS	MMB
25	avgE	1.001	1.000	1.012	1.024	1.000	1.000	1.000	1.008	1.017	1.040	1.000	1.148
	maxE	1.013	1.007	1.085	1.076	1.007	1.000	1.000	1.042	1.096	1.131	1.006	1.251
50	avgE	1.002	1.001	1.027	1.023	1.001	1.000	1.000	1.003	1.037	1.042	1.001	1.165
	maxE	1.013	1.007	1.092	1.058	1.003	1.000	1.000	1.025	1.085	1.144	1.006	1.244
100	avgE	1.001	1.000	1.039	1.032	1.000	1.000	1.000	1.002	1.053	1.031	1.000	1.166
	maxE	1.007	1.003	1.082	1.061	1.002	1.010	1.010	1.020	1.096	1.085	1.003	1.260
150	avgE	1.002	1.001	1.046	1.039	1.000	1.003	1.003	1.003	1.055	1.040	1.001	1.168
	maxE	1.012	1.006	1.070	1.068	1.002	1.013	1.013	1.014	1.075	1.131	1.006	1.262
200	avgE	1.001	1.000	1.046	1.041	1.000	1.005	1.005	1.003	1.051	1.030	1.000	1.172
	maxE	1.008	1.004	1.069	1.067	1.001	1.012	1.012	1.011	1.068	1.107	1.004	1.250

To continue

MMM	MMS	MSB	MSM	MSS	SBB	SBM	SBS	SMB	SMM	SMS	SSB	SSM
1.146	1.000	1.019	1.019	1.023	1.003	1.003	1.020	1.004	1.004	1.039	1.150	1.149
1.241	1.006	1.096	1.095	1.077	1.071	1.069	1.065	1.133	1.129	1.123	1.269	1.263
1.162	1.001	1.034	1.034	1.024	1.014	1.014	1.021	1.023	1.023	1.039	1.163	1.162
1.241	1.003	1.085	1.085	1.063	1.078	1.076	1.072	1.078	1.077	1.087	1.240	1.239
1.164	1.000	1.047	1.047	1.035	1.012	1.012	1.015	1.018	1.017	1.026	1.166	1.165
1.258	1.002	1.096	1.096	1.064	1.044	1.043	1.042	1.087	1.085	1.082	1.251	1.250
1.166	1.000	1.051	1.051	1.036	1.018	1.018	1.020	1.021	1.021	1.026	1.161	1.161
1.262	1.002	1.080	1.079	1.063	1.070	1.068	1.065	1.059	1.058	1.060	1.263	1.262
1.171	1.000	1.052	1.052	1.041	1.014	1.013	1.015	1.015	1.015	1.019	1.169	1.169
1.246	1.001	1.072	1.072	1.063	1.056	1.055	1.053	1.044	1.044	1.046	1.247	1.246