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A Simplified Blind Carrier Frequency Offset Estimation Algorithm Based on the Power of Zero-Subcarriers for CO-OFDM Systems

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Abstract: A simplified blind CFO estimation algorithm is proposed for CO-OFDM systems based on the power of zero-subcarriers, which models the cost function as a cosine function, then easily estimating CFO by calculating three test functions.

1. Introduction

Coherent optical orthogonal frequency-division multiplexing (CO-OFDM) has been an attractive technology for long-haul optical communication systems due to its high spectral efficiency and excellent tolerance to the fiber chromatic dispersion (CD) and polarization mode dispersion (PMD). However, CO-OFDM is sensitive to the carrier frequency offset (CFO) which is induced by the incoherence between the transmitter laser and the local oscillator (LO). The frequency offset normalized by the subcarrier spacing can be divided into a fractional part and an integral part.

Various approaches have been proposed to estimate and compensate for the CFO, which can be generally classified into two categories: data-aided and blind CFO estimation algorithms [1–4]. In [5], two training symbols are used to estimate the integeral part of the CFO but this estimation involves an exhaustive search, while the fractional part is obtained by taking correlation of the two identical halves in the first training symbol. However, an overhead is usually inserted for a data-aided algorithm, which reduces the effective transmission bit rate. Blind CFO estimation algorithms can not only solve this problem, but also increase the estimation accuracy. In [6], cyclic prefix (CP) was used to estimate CFO blindly, however, the estimation performance is highly dependent on the length of CP. Wu [3] proposed to utilize the subcarrier residual power to estimate the CFO, but the estimation range is limited to the fractional part.

In this paper, we propose a simplified blind CFO estimation algorithm that is based on the power of zero-subcarriers, with full-range CFO estimation. Exhaustive search complexity in [7] can be avoided by modelling cost function with a simple cosine function. In order to increase the tolerance to additive white Gaussian noise (AWGN) and laser phase noise (LPN), we further modified the algortihm by taking the power average through a series of OFDM symbols. The performance of the proposed algorithms is compared with the conventional Schmidl's algorithm [5] and demonstrated through simulations.

2. Methodology

The n-th received time domain OFDM sample $r(n)$ with the appearance of CFO and LPN is given by [8]:

$$r(n) = x(n) \otimes h(n) e^{j[2\pi\epsilon n/N + \theta(n)]} + w(n), \tag{1}$$

where we have $n = 0, 1, \cdots, N-1$. Term $x(n)$ is the n-th transmitted time-domain sample, and $h(n)$ is the channel impulse response. Term ϵ is the CFO normalized by the OFDM subcarrier spacing and can be divided into the integral and fractional parts: $\epsilon = \epsilon_i + \epsilon_f$, with ϵ_i being an integer and $\epsilon_f \in [-0.5, 0.5)$. Term N is the size of discrete Fourier transform (DFT) and $\theta(n)$ represents the LPN, which is a Wiener process modelled by $\theta(n) = \theta(n-1) + \nu(n)$, where $\{\nu(n)\}_{n=0}^{N-1} \sim \mathcal{N}(0, \sigma_p^2)$ with variance $\sigma_p^2 = 2\pi\Delta\nu T$, where $\Delta\nu$ accounts for the combined laser linewidth and T is the sample time interval. Term $w(n)$ is the complex additive white Gaussian noise (AWGN) with mean zero and variance σ^2.

The fractional part and the integral part of the CFO have different effects on the received signal spectrum. In the presence of fractional CFO, the ZSP increases due to the ICI, while the integral part causes a shift of the amplitude spectrum. By utilizing the features stated above, we proposed a blind search method for full range CFO estimation in [7] . The integral and fractional CFO estimates are [7]:

$$\hat{\epsilon} = \arg\min_{\epsilon_s} \Gamma(\epsilon_s) = \arg\min_{\epsilon_s} \sum_{k \in \{\text{ZS index}\}} \left| \sum_{n=0}^{N-1} r(n) e^{-j2\pi n(\epsilon_s + k)/N} \right|^2. \quad (2)$$

Here, ϵ_s represents the CFO candidate. For integral part estimation, ϵ_s is taken from the set $S_i = \{\epsilon_s : \epsilon_s = -N/2 + s\Delta\epsilon, s = 0, 1, \cdots, N-1, \Delta\epsilon = 1\}$. After obtaining the integral estimate $\hat{\epsilon}_i$ and making the compensation, to estimate the fractional CFO, ϵ_s is chosen from the set $S_f = \{\epsilon_s : \epsilon_s = \hat{\epsilon}_i - 1/2 + s\Delta\epsilon, s = 0, 1, \cdots, M-1, \Delta\epsilon = 1/M\}$. Here $1/M$ is the step size and the choice of M is a trade-off between the CFO estimation accuracy and computational complexity.

To simplify the algorithm, here we assume that the LPN can be regarded as a constant phase offset within one OFDM symbol, i.e., $\theta(n) = \theta$. Then the cost function $\Gamma(\epsilon_s)$ can be written as:

$$\Gamma(\epsilon_s) = \sum_k \left| \sum_{n=0}^{N-1} \frac{1}{N} \sum_{l=-N/2}^{N/2-1} X(l) H(l) e^{j\left[\frac{2\pi n(\epsilon - \epsilon_s + l - k)}{N} + \theta\right]} + V(k, \epsilon_s) \right|^2 = \sum_k |M(k, \epsilon_s) + V(k, \epsilon_s)|^2. \quad (3)$$

where $X(l)$ and $H(l)$ are the l-th sample of the transmitted spectrum and channel response, respectively. Term ϵ_s is the known test value for CFO compensation, and we have $V(k, \epsilon_s) = \sum_{n=0}^{N-1} w(n) e^{-j2\pi n(\epsilon_s + k)/N}$, $V(k, \epsilon_s) \sim \mathcal{N}(0, N\sigma^2)$. In addition, the term $M(k, \epsilon_s)$ is defined as:

$$M(k, \epsilon_s) = \frac{1}{N} \sum_{l=-N/2}^{N/2-1} X(l) H(l) \sum_{n=0}^{N-1} e^{j\left[\frac{2\pi n(\epsilon - \epsilon_s + l - k)}{N} + \theta\right]} = \frac{1}{N} \sum_{l=-N/2}^{N/2-1} X(l) H(l) e^{j\theta} \frac{1 - e^{j2\pi(\epsilon - \epsilon_s + l - k)}}{1 - e^{j2\pi(\epsilon - \epsilon_s + l - k)/N}}. \quad (4)$$

In (4), since l and k are both integers, thus $M(k, \epsilon_s)$ can be rewritten as $M(k, \epsilon_s) = \frac{1}{N} \sum_{l=-N/2}^{N/2-1} X(l) H(l) e^{j\theta} \frac{1 - e^{j2\pi(\epsilon - \epsilon_s)}}{1 - e^{j2\pi(\epsilon - \epsilon_s + l - k)/N}}$. Now we define: $P(\epsilon, \epsilon_s, l, k) = \frac{1}{1 - e^{j2\pi(\epsilon - \epsilon_s + l - k)/N}}$, where we have $\epsilon \in [-0.5, 0.5)$ and $\epsilon_s \in [-0.5, 0.5)$, thus $2\pi(\epsilon - \epsilon_s)/N << 1$. Thus we have:

$$P(\epsilon, \epsilon_s, l, k) = \frac{1}{1 - e^{j2\pi(l-k)/N}(1 + j2\pi(\epsilon - \epsilon_s)/N)} \approx \frac{1}{1 - e^{j2\pi(l-k)/N}}. \quad (5)$$

Here, based on the approximations, $P(\epsilon, \epsilon_s, l, k)$ is unrelated to ϵ and ϵ_s. Then we express (5) as: $P(l, k) = a(l, k) + jb(l, k)$, where $a(l, k)$ and $b(l, k)$ are the real and imaginary parts of $P(l, k)$ which varies with l and k. Therefore, $M(k, \epsilon_s)$ can be expressed as:

$$M(k, \epsilon_s) = \frac{1}{N} \sum_{l=-N/2}^{N/2-1} X(l) H(l) e^{j\theta} [a(l, k) + jb(l, k)] \cdot \left(1 - e^{j2\pi(\epsilon - \epsilon_s)}\right) = [\alpha(k) + j\beta(k)] \left(1 - e^{j2\pi(\epsilon - \epsilon_s)}\right), \quad (6)$$

where $\alpha(k) = \Re\left\{\frac{1}{N} \sum_{l=-N/2}^{N/2-1} X(l) H(l) e^{j\theta} [a(l, k) + jb(l, k)]\right\}$, $\beta(k) = \Im\left\{\frac{1}{N} \sum_{l=-\frac{N}{2}}^{\frac{N}{2}-1} X(l) H(l) e^{j\theta} [a(l, k) + jb(l, k)]\right\}$.

Finally, the cost function $\Gamma(\epsilon_s)$ can be written as:

$$\Gamma(\epsilon_s) = \sum_{k \in \{\text{ZS index}\}} |M(k, \epsilon_s) + V(k, \epsilon_s)|^2 = \sum_{k \in \{\text{ZS index}\}} \left| [\alpha(k) + j\beta(k)] \left(1 - e^{j2\pi(\epsilon - \epsilon_s)}\right) + V(k, \epsilon_s) \right|^2 = A[1 - \cos(2\pi(\epsilon - \epsilon_s))] + C, \quad (7)$$

where we have $A = 2\sum_{k \in \{\text{ZS index}\}} \left\{\alpha^2(k) + \beta^2(k)\right\}$ and C denotes the DC component caused by the term $\sum_{k \in \{\text{ZS index}\}} |V(k, \epsilon_s)|^2$. As can be seen in (7), there are three unknown parameters, A, C and ϵ. Thus, by taking three test values of ϵ_s, we can solve for the three unknown parameters. For simplicity, we calculate three values of cost function, $\Gamma(0)$, $\Gamma(0.25)$ and $\Gamma(0.5)$, i.e., $\epsilon_s = 0, 0.25, 0.5$, as:

$$\Gamma(0) = A[1 - \cos(2\pi\epsilon)] + C, \Gamma(0.25) = A[1 - \sin(2\pi\epsilon)] + C, \Gamma(0.5) = A[1 + \cos(2\pi\epsilon)] + C. \quad (8)$$

By solving (8), we have $A\cos(2\pi\epsilon) = \frac{\Gamma(0.5) - \Gamma(0)}{2}$, $A\sin(2\pi\epsilon) = \frac{\Gamma(0) + \Gamma(0.5)}{2} - \Gamma(0.25)$. By defining a complex variable z as: $z = A\cos(2\pi\epsilon) + jA\sin(2\pi\epsilon)$, the CFO estimate can be obtained by $\hat{\epsilon}_f = \frac{\angle\{z\}}{2\pi}$. To further improve the estimation performance, here we propose to take the power average through several OFDM symbols, and call this method as modified ZSP method (MZSP). The we use MSZSP to represent the modified simplified ZSP method.

3. Simulation Results and Discussion

In our simulations, the length of serial input data bits is $2^{15} - 1$ and the DFT size is 256, with 128 effective data subcarriers and the CP length of 32 samples. The data is modulated onto 16-QAM, and the sample rate of the arbitrary waveform generator (AWG) is 25 GSa/s.

In Fig. 1, we can observe that the modelled function matches quite well with the real cost function, even at 10 dB SNR. This illustrates the feasibility of the proposed method. In Fig. 2, we investigate the effect of the number of ZS on CFO estimation performance. It is obvious that the CFO estimation becomes more accurate with the increase of the number of ZS. Because the presence of AWGN will cause the increase of ZSP, by using more ZS, the estimation error can be reduced. We use 128 ZS for CFO estimation in following simulations, whose length is equal to half of the DFT size. In Fig.3, SZSP method performs slightly worse than ZSP method, but both of them show worse performance than that of Schmidl's method. This is mainly because Schmidl's method utilizes the training symbol for CFO estimation, which contains two identical halves, while the proposed SZSP method is totally blind and the number of ZS has a significant influence on the estimation performance as shown in Fig. 2. However, by utilizing the MZSP, the performance can be improved to be nearly the same as Schmidl's method by using 32 OFDM symbols. In Fig. 4, the proposed methods show quite good LPN tolerance, since the LPN in the frequency domain can be divided into a common phase error (CPE) and an ICI part which is in general regarded as AWGN at low LPN. In our proposed methods, when calculating the ZSP, the CPE can be filtered out. This is why when the CLW is lower than 100 kHz, the estimation does not change significantly. In the MSZSP algorithm, when $M = 64$, the estimation error variance is nearly one order smaller than that of Schmidl's method, which illustrates the robustness of our proposed methods.

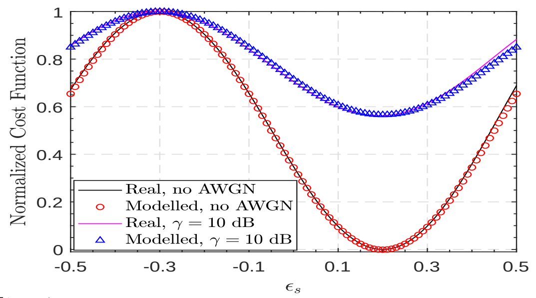

Fig. 1: Normalized real and modelled cost function under $\epsilon = 0.2$.

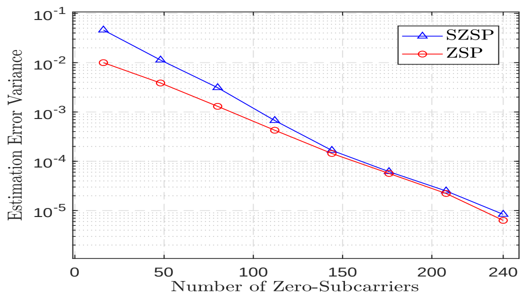

Fig. 2: The estimation error variance vs. the number of zero-subcarriers, with the CFO of 5 GHz, at 20 dB SNR.

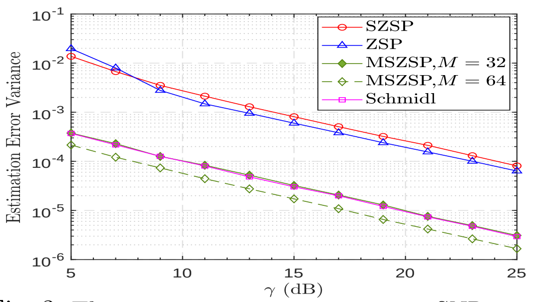

Fig. 3: The estimation error variance vs. SNR, with the CFO of 5 GHz.

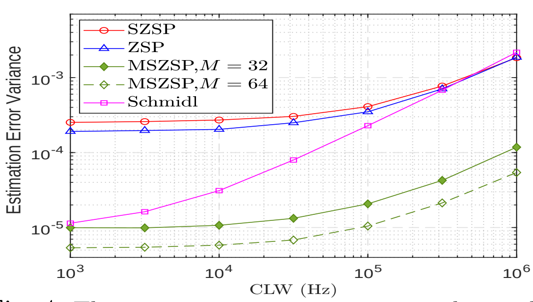

Fig. 4: The estimation error variance vs. the combined laser linewidth with the CFO of 5 GHz, at 20 dB SNR.

4. Conclusion

In this paper, we propose a simplified blind CFO estimation algorithm that utilizes only three test values to estimate CFO which is simple and easy to implement. By taking the power average through several OFDM symbols, the MSZSP method can obtain a more accurate result. The simulation results verify the effectiveness of our proposed algorithms, and its robustness to the laser phase noise.

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