

# Shipping emission control area optimization considering carbon emission reduction

Dan Zhuge

School of Management, Shanghai University, Shanghai, China, dan\_zhuge@shu.edu.cn

Shuaian Wang

Department of Logistics & Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Hong Kong, wangshuaian@gmail.com

Lu Zhen\*

School of Management, Shanghai University, Shanghai, China, lzhen@shu.edu.cn

Sulfur emission control areas (ECAs) are crucial for reducing global shipping emissions and protecting the environment. The main plank of an ECA policy is usually a fuel sulfur limit. However, the approaches to setting sulfur limits are relatively subjective and lack scientific support. This paper investigates the design of ECA policies, especially sulfur limits, for sailing legs with ECAs. The objective is to minimize the social costs of shipping operations, local sulfur oxides ( $\text{SO}_x$ ) emissions, and global carbon dioxide ( $\text{CO}_2$ ) emissions. First, a case with a no-ECA policy and a case with the current ECA policy are analyzed. Then, two new voyage-dependent ECA policies with sulfur limits, designated sailing paths, and speed limits are proposed. Stackelberg game models are developed to solve the research problem with the two proposed policies and two players: the ECA regulator and a shipping company, aiming to minimize social costs and company costs, respectively. The ECA regulator determines the sulfur limit, sailing path, and speed limit, and the shipping company optimizes the sailing speed accordingly. We also compare and analyze each type of cost under different ECA policies, i.e., no ECA, the current ECA policy, and the proposed ECA policies. The research problem is then extended from a sailing leg to a shipping network to improve the practicality of the findings. A dynamic programming based algorithm is developed to optimize the ECA policies for the shipping network from the perspective of the ECA regulator. Mathematical derivation shows that the proposed ECA policies can reduce the social costs of shipping. The results of extensive numerical experiments further demonstrate the ability of the proposed policies to reduce social costs, providing important insights for voyage-dependent ECA policy design.

*Key words:* Local sulfur oxides emissions; global carbon dioxide emissions; emission control areas; voyage-dependent ECA policy design

## 1. Introduction

Maritime transportation plays a central role in the global supply chain, but increased shipping volume has caused severe environmental problems. Shipping activities generate substantial exhaust gases, such as sulfur oxides ( $\text{SO}_x$ ) and carbon dioxide ( $\text{CO}_2$ ). Extensive  $\text{SO}_x$  emissions

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\* Corresponding author

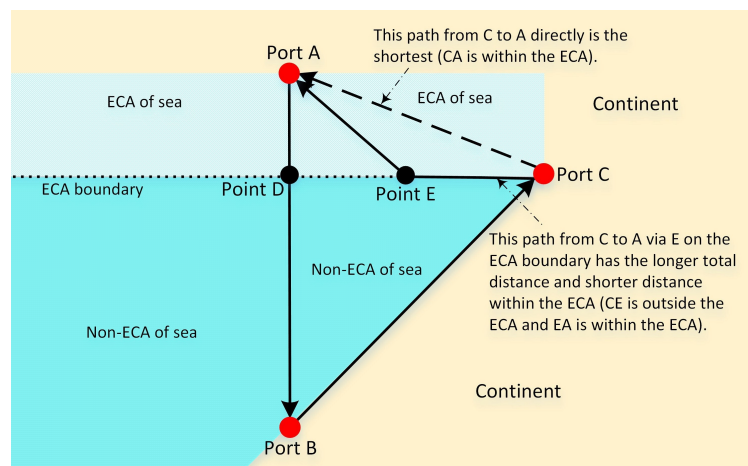
can seriously harm the environment and human health, causing conditions such as cardiovascular disease, lung cancer, and birth defects (Dai et al. 2019). Sofiev et al. (2018) reported that globally, shipping emissions indirectly cause at least 400,000 premature deaths per year. Emissions of CO<sub>2</sub>, the dominant greenhouse gas (GHG), may lead to rising sea levels, a shrinking ozone layer, extreme weather, and changing ecosystems. The International Maritime Organization (IMO) reported that maritime activities were responsible for approximately 2.89% of global anthropogenic CO<sub>2</sub> emissions in 2018, an increase of 5% from 2012 (IMO 2020). Therefore, mitigating the environmental effects of maritime transportation while maintaining normal operations is essential.

To control ship emissions, the 72nd session of the Marine Environment Protection Committee of the IMO adopted a strategy to reduce CO<sub>2</sub> emissions by at least 40% by 2030, relative to 2008. Additionally, the International Chamber of Shipping (ICS 2021) has presented several urgent measures that governments must implement to deliver net zero CO<sub>2</sub> emissions by 2050. The IMO encourages improvements in energy efficiency during ship design and operations, and has implemented various tools, e.g., the Energy Efficiency Design Index, Energy Efficiency Existing-Ship Index, and Ship Energy Efficiency Management Plan, to monitor such improvements.

In recent years, sulfur emissions have been one of the most intensively discussed environmental concerns in the shipping industry. Among the various measures implemented by the IMO and governments, the designation of emission control areas (ECAs) is particularly important and has received considerable attention. Since January 1, 2015, four ECAs, i.e., the Baltic Sea, North Sea, North American, and United States Caribbean Sea ECAs, have required that ships within their boundaries use marine fuel with no more than 0.1% sulfur or take equivalent measures to reduce emissions. South Korea has announced several ECAs with a 0.1% sulfur limit: Incheon, Pyeongtaek-Dangjin, Yeosu-Gwangyang, Busan, and Ulsan. China has had a 0.1% sulfur limit in Hainan waters since January 1, 2022, and is evaluating whether to extend the regulation to all Chinese waters. An IMO-designated ECA in the Mediterranean Sea will be effective in 2025. Several European countries, including Spain, Portugal, and the United Kingdom, plan to propose an ECA in the North-East Atlantic Ocean, and Canada also pledges to designate an ECA in its Arctic waters. The enforcement of ECA regulations is effective in controlling SO<sub>x</sub> emissions in coastal areas (Svindland 2018, Zhang et al. 2020). To further reduce SO<sub>x</sub> emissions, the IMO has enforced a 0.5% global sulfur limit for ocean-going ships since January 1, 2020, which has been analyzed by several scholars, such as Zhu et al. (2020) and Wang et al. (2021).

This paper addresses the ECA policy design problem—first for a sailing leg (i.e., a sailing voyage from one port to another, with two types of legs involving ECAs) and then for a shipping network.

Sulfur emission regulations restrict traditional ships without scrubbers to use marine gas oil (MGO) with a sulfur content of no more than 0.1% within ECAs, whereas very low-sulfur fuel oil (VLSFO) that is of no more than 0.5% sulfur and has a lower unit price can be used outside ECAs. Fig. 1 illustrates the problem. A ship that plans to sail from port A to port B within a given time will sail at a lower speed from port A to point D and a higher speed from point D to port B to reduce the consumption of high-price MGO, given the approximately cubic relationship between daily fuel consumption and sailing speed. Thus, the total fuel cost for the voyage may be reduced. Meanwhile, a ship that plans to sail from port C to port A will sail via point E on the ECA boundary to reduce the sailing distance within ECAs. Compared with sailing directly from port C to port A, sailing via point E results in a longer total distance but a shorter distance within the ECA, reducing the use of MGO, and thus the total fuel cost may also be reduced. Both the speed differentiation and



**Figure 1** Illustration of rerouting/speed differentiation

detour decisions, made by the shipping companies, increase the total fuel burned. Burning equal amounts of MGO and VLSFO produces the same amount of  $\text{CO}_2$  emissions but different amounts of  $\text{SO}_x$  emissions (Fagerholt and Psaraftis 2015). A survey of ship traffic in Baltic Sea, North Sea, and North American ECAs indicated that ECA policies may increase global  $\text{CO}_2$  emissions (Eason 2015). Quantitative analyses of speed differentiation and detours motivated by ECA policies show that the resulting increase in  $\text{CO}_2$  emissions can exceed 5% (Doudnikoff and Lacoste 2014) or even 10% in some scenarios (Fagerholt et al. 2015).

The current sulfur limits can have the unintended consequence of increasing  $\text{CO}_2$  emissions because they are not based on scientific calculations and quantitative decision making. Therefore, this study addresses the problem of minimizing the social costs of shipping operations, local  $\text{SO}_x$  emissions (i.e.,  $\text{SO}_x$  emissions within ECAs), and global  $\text{CO}_2$  emissions. Thus, it is of practical importance for reducing global  $\text{CO}_2$  emissions and realizing carbon neutrality. A Stackelberg game

model is used to optimize the decisions of the ECA regulator and a shipping company. The ECA regulator is concerned about both environmental problems and economic development, and the shipping company is concerned about its shipping operations. In a scenario with a given sailing time between ports, four cases are analyzed: no ECA, the current ECA policy, and two proposed ECA policies. Under the proposed policies, Stackelberg game models with the ECA regulator and a shipping company are presented, in which the shipping company aims to minimize its costs (henceforth referred to as the company cost) and the ECA regulator aims to minimize social costs (including company,  $\text{SO}_x$ , and  $\text{CO}_2$  costs). The ECA regulator sets the sulfur limit, sailing path, and speed limit, and the shipping company optimizes sailing speeds within and outside ECAs accordingly. To improve the practicality of the research problem, the problem is then extended from a sailing between two ports (i.e., a leg) to a shipping company's network with numerous service routes each including several legs, and the company's ship deployment decision for a route is the input of the sailing time for the legs of the route. A dynamic programming based algorithm is developed to optimize ECA policies for the shipping network. Numerical experiments are conducted to validate the effectiveness of the proposed policies in minimizing social costs.

This paper makes three main contributions. First, it is the first study on how to design ECA policies to minimize the social costs of shipping operations, local  $\text{SO}_x$  emissions, and global  $\text{CO}_2$  emissions. To the best of our knowledge, no studies have focused on controlling increased global  $\text{CO}_2$  emissions under the current ECA policy, including those that have observed the increase. Moreover, unlike current ECA policies and previous research, which assume a unified ECA policy for all ships, this paper creatively designs specific ECA policies for each voyage to further reduce social costs. Second, we identify many important properties relevant to the design of voyage-dependent ECA policies that can provide practical guidance for ECA policy redesign, e.g., that unit fuel prices within and outside ECAs and costs of  $\text{SO}_x$  emissions from burning one ton of fuel with different sulfur contents are relevant to the design of sulfur limits, that the optimal sulfur limits in different ECAs in a single voyage are the same, and that the sailing speed limit within ECAs may be bounded from below instead of above. Third, the effectiveness of the proposed ECA policies in saving social costs is identified using mathematical derivation and validated by extensive numerical experiments (including sensitivity analyses of the important parameters).

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the ECA policy design problem for two types of sailing legs and presents a Stackelberg game model with the ECA regulator and a shipping company to solve it. Section 4 discusses how to optimize voyage-dependent ECA policies with respect to the sulfur limit, sailing path, and speed limit for two types of legs with a given sailing time. In Section 5, the study is extended to a shipping network, and a dynamic programming based algorithm is developed to optimize the



shipping network problem. Section 6 discusses the numerical experiments, and Section 7 concludes the paper. The online supplement contains four electronic companions (ECs): EC.1 addresses a voyage-dependent ECA design problem for two types of legs with sailing time optimization; the proofs and mathematical derivation are presented in EC.2; all of the algorithms are developed in EC.3; EC.4 conducts numerical experiments on current policies and newly added cases.

## 2. Literature review

Considerable research on maritime transportation has been published, e.g., Agarwal and Ergun (2010), Fransoo and Lee (2013), Lu et al. (2017), Wang et al. (2018), Lübbecke et al. (2019), Qi et al. (2021), and Lee et al. (2021). We focus on the topics relevant to this study: shipping speed limits, optimizing ship operations in ECAs, and the effects of ECA policies.

Maritime regulators usually enforce shipping speed limits to control an increase in airborne emissions (Corbett et al. 2009), and numerous researchers have studied the effect of regional maximum speed limits (RMSLs) on shipping emissions in light of the decisions of both maritime regulators and shipping companies. Khan et al. (2012), Zis et al. (2014), Chang and Jhang (2016), and López-Aparicio et al. (2017) assessed speed limits in near-shore speed reduction zones and observed significant emission reductions in those zones. Cariou and Cheaitou (2012) optimized both ship speeds and schedules based on an RMSL. Their findings indicated that given a fixed number of ships, CO<sub>2</sub> emissions decrease in areas with an RMSL, but global emissions increase, as ships respond by sailing faster in other areas. Zis et al. (2015) also found that an RMSL leads to an increase in global CO<sub>2</sub> emissions.

Numerous studies have also explored changes in sailing patterns under ECA regulations. Doudnikoff and Lacoste (2014) demonstrated that differences in sailing speeds inside and outside ECAs should not be ignored, as they can slightly reduce costs but increase CO<sub>2</sub> emissions. Fagerholt et al. (2015) and Fagerholt and Psaraftis (2015) found that shipping companies purposely design detours and vary speeds within and outside ECAs. Gu and Wallace (2017) calculated the lifespan cost of fuel switching and scrubbers with optimized sailing patterns. Zhen et al. (2018) reported that ships sailing at lower speeds within ECAs and choosing a longer path can reduce the use of high-price fuel. Li et al. (2020) and Wang et al. (2021) obtained similar results for sailing pattern optimization.

Most studies on optimizing the efficiency of shipping networks have found that optimization considerably reduces SO<sub>x</sub> emissions, even if that is not the intended outcome (Xia et al. 2015, Akyüz and Lee 2016, Karsten et al. 2018, Ng and Lin 2018, Dong et al. 2020). This is reasonable because SO<sub>x</sub> emissions are proportional to fuel consumption given a fuel with a fixed sulfur content (Song and Xu 2012). ECAs mandate that ships use lower-sulfur fuel (e.g., MGO) within their

boundaries, thus reducing local  $\text{SO}_x$  emissions. Chen et al. (2018) investigated how a (at that time) hypothetical ECA in the Mediterranean Sea would affect route choices and shipping emissions. Zhen et al. (2020) analyzed the effects of ECA boundaries, decision-makers, and fuel cost on  $\text{SO}_x$  emissions. Browning et al. (2012), Chang et al. (2014), Svindland (2018), and Zhang et al. (2020) demonstrated the effectiveness of ECAs in reducing  $\text{SO}_x$  emissions. However, Sheng et al. (2019) and Ma et al. (2020) concluded that ECA regulations increase global  $\text{CO}_2$  emissions. Maritime regulators did not anticipate this unintended consequence in their cost-benefit analyses of ECAs. Hence, determining how to manage ECAs to curb the increase in  $\text{CO}_2$  emissions is urgently needed.

As noted, almost no research has simultaneously addressed local sulfur limits and global  $\text{CO}_2$  emissions. Therefore, this study uses quantitative analysis to investigate the design of ECA policies while considering the response of shipping companies and the effects on  $\text{SO}_x$  and  $\text{CO}_2$  emissions.

### 3. Problem background

We investigate the design of voyage-dependent ECA policies to minimize the social costs of shipping operations,  $\text{SO}_x$  emissions, and  $\text{CO}_2$  emissions. Studies using numerical experiments have found that the current ECA policy may increase global  $\text{CO}_2$  emissions from shipping. The main reason for the increase in  $\text{CO}_2$  emissions is that ECA policy and shipping operation decisions are made separately by the ECA regulator and shipping companies, respectively. A unified ECA policy may not be optimal for different sailing legs. Therefore, we consider an ECA regulator that is the central regulatory authority (e.g., the IMO) that designs ECA policies for all sailing legs and considers the costs of  $\text{SO}_x$  and  $\text{CO}_2$  emissions and shipping company costs. Accordingly, the ECA regulator designs voyage-dependent ECA policies for each leg that include a sulfur limit, sailing path, and speed limit, after considering the shipping company's response regarding sailing speed, to mitigate the sum of the company cost and  $\text{SO}_x$  and  $\text{CO}_2$  emission costs.

We focus on two types of legs (see Fig. 2): a leg with only one port covered by an ECA (Type-1) and a leg with two ports covered by ECAs (Type-2). Each leg has multiple stretches. As shown in Fig. 2, the Type-1 leg has one ECA stretch and one non-ECA stretch, and the Type-2 leg has two ECA stretches and one non-ECA stretch. To simplify the construction of the model, we prove that the optimal sulfur and speed limits designed by the ECA regulator and the optimal sailing speeds decided by the shipping company for all ECA stretches of a leg are identical in Section 4. According to this analysis, we only need to consider the total distance of all ECA stretches (denoted by  $L_E$ ) and the total distance of all non-ECA stretches (denoted by  $L_N$ ) on a leg. Therefore, the same models (see Eqs. (1) and (4)) for the company and social costs and the same functions (see Eqs. (2) and (3)) for  $\text{SO}_x$  and  $\text{CO}_2$  costs can be used for both types of legs. Frequently used notations are described below.

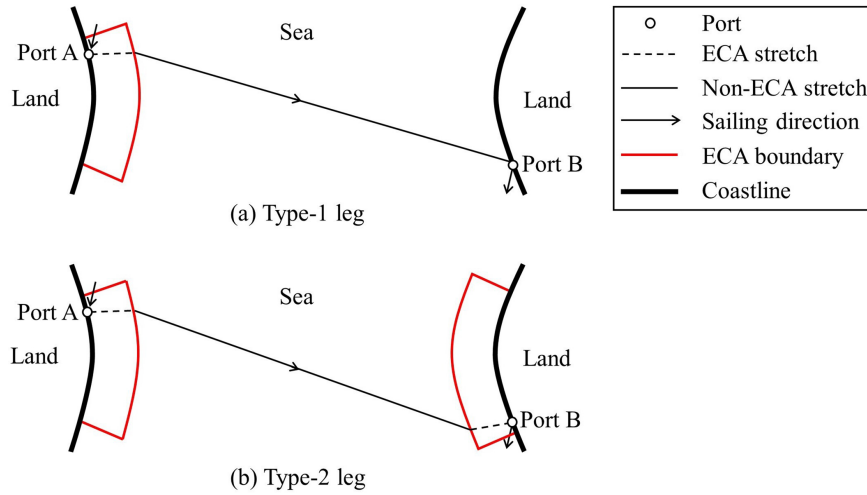


Figure 2 Two types of sailing legs

### Parameters

$\alpha_E$	Unit price (US\$/ton) of 0.1% sulfur fuel
$\alpha_N$	Unit price (US\$/ton) of 0.5% sulfur fuel, $\alpha_N < \alpha_E$
$\rho_E$	Cost of $\text{SO}_x$ emissions from burning one ton of 0.1% sulfur fuel (US\$/ton), which equals the unit cost of $\text{SO}_x$ times the amount of $\text{SO}_x$ generated
$\rho_N$	Cost of $\text{SO}_x$ emissions from burning one ton of 0.5% sulfur fuel (US\$/ton); $\rho_N = 5\rho_E$ because the $\text{SO}_x$ generated is five times that generated from burning one ton of 0.1% sulfur fuel
$\kappa$	Cost of $\text{CO}_2$ emissions from burning one ton of fuel (within or outside ECAs) (US\$/ton)
$a, b$	Conversion factors between fuel consumption per unit distance and sailing speed: fuel consumption per unit distance is $a \cdot v^b$ (ton/nm), where $v$ is sailing speed (knots); $a > 0$ and $b > 1$
$c$	Hourly operating and capital costs of each ship (US\$/hour)

### Decision variables

$L_E$	Sailing distance (nm) within ECAs; note that the same sulfur limit within and outside ECAs is treated as a no-ECA policy
$L_N$	Sailing distance (nm) outside ECAs
$T$	Total sailing time (hour) within and outside ECAs
$\theta$	Proposed sulfur rule for fuel used within ECAs, which is a mixture of $100\theta\%$ fuel with 0.1% sulfur and $100(1 - \theta)\%$ fuel with 0.5% sulfur ( $0 \leq \theta \leq 1$ ); therefore, its sulfur content is $100\theta \times 0.1\% + 100(1 - \theta) \times 0.5\%$ percent, and its price is $\theta\alpha_E + (1 - \theta)\alpha_N$
$v_E$	Sailing speed (knot) within ECAs

### Derived variables

$\alpha_{E\theta}$	Unit price (US\$/ton) of mixed fuel burned within ECAs; $\alpha_{E\theta} = \theta\alpha_E + (1 - \theta)\alpha_N$
$\rho_{E\theta}$	Cost of $\text{SO}_x$ emissions from burning one ton of mixed fuel within ECAs (US\$/ton); $\rho_{E\theta} = \theta\rho_E + (1 - \theta)\rho_N$

The shipping company aims to minimize its costs, which consist of operating and capital costs for a given sailing time and the fuel costs for the ECA and non-ECA stretches of the leg. The sailing time for the ECA stretch(es) is  $\frac{L_E}{v_E}$ , and the sailing time for the non-ECA stretch is  $\frac{v_E T - L_E}{v_E}$ . Therefore, the sailing speed for the non-ECA stretch is  $\frac{v_E L_N}{v_E T - L_E}$ . The fuel consumption rate of a stretch is  $a \cdot v^b$  (ton/nm), where  $a > 0$ ,  $b > 1$ , and  $v$  is the sailing speed on the stretch. The company cost is

$$\min C^{\text{company}}(L_E, L_N, v_E, \theta, T) = \alpha_{E\theta} \cdot a(v_E)^b L_E + \alpha_N \cdot a \left( \frac{v_E L_N}{v_E T - L_E} \right)^b L_N + cT, \quad (1)$$

subject to the constraints of the sulfur limit, sailing path, and speed limit determined by the ECA regulator, as discussed in Section 4. The ECA regulator focuses on local sulfur emissions; therefore, the  $\text{SO}_x$  cost is limited to the sulfur emissions from fuel burned *within* ECAs.

$$C^{\text{SO}_x}(L_E, v_E, \theta) = \rho_{E\theta} \cdot a(v_E)^b L_E. \quad (2)$$

The  $\text{CO}_2$  cost is determined by total  $\text{CO}_2$  emissions, i.e., the  $\text{CO}_2$  emissions of the whole leg (within and outside ECAs).

$$C^{\text{CO}_2}(L_E, L_N, v_E, \theta, T) = \kappa \cdot \left[ a(v_E)^b L_E + a \left( \frac{v_E L_N}{v_E T - L_E} \right)^b L_N \right]. \quad (3)$$

The ECA regulator aims to minimize social costs, which encompass the three types of costs previously noted.

$$\begin{aligned} \min C^{\text{social}}(L_E, L_N, v_E, \theta, T) &= C^{\text{company}}(L_E, L_N, v_E, \theta, T) + C^{\text{SO}_x}(L_E, v_E, \theta) \\ &\quad + C^{\text{CO}_2}(L_E, L_N, v_E, \theta, T) \\ &= (\alpha_{E\theta} + \rho_{E\theta} + \kappa) \cdot a(v_E)^b L_E + (\alpha_N + \kappa) \cdot a \left( \frac{v_E L_N}{v_E T - L_E} \right)^b L_N + cT. \end{aligned} \quad (4)$$

To simplify the notation, we provide the following definitions:

**Definition 1.** Given ship fuel consumption parameters  $a$  and  $b$  and the unit prices of the fuel burned within and outside ECAs  $\alpha_{E\theta}$  and  $\alpha_N$ ,

$$\gamma_\theta := \left( \frac{\alpha_{E\theta}}{\alpha_N} \right)^{\frac{1}{b+1}}. \quad (5)$$

**Definition 2.** Given ship fuel consumption parameters  $a$  and  $b$ , the unit prices of the fuel burned within and outside ECAs  $\alpha_{E\theta}$  and  $\alpha_N$ , the costs of  $\text{SO}_x$  emissions from burning one ton of fuel within ECAs  $\rho_{E\theta}$ , and the cost of  $\text{CO}_2$  emissions from burning one ton of fuel  $\kappa$ ,

$$\xi_\theta := \left( \frac{\alpha_{E\theta} + \rho_{E\theta} + \kappa}{\alpha_N + \kappa} \right)^{\frac{1}{b+1}}. \quad (6)$$

We also define  $\alpha_{E\theta} + \rho_{E\theta} + \kappa$  and  $\alpha_N + \kappa$  as the unit social costs of fuels burned within and outside ECAs, respectively. Hence,  $(\gamma_\theta)^{b+1}$  is the ratio between the unit fuel prices within and outside ECAs, and  $(\xi_\theta)^{b+1}$  is the ratio between the unit social costs of fuels within and outside ECAs.

Using these models and functions, we analyze in Sections 4.1 and 4.2 the two types of legs with a no-ECA policy and under the current ECA policy, and then in Sections 4.3 and 4.4, we propose two ECA policies that consider the effects of ECA policies on the company,  $\text{SO}_x$ ,  $\text{CO}_2$ , and social costs of each leg. In Section 5, the research problem is extended to the shipping network of a shipping company, and the sailing time for each leg is updated according to the company's ship deployment decision. Fig. 3 illustrates this study's framework.

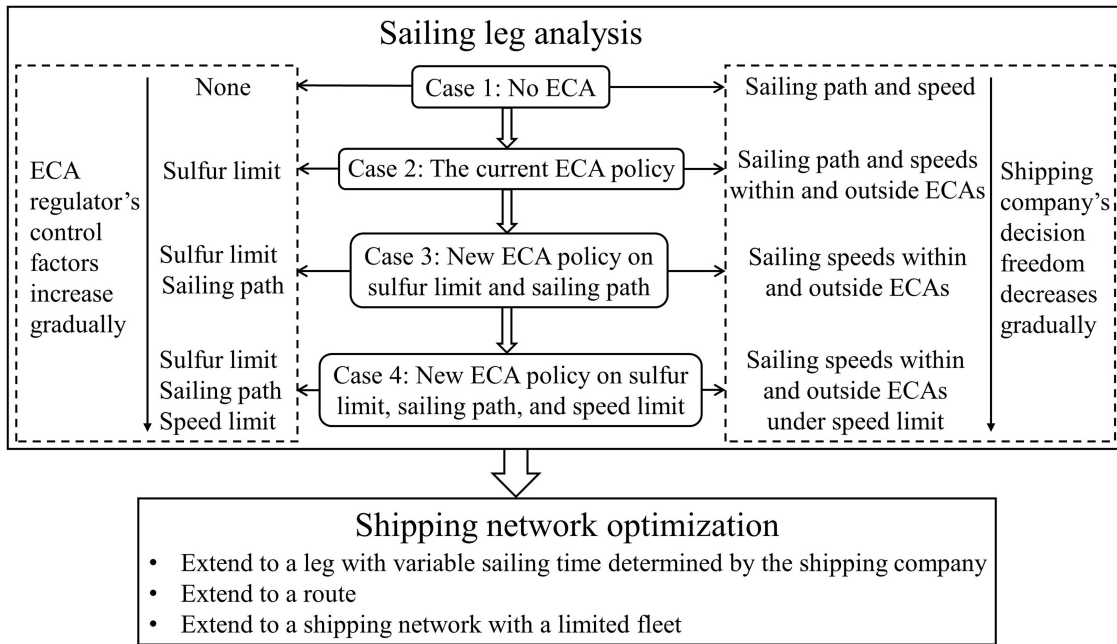


Figure 3 Framework diagram

#### 4. Voyage-dependent ECA policy design

In this section, we design voyage-dependent ECA policies for each leg given sailing time  $\bar{T}$ . The sailing path and speed are optimized by the shipping company, and the company,  $\text{SO}_x$ ,  $\text{CO}_2$ , and social costs are calculated for each leg with a no-ECA policy (Case 1). The detour and speed decisions are then made by the shipping company for the Type-1 and Type-2 legs under the current ECA policy (Case 2), and the four types of costs are computed for comparison. Different from the literature, this study derives analytical solutions for the optimal sailing path and speed for a sailing leg in the first two cases, which we can use to identify the increase in  $\text{CO}_2$  emissions over the whole leg and even the possible increase in social costs under the current ECA policy. Thus, the need to redesign the current ECA policy is validated. We then propose an ECA policy with a sulfur

limit and specified sailing path (Case 3) and another ECA policy that further incorporates a speed limit (Case 4) for each type of leg. Stackelberg game models with two players (the ECA regulator and a shipping company) are developed to study the effects of the proposed policies. The ECA regulator takes the leader role and considers the response of the shipping company before enacting a voyage-dependent sulfur limit and designated sailing path in Case 3 and a voyage-dependent sulfur limit, designated sailing path, and voyage-dependent speed limit in Case 4 to minimize the social costs of a sailing leg. The shipping company takes the follower role and aims to minimize its costs.

#### 4.1. Case 1: Same sulfur limit within and outside ECAs (No ECA)

In Case 1, the sulfur limit is the same everywhere, i.e.,  $\theta = 0$ , and there are no limits on sailing path or speed. As the same fuel can be used within and outside ECAs, the shortest path, whose lengths within and outside ECAs are denoted by  $\hat{L}_{E1}$  and  $\hat{L}_{N1}$ , respectively (subscript “1” indicates Case 1, and “^” in  $\hat{L}_{E1}$  and  $\hat{L}_{N1}$  indicates that the two distances are selected by the shipping company), is optimal for the shipping company, and the optimal sailing speeds within and outside ECAs, denoted by  $\hat{v}_{E1}$  and  $\hat{v}_{N1}$ , respectively, both equal  $\frac{\hat{L}_{E1} + \hat{L}_{N1}}{\bar{T}}$ . As a result, the minimum company cost is

$$C^{\text{company}}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T}) = \alpha_N \cdot a \bar{T}^{-b} (\hat{L}_{E1} + \hat{L}_{N1})^{b+1} + c \bar{T}, \quad (7)$$

the  $\text{SO}_x$  cost is

$$C^{\text{SO}_x}(\hat{L}_{E1}, \hat{v}_{E1}, 0) = \rho_N \cdot a \bar{T}^{-b} (\hat{L}_{E1} + \hat{L}_{N1})^b \hat{L}_{E1}, \quad (8)$$

the  $\text{CO}_2$  cost is

$$C^{\text{CO}_2}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T}) = \kappa \cdot a \bar{T}^{-b} (\hat{L}_{E1} + \hat{L}_{N1})^{b+1}, \quad (9)$$

and the social cost is

$$C^{\text{social}}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T}) = (\alpha_N + \kappa) \cdot a \bar{T}^{-b} (\hat{L}_{E1} + \hat{L}_{N1})^b (\xi_0^{b+1} \hat{L}_{E1} + \hat{L}_{N1}) + c \bar{T}. \quad (10)$$

#### 4.2. Case 2: Current ECA policy

This subsection considers the current ECA policy ( $\theta = 1$ ), i.e., the sulfur limit within ECAs is 0.1% but there are no limits on sailing path or speed. The fuel consumption rate  $a \cdot v^b$  is a convex function of speed  $v$ , as  $a > 0$  and  $b > 1$ . Therefore, the optimal speed in areas with the same sulfur limit should be the same, i.e., the sailing speeds for the two ECA stretches of the Type-2 leg (see Fig. 2(b)) are identical and the optimal speeds outside ECAs are also identical. Hence, each type of leg can be regarded as having one ECA part and one non-ECA part. Therefore, the company and social cost models, i.e., Eqs. (1) and (4), and the  $\text{SO}_x$  and  $\text{CO}_2$  cost functions,

i.e., Eqs. (2) and (3), proposed in Section 3 are appropriate for each type of leg. The company cost is convex on the sailing speed within ECAs, i.e.,  $v_E$ . Given a path with distances of  $L_E$  within ECAs and  $L_N$  outside ECAs, denote by  $\hat{v}_{E2}$  (subscript “2” indicates Case 2) the optimal solution that minimizes  $C^{\text{company}}(L_E, L_N, v_E, 1, \bar{T})$ . When  $v_E = \hat{v}_{E2}$ , the first-order partial derivative of  $C^{\text{company}}(L_E, L_N, v_E, 1, \bar{T})$  with respect to  $v_E$  equals 0. We hence calculate the following:

$$\hat{v}_{E2} = \frac{L_E + \frac{L_N}{\gamma_1}}{\bar{T}}. \quad (11)$$

(Note that  $\hat{v}_{E2}$  is actually a function of  $L_E$  and  $L_N$ .) The optimal speed outside ECAs is denoted by  $\hat{v}_{N2}$ , and

$$\frac{\hat{v}_{E2}}{\hat{v}_{N2}} = \frac{1}{\gamma_1}. \quad (12)$$

Eq. (12) shows that  $\hat{v}_{E2} < \hat{v}_{N2}$  as  $\gamma_1 = \left(\frac{\alpha_{E1}}{\alpha_N}\right)^{\frac{1}{b+1}} = \left(\frac{\alpha_E}{\alpha_N}\right)^{\frac{1}{b+1}}$  and  $\alpha_N < \alpha_E$ . For instance, if  $b = 2$  (i.e., the fuel consumption per unit distance  $a \cdot v^b$  is proportional to the speed squared, and the daily fuel consumption  $24a \cdot v^{b+1}$  is proportional to the speed cubed) and  $\alpha_E = 2\alpha_N$ , then  $\hat{v}_{E2} \approx 0.8\hat{v}_{N2}$ . In summary, the ship slows down within ECAs to minimize its fuel cost.

Plugging Eq. (11) into  $C^{\text{company}}(L_E, L_N, v_E, 1, \bar{T})$ , we obtain the minimum company cost for a path with distance  $L_E$  within ECAs and  $L_N$  outside ECAs under the current ECA policy:

$$C^{\text{company}}(L_E, L_N, \hat{v}_{E2}, 1, \bar{T}) = \alpha_N \cdot a\bar{T}^{-b}(\gamma_1 L_E + L_N)^{b+1} + c\bar{T}. \quad (13)$$

We use Eq. (13) to examine the choice of sailing path. Comparing Eqs. (13) and (7) shows that the company cost to sail one mile within an ECA is equivalent to sail  $\gamma_1$  miles outside ECAs. Therefore, the optimal sailing path has the smallest  $\gamma_1 L_E + L_N$  (denoted by  $\gamma_1 \hat{L}_{E2} + \hat{L}_{N2}$ ), so a detour decision is often used to select the sailing path. (Note that the optimal sailing path chosen by the shipping company in Case 1 is that with the smallest  $L_E + L_N$ .)

Therefore, the corresponding results for the current ECA policy are as follows:

$$C^{\text{company}}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T}) = \alpha_N \cdot a\bar{T}^{-b}(\gamma_1 \hat{L}_{E2} + \hat{L}_{N2})^{b+1} + c\bar{T} \quad (14)$$

$$C^{\text{SO}_x}(\hat{L}_{E2}, \hat{v}_{E2}, 1) = \rho_E \cdot a\bar{T}^{-b}(\gamma_1 \hat{L}_{E2} + \hat{L}_{N2})^b \gamma_1^{-b} \hat{L}_{E2} \quad (15)$$

$$C^{\text{CO}_2}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T}) = \kappa \cdot a\bar{T}^{-b}(\gamma_1 \hat{L}_{E2} + \hat{L}_{N2})^b (\gamma_1^{-b} \hat{L}_{E2} + \hat{L}_{N2}) \quad (16)$$

$$C^{\text{social}}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T}) = (\alpha_N + \kappa) \cdot a\bar{T}^{-b}(\gamma_1 \hat{L}_{E2} + \hat{L}_{N2})^b (\xi_1^{b+1} \gamma_1^{-b} \hat{L}_{E2} + \hat{L}_{N2}) + c\bar{T}. \quad (17)$$

Case 2 has a higher company cost than Case 1 because  $\gamma_1 \hat{L}_{E2} + \hat{L}_{N2} > \hat{L}_{E1} + \hat{L}_{N1}$ . The current ECA policy's 0.1% sulfur limit within ECAs requires the use of high-price marine fuel (e.g., MGO), which increases the company cost. Compared with speeds and distances outside ECAs, the lower speed and shorter sailing distance within ECAs reduce fuel consumption within ECAs, while the

higher speed and longer distance outside ECAs will increase the fuel consumption outside ECAs. In summary, speed decisions and detours can mitigate but not completely offset the company cost increase caused by the current ECA policy. We also find that the  $\text{SO}_x$  cost in Case 2 is lower than that in Case 1 because the stricter sulfur limit, lower speed, and shorter sailing distance within ECAs effectively reduce  $\text{SO}_x$  emissions in the area. Proposition 1 compares  $\text{CO}_2$  costs between Cases 2 and 1.

**Proposition 1.** *We have  $C^{\text{CO}_2}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T}) > C^{\text{CO}_2}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T})$ .*

Proposition 1 mathematically proves that the  $\text{CO}_2$  cost is higher in Case 2 than in Case 1. There are two main reasons for this result: (i) the different speeds within and outside ECAs in Case 2 increase the  $\text{CO}_2$  cost because the fuel consumption rate is convex on sailing speed, and (ii) the longer total distance in Case 2 leads to a higher average speed for the voyage with a given sailing time, and thus more fuel is consumed and more  $\text{CO}_2$  is emitted. Whether the social cost in Case 2 is higher than, lower than, or equal to that in Case 1 depends on the difference between the increases in company and  $\text{CO}_2$  costs and the reduction in  $\text{SO}_x$  cost (see the cost comparison between Cases 1 and 2 in Table 1). If the decrease in  $\text{SO}_x$  cost does not offset the increase in company and  $\text{CO}_2$  costs after the implementation of the current ECA policy, the social cost in Case 2 may be higher than that in Case 1.

**Proposition 2.** *Social costs may increase when an ECA is set up.*

An increase in social costs violates the purpose of an ECA policy. Therefore, ECA policies should be redesigned to guarantee a decrease in social costs.

**Table 1** Comparison of the four types of costs between Cases 1 and 2

Type of cost	Comparison result
Social cost	$C_2^{\text{social}} >, =, \text{ or } < C_1^{\text{social}}$
Company cost	$C_2^{\text{company}} > C_1^{\text{company}}$
$\text{SO}_x$ cost	$C_2^{\text{SO}_x} < C_1^{\text{SO}_x}$
$\text{CO}_2$ cost	$C_2^{\text{CO}_2} > C_1^{\text{CO}_2}$

Notes: “ $C_1^{\text{social}}$ ”, “ $C_1^{\text{company}}$ ”, “ $C_1^{\text{SO}_x}$ ”, and “ $C_1^{\text{CO}_2}$ ” indicate  $C^{\text{social}}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T})$ ,  $C^{\text{company}}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T})$ ,  $C^{\text{SO}_x}(\hat{L}_{E1}, \hat{v}_{E1}, 0)$ , and  $C^{\text{CO}_2}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T})$ , respectively; “ $C_2^{\text{social}}$ ”, “ $C_2^{\text{company}}$ ”, “ $C_2^{\text{SO}_x}$ ”, and “ $C_2^{\text{CO}_2}$ ” indicate  $C^{\text{social}}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T})$ ,  $C^{\text{company}}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T})$ ,  $C^{\text{SO}_x}(\hat{L}_{E2}, \hat{v}_{E2}, 1)$ , and  $C^{\text{CO}_2}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T})$ , respectively.

#### 4.3. Case 3: Proposed ECA policy on sulfur limit and sailing path

A new ECA policy is proposed in Case 3, in which the ECA regulator simultaneously optimizes the sulfur limit and sailing path, and the shipping company determines the sailing speed. The optimal



sulfur limit is in the range of 0.1% to 0.5% (including 0.1% and 0.5%). We first examine the Type-2 leg (as shown in Fig. 2(b)) in which both ports are within ECAs. The optimal sulfur limits of the two ECAs are summarized in the following proposition.

**Proposition 3.** *The sulfur limits on the two ECA stretches of a Type-2 leg that minimize social costs are the same.*

The same sulfur limit for both ECA stretches of a Type-2 leg yields the same optimal sailing speed choice by the shipping company, indicating that each type of leg can be treated as having one ECA part and one non-ECA part. The company and social cost models and the  $\text{SO}_x$  and  $\text{CO}_2$  cost functions proposed in Section 3 are therefore used for both types of legs.

We fix the sailing distances within and outside ECAs, denoted by  $\bar{L}_E$  and  $\bar{L}_N$ , to analyze the relationship between the sulfur limit and sailing speed and then design the optimal sulfur limit and sailing path to minimize social costs. According to Eq. (1), the optimal sailing speed for the shipping company  $\hat{v}_{E3}$  (subscript “3” indicates Case 3) is

$$\hat{v}_{E3} = \frac{\bar{L}_E + \frac{\bar{L}_N}{\gamma_\theta}}{\bar{T}}. \quad (18)$$

Substituting Eq. (18) into the social cost function  $C^{\text{social}}(L_E, L_N, v_E, \theta, \bar{T})$ , we have

$$C^{\text{social}}(\bar{L}_E, \bar{L}_N, \hat{v}_{E3}, \theta, \bar{T}) = (\alpha_N + \kappa) \cdot a \bar{T}^{-b} (\gamma_\theta \bar{L}_E + \bar{L}_N)^b (\xi_\theta^{b+1} \gamma_\theta^{-b} \bar{L}_E + \bar{L}_N) + c \bar{T}. \quad (19)$$

**Proposition 4.** *According to Eq. (19), the optimal sulfur rule within ECAs (denoted by  $\theta_3^*$ ) and the optimal sailing path (denoted by  $(L_E^*, L_N^*)$ ) to minimize social costs are independent on the given sailing time  $\bar{T}$ .*

The first-order derivative of  $C^{\text{social}}(\bar{L}_E, \bar{L}_N, \hat{v}_{E3}, \theta, \bar{T})$  on  $\theta$  is

$$\frac{dC^{\text{social}}(\bar{L}_E, \bar{L}_N, \hat{v}_{E3}, \theta, \bar{T})}{d\theta} = a \bar{T}^{-b} \bar{L}_E \left( \bar{L}_E + \frac{\bar{L}_N}{\gamma_\theta} \right)^{b-1} \alpha_{E\theta}^{-\frac{b+2}{b+1}} f(\theta), \quad (20)$$

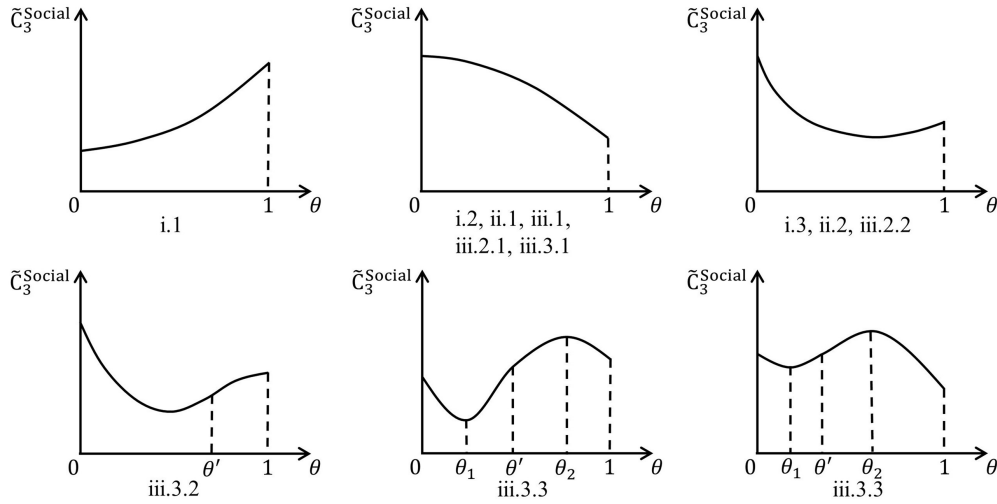
and the function  $f(\theta)$  and its first- and second-order derivatives are displayed in EC.2.4.

The tendency of  $C^{\text{social}}(\bar{L}_E, \bar{L}_N, \hat{v}_{E3}, \theta, \bar{T})$  to change with an increase in  $\theta$  is related to the term  $\alpha_E - \alpha_N + \rho_E - \rho_N$ . Hence, using different values of  $\alpha_E - \alpha_N + \rho_E - \rho_N$ , we analyze the optimal sulfur limit  $\bar{\theta}_3^*$  (“\*” in  $\bar{\theta}_3^*$  indicates that the sulfur rule within ECAs is set by the ECA regulator, and “-” in  $\bar{\theta}_3^*$  indicates that the optimal sulfur rule within ECAs is designed for a given sailing path with distances  $\bar{L}_E$  and  $\bar{L}_N$ ) in Proposition 5. We plot the corresponding increase or decrease in social costs with an increase in  $\theta$  in Fig. 4.

**Proposition 5.** *The optimal fuel sulfur limit within ECAs that minimizes  $C^{\text{social}}(\bar{L}_E, \bar{L}_N, \hat{v}_{E3}, \theta, \bar{T})$  in Eq. (19), i.e., the design of  $\theta$ , can be determined as follows.*

(i) If  $\alpha_E - \alpha_N + \rho_E - \rho_N > 0$ ,

- (i.1) and if  $f(0) \geq 0$ , then  $\bar{\theta}_3^* = 0$ ;
- (i.2) and if  $f(1) \leq 0$ , then  $\bar{\theta}_3^* = 1$ ;
- (i.3) and if  $f(0) < 0$  and  $f(1) > 0$ , then  $\bar{\theta}_3^* \in (0, 1)$ .
- (ii) If  $\alpha_E - \alpha_N + \rho_E - \rho_N = 0$ ,
- (ii.1) and if  $f(1) \leq 0$ , then  $\bar{\theta}_3^* = 1$ ;
- (ii.2) and if  $f(1) > 0$ , then  $\bar{\theta}_3^* \in (0, 1)$ .
- (iii) If  $\alpha_E - \alpha_N + \rho_E - \rho_N < 0$ ,
- (iii.1) and if  $\frac{df(0)}{d\theta} \leq 0$ , then  $\bar{\theta}_3^* = 1$ ;
- (iii.2) and if  $\frac{df(1)}{d\theta} \geq 0$ , (iii.2.1) and  $f(1) \leq 0$ , then  $\bar{\theta}_3^* = 1$ ; (iii.2.2) and  $f(1) > 0$ , then  $\bar{\theta}_3^* \in (0, 1)$ ;
- (iii.3) and if  $\frac{df(0)}{d\theta} > 0$ ,  $\frac{df(1)}{d\theta} < 0$  (note that there exists a unique  $\theta' \in (0, 1)$  satisfying  $\frac{df(\theta')}{d\theta} = 0$ ),
- (iii.3.1) and  $f(\theta') \leq 0$ , then  $\bar{\theta}_3^* = 1$ ; (iii.3.2)  $f(\theta') > 0$ , and  $f(1) \geq 0$ , then  $\bar{\theta}_3^* \in (0, \theta')$ ; (iii.3.3)  $f(\theta') > 0$ , and  $f(1) < 0$ , then  $\bar{\theta}_3^* \in (0, \theta')$  or  $\bar{\theta}_3^* = 1$ .



**Figure 4** Change tendency of the social cost function on  $\theta$  in Case 3

Note: " $\tilde{C}_3^{\text{social}}$ " indicates  $C_3^{\text{social}}(\bar{L}_E, \bar{L}_N, \hat{v}_{E3}, \theta, \bar{T})$ .

Proposition 5 shows that when the unit social cost for sailing within ECAs with a 0.1% sulfur limit (i.e.,  $\alpha_E + \rho_E + \kappa$ ) is lower than or equal to that of ECAs with a 0.5% sulfur limit (i.e.,  $\alpha_N + \rho_N + \kappa$ ), the optimal sulfur limit cannot be 0.5%. However, an interesting finding is that

**Paradox 1.** When  $\alpha_E + \rho_E + \kappa > \alpha_N + \rho_N + \kappa$ , the optimal sulfur limit can be 0.1%.

Note that the same sulfur limit within and outside ECAs results in the shipping company making the same sailing speed choice throughout the leg. Different sulfur limits within and outside ECAs lead to the shipping company choosing different speeds in the two areas, and thus the lower speed within ECAs may reduce the social cost of sailing within ECAs and the whole leg. Therefore, if  $\alpha_E + \rho_E + \kappa > \alpha_N + \rho_N + \kappa$ , the sulfur limit can be any value depending on the trade-off between

the speed difference and the unit social cost within ECAs; otherwise, the ECA regulator chooses a sulfur limit stricter than 0.5% to take advantage of the lower or equivalent unit social cost and speed difference.

According to the optimal sulfur limit for a given sailing path, its social cost can be expressed as

$$C^{\text{social}}(\bar{L}_E, \bar{L}_N, \hat{v}_{E3}, \bar{\theta}_3^*, \bar{T}) = (\alpha_N + \kappa) \cdot a\bar{T}^{-b} (\gamma_{\bar{\theta}_3^*} \bar{L}_E + \bar{L}_N)^b (\xi_{\bar{\theta}_3^*}^{b+1} \gamma_{\bar{\theta}_3^*}^{-b} \bar{L}_E + \bar{L}_N) + c\bar{T}. \quad (21)$$

The ECA regulator then optimizes the sailing path by discretizing the ECA boundary and constructing numerous paths. We define a set  $\mathcal{L}$  that includes the distances of all such paths within and outside ECAs. The optimal sulfur rule within ECAs  $\bar{\theta}_3^*$  for each path in  $(\bar{L}_E, \bar{L}_N) \in \mathcal{L}$  is calculated by Proposition 5. The optimal sailing path  $(L_{E3}^*, L_{N3}^*)$  and optimal sulfur rule within ECAs  $\theta_3^*$  can be determined by Proposition 6.

**Proposition 6.** *The optimal sailing path  $(L_{E3}^*, L_{N3}^*)$  for minimizing the social cost of a leg is the path with the smallest  $(\gamma_{\bar{\theta}_3^*} \bar{L}_E + \bar{L}_N)^b (\xi_{\bar{\theta}_3^*}^{b+1} \gamma_{\bar{\theta}_3^*}^{-b} \bar{L}_E + \bar{L}_N)$ ; the optimal sulfur rule within ECAs  $\theta_3^*$  for the optimal path can be obtained from Proposition 5.*

Then the shipping company determines the sailing speed according to the sulfur limit, and follows the sailing path designated by the ECA regulator. The four types of costs in Case 3 are

$$C^{\text{social}}(L_{E3}^*, L_{N3}^*, \hat{v}_{E3}, \theta_3^*, \bar{T}) = (\alpha_N + \kappa) \cdot a\bar{T}^{-b} (\gamma_{\theta_3^*} L_{E3}^* + L_{N3}^*)^b (\xi_{\theta_3^*}^{b+1} \gamma_{\theta_3^*}^{-b} L_{E3}^* + L_{N3}^*) + c\bar{T} \quad (22)$$

$$C^{\text{company}}(L_{E3}^*, L_{N3}^*, \hat{v}_{E3}, \theta_3^*, \bar{T}) = \alpha_N \cdot a\bar{T}^{-b} (\gamma_{\theta_3^*} L_{E3}^* + L_{N3}^*)^{b+1} + c\bar{T} \quad (23)$$

$$C^{\text{SO}_x}(L_{E3}^*, \hat{v}_{E3}, \theta_3^*) = \rho_{E\theta_3^*} \cdot a\bar{T}^{-b} (\gamma_{\theta_3^*} L_{E3}^* + L_{N3}^*)^b \gamma_{\theta_3^*}^{-b} L_{E3}^* \quad (24)$$

$$C^{\text{CO}_2}(L_{E3}^*, L_{N3}^*, \hat{v}_{E3}, \theta_3^*, \bar{T}) = \kappa \cdot a\bar{T}^{-b} (\gamma_{\theta_3^*} L_{E3}^* + L_{N3}^*)^b (\gamma_{\theta_3^*}^{-b} L_{E3}^* + L_{N3}^*). \quad (25)$$

**Proposition 7.** *We have  $C^{\text{social}}(L_{E3}^*, L_{N3}^*, \hat{v}_{E3}, \theta_3^*, \bar{T}) \leq C^{\text{social}}(\hat{L}_{E1}, \hat{L}_{N1}, \hat{v}_{E1}, 0, \bar{T})$  and  $C^{\text{social}}(L_{E3}^*, L_{N3}^*, \hat{v}_{E3}, \theta_3^*, \bar{T}) \leq C^{\text{social}}(\hat{L}_{E2}, \hat{L}_{N2}, \hat{v}_{E2}, 1, \bar{T})$ .*

Proposition 7 indicates that in light of the shipping company's speed choice, the ECA regulator can optimize both the sulfur limit and sailing path to ensure that the social cost under the proposed ECA policy is lower than or equal to those under a no-ECA policy and the current ECA policy. Thus, concerns about the current ECA policy potentially increasing social costs are addressed.

The company cost is not lower in Case 3 than in Case 1 because a stricter sulfur limit within ECAs and a longer sailing path may be designed in Case 3. If  $\theta_3^* = 1$  (i.e., the sulfur limits within ECAs in Cases 3 and 2 are identical), the sailing paths in Cases 3 and 2 are determined by the ECA regulator and the shipping company to minimize the social and the company costs, respectively, and we have  $\gamma_{\theta_3^*} L_{E3}^* + L_{N3}^* \geq \gamma_1 \hat{L}_{E2} + \hat{L}_{N2}$ , i.e., the company incurs the same or higher cost in Case 3 compared with Case 2.