https://doi.org/10.1016/j.proci.2016.06.063

On the flame height of circulation-controlled firewhirls with variable density

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Abstract:

A theoretical analysis on the flame height of circulation-controlled firewhirls is presented with emphasis on studying the influence of variable density that was ignored in most previous studies. A circulation-controlled firewhirl is approximated as a non-premixed flame, with variable density and temperature-dependent diffusivities in a steady and axisymmetric Burgers vortex. A coupling-function-based formulation is established to describe the firewhirl by invoking the assumption of unity Lewis number. By virtue of a Howarth-Dorodnitsyn-like coordinate transformation, the transport equations for the coupling functions can be solved analytically to result in the well-known linear relation between the normalized flame height and the modified Peclet number. The variable density effect in enhancing the flame height is characterized by a dimensionless temperature factor multiplied to the linear relation. For the firewhirls supplied by a fuel in condensed phase, the boundary conditions at the fuel surface are specified by considering the physical mechanism of Stefan flow.

Keywords: Firewhirl; Variable density; Burgers vortex; Flame height; Coupling function

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Nomenclature

Physical quantities

a	strain rate of Burgers vortex
c_p	constant pressure specific heat
$d_0(r_0)$	diameter (radius) of fuel pool
D	mass diffusivity
p	pressure
q_c	heat of combustion per unit mass of fuel
q_v	latent heat of vaporization per unit mass of fuel
r, x, θ	cylindrical coordinates
T	temperature
u, v, w	velocity components in x, r, θ directions
W	molar weight
Y	mass fraction
Z_{st}	stoichiometric mixture fraction
Γ	circulation of Burgers vortex
λ	thermal conductivity
ν	kinematic viscosity
ρ	density
σ_{FO}	ratio of molar weights $\sigma_{FO} = W_O/W_F$
Ω	angular velocity

Average quantities at $x = 0$

$Q_0 = \frac{1}{r_0}\int_0^{r_0} Q(0,r)dr,\ \ Q = (p, T, u, Y_F, \rho)$

T_0'	modified temperature $T_0' = T_0 - q_v/c_p$

Non-dimensional and normalized variables

$\tilde{X} = X/X_0,\ \ X = (D, p, r, x, u, v, w, Y_F, \rho, \nu)$

$\tilde{a}$	$\tilde{a} = ar_0^2/\nu_0$

$\tilde{T}$ $\tilde{T} = c_p T / q_c Y_{F0}$

$\tilde{Y}_O$ $\tilde{Y}_O = Y_O / \sigma_{FO} Y_{F0}$

β_{FO} $\beta_{FO} = \tilde{Y}_F - \tilde{Y}_O$

β_{FT} $\beta_{FT} = \tilde{Y}_F + \tilde{T}$

$\tilde{\Gamma}$ $\tilde{\Gamma} = \Gamma / u_0 r_0$

Transformed coordinates

χ, ζ stream function coordinates

ξ, η density-mass diffusivity-weighted coordinates

Subscripts

O quantities on the oxidizer side

F quantities on the fuel side

N quantities for inert gas

0 quantities at $x = 0$

∞ quantities in far field

1. Introduction

Firewhirls are spinning non-premixed flames that often occur in wild and urban fires and may cause severe destructions. A number of experimental and theoretical studies have been conducted to understand the occurrence and characteristics of firewhirls[1-20]. Flame length or flame height is an important parameter for characterizing a firewhirl[6,9-12,15,21], as the increasing rotational velocity and entrainment of ambient air result in an appreciable lengthening of the flame due to the conservation of angular momentum[4,5,11,15]. Moreover, the increase of flame length may also enhance the radiant energy flux transmitted to the ambience which in turn leads to the spread of spot fires in distance[4,5,11,15].

It was commonly believed that the flame length of a firewhirl is mainly controlled by the buoyancy effect, which can be characterized by Froude number (Fr) measuring the relative importance of the inertial force compared with the gravitational force. An upwardly convective flow will be induced by the buoyancy when a substantial density variation is caused by the large heat release from the firewhirl [1, 3-6, 9, 10]. Since a firewhirl must involve a rotating fluid motion, its characterization often needs another non-dimensional parameter, the Rossby number (Ro), to measure the ratio of the inertial force to the Coriolis force.

Compared with a large number of experimental investigations reported in the literature, the theoretical analysis of firewhirls has not been attempted sufficiently. By assuming firewhirl as an inviscid and incompressible flow with buoyancy, Battaglia et al.[5] obtained the numerical solutions of velocity components near the axis for Ro>0.5. It is noted that a "non-Boussinesq" model was proposed in their study to account for the large density variation through a density-scaled velocity transformation, which was introduced by Yih [22] for incompressible stratified flows. Recently, Kuwana et al.[12] experimentally observed that the presence of a weak circulation in buoyancy-controlled firewhirls can have a considerable influence on the flame height by enhancing the burning rate. By introducing a modified "Top-hat" approximation for the axial velocity distribution near the axis, they obtained a linear relation between the dimensionless flame height and the Peclet number, which is the ratio of the flow velocity at the vaporizing liquid surface to the characteristic gaseous diffusion velocity.

In the recent study of Chuah et al.[11] on inclined firewhirls, the flame heights were found to be independent of the inclination angles and the burning rates, and thereby confirming the existence of circulation-dominated firewhirls. The physical explanation to the phenomena is that the inclined firewhirls remain to be inclined rather than vertical when sufficiently strong circulation was imposed onto the firewhirl. The strong vortex in the firewhirl stretches the flame along its axis of rotation and preserves its direction under the constraint of the conservation of angular momentum. In their study, a linear relation between the normalized flame height and the Peclet number was obtained theoretically by approximating the firewhirl as a constant-density Burgers vortex. Although Chuah et al.'s linear relation predicts the right trend of the flame heights for the concerned firewhirls with or without whirls, it significantly underestimates the flame heights in the presence of strong rotational flow[11]. Considering that the vortex-stretching effect in Burgers vortex may not be sufficiently strong to account for the strong vorticity in a firewhirl, Klimenko and Williams[15] proposed an theoretical model by employing a strong-vortex approximation[23] and its compensating regime[24]. A modified linear relation between the normalized flame height and the modified Peclet number was obtained and can produce good agreements with the experimental data by adjusting an effective exponent responsible for flame elongation.

In spite of these worthy advances in understanding the factors affecting the firewhirl lengths, the approximation of constant density that is always invoked in the previous studies[11,15] has not been adequately justified. The influence of density variation arising from the large temperature nonuniformity of firewhirl cannot be simply neglected. Because the decrease of density and hence the flow inertia within the high-temperature vortex core of a firewhirl flow renders the fuel mass more readily to be advected to a larger height. The Boussinesq assumption is often applied to account for the effect of variable density in body force, but it is not applicable for circulation-controlled firewhirls[15]. It is noted that the "non-Boussinesq" approximation adopted in Battaglia et al.[5] for large temperature and density variations is essentially a low Mach number approximation that is however inappropriate when the circulation became sufficiently intense, corresponding to Ro being less than a threshold value 0.5.

In the present study, we established a "variable density and diffusivity" model in order to propose an alternative interpretation to the experimentally observed flame height of the circulation-controlled firewhirls. In section 2, a steady, axisymmetric firewhirl system is mathematically formulated in terms of the coupling functions by assuming unity Lewis number. In section 3, a Howarth-Dorodnitsyn-like coordinate transformation[25,26] is used to reproduce a formulation that is formally the same as the "constant-density" one obtained in the previous studies[11,15]. In section 4 and 5, the influence of density variation on the flame height is discussed, followed by the specification of the boundary conditions at the vaporizing surface when the firewhirl is supplied by a fuel in condensed phase, in section 6.

2. Coupling-function Formulation

A firewhirl is called circulation-controlled when the vortex-stretching effect plays a dominant role in determining the flame height and the buoyancy effect is negligible. We consider such a firewhirl as a steady non-premixed flame in a forced, axisymmetric rotating flow with chemical reaction in place[6,11,12,15]. The schematic of such a firewhirl is shown in Figure 1.

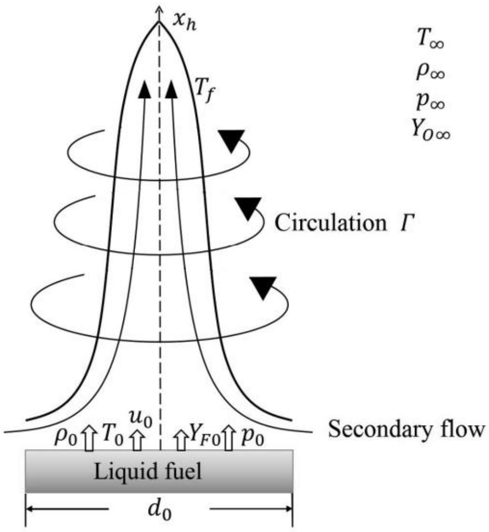

Figure 1. Schematic of circulation-controlled firewhirls

The flow temperature varies considerably in space due to the large heat release from the flame and therefore causes a substantial variation in density as the pressure change insignificantly in low Mach number flows[26]. To analytically characterize the firewhirl, we need to solve the spatial distributions of fuel, oxidizer mass fractions and temperature, which constitute two coupling functions β_{FO} and β_{FT} by assuming that the fuel and oxidizer have equal density-weighted mass diffusivity ρD and the Lewis number $\mathrm{Le} = \lambda/\rho c_p D$ is unity.

The dimensional transport equation for the coupling functions can be written as

$$\rho ur\frac{\partial \beta_i}{\partial x} + \rho vr\frac{\partial \beta_i}{\partial r} = \frac{\partial}{\partial x}\left(\rho Dr\frac{\partial \beta_i}{\partial x}\right) + \frac{\partial}{\partial r}\left(\rho Dr\frac{\partial \beta_i}{\partial r}\right) \quad (1)$$

where $\beta_i = \beta_{FO}$ or β_{FT}. The boundary conditions (BC for short hereinafter) for Equation (1) are specified as follows: the axisymmetric BC(i) at $r = 0$, $\partial\beta_i/\partial r = 0$; the far-field BC(ii) at $r \to \infty$, $\partial\beta_i/\partial r = 0$; BC(iii) at $x = 0, r \le r_0$, $\rho u \beta_{FO} - \rho D\, \partial\beta_{FO}/\partial x = \rho u / Y_{F0}$, $\rho u \beta_{FT} - \rho D \partial\beta_{FT}/\partial x = \rho u\left(1/Y_{F0} + \tilde{T} + q_v/q_c Y_{F0}\right)$, at $x = 0, r > r_0$, $\partial\beta_i/\partial x = 0$; BC(iv) at $x \to \infty$, $\beta_{FO} = -\tilde{Y}_{O\infty}$, $\beta_{FT} = \tilde{T}_\infty$. BC(iii) describes the additional physical conditions on a liquid fuel pool of radius r_0: the diffusive and convective transport of fuel is balanced by the fuel evaporation[11,26], which is driven by heat transport from the flame [26]. Being compatible with BC (ii), BC(iv) literally indicates that no fuel is present in the far field of the fuel source, as the result of the assumption of infinitely fast reaction rate or equivalently the flame sheet assumption.

3. Coordinate Transformation

To transform Equation (1) into an analytically tractable form, we shall first rewrite it in terms of the non-dimensional variables defined in the nomenclature, and then introduce a density-mass diffusivity-weighted coordinate system defined by

$$\xi = \frac{D_0}{u_0 r_0} \int_0^{\tilde{x}} \tilde{\rho}^2 \tilde{D}\, dx', \qquad \eta = \int_0^{\tilde{r}} \tilde{\rho} dr' \tag{2}$$

which is an analog of the well-known Howarth-Dorodnitsyn transformation[25] for compressible boundary layers, in spite of the latter is particularly useful for self-similar flow fields. In the present problem, the flow is not self-similar because of the characteristic length scale r_0. It is noted that the strong-vortex solution of Klimenko and Williams[15] was obtained by forcing self-similarity in the firewhirl flow.

Applying the transformation (2) to Equation (1), we can have

$$\tilde{u}\frac{\partial\beta_i}{\partial\xi} + \tilde{v}'\frac{\partial\beta_i}{\partial\eta} = \frac{4}{\mathrm{Pe}^2}\frac{1}{\tilde{\rho}}\frac{\partial}{\partial\xi}\left(\tilde{\rho}^3\tilde{D}^2\frac{\partial\beta_i}{\partial\xi}\right) + \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\beta_i}{\partial\eta}\right) \tag{3}$$

where the Peclet number is defined as $\mathrm{Pe} = u_0 d_0 / D_0$ and $\tilde{v}' = \tilde{v}\mathrm{Pe}\left(2\tilde{\rho}\tilde{D}\right)^{-1}$ an intermediate variable for mathematical conciseness. To derive Equation (3), we have invoked the well-known Chapman-Rubesin approximation[26] by $C = \tilde{\rho}^2\tilde{D}\tilde{r}\left(\int_0^{\tilde{r}} \tilde{\rho} dr'\right)^{-1}$ = const. It is seen that, although the density and mass diffusivity do not appear explicitly in Equation (3), their variations are taken into account in the new coordinates. Accordingly, the boundary conditions BC(i-iv) are rewritten as follows: BC(i') at $\eta = 0$, $\partial\beta_i/\partial\eta = 0$; BC(ii') at $\eta \to \infty$, $\partial\beta_i/\partial\eta = 0$; BC(iii') at $\xi = 0, \eta \le 1$, $\beta_{FO} = Y_{F0}^{-1} + \frac{4}{Pe^2}\left(\frac{\partial\beta_{FO}}{\partial\xi}\right)$ and $\beta_{FT} = Y_{F0}^{-1} + \tilde{T}_0 - \frac{q_v}{q_c Y_{F0}} + \frac{4}{Pe^2}\left(\frac{\partial\beta_{FT}}{\partial\xi}\right)$, and at $\xi = 0, \eta > 1$, $\frac{\partial\beta_i}{\partial\xi} = 0$; BC(iv') at $\xi \to \infty$, $\beta_{FO} = -\tilde{Y}_{O\infty}$, $\beta_{FT} = \tilde{T}_\infty$.

In the present study on circulation-controlled firewhirls, we assume the Peclet number is sufficiently large so that the axial diffusion is limited in a thin evaporation layer adjacent to the liquid pool and it can be negligible outside the layer. Consequently, Equation (3) can be written as

$$\tilde{u}\frac{\partial\beta_i}{\partial\xi} + \tilde{v}'\frac{\partial\beta_i}{\partial\eta} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\beta_i}{\partial\eta}\right) \tag{4}$$

In the large Peclet number approximation, BC (i') and BC(ii') remain intact; the simplified BC(iii') is that at $\xi = 0, \eta \le 1$, $\beta_{FO} = 1/Y_{F0}$, $\beta_{FT} = 1/Y_{F0} + \tilde{T}_0'$ and at $\xi = 0, \eta > 1$, $\beta_{FO} = -\tilde{Y}_{O\infty}$, $\beta_{FT} = \tilde{T}_\infty$; BC(iv') is not required by Equation (4).

Formally, we can regard the coordinates (ξ, η) as "constant density" ones, in which we approximate the firewhirl flow as a Burgers vortex[11]. Burgers vortex[27] is a generalization of two-dimensional Oseen vortex[28] by introducing an additional linear radial and axial secondary flow to account for the vortex-stretching effect, which plays a crucial role in determining the flame heights of circulation-controlled firewhirls. The stream function of the Burgers vortex is given by $\psi = \left(\frac{\tilde{\alpha}}{2}\xi + \frac{1}{2}\right)\eta^2$. In terms of the stream function ψ, the axial velocity $\tilde{u}$, the scaled radial velocity $\tilde{v}'$ and the azimuthal velocity $\tilde{w}$ can be expressed by

$$\tilde{u} = \frac{1}{\eta}\frac{\partial\psi}{\partial\eta} = \tilde{\alpha}\xi + 1, \qquad \mathrm{Pe}\frac{\tilde{v}}{2\tilde{\rho}\tilde{D}} = -\frac{1}{\eta}\frac{\partial\psi}{\partial\xi} = -\frac{1}{2}\tilde{\alpha}\eta, \qquad \tilde{w} = \frac{\tilde{\Gamma}}{2\pi\eta}\left[1 - \exp\left(-\frac{\tilde{\alpha}\eta^2}{4\tilde{\nu}}\right)\right]$$

(5)

In the physical coordinates, Equation (5) is written by

$$u = a\int_0^x \left(\frac{\rho}{\rho_0}\right)^2 \frac{D}{D_0} dx' + u_0, \quad v = -\frac{1}{2} a \frac{\rho}{\rho_0}\frac{D}{D_0}\int_0^r \left(\frac{\rho}{\rho_0}\right) dr', \quad w = \frac{\Gamma}{2\pi\int_0^r \rho/\rho_0 dr'}\left[1 - e^{-\frac{a}{4\nu}\left(\int_0^r \frac{\rho}{\rho_0} dr'\right)^2}\right]$$

(5')

where the integrals in the velocity components represent the stretching effects of the density variation. Given that the density is a slowly-varying function of x and Chapman-Rubesin number is constant, it is readily seen that Equation (5') satisfies the equation of continuity as does the primitive model[28]. The direct validation of Equation (5) is not available in the present study while it merits future study when experimental measurements are available.

Making use of the stream function ψ, we can introduce the stream function coordinate defined as $\chi = \xi, \zeta = \sqrt{2\psi}$, which was also used by Klimenko and Williams[15] in a dimensional form. Equation (4) can be rewritten in the stream function coordinate (χ, ζ) as

$$\frac{\partial \beta_i}{\partial \chi} = \frac{1}{\zeta}\frac{\partial}{\partial \zeta}\left(\zeta \frac{\partial \beta_i}{\partial \zeta}\right)$$

(6)

Correspondingly, the BCs (i') to (iii') can be rewritten as BC(a) at $\zeta = 0, \partial\beta_i/\partial\zeta = 0$; BC(b) at $\zeta \to \infty$, $\partial\beta_i/\partial\zeta = 0$; BC(c) at $\chi = 0, \zeta \leq 1$, $\beta_{FO} = Y_{F0}^{-1}$, $\beta_{FT} = Y_{F0}^{-1} + \tilde{T}_0'$ and at $\chi = 0, \zeta > 1$, $\beta_{FO} = -\tilde{Y}_{O\infty}$, $\beta_{FT} = \tilde{T}_\infty$. Equation (6) together with BCs (a)-(c) constitute an analytically solvable formulation describing the firewhirl problem, to be solved in the following section.

4. Solutions for Coupling Functions

Equation (6) can be solved by means of separation of variables which results in the Bessel function of the first kind[29]. After taking into account of BCs (a)-(c), we have

$$\beta_{FO} = -\tilde{Y}_{O\infty} + \left(Y_{F0}^{-1} + \tilde{Y}_{O\infty}\right)\int_0^\infty J_1(\omega) J_0(\omega\zeta) e^{-\omega^2\chi} d\omega$$

(7)

$$\beta_{FT} = \tilde{T}_\infty + \left(\tilde{T}_0' + Y_{F0}^{-1} - \tilde{T}_\infty\right) \int_0^\infty J_1(\omega) J_0(\omega\zeta) e^{-\omega^2 \chi} d\omega \tag{8}$$

where J_0 and J_1 are the zeroth- and the first-order Bessel functions of the first kind, respectively.

The flame location, expressed by $\chi_f = f(\zeta)$, can be determined by using $\beta_{FO} = 0$ in Equation (7) because the reactants vanish on the non-premixed flame sheet. Since oxidizer is absent on the fuel side in the flame-sheet assumption, $\tilde{Y}_F$ is determined from Equation (7) by

$$\tilde{Y}_F = \beta_{FO} = -\tilde{Y}_{O\infty} + \left(Y_{F0}^{-1} + \tilde{Y}_{O\infty}\right) \int_0^\infty J_1(\omega) J_0(\omega\zeta) e^{-\omega^2 \chi} d\omega \tag{9}$$

and $\tilde{T}$ from Equations (7) and (8) by

$$\tilde{T} = \beta_{FT} - \beta_{FO} = \tilde{T}_\infty + \tilde{Y}_{O,\infty} + \left(\tilde{T}_0' - \tilde{T}_\infty - \tilde{Y}_{O\infty}\right) \int_0^\infty J_1(\omega) J_0(\omega\zeta) e^{-\omega^2 \chi} d\omega \tag{10}$$

Similarly, fuel vanishes on the oxidizer size and we hence have

$$\tilde{Y}_O = -\beta_{FO} = \tilde{Y}_{O\infty} - \left(Y_{F0}^{-1} + \tilde{Y}_{O\infty}\right) \int_0^\infty J_1(\omega) J_0(\omega\zeta) e^{-\omega^2 \chi} d\omega \tag{11}$$

and

$$\tilde{T} = \beta_{FT} = \tilde{T}_\infty + \left(\tilde{T}_0' + Y_{F0}^{-1} - \tilde{T}_\infty\right) \int_0^\infty J_1(\omega) J_0(\omega\zeta) e^{-\omega^2 \chi} d\omega \tag{12}$$

Transforming χ and ζ in the Equations (9)-(12) back to $\tilde{x}$ and $\tilde{r}$ and expressing $\tilde{T}$ and $\tilde{Y}$ in their dimensional forms, we can obtain the spatial distributions of T, Y_F and Y_O in physical coordinates and hence the firewhirl is completely determined.

5. Flame Height and Variable Density Effect

The flame height of the firewhirls is defined as the highest flame location on the χ-axis and denoted by χ_h, which can be determined by setting both ζ and β_{FO} equal to zero in Equation (7), yielding

$$\int_0^\infty J_1(\omega)e^{-\omega^2\chi_h}d\omega = \frac{\tilde{Y}_{O\infty}}{Y_{F0}^{-1}+\tilde{Y}_{O\infty}} = Z_{st} \tag{13}$$

In actual experimental conditions, the flame height is usually significantly larger than the radius of fuel source pool, namely, $\chi_h \gg 1$. Therefore, the integral on the LHS of Equation (13) is mainly attributed to a small region around $\omega = 0$ [30]. By using the Taylor expansion of the Bessel function $J_1(\omega)$ around $\omega = 0$, i.e., $J_1(\omega) = \omega/2 - \omega^3/16 + \cdots$, substituting it into Equation (13), and using integration by parts, we have $(4\chi_h)^{-1}[1-(8\chi_h)^{-1}] = Z_{st}$ and therefore

$$\chi_h = \frac{1}{4Z_{st}} - \frac{1}{8} \tag{14a}$$

where only the first two terms in the Taylor expansion were used because of $8\chi_h \gg 1$. To facilitate the comparison with previous experimental observations, we write χ_h in its dimensional form by

$$\chi_h = \frac{1}{\rho_0^2 u_0 r_0^2}\int_0^{x_h}\rho^2 D dx = \frac{T_0^2}{u_0 r_0^2}\int_0^{x_h}\frac{D}{T^2}dx \tag{14b}$$

To derive the above equation, we have adopted the isobaric approximation [11,15,26] which has been well justified in low Mach number flows. Substituting (14b) to the RHS of Equation (14a), we have

$$\int_0^{x_h} T^{a-2}dx = \mathrm{Pe}T_0^{a-2}d_0\left[\frac{1}{16Z_{st}} - \frac{1}{32}\right] \tag{15}$$

where the mass diffusivity in arbitrary axial location is related to D_0 at $x = 0$ through the relation $D = D_0(T/T_0)^\alpha$, in which α is usually less than 2[11,31] and equal to 1.5 in the kinetic theory of gases employing the rigid-sphere model[26].

Because Z_{st} is around 0.1 under the actual experimental conditions[11], we can neglect 1/32 in Equation (15) and formally rewrite the equation as

$$\frac{x_h}{d_0} = \left(\frac{T_m}{T_0}\right)^{2-\alpha} \frac{\mathrm{Pe}}{16Z_{st}} \tag{16}$$

which has exactly the same form as that given by Klimenko and Williams[15] except for the additional term $(T_m/T_0)^{2-\alpha}$ describing variable density effect on the flame heights. Introduced for mathematical convenience, T_m denotes the integral

$$T_m = \left[\frac{1}{x_h}\int_0^{x_h} T^{\alpha-2} dx\right]^{1/(\alpha-2)} \tag{17}$$

and is determined, with the help of Equation (10), by

$$T_m = \left\{\frac{1}{x_h}\int_0^{x_h}\left[T_\infty + \frac{q_c Y_{O\infty}}{\sigma_{FO} c_p} + \left(T_0 - \frac{q_v}{c_p} - \frac{q_c Y_{O\infty}}{\sigma_{FO} c_p}\right)\int_0^\infty J_1(\omega)\exp\left(-\omega^2\int_0^{x/r_0}\tilde{\rho}^2\tilde{D}\,dx'\right)d\omega\right]^{\alpha-2} dx\right\}^{1/(\alpha-2)} \tag{18}$$

Considering that directly evaluating T_m by means of (18) is mathematically challenging due to multiple-fold integrations of a product of Bessel function with exponential function, we can roughly estimate T_m by the arithmetic mean, the geometric average or the well-known $1/3$-rule[26] to facilitate the interpretation of variable density effects and the comparison between theory and experiments.

For the following illustrations, both the $1/3$-rule, $T_m \cong (T_0 + 2T_f)/3$, and the arithmetic mean, $T_m \cong (T_0 + T_f)/2$, are adopted to estimate T_m, where the quantity T_f denotes the flame temperature. In Chuah et al.'s experiments, T_f of alcohols (methanol, ethanol and 2-propanol) are estimated at 1300 K[11] rather than 1500K[31] since the alcohols burn cooler than hydrocarbons. T_0 is approximated as 337K, the boiling point temperature of methanol at atmospheric pressure, although T_0 should be lower than the actual boiling point temperature. The slight difference of the boiling point temperatures between methanol (337K) and 2-propanol (370K) does not cause any significant change to the temperature factor $(T_m/T_0)^{2-\alpha}$. Two typical values for the exponent α, such as 1.5 from the kinetic theory of gases[26] and 1.8 suggested by Chuah et al. [31], will be used and compared to minimize the uncertainty caused by the choice of the exponent.

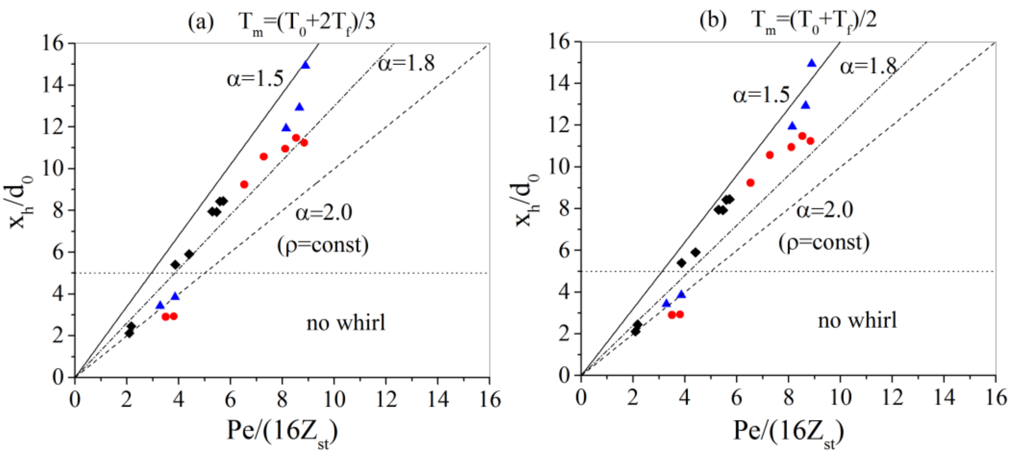

Figure 2. Normalized flame length plotted against normalized Peclet number. Solid symbols represent experimental data given by Chuah et al[11]. Fuel types are distinguished by solid symbols, methanol (♦), ethanol (•), 2-propanol (7).

Figure 2 shows the present theoretical predictions for different values of α and different estimates of the mean temperature. For comparison, the previous theoretical results based on the constant density assumption are also shown and can be readily reproduced by using a physically unrealistic value of $\alpha = 2$ in the present theory. It is seen that the constant density predictions result in a linear relation with a unity slope in the parameter space of the normalized flame height, x_h/d_0, and the modified Peclet number, $\mathrm{Pe}/16Z_{st}$. When the variable density effect is taken into account in the present theory, the linearity between x_h/d_0 and $\mathrm{Pe}/16Z_{st}$ remains while the slope is modified by a factor $(T_m/T_0)^{2-\alpha} > 1$ because $T_m > T_0$ and $\alpha < 2$. This result implies that the flame height undergoes an additional increase in circulation-controlled firewhirls when the variable density effect is taken into account. The underlying physics can be understood by that the decrease of density and hence the flow inertia within the high-temperature vortex core of a firewhirl flow renders the fuel mass more readily to be advected to a larger height.

It is also seen by comparing Figure 2(a) with 2(b) that the different approximation methods for estimating the mean temperature T_m do not cause any qualitative changes to the theoretical predictions. Furthermore, the good agreements with the experimental data have been achieved by the present theory with either $\alpha = 1.5$ or $\alpha = 1.8$, while the latter seems better and more physically realistic. It is noted that the present theory seems to moderately overestimate the experimental data below the horizontal dot line, which however represent the experimental cases without whirl and therefore should not be used for a direct comparison. As circulation decreases, the buoyancy effect may become a nontrivial or even dominant factor in determining the flame height.

An alternative way to estimate T_f is by evaluating Equation (10) at x_h and setting ζ equal to zero. Neglecting the slight difference between $\tilde{T}_\infty$ and $\tilde{T}_0'$ in most actual situations and evaluating T_m and using the arithmetic mean as an example, we have

$$\frac{x_h}{d_0} = \left(\frac{1+\kappa}{2} + \kappa\frac{q_c}{2c_pT_0}Z_{st} - \kappa\frac{q_v}{2c_pT_0}\right)^{2-\alpha}\frac{\mathrm{Pe}}{16Z_{st}} \tag{19}$$

where the inclusion of the coefficient κ is to account for possible heat loss due to radiation and other factors, which might play important role and therefore merits further studies. Although it is noted that q_c and Z_{st} do not vary significantly for common hydrocarbon fuels, Equation (19) also indicates the influence of other physical-chemical properties, such as c_p and T_0 on the flame height. Detailed investigation on these properties is beyond the scope of the present study due to the currently unavailable experimental data.

6. Specification of boundary conditions for condensed fuels

If the fuel is in condensed phase, we need to consider the physics of Stefan flow at the evaporating surface of the fuel source to determine the fuel vapor mass fraction $Y_F(0,r)$, the fuel convection velocity, $u(0,r)$ and temperature, $T(0,r)$, which requires analyzing the thin evaporation layer in the large Peclet number approximation. By applying the stretched coordinates such as $\tilde{x} = 2\tilde{x}'/Pe,\ \tilde{r} = \tilde{r}'$ to the transport equation for Y_F, we have

$$\frac{\partial}{\partial\tilde{x}'}(\tilde{\rho}\tilde{u}\tilde{r}'Y_F) + \frac{2}{\mathrm{Pe}}\frac{\partial}{\partial\tilde{r}'}(\tilde{\rho}\tilde{v}\tilde{r}'Y_F) = \frac{\partial}{\partial\tilde{x}'}\left(\tilde{\rho}\widetilde{D}\tilde{r}'\frac{\partial Y_F}{\partial\tilde{x}'}\right) + \frac{4}{\mathrm{Pe}^2}\frac{\partial}{\partial\tilde{r}'}\left(\tilde{\rho}\widetilde{D}\tilde{r}'\frac{\partial Y_F}{\partial\tilde{r}'}\right)$$

(20)

Neglecting the radial convection and diffusion terms in Equation (20) because of the large Peclet number and integrating the equation over $\tilde{x}'$ across the evaporation layer, we have $(\tilde{\rho}\tilde{u}Y_F) - \left(\tilde{\rho}\widetilde{D}\frac{\partial Y_F}{\partial\tilde{x}'}\right) = c_F$, which indicates that the convective and diffusive transport of fuel is balanced by the evaporation. Since both the inert gas and the oxidizer gas are non-condensable, the total mass flux at the evaporating surface is completely attributed to the fuel vapor. We hence have

$$\rho u Y_F\,|_{x=0} - \rho D\frac{dY_F}{dx}\,|_{x=0} = \rho u\,|_{x=0}$$

(21)

Similarly, by considering the heat transport in the evaporation layer, we have

$$\lambda\frac{dT}{dx}\,|_{x=0} = q_v\rho u\,|_{x=0}$$

(22)

Combining Equations (21) and (22), we can solve for $Y_F(0,r)$ and $u(0,r)$ as

$$u(0,r) = \frac{D_0}{r_0}\sqrt{\frac{c_pT_\infty - c_pT(0,r) + q_c\,Y_{O\infty}/\sigma_{FO} + q_v}{q_v}\int_0^\infty \omega J_1(\omega)\left[\omega J_0\left(\frac{\omega r}{r_0}\right) + \frac{a}{2}\frac{r}{r_0}J_1\left(\frac{\omega r}{r_0}\right)\right]d\omega}$$

(23)

$$Y_F(0,r) = 1 - \frac{q_v(1 + Y_{O\infty}/\sigma_{FO})}{c_pT_\infty - c_pT(0,r) + q_c\,Y_{O\infty}/\sigma_{FO} + q_v}$$

(24)

In Equations (23) and (24), $T(0,r)$ is related to $p(0,r)$ by the Clausius-Clapeyron relation,

$$\frac{Y_F(0,r)/W_F}{Y_F(0,r)/W_F + [1 - Y_F(0,r)]/W_N} = \frac{p_\infty}{p(0,r)}\exp\left\{\frac{q_v}{R}\left[\frac{1}{T_{b,\infty}} - \frac{1}{T(0,r)}\right]\right\}$$

(25)

Furthermore, $p(0,r)$ can be approximately determined by using the body-force-free Bernoulli's equation and the velocity components of the Burgers vortex as follows

$$p(0,r) = p_\infty - \frac{1}{2}\rho(0,r)\left[u_0^2 + \frac{1}{4}a^2r^2 + \frac{\Gamma^2}{4\pi^2r^2}\left(1 - e^{-ar^2/4\nu}\right)^2\right]$$

(26)

In addition, the equation of state gives another relation

$$\rho(0,r) = \frac{p(0,r)}{RT(0,r)} = \frac{p_\infty}{RT(0,r)(1+\gamma Ma^2/2)}$$

(27)

According to Equation (26), increasing the vortex circulation, the static pressure $p(0,r)$ over the fuel pool becomes smaller and so does the temperature $T(0,r)$ due to Equation (25) because the temperature is close to the boiling point temperature under the reduced pressure[26]. As a result, both $u(0,r)$ and $\rho(0,r)$ and therefore the evaporation flux $\rho(0,r)u(0,r)$ increases with the circulation with the circulation, given that the vortex Mach number is sufficiently small.

By solving Equations (23)-(27), we can obtain Y_F, u, T, p and ρ at the evaporating fuel surface. Therefore, the firewhirl system considered in the present problem is closured with the boundary conditions specified in the section.

7. Concluding Remarks

The variable density effect on the flame height of circulation-controlled firewhirls was investigated theoretically by assuming the flow field as steady, axisymmetric Burgers vortex. Similar to the previous studies based on the constant density assumption, the normalized flame height x_h/d_0 can be expressed as a linear function of the modified Peclet number $\mathrm{Pe}/16Z_{st}$. An additional increase in flame height due to the variable density effect is determined by a multiplication factor $(T_m/T_0)^{2-\alpha}$, in which T_m is a temperature integral within the core of firewhirl, T_0 the average temperature of fuel vapor on the surface, and α determines the temperature dependence of mass diffusivity. Either the physically realistic $\alpha = 1.8$ or the approximate value $\alpha = 1.5$ from the kinetic theory of gases predict well the previous experimental data by Chuah et al. [11]. Consequently, the present theory provides an alternative interpretation to the increased flame heights of circulation-controlled firewhirls, which was attributed by Klimenko and Williams to the deficiency of constant-density Burgers vortex in accounting for strong rotation. Future studies are still merited to unify these theories and to consider other important albeit previously ignored factors such as the non-unity Lewis number effect, the finite-rate flame chemistry, and the radiative heat loss.

Acknowledgement

The authors thank Professor Chung K. Law for arousing their interest in fire research and for his constant encouragement through the course of the study. This work was supported by the Hong Kong RGC/ECS(operating under contract number PolyU 5380/13E) and RGC/GRF (operating under contract number PolyU 152217/14E).

Reference

[1] H.W. Emmons, S.-J. Ying, The fire whirl, Symposium (International) on Combustion. Elsevier, 1967, p. 475.

[2] F. Roper, Combustion and Flame 29 (1977) 219.

[3] F.A. Williams, Progress in Energy and Combustion Science 8 (1982) 317.

[4] F. Battaglia, K.B. McGrattan, R.G. Rehm, H.R. Baum, Combustion Theory and Modelling 4 (2000) 123.

[5] F. Battaglia, R.G. Rehm, H.R. Baum, Physics of Fluids 12 (2000) 2859.

[6] K.H. Chuah, G. Kushida, Proceedings of the Combustion Institute 31 (2007) 2599.

[7] K. Kuwana, K. Sekimoto, K. Saio, F.A. Williams, Y. Hayashi, H. Masuda, Aiaa Journal 45 (2007) 16.

[8] R. Zhou, Z.N. Wu, Journal of Fluid Mechanics 583 (2007) 313.

[9] K. Kuwana, K. Sekimoto, K. Saito, F.A. Williams, Fire Safety Journal 43 (2008) 252.

[10] K.H. Chuah, K. Kuwana, K. Saito, Combustion and Flame 156 (2009) 1828.

[11] K.H. Chuah, K. Kuwana, K. Saito, F.A. Williams, Proceedings of the Combustion Institute 33 (2011) 2417.

[12] K. Kuwana, S. Morishita, R. Dobashi, K.H. Chuah, K. Saito, Proceedings of the Combustion Institute 33 (2011) 2425.

[13] J.A. Lei, N.A. Liu, L.H. Zhang, H.X. Chen, L.F. Shu, P. Chen, Z.H. Deng, J.P. Zhu, K. Satoh, J.L. de Ris, Proceedings of the Combustion Institute 33 (2011) 2407.

[14] Y. Hayashi, K. Kuwana, T. Mogi, R. Dobashi, Journal of Chemical Engineering of Japan 46 (2013) 689.

[15] A.Y. Klimenko, F.A. Williams, Combustion and Flame 160 (2013) 335.

[16] K. Kuwana, K. Sekimoto, T. Minami, T. Tashiro, K. Saito, Proceedings of the Combustion Institute 34 (2013) 2625.

[17] A.Y. Klimenko, Journal of Fluid Mechanics 738 (2014) 268.

[18] L.H. Hu, J.J. Hu, S. Liu, W. Tang, X.Z. Zhang, Proceedings of the Combustion Institute 35 (2015) 2511.

[19] J. Lei, N. Liu, K. Satoh, Proceedings of the Combustion Institute 35 (2015) 2503.

[20] J. Lei, N.A. Liu, L.H. Zhang, K. Satoh, Combustion and Flame 162 (2015) 745.

[21] P.B. Sunderland, J.E. Haylett, D.L. Urban, V. Nayagam, Combustion and Flame 152 (2008) 60.

[22] C.-S. Yih, Dynamics of nonhomogeneous fluids, Macmillan New York, 1965.

[23] A. Klimenko, Physics of Fluids (1994-present) 13 (2001) 1815.

[24] A. Klimenko, Theoretical and Computational Fluid Dynamics 14 (2001) 243.

[25] P.M. Chung, Advances in heat transfer 2 (1965) 109.

[26] C.K. Law, Combustion physics, Cambridge university press, 2006.

[27] J.M. Burgers, Adv. in Appl. Mech. 1 (1948) 171.

[28] P.G. Saffman, Vortex dynamics, Cambridge university press, 1992.

[29] F. Williams, Combustion Theory 2nd. The Benjamin/Cummings, 1985.

[30] A.H. Nayfeh, Introduction to perturbation techniques, John Wiley & Sons, 2011.

[31] R.A. Yetter, I. Glassman, H.C. Gabler, Proceedings of the combustion institute 28 (2000) 1265.