Chapter 10
Summary

In this book, the space–time CESE method is introduced as a novel and distinctive numerical approach to solving conservation equations in engineering sciences. Also, from a historical perspective, the development of the CESE method since the 1990s has been reviewed and the remarkable improvements and extensions of the CESE method in the past few years has been summarized. Various applications of the CESE method have been presented to emphasize its numerical performance under different scenarios, including many research topics in engineering, such as compressible multi-fluid flows and detonations.

The most elegant part of the CESE method might be the non-dissipative a scheme described in Chap. 2, where the unknown variable u and its spatial derivative u_x can be obtained directly from the conservation laws. However, for the purpose of shock capturing, two families of CESE schemes have been developed on the basis of a scheme. The first family is called the central CESE scheme, including the a–α and CNI schemes. In these schemes, the spatial derivatives are updated by specially designed procedures, where artificial dissipation can be added. The central CESE schemes avoid any characteristic-based techniques (i.e., they are free of Riemann solver). The second family is the upwind CESE schemes, in which the upwind flux solver (e.g., Riemann solver) can be utilized, but it just plays a supporting role. By combining the ideas in the original CESE method and the conventional upwind FVM, the upwind CESE scheme managed to retain the forms of discretized equations for u and u_x in the a scheme, while the numerical dissipation can be introduced in a reasonable manner. It is worth noting that the dissipation of the upwind CESE scheme is naturally insensitive to the CFL number.

Two important issues about the CESE method, namely the CESE schemes on unstructured meshes and the high-order CESE schemes, have been discussed in Chaps. 4 and 5, respectively. The CESE method is naturally multi-dimensional and compatible with unstructured meshes. A high-order extension of the CE/SE method in both space and time can be achieved by storing and updating high-order derivatives

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(e.g., u_{xx} and u_{xxx}) at each solution point, without enlarging the stencil or adding stages in time integration.

The CESE method has low dissipation and high compactness. When applied to the simulations of complex physical processes, the CESE method can catch shock waves, contact discontinuities, fine structures, and small disturbances, with high resolution and strong robustness. Therefore, the CESE method demonstrates good performances in the numerical simulations of wave-propagation problems (e.g., shock waves, acoustic waves, detonation waves, stress waves in solid, and the electromagnetic waves), interfacial instabilities, and shock–bubble/droplet interactions. In many hot research areas including hypersonic aerodynamics, shock dynamics, detonation, aeroacoustics, MHD, and solid dynamics, the CESE method proves to be suitable and shows a good development prospect.

In recent years, an important topic in CESE research has been its application to highly nonuniform and high-aspect-ratio computational meshes, which are often required in simulations of boundary layers. In particular, the CESE method has been recently applied to aerodynamic heating problems and direct numerical simulation of laminar or turbulent flows. It is also worth noting that the CESE schemes are mainly applied to unsteady problems due to their explicit nature in time with the associated restriction on time step size. In order to solve steady problems efficiently, the extensions to implicit time-stepping CESE schemes are under developing. Further analyses of the numerical properties of CESE schemes will be very useful to solidify their mathematical foundation. The modified equation analysis and the spectral property analysis for the upwind CESE scheme are in progress now.

Along with the continuous improvement of high-performance computing, numerical methods emerge one after another. Many challenging numerical simulations can be accomplished today. However, none of the existing numerical methods can be accurate, robust, and efficient under all situations. In this sense, the CESE method may not be superior to the existing methods, but it does provide an alternative approach which deserves more evaluations and further investigations. The authors wish this book can familiarize the readers with the CESE method.