

# A Lamb Wave-based Crack Diagnosis Method Using an Improved RAPID Algorithm

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## Abstract

Crack is one of the most typical damage types in engineering structures, and therefore is a research focus in the field of structural health monitoring (SHM). The monitoring and evaluation methods for crack-type damage are studied using a Lamb wave-based RAPID algorithm (Reconstruction Algorithm for the Probabilistic Inspection of Damage) in this study, which is based on a correlation analysis. RAPID algorithm can overcome the shortcomings of the complex signal analysis affected by multimode nature of Lamb waves. The improved RAPID algorithm is proposed to reconstruct the tomographic image of the crack-type damage by correcting the Signal Difference Coefficient (SDC) of the sensing path at the crack direction based on the Lamb wave reflection and scattering principle at the crack. The orientation of the crack can be enriched after the correction so that it can be highlighted in the reconstructed image. Furthermore, the length of the crack can be evaluated by the SDC distribution map. The experiments were carried out to validate the improved method and results shown that the reconstructed tomographic images can indicate the crack damage quantitatively. The orientation of artificial crack damages were revealed with minor errors and the length of the damages were also evaluated. The experimental results have stressed that the proposed method is effective for crack damage quantitative evaluation and monitoring of crack growth.

## 1 INTRODUCTION

Structural Health Monitoring (SHM) is one of the great interests for critical aerospace and civil infrastructure applications <sup>[1]</sup>. The general goal of SHM is to identify, locate and characterize defects as they initiate and grow to a critical size



during the service lifetime of structures. Conventional nondestructive evaluation (NDE) methods, such as ultrasonic testing, eddy current testing and X-rays can be used with great effectiveness for identifying and locating defects at the component level, but their usages on structural assemblies are usually limited due to the restricted access conditions and the high costs of field testing instruments. Lamb wave technology has been seen as one of the most promising technologies in the existing structural health monitoring technology, due to its excellent propagation capability, high sensitivity to changes of material and structural properties <sup>[2-5]</sup>.

Lamb wave tomography method is effective for damage detection and illustration. Tomography technique originated from medical science. In recent years it was led into SHM domain and has been investigated for more than one decade. Hildebrand <sup>[6]</sup> and Leonard et.al <sup>[7]</sup> studied Lamb wave tomography damage detection technique for pipes or pipe-like structures. Jansen and Hutchins <sup>[8]</sup> utilized Lamb wave to reconstruct defect image in metallic and composite materials. McKeon and Hinder <sup>[9]</sup> combined the algebraic reconstruction technique (ART) with fan beam projection tomography technique (FBP) to inspect damages in aircraft materials. Xu C and J L Rose <sup>[10]</sup> found that the damage in a plate can be simulated by an ellipse, and its location can be detected by Lamb wave tomography. T. R. Hay and R L Royer <sup>[11]</sup> creatively presented a novel algorithm called RAPID (Reconstruction Algorithm for the Probabilistic Inspection of Damage) that was extremely sensitive to material loss. It should be noted that few of Lamb wave tomography techniques has been applied for tomographic <sup>[11]</sup> imaging of the crack damages. However, crack is one of the most typical damages in real-world structures. Actually, when the Lamb wave encounters the crack, the interaction may lead to the change of Lamb wave modes <sup>[12]</sup>, which makes it difficult to judge the direction of the crack and to reconstruct the image of crack damage accurately.

In this paper, a new approach is proposed and investigated to judge the orientations of the cracks and the traditional RAPID algorithm is improved for tomographic imaging of the crack.

## **2 RAPID FOR TOMOGRAPHIC IMAGING OF THE CRACK**

Many of traditional Lamb wave tomography techniques employ wave velocity <sup>[13]</sup> and energy attenuation as the parameters for damage detection and image reconstruction. However, in real-world applications, it is unpractical to extract the features by separating modes from each other, due to its complex waveforms including scattering, reflection and mode conversion caused by structural complicity and damages. In order to obtain accurately reconstructed tomographic image, Lamb wave tomography techniques usually need a large number of sensors. Moreover, it is time-consuming <sup>[14]</sup>. Unlike other techniques counterparts, such as fan beam projection and algebraic reconstruction, they aimed to calculate a line integral of direct path <sup>[15-16]</sup>. RAPID algorithm based on the correlation analysis. The basic idea of RAPID is that the damage can be detected by analyzing differences <sup>[17]</sup> between reference signals (no damage) and current signals. Signal difference coefficient (SDC) is used to represent differences in the statistical features of the signals, and it is more sensitive to the

changes of the waveforms. SDC values are defined as follows:

$$SDC_{ij} = 1 - \frac{\left| \int_{t_0}^{t_0+\Delta T} [x_{ij} - \mu][y_{ij} - \mu] dt \right|}{\sqrt{\int_{t_0}^{t_0+\Delta T} [x_{ij} - \mu]^2 dt \int_{t_0}^{t_0+\Delta T} [y_{ij} - \mu]^2 dt}} \quad (1)$$

Where  $i, j$  denote actuator and receiver numbers respectively,  $t_0$  denotes arriving time;  $\mu$  denotes average value of the response signal;  $\Delta T$  denotes time window.

In the experimental environments, structural damage should be the only reason to induce the change of the received signal. For any sensing path, the value of SDC reflects the degree of damage and the relative position between damage and the sensing path. Obviously, SDC value ranges from 0 to 1. Specifically, if the signals are entirely out of phase, the SDC will be equal to its maximum value of 1. On the contrary, if the signals are the same, SDC will be its minimum value of 0.

Therefore, according to the SDC value of the corresponding sensing path, the damage probability distribution in the near area can be reconstructed. In the reconstructed image, each SDC values are arranged in an ellipse, actuator  $i$  and receiver  $j$  on the corresponding sensing path are the two focal points of the ellipse.

SDC value space distribution function is defined as follows:

$$\begin{cases} s_{ij}(x, y) = \frac{\beta - R_{ij}(x, y)}{1 - \beta}, & \beta > R_{ij}(x, y) \\ s_{ij}(x, y) = 0, & \beta \leq R_{ij}(x, y) \end{cases} \quad (2)$$

$R_{ij}(x, y)$  denotes the radio between the distance of the point  $(x, y)$  to the actuator and to the receiver and the length of sensing path;  $\beta$  denotes form factor which controls the size of the ellipse, and its value is greater than 1.

$$R(x, y, x_{ik}, y_{ik}, x_{jk}, y_{jk}) = \frac{\sqrt{(x - x_{ik})^2 + (y - y_{ik})^2} + \sqrt{(x - x_{jk})^2 + (y - y_{jk})^2}}{\sqrt{(x_{ik} - x_{jk})^2 + (y_{ik} - y_{jk})^2}} \quad (3)$$

From the Eq. (3), probability value of each point would form an elliptical area. From the Eq. (2), damage probability distribution can be obtained.

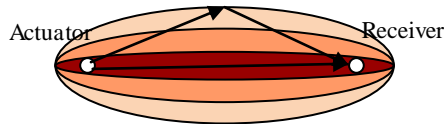


Figure 1: Damage probability distribution ellipse area

For the single sensing path, damage probability distribution can only indicate the damage vertical to the sensing path. In order to locate the damage location accurately, the probability distribution of all sensing paths should be superimposed. Therefore, the damage probability distribution of any point  $(x, y)$  in the testing area is

formulated as follows:

$$P(x, y) = \sum_{i=1}^{N-1} \sum_{j=i+1}^N \text{SDC}_{ij}^{s_{ij}}(x, y) \quad (4)$$

Where, N is the number of sensing path.

### 3 CRACK DETECTION BASED ON LAMB WAVE TOMOGRAPHY

#### 3.1 Detection of orientation of the crack

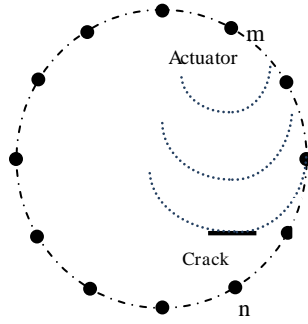
As shown in Figure 2 (a), 12 sensors are evenly mapped around the circumference, and the crack is located at the sensing path (sensor m-sensor n). Sensor m is set as the actuator and the rest 11 sensors are set as receivers. Excite actuator m and collect the signals received by each receiver, then the information between reference and received signals will yield the eleven SDC values. It can be observed that there would be some loss to the wave when it passes through the crack. Obviously, the sensing path (sensor m-sensor n) that yields the maximum value of the eleven SDC values should be vertical to the crack. In another case, shown in Figure 2 (b), similarly actuator p excites and the signals are received by other 11 receivers. In this case, there is little loss to the waveform of the actuating signal when the actuating wave passes through the crack. After the eleven SDC values are calculated, comparing them with the former case, the maximum value of eleven SDCs related with actuator p should much smaller than that related with actuator a.

In order to obtain better damage reconstruction, the orientation of crack can be determined and the crack damage can be reconstructed by following the correction: the SDC on sensing path of the detected orientation of the crack is set as 1. Equation (1) after the correction should be:

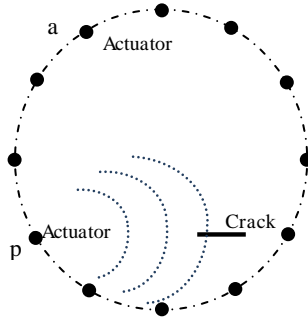
$$\text{SDC}_{ij} = \begin{cases} 1 & , \text{ if path } i, j \text{ in the direction of the crack} \\ 1 - \frac{\sum_{k=1}^K (X_k - \mu_x)(Y_k - \mu_y)}{\sqrt{\sum_{k=1}^K (X_k - \mu_x)^2} \sqrt{\sum_{k=1}^K (Y_k - \mu_y)^2}} & , \text{ other} \end{cases} \quad (5)$$

Finally, the procedure of the improved tomography technique is summarized as following steps:

- (1) One actuator excites, meanwhile the rest sensors receive signals. These 11 sensing paths are viewed as a group;
- (2) Then calculate SDC value of each sensing path, and compare all the SDC values to find the maximum one in each group;
- (3) 12 actuators excite in turn. Thus, 12 maximum SDC values of all groups will be obtained. List the 12 values in the descending order, and find the last two ones. In fact, the sensing path, connected the two sensors corresponding to the actuators of these last two values, will be the detected orientation of the crack. So, the SDC of this sensing path should be corrected to be 1.
- (4) Reconstruct the crack damage by using the corrected SDC values.



(a) Propagation direction of the exciting signal is parallel to the orientation of the crack



(b) Propagation direction of the exciting signal vertical to the orientation of the crack

Figure 2: The diagram of determining the orientation of the crack

### 3.2 Principle of evaluation of the length of the crack

As shown in Figure 3, it can be assumed that the orientation of the crack has been detected as the line between sensor A and B. In the circular sensor array, the sensor locate at the center of minor arc AB viewed as an actuator while all of the sensors on the major arc AB as receivers. In the center O establish the origin of coordinates Cartesian coordinate system and then the central angle of each sensor can be obtained. Calculate SDC value of each sensing path, and set radius angle of receiving sensor as x axis, SDC value as y axis to draw the distribution of the value of SDC. Finally, the crack length is determined based on a threshold. In practice, considering the influence of endpoint and based on the results of several tests, the path whose SDC value is greater than 0.4 should pass through injury.

In the distribution of the SDC value, assuming that the SDC value greater than 0.4 is located between the sensing path MP and MQ, the length of crack could be obtained by the following steps: computing the intersection of straight line AB and MP, as well as the intersection of straight line AB and MQ, the distance between the two intersections is the length of crack.

## 4 EXPERIMENTAL VALIDATION

### 4.1 Experimental Setup

The experimental specimen was the aluminum plate of 400mm\*400mm\*2.0mm. The artificial crack zone was located at the direction of sensor4–sensor10 with the length of 40 mm and the width of 1 mm. 12 sensors were located at a circular

array with a diameter of 320 mm, as shown in Figure 4.

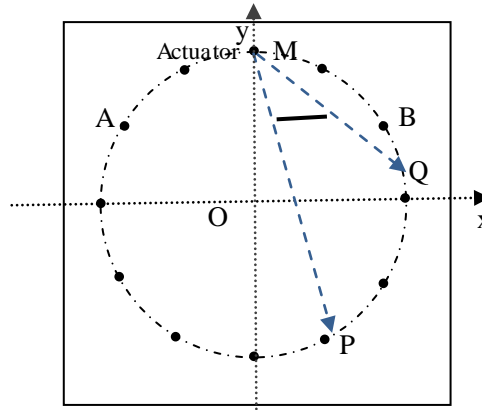


Figure 3: The diagram of determining the length of the crack

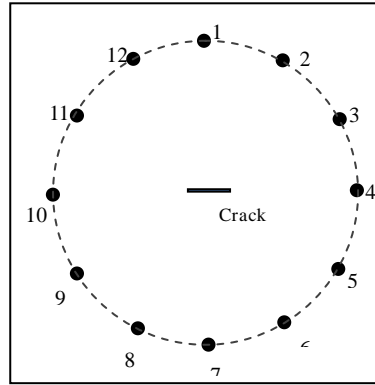


Figure 4: The experimental specimen

In experiments, single-mode Lamb wave signal was excited by the one actuator, meanwhile the rest sensors were adopted to collect Lamb wave structural responses. The exciting signal used in the experiment was narrowband modulated 5-cycle sinusoidal wave, which is expressed as follows:

$$I(t) = [H(t) - H(t - M / f_c)] \times (1 - \cos(2\pi f_c t / M)) \sin 2\pi f_c t \quad (6)$$

Where,  $f_c$  is the center frequency of the generation signal,  $H(t)$  is the Heaviside step function,  $N$  is the number of wave peaks of sine modulate signal. The more the number of wave peaks are, the bigger the energy of the actuation is, and the narrower the frequency band is. Therefore, we adopt the actuating signal with the center frequency of 200kHz. The actuating signal is loaded in the direction of perpendicular to the plate plane. The excited Lamb wave mode is  $A_0$ , and such a single mode situation can simplify subsequent signal processing. The time-domain waveform is shown as Figure 5.

#### 4.2 Simulation and Experimental Results Analysis

Abaqus<sup>®</sup> finite element simulation software was adopted for the simulation according to the experiment scheme above. First, every actuator was excited in turn,

and meanwhile Lamb wave structural responses were collected as shown in Figure 6. In Figure 6 (a), the red line is damaged signal, and the blue line is reference signal. Damaged signal and reference signal were obviously out of phase, and the amplitude of damaged signal reduced a lot in comparison with reference signal. So their relevance was very small and the SDC value should be big. The propagation direction of the exciting signal was close to the vertical orientation of crack according to the theory mentioned above. However, in Figure 6 (b), damaged signal was only a little delayed. The amplitude of damaged signal was almost as same as the reference signal. Then SDC value of 11 sensing paths of 12 groups is calculated.

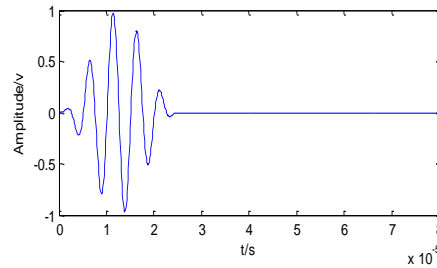
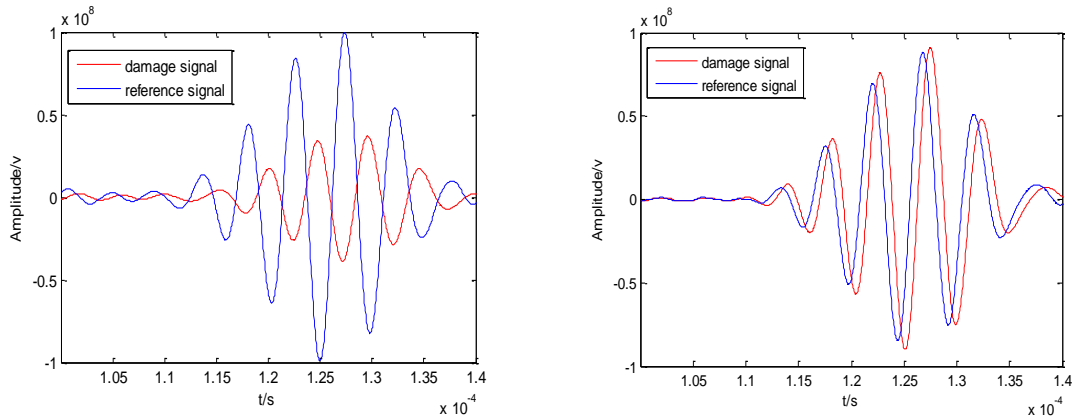


Figure 5: The time-domain waveform of actuating signal



(a) Waveforms of sensing path (actuator 1-receiver7) (b) Waveforms of sensing path (actuator 4-receiver10)

Figure 6: Waveforms of two sensing paths (1-7 and 4-10)

The maximum of 11 sensing paths in each group was shown in Figure 7. The maximum of SDC values of group 4 and group 10 were less than the rest maximums of groups. Therefore, the direction of the connection between actuator 4 and actuator 10 was the detected as the orientation of crack according to principle mentioned in Chapter 4.2. Then the SDC value of the sensing path (sensor 4 and sensor 10) was corrected to 1. The corrected SDC values were employed to reconstruct tomographic image of the crack by the improved RAPID algorithm. According to the Eq. (2), the tomographic image of the crack can be reconstructed accurately by adjusting the parameter  $\beta$ . The reconstructed tomographic image of the crack damage was shown in Figure 8. The deeper the color was, the bigger the probability of existing damage was. Figure 9 was the tomographic image after a threshold value, and the red area represented the crack damage. The length of the reconstructed crack was 39mm, which was consistent with the real damage.

After detecting the crack orientation as the straight line between the sensor 3 and the sensor 11, the sensor 1 was set as actuator, more than half of the arc sensors between the sensor No. 3 to the sensor No.11 were set as receivers. And establish a

Cartesian coordinate system with the center of circle as Origin of coordinates. Set central angle of actuator 1 as 0, each receiver's (sensor No.3 to sensor No.11) central angle could be obtained. Calculated SDC values of each sensing path and the distribution of SDC values were shown in Figure 10.

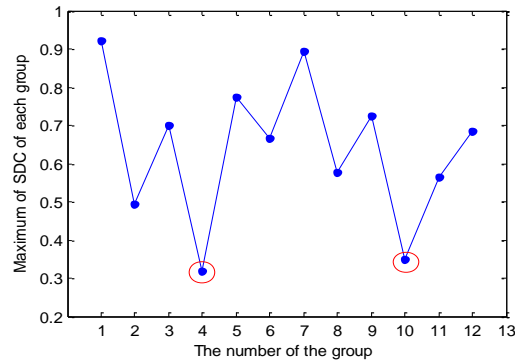


Figure 7: The maximum SDC values of 11 sensing paths in each group

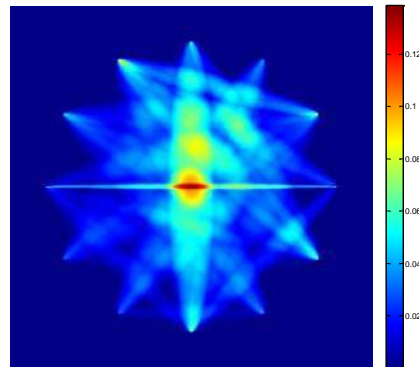


Figure 8: Tomographic image reconstruction by the improved RAPID algorithm

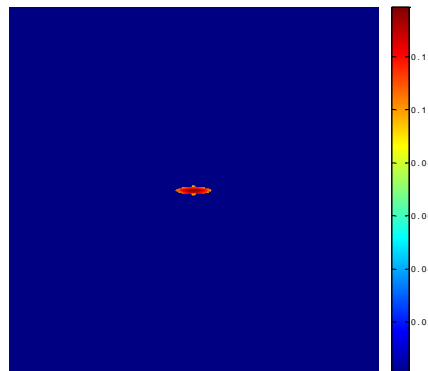


Figure 9: Tomographic image reconstruction by the improved RAPID algorithm after a threshold value

Between 107 degrees to 171 degrees, SDC value was greater than 0.4 as shown in Figure 10. According to the principle mentioned in section 3.2, the length of crack can be calculated 50.7mm, with the error 9.7mm. Using the same experimental setup and steps, cracks with different angles and lengths were detected as shown in Table 1.

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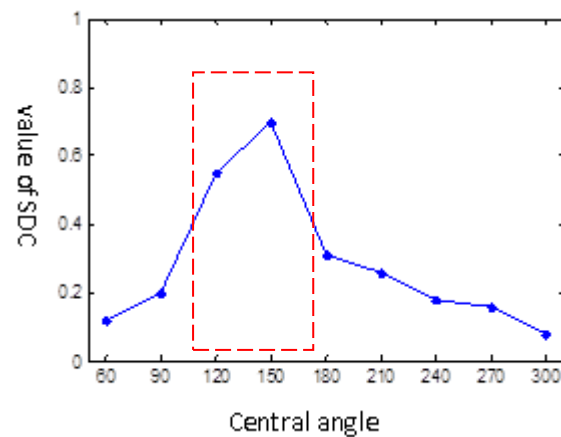


Figure 10: Distribution of SDC values

The true value of the crack length mm	The monitoring value of the crack length mm	Absolute error mm	Relative error mm
40mm	50.7mm	9.7mm	24.25%
80mm	95.4mm	15.4mm	19.25%
120mm	138.7mm	18.7mm	15.58%

Table 1: Experimental results

## 5 CONCLUSION

In this paper, an improved Lamb wave based crack tomography RAPID algorithm was presented to reconstruct tomographic image to indicate the information of the crack. Experimental results show that this improved RAPID algorithm could detect and evaluate crack damage and reconstruct tomographic image successfully in large scale plate-like structure. Future work will focus on the reconstruction of multiple crack damages.

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