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Optimal Dispatcher Number for One-way Carsharing Services Considering Break Requirement

Lu Yang, Min Xu, Shi An, and Xiaowei Hu

***Abstract*—This study investigates the dispatcher number (DN) of one-way carsharing services (CSSs), considering the critical yet overlooked aspect of dispatchers' break requirement with vehicle relocation and dispatcher assignments. The DN problem aims to minimize the daily cost of one-way CSS operators by determining the fleet size, the number of dispatchers, vehicle relocation, and dispatcher movement under the restriction on the maximum accumulative working time of dispatchers. The novelty of the study lies in the incorporation of the break requirement consideration of dispatchers into the personnel assignment during the operation period. A nonlinear integer programming (NLIP) model is first developed for the DN problem. By exploring the structure of the proposed model, an effective heuristic solution method consisting of optimization and simulation modules is proposed to obtain the optimal solution to the problem. Numerical experiments based on EVCARD, a popular one-way CSS operator in China, are conducted to demonstrate the effectiveness of the proposed models and solution method and the rationality of incorporating the break requirement into the decision-making process. Furthermore, we investigated the impacts of critical parameters, such as payment for dispatchers and relocation cost, on the performance of the one-way CSS, providing valuable insights for carsharing service operators.**

***Index Terms*—Dispatcher number, one-way carsharing services, break requirement, optimization-simulation method.**

I. INTRODUCTION

A. Motivation

CARSHARING services (CSSs) are considered a promising transportation mode, providing users with an attractive alternative to using private cars without processing the cost of owning them [1], [2], which are divided into two kinds based on the operation modes. Compared to traditional two-way CSSs, users can return rented vehicles to different stations in the one-way CSSs. One-way carsharing services can relieve the pressure on transportation systems. However, the uneven distribution of demands inevitably causes temporal and spatial vehicle imbalance issues across stations. To maintain balance, some operators choose to conduct operator-based intervention strategies, i.e., employ dispatchers to conduct vehicle relocation tasks across stations in station-based one-way CSSs [3], [4]. Dispatchers take other transport modes to move across predesigned stations to take two separate relocation operations [5], whereas inappropriate movements of employed dispatchers could also result in an imbalance of dispatchers' distribution [6]. Thus, dispatchers' movements are also vital in solving the imbalance problem of one-way CSSs.

Although decision problems related to vehicle relocation and dispatcher rebalancing in one-way CSS have been extensively investigated (see Subsection I.B), limited research attention has been given to the break requirements of dispatchers in the rebalancing problem of one-way CSS. In practice, some full-time dispatchers are likely to undergo frequent relocations and movements without sufficient breaks to minimize costs for CSS operators, which can lead to driving fatigue during relocation tasks [7], [8]. Fatigue driving has been a major cause of traffic accidents worldwide [9]. Therefore, apart from the rebalancing of vehicles and dispatchers, additional attention should be given to the full-time dispatcher arrangement and their break requirement to avoid fatigue driving in the decision-making of one-way CSS operators.

This study focuses on station-based one-way carsharing services and aims to minimize operation costs for one-way CSS operators by jointly determining the number of employed dispatchers and the optimal fleet size, considering vehicle relocation, dispatcher assignment, and break requirement of each dispatcher, which is critical to ensuring road safety and dispatcher well-being.

B. Related studies

Motivated by the increasing popularity of one-way carsharing services, various models have been proposed to address the vehicle relocation problem, with the objective of maximizing the profit or minimizing the cost of CSS operators to eliminate the vehicle imbalance issue in one-way carsharing services [10-14]. These studies devoted to vehicle relocation are often integrated with strategic or tactical decisions, such as the

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number or size of stations [10], [11], fleet size [11-13], and travel price [14]. For example, in [11], a frame for optimization of fleet size and parking capacities was proposed to maximize operating profit. A mixed-integer nonlinear programming model was developed to determine vehicle fleet size and relocation tasks [13]. Elastic demand and passenger waiting time were jointly incorporated into the vehicle rebalancing problem [14]. Interested readers may refer to review articles [15] and [16] for more details.

Majority of previous studies on vehicle relocation problems overlooked dispatcher movements [2], [17]. That is, they assume that dispatchers are sufficient to relocate imbalanced vehicles. In reality, CSS operators must consider movements of dispatchers which inevitably affect fleet size and vehicle relocation arrangement due to their interactive relationship regarding CSS operation cost. Only a few studies have jointly taken dispatchers' movement and vehicle relocation into account [18-20]. For example, a joint optimization method was proposed to rebalance vehicles and dispatchers with the consideration of charging scheduling [19]. In [20], a fleet size and trip pricing problem was proposed with the consideration of vehicle relocation and personnel assignment. Although many studies have considered related problems in CSSs, none of them considered the working time and break requirement of dispatchers during the operation period. In other words, those studies implicitly assumed that dispatchers would always be able to conduct tasks as long as they stayed at stations, disregarding the number of tasks conducted or their well-being. It is commonly known that one-way CSS operators tend to employ fewer dispatchers to conduct tasks for cost minimization, which leads to frequent relocation and movement assigned to employed dispatchers. According to Crawford [21], driving for a prolonged period and other excessive physical exertion before driving could cause driving fatigue, posing a threat to road safety. So the above assumption may lead to overload working and driving fatigue, and, accordingly, safety threat of full-time dispatchers and accident loss of CSSs.

As a kind of driving, the relocations conducted by dispatchers were regarded as skillful tasks that required sustained attention and quick reactions to respond to emergencies [8]. To eliminate the negative effect of fatigue, drivers should take a certain amount of breaks after long-time work to recover their attention, reactions, operating ability, and perceptions before continuing driving [22]. Many regulations on breaks have been set considering drivers’ fatigue. For example, in Brazil, drivers are regulated to take at least 15 minutes after driving for 5.5 hours to improve road safety [23], [24]. Although there are still no specialized regulations on the break requirement of dispatchers in cities, it is essential to take the break requirement of dispatchers into account for road safety, especially in the context of the increasing popularity of one-way carsharing services worldwide.

C. Our contribution

The paper aims to propose the cost minimization for one-way carsharing operators by jointly determining the optimal fleet size, and dispatcher number with the consideration of break requirement, vehicle relocation, dispatcher assignment, and multiple movement modes. Our study presents a novel approach to optimizing one-way CSS operations, which considers both operation cost and the well-being of dispatchers, ultimately contributing to the sustainability and safety of these services.

The contributions of the paper are given below:

(1) We make the first attempt to define the break requirement of dispatchers and incorporate it into tactical decision-making of one-way carsharing systems.

(2) A nonlinear integer programming model considering the break requirement of dispatchers, parking resources, and multiple movement manners is proposed for the proposed problem.

(3) The nonlinearity of the accumulative working time function and model flexibility make it unable to be efficiently solved by state-of-the-art solvers. A two-stage optimization-simulation solution algorithm is thus proposed to find the optimal solution considering the structure of the proposed model. The impact of break requirement consideration and parameters on system performance are evaluated.

The remainder of this study is organized as follows. Assumptions, notations, problem descriptions, and the break requirement description are elaborated on in Section II. A nonlinear integer programming model for the DN problem is formulated in Section III. The optimization-simulation algorithm is proposed in Section IV. The efficiency of the proposed model and algorithm is demonstrated with a case study of a CSS operator in Suzhou named EVCARD in Section V. Finally, conclusions and future study directions are described in Section VI.

II. Assumptions and Problem Description

Considering a one-way carsharing service operator who fulfils reserved rental demand of users using vehicles among several predetermined stations in an urban area. The number of parking spaces at each station is determined in advance. It is assumed that the details of users’ rental demands, i.e., origin, destination, and the starting time of trips, are known in advance. Users could pick up vehicles based on their reservations and return them to other stations different from origin or pickup stations after rental. Vehicles could be available after arriving at any stations driven by users or dispatchers during the operation period. Due to the imbalanced distribution of vehicle rental demands and limited vehicle resources, a few dispatchers are employed to relocate vehicles between stations to address vehicle imbalance problems. Each dispatcher moves among stations by other transport modes to conduct two separate relocations. We assume that an individual dispatcher must take a compulsory break to avoid overloading work after a maximum accumulative working time according to the break requirement, defined in Subsection II.B.

A directed, connected traffic network (N, A) is proposed to illustrate the topological structure in the DN problem, where N is a finite set of nodes that represent pick-up and drop-off stations and A is the set of directed links between two different stations, i.e., station $i \in \boldsymbol{N}$ and station $j \in \boldsymbol{N} \setminus \{i\}$. Set A contains all possible non-identical pairs of stations, each pair connecting two stations could represent several predetermined paths between these stations in the physical transport network. The daily operation period is discretized into several timestamps with the same time intervals, denoted by Δ, between two adjacent ones. For instance, 15-minute time intervals, i.e., Δ=15minutes, during the operation period from 8:00 to 18:00. We assume that all the trips, relocation of vehicles, and movement of dispatchers start and end at the beginning of time intervals, that is, relocation time, movement time, and break time, will be integer multiples of the given time interval. The total number of time intervals in the time horizon is denoted by T. The set of time intervals is denoted by $\boldsymbol{T} = \{1, 2, \cdots T-1, T\}$. To illustrate the DN problem, the following subsections will cover network representation, the flow conservation of vehicles and dispatchers, the interpretation of break requirement, and the calculation of accumulative working time.

A. Flow conservation of vehicles and dispatchers

Users' rental demands for the one-way carsharing services are aggregated in the manner that their origins and destinations are rightly at the designated stations. Rental demands with the same starting time $t \in \boldsymbol{T}$ and O-D station pair ij are classified into different groups $g \in \boldsymbol{G}_{ij}^{t}$, according to their rental duration. That is, rentals in the same group have the same duration.

Available vehicles can be driven in/out of the origin/destination stations at the beginning/end of the operation period by users or dispatchers. There are three possible activities of vehicles, i.e., relocated by dispatchers, driven by users, and parked at stations. The number of vehicles at the station i, can be rented or relocated at the end of time interval t, denoted by n_i^t. Note that the variable n_i^0, n_i^t at $t=0$, indicates the initial number of vehicles at station i at the beginning of time interval 1.

There are also three activities of employed dispatchers: conducting vehicle relocations, movement by other transport modes, e.g., electric bikes or regular bikes, and taking breaks or waiting at stations. For the generality of operation, employed dispatchers are assumed to be assigned to conduct relocation tasks from a virtual origin hub, denoted by o, where dispatchers are initially assigned to conduct relocation tasks. The travel cost and time from the origin hub and any station for the initial assignment are assumed to be zero. The set of available dispatchers is denoted by $\boldsymbol{W}$. For each dispatcher $w \in \boldsymbol{W}$, we introduce several binary decision variables to denote the activity of individual dispatchers. Let binary variable z_{oi}^{tw} denote whether the dispatcher w from the origin hub to station i conducts the first relocation tasks at the beginning of time interval t. We further introduce $\tilde{z}_{ij}^{tw}$ to denote whether dispatcher w conducts relocation tasks from station i to station $j \neq i$ at the beginning of time interval t, and $\tilde{z}_{i*}^{tw}$, z_{ij}^{ktw}, z_{ii}^{tw} to express whether dispatcher w is at station i at the end of time interval t and will take actions, i.e., relocate vehicles from station i, move to the station $j \neq i$ by transport mode k, and wait at station i for one time interval, respectively.

B. Break requirement of dispatchers

In the context of the CSS system, there are currently no specific regulations addressing the break requirement of dispatchers. In light of regulations or laws pertaining to driver working hours in relevant literature [24], [25]. We have formulated the definition of the dispatcher break requirement in this research, by introducing accumulative working time and compulsory break time. Specifically, we assume that the total accumulative working time of the dispatcher w, including the relocation and movement time should not be more than a threshold, i.e., maximum accumulative working time, denoted by $\delta_{\max}$. That is, dispatcher $w \in \boldsymbol{W}$ with maximum accumulative working time should stay at the parking station for at least compulsory break time, denoted by $\theta_{\min}$ to conduct the subsequent assigned relocations or movement tasks during the operation period. To model the above practical constraints, we define integer variables $\delta^{tw}, \forall t \in \boldsymbol{T}, w \in \boldsymbol{W}$, integer multiples of given time interval Δ, to denote the accumulative working time of dispatcher w at the end of time interval t. Thus, we have the following constraints on accumulative working time:

$$\delta^{tw} \leq \delta_{\max}, \qquad \forall t \in \boldsymbol{T}, w \in \boldsymbol{W} \tag{1}$$

We then proceed to establish the relationship between δ^{tw} and other dispatcher variables. The value of δ^{tw} is continuously calculated with $\delta^{(t-1)w}$ and formerly assigned tasks, i.e., relocation and movement. Not that, the value of δ^{tw} will be updated to be 0 if dispatcher w takes a continuous break at a station for no less than $\theta_{\min}$. To model the above relationship, we define binary decision variables $\eta^{tw}, \forall t \in \boldsymbol{T}, w \in \boldsymbol{W}$, indicating whether the accumulative working time of dispatcher w at the end of time interval t will be updated to be 0. It can be seen that η^{tw} is determined by z_{ii}^{tw} as follows:

$$\eta^{tw} = \begin{cases} 1, & \text{if } \Delta \sum\limits_{\tau=\max\{t-\theta_{\min}/\Delta, 0\}}^{t-1} \sum\limits_{i \in \boldsymbol{N}} z_{ii}^{\tau w} < \theta_{\min} \\ 0, & \text{if } \Delta \sum\limits_{\tau=\max\{t-\theta_{\min}/\Delta, 0\}}^{t-1} \sum\limits_{i \in \boldsymbol{N}} z_{ii}^{\tau w} \geq \theta_{\min} \end{cases}, \quad \forall t \in \boldsymbol{T}, w \in \boldsymbol{W} \tag{2}$$

The above equation shows that the value of η^{tw} depends on z_{ii}^{tw} only. Specifically, $\eta^{tw}=0$ if dispatcher w waits/rests continuously at a station for no less than $\theta_{\min}$; $\eta^{tw}=1$ otherwise, and the accumulative working time at the end of time interval t is equal to that at the end of the last time interval. Considering that η^{tw} is a binary variable, the relationship between η^{tw} and z_{ii}^{tw} in (2) can be equivalently expressed by the following constraints:

$$\frac{\theta_{\min}/\Delta - \sum_{\tau=\max\{t-\theta_{\min}/\Delta,0\}}^{t-1}\sum_{i\in \boldsymbol{N}} z_{ii}^{\tau w}}{\theta_{\min}/\Delta} \le \eta^{tw} \le \theta_{\min}/\Delta - \sum_{\tau=\max\{t-\theta_{\min}/\Delta,0\}}^{t-1}\sum_{i\in \boldsymbol{N}} z_{ii}^{\tau w}, \quad \forall t\in \boldsymbol{T}, w\in \boldsymbol{W}$$

(3)

Different from drivers engaged in the carriage of goods and passengers by road, there are three activities of employed dispatchers: taking vehicle relocation, movement among stations, and taking breaks at stations. Aside from their resting intervals, the time that dispatchers conduct both vehicle relocation tasks and movements between stations should be recognized as working time. Given the decision variables z_{oi}^{tw}, $\tilde{z}_{ij}^{tw}$, z_{ij}^{ktw}, and η^{tw}, the accumulative working time of dispatcher w at the end of time interval t can thus be calculated by following equations:

$$\delta^{tw} = \Delta\sum_{i\in \boldsymbol{N}} z_{oi}^{1w}, \qquad \forall w\in \boldsymbol{W}, t=1 \tag{4}$$

$$\delta^{tw} = \left(\begin{array}{l} \delta^{(t-1)w} + \Delta\sum_{\tau=1}^{t}\sum_{i\in \boldsymbol{N}}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\sum_{\Delta\tau+\gamma_{ij}^{\tau}>=\Delta(t+1)} \tilde{z}_{ij}^{tw} \\ +\Delta\sum_{k\in \boldsymbol{K}}\sum_{\tau=1}^{t-1}\sum_{i\in \boldsymbol{N}}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\sum_{\Delta\tau+\xi_{ij}^{kt}>=\Delta t} z_{ij}^{ktw} \end{array} \right) \times \eta^{tw}, \quad \forall w\in \boldsymbol{W}, t\in \boldsymbol{T}\setminus\{1\} \tag{5}$$

Equations (4) and (5) update the accumulative working time of dispatcher $w\in \boldsymbol{W}$ at the end of time interval $\forall t\in \boldsymbol{T}$. The accumulative working time at the end of the first time interval is Δ if dispatcher w conducts the first relocation task at the beginning of time interval $t=1$ originating from any station $i\in \boldsymbol{N}$. The update of δ^{tw} at the end of time interval $t\in \boldsymbol{T}\setminus\{1\}$ can be equivalently expressed by (5), which indicates that the accumulative working time δ^{tw} can be adjusted by the $\delta^{(t-1)w}$ (i.e., the accumulative working time at the end of time interval $t-1$), the relocation tasks conducted by dispatcher w from time interval 1 to t, and the movement tasks conducted by dispatcher w from time interval 1 to $t-1$.

The consideration of the break requirement would significantly affect the optimal dispatcher assignment for the cost minimization of CSS operators. This can be illustrated by a simple example. Suppose there are one vehicle and two dispatchers, i.e., Dis A and Dis B, in the CSS with three stations, i.e., Station I, Station II, and Station III. The maximum accumulating working time $\delta_{\max}$ and compulsory break time $\theta_{\min}$ are set to be 120 minutes and 15 minutes, respectively. The time interval Δ is 15 minutes. The vehicle is available at Station I from the beginning of time interval 3 onwards. Dis A is at Station I from the beginning of time interval 3 with an accumulative working time of 120 minutes, while Dis B is at Station III at the beginning of time interval 1, whose

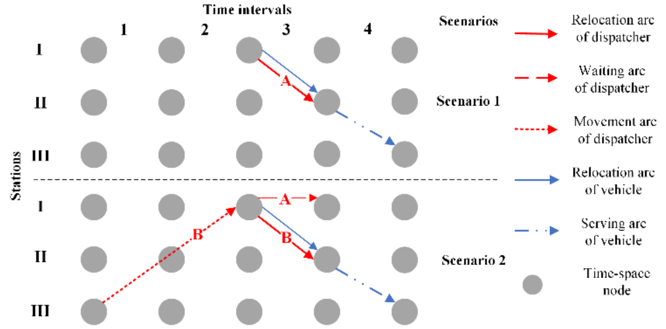


Fig. 1. Vehicle and dispatcher assignment under two scenarios.

accumulative working time is 15 minutes. There will be a rental from Station II to Station III at the beginning of time interval 4. To serve this demand, the vehicle must be relocated from Station I to Station II and arrive at Station II beforehand. We further assume that the relocation time from Station I to Station II is 15 minutes and the movement time from Station III to Station I is 30 minutes.

Without the consideration of the break requirement, illustrated in Scenario 1, Dis A will relocate the vehicle from Station I to Station II to fulfil the demand for cost minimization. But due to the constraints on $\delta_{\max}$, Dis A must take a break for at least 15 minutes at Station I. After the 15-minute rest, there would be insufficient time for Dis A to take the relocation of the vehicle. Dis B thus will first move from Station III to Station I to relocate the vehicle to Station II to fulfil the demand as Scenario 2 illustrates. The extra movement of Dis B inevitably results in more cost than the former arrangement performed by Dis A. Fig. 1. shows the vehicle and dispatcher trajectories of above two scenarios.

It is worth noting that although the definition of break requirement is currently based on accumulative working time and compulsory break time, it can be easily extended to further incorporate other factors related to driving fatigue, such as work intensity and individual dispatcher variables. For example, to further consider the work intensity, an equivalent conversion factor α^{k}, induced to obtain the equivalent working time when a dispatcher taking movements by transportation mode k, could be incorporated in the process of calculating the accumulative working time. The maximum accumulative working time and compulsory break time could be individually specified for each dispatcher w, i.e., $\delta_{\max}^{w}$ and $\theta_{\min}^{w}$, respectively, to further incorporate the individual dispatcher variability. Furthermore, similar to regulations of minimum overnight rest of drivers, after a non-consecutive long working time with several minimum short breaks, a longer break time could be guaranteed by extending the break requirement to enhance the flexibility of the break policy. For example, we could initially introduce a new set of parameters related to accumulative working time and compulsory break time with larger values, in which $\delta^*_{\max}$ denotes a larger maximum accumulative time, several-fold the value of $\delta_{\max}$, and $\theta^*_{\min}$ denotes a larger compulsory break time than $\theta_{\min}$. Constraints, similar to (1) and (3)-(5), can be formulated with newly introduced $\delta^*_{\max}$ and $\theta^*_{\min}$, and integrated with existing constraints to extend the definition of break requirement. Above extension of break requirement could be solved efficiently due to the proposed optimization-simulation method in Section IV.

TABLE I

DESCRIPTION OF NOTATIONS

Notation	Definition
Indices and sets	
t, τ	indices for time interval
$i.j$	indices for parking station
g	index for group of one-way carsharing users' rental demand
w	index for the dispatcher employed to conduct relocation task
k	index for transport mode available to dispatchers for moving between pairs of stations
o	origin hub
$\boldsymbol{N}$	set of pick-up and drop-off stations
$\boldsymbol{T}$	set of time intervals
$\boldsymbol{W}$	set of dispatchers
$\boldsymbol{K}$	set of transport modes
$\boldsymbol{G}_{ij}^{t}$	set of groups of one-way carsharing users who pick up vehicles from station i at the beginning of time interval t and drop off them at station j after using
Known parameters	
T	total number of time intervals during the operation period
Δ	fixed duration of each time interval (e.g., 15 minutes)
d_{ij}^{tg}	the number of demands in group $g \in \boldsymbol{G}_{ij}^{t}$
ε_{ij}^{tg}	rental duration of users in group $g \in \boldsymbol{G}_{ij}^{t}$
γ_{ij}^{t}	relocation time of vehicles by dispatchers from station i to station j at the end of time interval t
ξ_{ij}^{kt}	dispatcher movement time from station i to station j at the end of time interval t by transport mode k
VC	daily mixed cost of each vehicle measured in \$/ veh-day
RC_{ij}^{t}	relocation cost of dispatchers to relocate vehicles from station i at the beginning of time interval t to station j, i.e., cost of fuel/electricity consumption and time consumption for vehicle relocation operation
MC_{ij}^{kt}	movement cost of dispatchers to move from station i to station j at the end of time interval t by transport mode k
PC	daily payment for dispatchers measured in \$/ per-day
N_V	maximum number of vehicles operated by the one-way CSS operator
N_{per}	maximum number of dispatchers employed by the one-way CSS operator
Q_i	number of parking spots at station i
$\delta_{\max}$	maximum accumulative working time of dispatchers based on the break requirement
$\theta_{\min}$	compulsory break time of dispatchers based on the break requirement
Decision variables	
n_i^0	initial deployment of vehicles at station i
$\tilde{z}_{ij}^{tw}$	binary variable for individual dispatcher, taking the value 1 if dispatcher w is assigned to carry out relocation tasks from station i to station j at the beginning of time interval t, and 0 otherwise.
z_{oi}^{tw}	binary variable for individual dispatchers, taking the value 1 if dispatcher w is from the origin hub and begins to conduct relocation tasks at the beginning of time interval t, and 0 otherwise.
$\tilde{z}_{i*}^{tw}$	binary variable of individual dispatchers, taking value 1 if dispatcher w is rightly at station i at the end of time interval t, and will conduct relocation tasks from station i to station $j \neq i$ at the end of time interval t, and 0 otherwise.
z_{ij}^{ktw}	binary variable for individual dispatchers, taking the value 1 if dispatcher w is rightly at station i at the end of time interval t, and will move to station $j \neq i$ by transport mode k, and 0 otherwise.
z_{ii}^{tw}	binary variable for individual dispatchers, taking the value 1 if dispatcher w is rightly at station i at the end of time interval t, and will wait /rest at station i for one time interval, and 0 otherwise.
η^{tw}	Binary variable denoting whether the accumulative working time of dispatcher w at the end of time interval t will be updated to be 0
Auxiliary variables	
n_i^t	number of available vehicles at station i that can be rented out by users or relocated by dispatchers at the end of time interval t
δ^{tw}	accumulative working time of dispatcher w at the end of time interval t

III. OPTIMIZATION MODEL BUILDING

A. Model formulation

Given the notations summarized in Table I, the DN problem can be formulated as follows:

[DN]

$$\begin{aligned}\min_{\mathbf{n},\mathbf{z},\boldsymbol{\eta},\boldsymbol{\delta}} COST(\mathbf{n},\mathbf{z},\boldsymbol{\eta},\boldsymbol{\delta}) = VC\sum_{i\in \boldsymbol{N}} n_i^0 \\ +PC\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{w\in \boldsymbol{W}} z_{oi}^{tw} \\ +\sum_{t=1}^{T}\sum_{i\in \mathbf{N}}\sum_{j\in \mathbf{N}\setminus\{i\}}\Big[(RC_{ij}^{t}\sum_{w\in \boldsymbol{W}}\tilde{z}_{ij}^{tw}) \\ +\sum_{k\in \boldsymbol{K}}(MC_{ij}^{kt}\sum_{w\in \boldsymbol{W}} z_{ij}^{ktw})\Big]\end{aligned} \tag{6}$$

subject to (1), (3)-(5) and

$$\sum_{i\in \boldsymbol{N}} n_i^0 \le N_V \tag{7}$$

$$n_i^0 \le Q_i, \qquad \forall i\in \boldsymbol{N} \tag{8}$$

$$\begin{aligned} n_i^t = n_i^0 - \sum_{\tau=1}^{t}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\Big(\sum_{g\in \boldsymbol{G}_{ij}^{t}} d_{ij}^{\tau g} + \sum_{w\in \boldsymbol{W}}\tilde{z}_{ij}^{\tau w}\Big) \\ +\sum_{\tau=1}^{t}\sum_{j\in \boldsymbol{N}\setminus\{i\}, g\in \boldsymbol{G}_{ij}^{\tau}:\Delta(\tau-1)+\varepsilon_{ji}^{\tau g}\le \Delta t} d_{ji}^{\tau g} \\ +\sum_{\tau=1}^{t}\sum_{j\in \boldsymbol{N}\setminus\{i\}:\Delta(\tau-1)+\gamma_{ji}^{\tau}\le \Delta t}\sum_{w\in \boldsymbol{W}}\tilde{z}_{ji}^{\tau w}, \\ \forall i\in \boldsymbol{N}, t\in \boldsymbol{T}\end{aligned} \tag{9}$$

$$\sum_{j\in \boldsymbol{N}\setminus\{i\}} \Big(\sum_{g\in \boldsymbol{G}_{ij}^{t}} d_{ij}^{tg} + \sum_{w\in \boldsymbol{W}} \tilde{z}_{ij}^{tw}\Big) \le n_i^{t-1}, \qquad \forall i\in \boldsymbol{N}, t\in \boldsymbol{T} \tag{10}$$

$$n_i^{t-1} - \sum_{j\in \boldsymbol{N}\setminus\{i\}} \Big(\sum_{g\in \boldsymbol{G}_{ij}^{t}} d_{ij}^{tg} + \sum_{w\in \boldsymbol{W}} \tilde{z}_{ij}^{tw}\Big) \le Q_i, \quad \forall i\in \boldsymbol{N}, t\in \boldsymbol{T}\setminus\{1\} \tag{11}$$

$$\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{w\in \boldsymbol{W}} z_{oi}^{tw} \le N_{per} \tag{12}$$

$$\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}} z_{oi}^{tw} \le 1, \qquad \forall w\in \boldsymbol{W} \tag{13}$$

$$\sum_{i\in \boldsymbol{N}\setminus\{j\}:\gamma_{ij}^{1}=\Delta} \tilde{z}_{ij}^{1w} = \sum_{k\in \boldsymbol{K}}\sum_{i\in \boldsymbol{N}\setminus\{j\}} z_{ji}^{k1w} + \tilde{z}_{j*}^{1w} + z_{jj}^{1w}, \qquad \forall j\in \boldsymbol{N}, w\in \boldsymbol{W} \tag{14}$$

$$\sum_{\tau=1}^{t}\sum_{i\in \boldsymbol{N}\setminus\{j\}:\gamma_{ij}^{\tau}=\Delta(t-\tau+1)} \tilde{z}_{ij}^{\tau w} + z_{jj}^{(t-1)w} + \sum_{k\in \boldsymbol{K}}\sum_{\tau=1}^{t-1}\sum_{i\in \boldsymbol{N}\setminus\{j\}:\xi_{ij}^{k\tau}=\Delta(t-\tau)} z_{ij}^{k\tau w} = \sum_{k\in \boldsymbol{K}}\sum_{i\in \boldsymbol{N}\setminus\{j\}} z_{ji}^{ktw} + \tilde{z}_{j*}^{tw} + z_{jj}^{tw}, \quad \forall j\in \boldsymbol{N}, t\in \boldsymbol{T}\setminus\{1\}, w\in \boldsymbol{W} \tag{15}$$

$$\sum_{j\in \boldsymbol{N}\setminus\{i\}} \tilde{z}_{ij}^{1w} = z_{oi}^{1w}, \qquad \forall i\in \boldsymbol{N}, t=1, w\in \boldsymbol{W} \tag{16}$$

$$\sum_{j\in \boldsymbol{N}\setminus\{i\}} \tilde{z}_{ij}^{tw} = z_{oi}^{tw} + \tilde{z}_{i*}^{tw}, \qquad \forall i\in \boldsymbol{N}, t\in \boldsymbol{T}\setminus\{1\}, w\in \boldsymbol{W} \tag{17}$$

$$n_i^0, n_i^t, \delta^{tw} \in \mathbb{Z}^+, \qquad \forall i\in \boldsymbol{N}, t\in \boldsymbol{T}, w\in \boldsymbol{W} \tag{18}$$

$$z_{oi}^{tw}, \tilde{z}_{ij}^{tw}, z_{ij}^{ktw}, \tilde{z}_{i*}^{tw}, z_{ii}^{tw}, \eta^{tw} \in \{0,1\}, \quad \forall i\ne j\in \boldsymbol{N}, t\in \boldsymbol{T}, k\in \boldsymbol{K}, w\in \boldsymbol{W} \tag{19}$$

where $\mathbb{Z}^+$ denotes the set of non-negative integers.

The objective function shown by (6) is the daily cost of the CSS operator. The cost includes the daily cost of vehicles, the payment of dispatchers from the origin hub to conduct relocation tasks, the relocation cost of vehicles, and the movement cost of dispatchers across stations by other transport modes. In sequence, the terms of (6) represent the fixed cost of vehicles, payment for employed dispatchers, the relocation cost, and the movement cost.

Constraints (7)-(11) are the constraints imposed for the initial vehicle deployment and relocation task of vehicles and dispatchers. Constraint (7) sets the upper bound of the vehicle fleet size. It is noted that the value of the maximum fleet size of vehicles N_v is determined by external factors, i.e., the upper bound of the investment budget of CSS operators. The limitation on the maximum number of vehicles contributes to a more realistic solution; that is, the solution obtained will not be unaffordable for CSS operators. Note that the value N_v will be set as a sufficiently large positive integer value for the scenarios without restrictions on maximum fleet size. Constraint (8) ensures that the initial deployment of vehicles should not exceed the available parking spots at stations. Constraint (9) updates the number of available vehicles at station i at the end of time interval t, which indicates that the number of available vehicles is adjusted by the demands of users and relocation by dispatchers drop off and arrive at station i from time interval 1 to t. Constraint (10) guarantees that the available vehicles fulfil the demands of relocation and rental. Constraint (11) ensures enough parking spots for vehicles.

Constraint (12) sets the upper bound of the number of employed dispatchers like (7) on vehicle fleet size. Constraints (13)-(17) are constraints about dispatcher assignments. Constraints (13)-(15) are the flow balance constraints of individual dispatchers. Note that expressions on the left-hand side of the Constraints (14)-(15) express whether dispatcher w is available at station j at the end of time interval. The right-hand sides of them denote the possible following activities of dispatcher w. If dispatcher w is available at the end of time interval $t\in \boldsymbol{T}\setminus\{1\}$, it suggests that he has just completed one of three kinds of possible activities: relocates vehicles from station $i\in \boldsymbol{N}\setminus\{j\}$ to station j and arrives at the end of time interval t, waits/takes breaks at station j from the end of time interval $(t-1)$, and moves by other transport modes from station $i\in \boldsymbol{N}\setminus\{j\}$ to station j and arrives at the end of time interval t. Constraints (16)-(17) denote whether dispatcher w could take relocation tasks from station i to station $j\in \boldsymbol{N}\setminus\{i\}$ at the beginning of time interval t. Note that the available dispatcher w conducting relocation tasks has two possible sources, i.e., coming from the origin hub to station i at the beginning of time interval t; and the other one is arriving at station i at the end of time interval $(t-1)$ and would relocate vehicles from station i at the beginning of time interval t.

Constraints (18) and (19) define the domains of variables n_i^0, n_i^t, δ^{tw}, z_{oi}^{tw}, $\tilde{z}_{ij}^{tw}$, z_{ij}^{ktw}, $\tilde{z}_{i*}^{tw}$, z_{ii}^{tw} and η^{tw}.

B. Operational level vehicle assignment

It is noted that the model we proposed could arrange the assignment of each dispatcher, while the arrangement of each vehicle could not be obtained from the proposed model. Similar to the existing study by Xu, Meng and Liu [20], the details of the activity of each vehicle can be easily figured out with the current model, though the studies focus on the tactical decision problems of fleet size and dispatcher number.

The arrangement of a vehicle during the operation period is a combination of three activities (i.e., being rented, being relocated by a dispatcher, and waiting). We take the following criteria to obtain the consecutive activities of each vehicle.

(i) If a vehicle is waiting at a station, it will be assigned to fulfil the rental demand or be relocated.

(ii) If the former activities of all the vehicles are "waiting," then assign the vehicle with a longer consecutive waiting time to conduct relocation or rental tasks.

(iii) If the former consecutive parking time for all available vehicles at a station is equal, then the vehicle with less accumulative driving time will be assigned. Suppose several tasks (i.e., relocation tasks and rentals) originate from the same station at the beginning of the same time interval. In that case, the vehicle with less

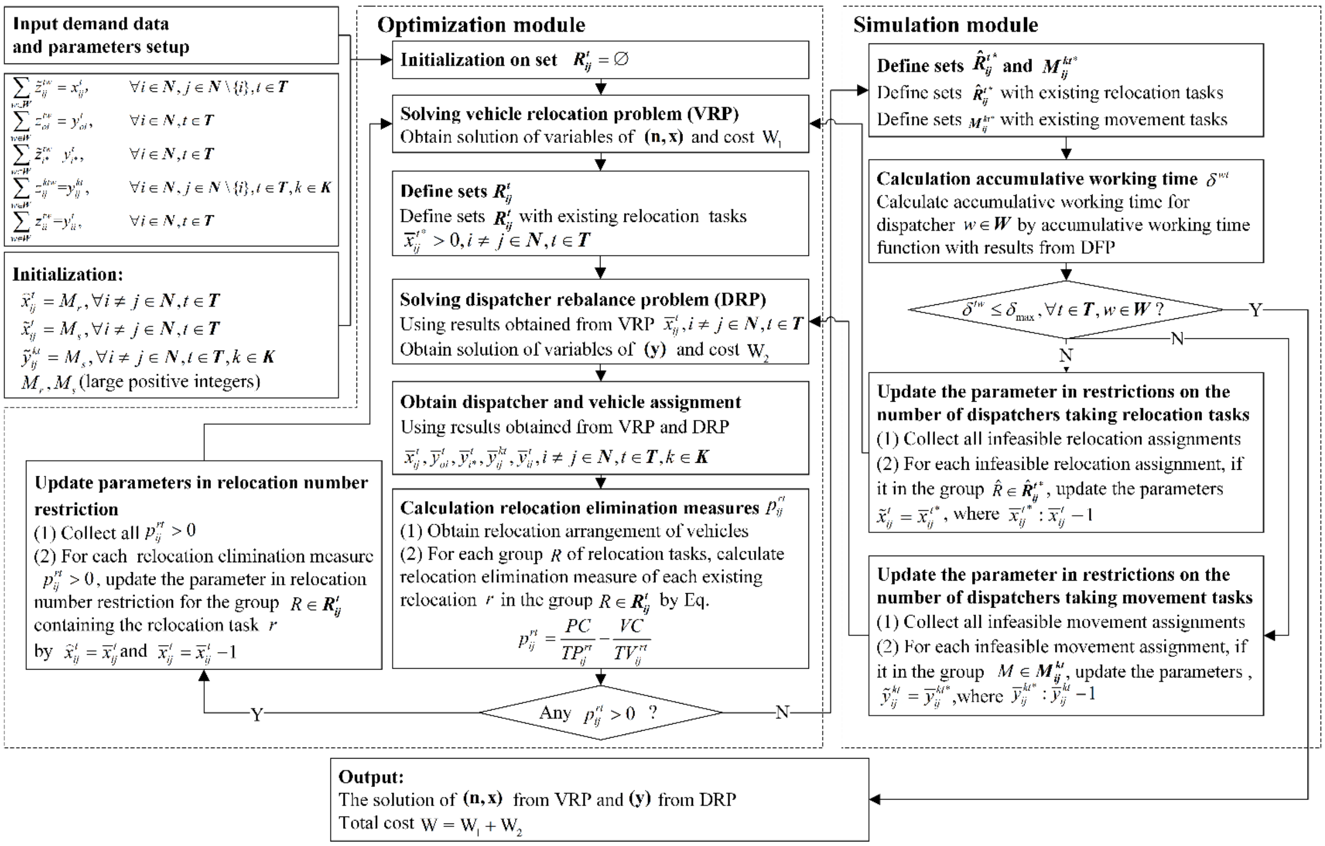


Fig. 2. Flowchart of the optimization-simulation algorithm.

accumulative time will be assigned to conduct the task.

(iv) In the case of a tie, assign the vehicle randomly.

IV. Optimization-simulation algorithm

The consideration of break requirement makes the model of the DN problem nonlinear with the consideration of vehicle relocation and individual dispatcher movement. Such a complex model is a challenge to solve directly with state-of-the-art solvers for large-scale situations in acceptance time.

A heuristic solution algorithm consisting of two modules is thus proposed in the section: (1) optimization module, and (2) simulation module. The former one is further decomposed into two sub-problems considering the structure of the model in this module. The restrictions on the break requirement of dispatchers will be disregarded in the optimization module, which can be addressed indirectly by the following simulation module. The solution obtained from the optimization module is further tested by the simulation module to verify the feasibility based on the dispatcher's break requirements. After an iteration of the optimization, if the simulation result shows that the solution is feasible, the process of solving will stop. Otherwise, constraints on the number of dispatchers who can conduct tasks (i.e., relocation or movement tasks) will be updated. The optimization models will be solved again with new constraints. The flowchart of the proposed optimization-simulation algorithm is illustrated in Fig. 2. To facilitate the description of the proposed algorithm, intermediate variables in the algorithm are summarized in Table II.

A. Optimization module

Although the break requirement is disregarded in the optimization module, the complex model in the module still fails to solve real-life instances within an acceptable time. The model in the optimization module is clear jointly considering both vehicle-related and dispatcher-related variables, in which two groups of decision variables with only a few coupling constraints. Hence, similar to the decomposition method in [5], the optimization module is further decomposed into two sub-problems, denoted by vehicle relocation problem (VRP), and dispatchers rebalance problem (DRP), respectively. The VRP aims to minimize vehicle-related costs without the consideration of the dispatchers' rebalance, and the DRP aims to minimize dispatcher-related costs using the results of the VRP. Individual dispatcher arrangements are obtained with the results of the former two problems. The solution to the original optimization model (without break requirement) is obtained by iteratively solving two sub-problems. New constraints on the number of relocations, named "relocation number constraints" in the VRP, will be updated to the VRP for further iteration until obtaining a good quality solution in the optimization module.

Considering the constraints of binary variables, to simplify the solving progress, we introduce aggregated variables to denote the number of dispatchers, as summarized in Table II. Specifically, let y_{oi}^{t} denote the number of dispatchers from the origin hub to station i to conduct the first relocation tasks at the beginning of time interval t. We further introduce x_{ij}^{t} to denote the number of dispatchers who conduct relocation tasks from station i to station $j \neq i$ at the beginning of time interval t, and y_{i*}^{t}, y_{ij}^{kt}, y_{ii}^{t} to express numbers of dispatchers who are at station i at the end of time interval t and will conduct relocation tasks, move to the station $j \neq i$ by transport mode k, and wait/break at station i for one time interval, respectively.

TABLE II
DESCRIPTION OF INDUCED NOTATIONS

Notation	Definition
Indices and sets	
r	index for relocation task
R	index for the group of existing relocation tasks from VRP with the same origin, destination, and starting time
$\hat{R}$	index for the group of existing relocation tasks from optimization module with the same origin, destination, and starting time
p_{ij}^{rt}	index for the difference between the average relocation cost of the dispatcher and the vehicle for the relocation r in the group $R \in \boldsymbol{R}_{ij}^{t}$.
k	index for transport mode available to dispatchers for moving between pairs of stations
M	index for the group of existing movement tasks from optimization module with the same origin, destination, and starting time
$\boldsymbol{R}_{ij}^{t}$	set of groups of existing relocation tasks (i.e., $\bar{x}_{ij}^{t} > 0, i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}$) obtained from VRP
$\hat{\boldsymbol{R}}_{ij}^{t}$	set of groups of existing relocation tasks (i.e., $\bar{x}_{ij}^{t} > 0, i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}$) obtained from optimization module
$\boldsymbol{M}_{ij}^{kt*}$	set of groups of existing movement tasks obtained from optimization module
Induced number variables	
x_{ij}^{t}	number of dispatchers who conduct relocation tasks from station i to station $j \neq i$ at the beginning of time interval t
y_{i*}^{t}	number of dispatchers who are at station i at the end of time interval t, and will conduct relocation tasks from station i to station $j \neq i$
y_{ij}^{kt}	number of dispatchers who are at station i at the end of time interval t, and will move to station $j \neq i$ by transport mode k
y_{ii}^{t}	number of dispatchers who are at station i at the end of time interval t, and will wait/break at station i for one time interval
$\hat{x}_{ij}^{t}$	The upper bound of possible relocation number originating from station i to station j at the beginning of time interval t, updated after each iteration in the optimization module
$\bar{x}_{ij}^{t}$	The known number of dispatchers who conduct relocation tasks from station i to station $j \neq i$ at the beginning of time interval t, obtained from VRP
$\tilde{x}_{ij}^{t}$	Upper bound of possible relocation number originating from station i to station j at the beginning of time interval t, updated after simulation module
$\tilde{y}_{ij}^{kt}$	Upper bound of possible movement number from station i and move to station $j \neq i$ by transport mode k at the end of time interval t, updated after simulation module

The relationship between binary variables of dispatcher $w \in \boldsymbol{W}$ (i.e., $\tilde{z}_{ij}^{tw}$, z_{oi}^{tw}, $\tilde{z}_{i*}^{tw}$, z_{ij}^{ktw}, and z_{ii}^{tw}) and integer variables denoting the number of dispatchers (i.e., x_{ij}^{t}, y_{oi}^{t}, y_{i*}^{t}, y_{ij}^{kt} and y_{ii}^{t}) can be indicated by following constraints.

$$\sum_{w \in \boldsymbol{W}} \tilde{z}_{ij}^{tw} = x_{ij}^{t}, \qquad \forall i \in \boldsymbol{N}, j \in \boldsymbol{N} \setminus \{i\}, t \in \boldsymbol{T} \tag{20}$$

$$\sum_{w \in \boldsymbol{W}} z_{oi}^{tw} = y_{oi}^{t}, \qquad \forall i \in \boldsymbol{N}, t \in \boldsymbol{T} \tag{21}$$

$$\sum_{w \in \boldsymbol{W}} \tilde{z}_{i*}^{tw} = y_{i*}^{t}, \qquad \forall i \in \boldsymbol{N}, t \in \boldsymbol{T} \tag{22}$$

$$\sum_{w \in \boldsymbol{W}} z_{ij}^{ktw} = y_{ij}^{kt}, \qquad \forall i \in \boldsymbol{N}, j \in \boldsymbol{N} \setminus \{i\}, t \in \boldsymbol{T}, k \in \boldsymbol{K} \tag{23}$$

$$\sum_{w \in \boldsymbol{W}} z_{ii}^{tw} = y_{ii}^{t}, \qquad \forall i \in \boldsymbol{N}, t \in \boldsymbol{T} \tag{24}$$

With the introduced integer variable x_{ij}^{t}, the VRP can be formulated as follows:

[VRP]

$$\min_{\mathbf{n},\mathbf{x}} COST(\mathbf{n},\mathbf{x}) = VC \sum_{i \in \boldsymbol{N}} n_i^0 + \sum_{t=1}^{T} \sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{N} \setminus \{i\}} RC_{ij}^{t} x_{ij}^{t} \tag{25}$$

subject to

$$\sum_{i \in \boldsymbol{N}} n_i^0 \leq N_V \tag{26}$$

$$n_i^0 \leq Q_i, \qquad \forall i \in \boldsymbol{N} \tag{27}$$

$$\begin{aligned} n_i^t = n_i^0 &- \sum_{\tau=1}^{t} \sum_{j \in \boldsymbol{N} \setminus \{i\}} \Big(\sum_{g \in \boldsymbol{G}_{ij}^{\tau}} d_{ij}^{\tau g} + x_{ij}^{\tau} \Big) \\ &+ \sum_{\tau=1}^{t} \sum_{j \in \boldsymbol{N} \setminus \{i\}, g \in \boldsymbol{G}_{ij}^{\tau} : \Delta(\tau-1) + \varepsilon_{ji}^{\tau g} \leq \Delta t} d_{ji}^{\tau g} \\ &+ \sum_{\tau=1}^{t} \sum_{j \in \boldsymbol{N} \setminus \{i\} : \Delta(\tau-1) + \gamma_{ji}^{\tau} \leq \Delta t} x_{ji}^{\tau}, \qquad \forall i \in \boldsymbol{N}, t \in \boldsymbol{T} \end{aligned} \tag{28}$$

$$\sum_{j \in \boldsymbol{N} \setminus \{i\}} \Big(\sum_{g \in \boldsymbol{G}_{ij}^{t}} d_{ij}^{tg} + x_{ij}^{t} \Big) \leq n_i^{t-1}, \qquad \forall i \in \boldsymbol{N}, t \in \boldsymbol{T} \tag{29}$$

$$n_i^{t-1} - \sum_{j \in \boldsymbol{N} \setminus \{i\}} \Big(\sum_{g \in G_{ij}^{t}} d_{ij}^{tg} + x_{ij}^{t} \Big) \leq Q_i, \quad \forall i \in \boldsymbol{N}, t \in \boldsymbol{T} \setminus \{1\} \tag{30}$$

$$x_{ij}^{t} \leq \hat{x}_{ij}^{t} - 1, \qquad \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T} \tag{31}$$

$$x_{ij}^{t} \leq \tilde{x}_{ij}^{t} - 1, \qquad \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T} \tag{32}$$

$$n_i^0, n_i^t, x_{ij}^{t} \in \mathbb{Z}^{+}, \qquad \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T} \tag{33}$$

where (26)-(30) and (33) are similar to (7)-(11) and (18), respectively. The objective function of VRP (25) only contains the two vehicle-related terms in the original objective function (6), i.e., the cost of fleet size and relocation cost. Constraint (31) is the relocation number restriction, where

$\widehat{x}_{ij}^{t}, \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}$ limits the number of relocation tasks originating from station i to station j at the beginning of time interval t. The value of $\widehat{x}_{ij}^{t}$ is updated after each iteration in the optimization module. Note that before the first iteration in the optimization module, the value of all $\widehat{x}_{ij}^{t}, \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}$ are initialized as M_r, a big positive integer, which demonstrates that (31) is invalid until the iteration in the optimization module starts. Constraints (32) are used to restrict the number of dispatchers conducting relocation tasks. As mentioned before, we will check the feasibility of the solution in the aspect of the break requirement by the simulation module. The output of the simulation module is used to update the value of $\tilde{x}_{ij}^{t}$. More detailed information will be shown in Subsection IV.B.

With integer variables y_{oi}^{t}, y_{i*}^{t}, y_{ij}^{kt}, and y_{ii}^{t}, the DRP can be formulated as follows:

[DRP]

$$\min_{\mathbf{y}} COST(\mathbf{y}) = PC\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}} y_{oi}^{t} + \sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\sum_{k\in \boldsymbol{K}} MC_{ij}^{kt} y_{ij}^{kt} \tag{34}$$

subject to

$$\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}} y_{oi}^{t} \leq N_{per} \tag{35}$$

$$\sum_{i\in \boldsymbol{N}\setminus\{j\}:\gamma_{ij}^{1}=\Delta} \overline{x}_{ij}^{1} = \sum_{k\in \boldsymbol{K}}\sum_{i\in \boldsymbol{N}\setminus\{j\}} y_{ji}^{k1} + y_{j*}^{1} + y_{jj}^{1}, \qquad \forall j \in \boldsymbol{N} \tag{36}$$

$$\sum_{\tau=1}^{t}\sum_{i\in \boldsymbol{N}\setminus\{j\}:\gamma_{ij}^{\tau}=\Delta(t-\tau+1)} \overline{x}_{ij}^{\tau} + y_{jj}^{t-1} + \sum_{k\in \boldsymbol{K}}\sum_{\tau=1}^{t-1}\sum_{i\in \boldsymbol{N}\setminus\{j\}:\xi_{ij}^{k\tau}=\Delta(t-\tau)} y_{ij}^{k\tau} = \sum_{k\in \boldsymbol{K}}\sum_{i\in \boldsymbol{N}\setminus\{j\}} y_{ji}^{kt} + y_{j*}^{t} + y_{jj}^{t}, \qquad \forall j \in \boldsymbol{N}, t \in \boldsymbol{T}\setminus\{1\} \tag{37}$$

$$\sum_{j\in \boldsymbol{N}\setminus\{i\}} \overline{x}_{ij}^{1} = y_{oi}^{1}, \qquad \forall i \in \boldsymbol{N}, t=1 \tag{38}$$

$$\sum_{j\in \boldsymbol{N}\setminus\{i\}} \overline{x}_{ij}^{t} = y_{oi}^{t} + y_{i*}^{t-1}, \qquad \forall i \in \boldsymbol{N}, t \in \boldsymbol{T}\setminus\{1\} \tag{39}$$

$$y_{ij}^{kt} \leq \tilde{y}_{ij}^{kt} - 1, \qquad \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}, k \in \boldsymbol{K} \tag{40}$$

$$y_{oi}^{t}, y_{i*}^{t}, y_{ij}^{kt}, y_{ii}^{t} \in \mathbb{Z}^{+}, \qquad \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}, k \in \boldsymbol{K} \tag{41}$$

where (35)-(39) and (41) are similar to (12)-(17) and (19), respectively, based on (21)-(24), except that variables x_{ij}^{t} are replaced with known values $\overline{x}_{ij}^{t}$, which are obtained from the VRP. Notably, (12) and (13) are both similar to (35) based on (21). The objective function of DRP (34) only contains two kinds of variables of the original objective function in Subsection III.A, concerned with the payment of dispatchers and movement cost. Constraints (40) are used to restrict the number of dispatchers conducting movement tasks. The output of the simulation module is used to update the value of $\tilde{y}_{ij}^{kt}$, which could be obtained as $\tilde{x}_{ij}^{t}$ in VRP. A more detailed introduction will be shown in Subsection IV.B.

For the dispatcher flow problem (DFP), given the solution of variables of x_{ij}^{t}, y_{oi}^{t}, y_{i*}^{t}, y_{ij}^{kt}, and y_{ii}^{t} from the VRP and DRP, the operation-level arrangement of individual dispatchers can be easily obtained with the following rules:

(i) A dispatcher waiting at a station will be assigned to conduct relocation or movement tasks.

(ii) The dispatcher with a longer consecutive waiting time will be assigned to conduct relocation or movement task.

(iii) If the former consecutive waiting time for all available dispatchers at a station is equal, then the dispatcher with less accumulative working time will be assigned.

(iv) In the case of a tie, assign the vehicle randomly.

The decomposition method can be explained in the following steps. First, after solving the VRP, we define the sets $\boldsymbol{R}_{ij}^{t}$ and $\boldsymbol{P}$. The set $\boldsymbol{R}_{ij}^{t}$ denotes the groups of existing relocation tasks (i.e., $\overline{x}_{ij}^{t} > 0, i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}$) obtained from VRP, originating from station i to station j at the beginning of time interval t. The numbers of total relocation tasks in the group $R \in \boldsymbol{R}_{ij}^{t}$ are $\overline{x}_{ij}^{t}$, where R denotes a group of relocation tasks with the same origin, destination, and starting time. Let $\{1, 2, \cdots, \overline{x}_{ij}^{t}\}$ be the set of relocation tasks and r be the indices for a single relocation task in the group $R \in \boldsymbol{R}_{ij}^{t}$. The set $\boldsymbol{P}$ is composed of the relocation elimination measures $p_{ij}^{rt} \in \boldsymbol{P}$, which estimate the cost of relocation task $r \in \{1, 2, \cdots, \overline{x}_{ij}^{t}\}$ in the group $R \in \boldsymbol{R}_{ij}^{t}$ imposing on the CSS. Hence, the purpose of the proposed decomposition algorithm is to find the relocations with a high cost p_{ij}^{rt} in the group $R \in \boldsymbol{R}_{ij}^{t}$ and remove them from the VRP. If $p_{ij}^{rt} \geq 0$, conduct the relocation task r in the group $R \in \boldsymbol{R}_{ij}^{t}$ would impose a higher cost on the CSS than relocation elimination. Thus, the relocation r will be eliminated from group $R \in \boldsymbol{R}_{ij}^{t}$ by updating the value of $\widehat{x}_{ij}^{t}$, and corresponding constraints in the VRP.

In this section, the values p_{ij}^{rt} are defined as the difference between the average relocation cost of the dispatcher and the vehicle for the relocation r in the group $R \in \boldsymbol{R}_{ij}^{t}$. The relocation assignments of each employed dispatcher could be obtained from consecutive execution of the VRP, DRP, and DFP. The relocation assignments of each vehicle could be obtained from the operational-level vehicle arrangement in Subsection III.B. We introduce TP_{ij}^{rt} and TV_{ij}^{rt} to denote the total relocation time performed by the dispatcher and vehicle, which is assigned to conduct relocation task r in the group $R \in \boldsymbol{R_{ij}^t}$, respectively.

$$p_{ij}^{rt} = \frac{PC}{TP_{ij}^{rt}} - \frac{VC}{TV_{ij}^{rt}} \tag{42}$$

The definition of p_{ij}^{rt} can be explained as follows:

(i) If $PC \gg VC$, the value of $p_{ij}^{rt} \geq 0$, which illustrates that the cost of employing a dispatcher is much higher than that of a vehicle. Thus, it is accepted that the VRP should increase the fleet size and eliminate the relocation in the following iteration.

(ii) If TP_{ij}^{rt} is small, the dispatcher who conducts the relocation has fewer other relocation tasks and corresponding movement tasks during operation periods. Thus, it is accepted that the relocation task should be eliminated so that the dispatcher who conducts the relocation task will be eliminated in the future to reduce the total cost in DRP.

(iii) A large TV_{ij}^{rt} implies that the vehicle conducting the relocation task r takes too much time on relocation, which demonstrates fewer rental demands fulfilled by the vehicle.

For all relocation groups, after calculating p_{ij}^{rt} of all relocation tasks in the group $R \in \boldsymbol{R_{ij}^t}$, if there exists relocation task r such that $p_{ij}^{rt} > 0$, the value of $\widehat{x}_{ij}^t$ will be updated by $\widehat{x}_{ij}^t = \overline{x}_{ij}^t$, where $\overline{x}_{ij}^t : \overline{x}_{ij}^t - 1$. Then constraints on the number of relocation tasks from station i to station j at the beginning of the time interval t in VRP will be updated.

The steps of the decomposition method in the optimization module are presented as follows:

Step 1: Initialize by setting $\boldsymbol{R}_{ij}^t = \varnothing$.

Step 2: Solve the vehicle relocation problem (VRP) and define sets $\boldsymbol{R}_{ij}^t$ with the existing relocation tasks.

Step 3: Solve the dispatcher rebalance problem (DRP) and obtain relocation arrangements for dispatchers and vehicles.

Step 4: For all groups of existing relocation tasks, calculate p_{ij}^{rt} of existing relocation task r in the group $R \in \boldsymbol{R}_{ij}^t$ using (42).

Step 5: Judge p_{ij}^{rt}. If all $p_{ij}^{rt} \leq 0, p_{ij}^{rt} \in \boldsymbol{P}$, terminate and final solution is obtained. Otherwise, if any $p_{ij}^{rt} > 0$, update $\widehat{x}_{ij}^t = \overline{x}_{ij}^t$, where $\overline{x}_{ij}^t : \overline{x}_{ij}^t - 1$. After judging all $p_{ij}^{rt} \in \boldsymbol{P}$, return to Step 2.

B. Simulation module

For the model without consideration of working time and compulsory resting, the solutions obtained by the optimization module are always feasible, which is, however, not the case in our study. The accumulative working time of employed dispatcher w must be constrained to satisfy the break requirement (i.e. (1) in Subsection II.B), which is disregarded in the optimization module. For this purpose, a simulation module is used to check the feasibility of the solution generated by the optimization module based on the constraints on break requirements. If the solution is feasible, that is, accumulative working time of each dispatcher during the whole operation period fulfills the break requirement constraint (1), the results from the optimization module are accepted. Otherwise, the simulation module returns updated constraints based on infeasible assignments. Specifically, if the infeasible arrangement is a relocation task, the generated constraint will be updated in the vehicle relocation problem (i.e., updating (32)); otherwise, (40) in the dispatcher rebalance problem will be updated.

Note that before the first iteration in the simulation module, the value of all $\tilde{x}_{ij}^t, \tilde{y}_{ij}^{kt}, \forall i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}, k \in \boldsymbol{K}$ are initialized as M_s, a big positive integer, which demonstrates that (32) and (40) are not valid until the iteration in the simulation module starts. The steps of the simulation module can be explained as follows. Firstly, Let $\boldsymbol{\hat{R}_{ij}^{t*}}$ and $\boldsymbol{M_{ij}^{kt*}}$ denote the groups of existing relocation tasks (i.e., $\overline{x}_{ij}^t > 0, i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}$) and movement tasks (i.e., $\overline{y}_{ij}^{kt} > 0, i \neq j \in \boldsymbol{N}, t \in \boldsymbol{T}, k \in \boldsymbol{K}$), respectively, obtained from the optimization module. Then the unfeasible assignment of each dispatcher is found and collected by simulation, if any.

The accumulative working time of dispatcher $w \in \boldsymbol{W}$ at the end of time interval $t \in \boldsymbol{T}$ can be obtained by (3)-(5) with the dispatcher assignment solution. For every dispatcher $w \in \boldsymbol{W}$, if $\delta^{tw} \leq \delta_{\max}, \forall t \in \boldsymbol{T}$, that is, the results obtained from the optimization module are feasible, the simulation ends. Otherwise, the simulation module generates the infeasible assignment, including infeasible relocation tasks or movement tasks of dispatcher w, which is defined as the assignment after taking which the dispatcher w with the accumulative working time of no more than $\delta_{\max}$ at the end of time interval t (i.e., $\delta^{tw} \leq \delta_{\max}$) will exceed the maximum accumulative time. Updated constraints are generated based on the infeasible assignment. Specifically, if the infeasible assignment is in the group $\hat{R} \in \boldsymbol{\hat{R}_{ij}^{t*}}$, we update the value of $\tilde{x}_{ij}^t$ by $\tilde{x}_{ij}^t = \overline{x}_{ij}^{t*}$, where $\overline{x}_{ij}^{t*} : \overline{x}_{ij}^t - 1$. If the infeasible assignment is in the group $M \in \boldsymbol{M_{ij}^{kt*}}$, then we update the value of $\tilde{y}_{ij}^{kt}$ by $\tilde{y}_{ij}^{kt} = \overline{y}_{ij}^{kt*}$, where $\overline{y}_{ij}^{kt*} : \overline{y}_{ij}^{kt} - 1$. After the simulation of each dispatcher $w \in \boldsymbol{W}$, if infeasible arrangements exist, (32) and (40) will be updated, and the optimization module will be solved again.

V. NUMERICAL EXPERIMENTS

The section presents the numerical experiments to validate the performance of the proposed model and the optimization-simulation algorithm. The proposed algorithm is coded in C++, calling Gurobi 9.0.0 by a personal computer with 3.0 GHz and 16 GB RAM. In Subsection V.A, a CSS company, EVCARD in Suzhou, China, used to generate random demand data, and parameters setup will be introduced. In Subsection V.B, computational performance of the proposed algorithm will be assessed by comparison with the linearization method and genetic algorithm. Finally, sensitivity analysis will be conducted to explore the impacts of the break requirement consideration, payment for dispatchers, and relocation cost on the system performance of one-way CSS in Subsection V.C.

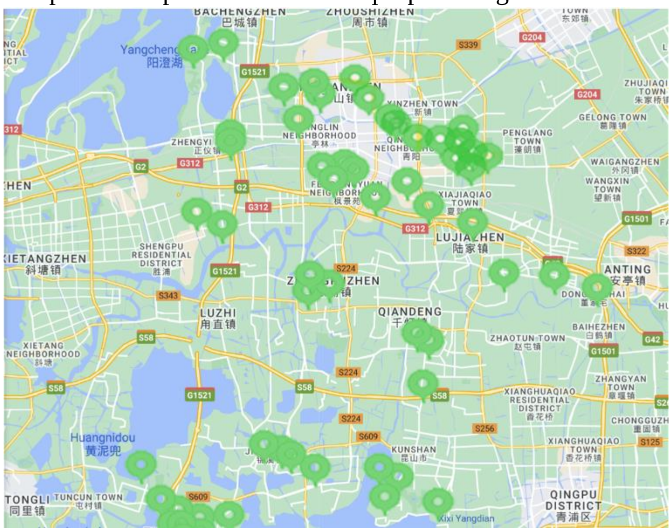


Fig. 3. Stations deployment in Kunshan, Suzhou.

TABLE III
THE SETTING OF TRAVEL TIME AND COST

	Travel Time	Cost
Vehicle relocation	$\gamma_{ij}^{t}=t_{ij}$	$RC_{ij}^{t}=0.6\gamma_{ij}^{t}$
Dispatcher movement by electric bikes	$\xi_{ij}^{1t}=1.5t_{ij}$	$MC_{ij}^{1t}=0.12\xi_{ij}^{1t}$
Dispatcher movement by regular bikes	$\xi_{ij}^{2t}=2t_{ij}$	$MC_{ij}^{2t}=0.06\xi_{ij}^{2t}$

Note that t_{ij} denotes the shortest travel time from station i to station j

A. EVCARD in China and parameter setup

EVCARD, a popular one-way carsharing operator in China is mainly engaged in vehicle time-sharing rental. By the end of 2023, the company will provide one-way carsharing service in 28 major cities in China, with over 30,000 vehicles. The users can rent vehicles through a smartphone application. The stations used in the study are located in a district of Suzhou named Kunshan. The distribution of the 70 stations in Kunshan is shown in Fig. 3. If the shortest travel time between nearby stations is less than 5 minutes, these stations will be merged into one based on the assumption that there would be no rentals between them. The travel time data are obtained by Google Maps [26] with the model of 'driving' without traffic congestion. After combining processing, 23 stations are obtained. Let $\{1,2,\cdots,23\}$ be the set of stations in numerical experiments. The origin and destination stations (i.e., $i,j\in\boldsymbol{N}$) of rentals are generated randomly from these 23 stations.

The operation period is assumed to be from 8:00 to 18:00, which is considered the period when the most rentals generate, with 15 minutes for each interval, i.e. Δ=15minutes. The total number of time intervals during the operation period is 40, that is, the set of time intervals $\boldsymbol{T}=\{1,2,\cdots,40\}$. Specifically, 8:00 (i.e., the beginning of time interval 1) is considered the time benchmark. The department time of each rental is randomly generated from the beginning of time interval $\{1,2,\cdots,40\}$. The rental duration of each rental demand at the beginning of time interval $t\in\boldsymbol{T}$ is chosen as a random integer from the set $\{\Delta T_{min},\Delta T_{min}+15,...,\Delta T_{max}\}$, where the ΔT_{min} and ΔT_{max} are the minimum and maximum rental duration, respectively. We assume that rented vehicles will be returned to a station distinct from the pick-up station on the same working day. The ΔT_{min} is set to be the shortest travel time between the origin station and the destination station of a rental demand, obtained by Google Maps [26]. ΔT_{max} is set to be 600 min with an average speed of 35 km/h, that is, the maximum rental duration equals the operation duration. The values of d_{ij}^{tg}, $g\in\boldsymbol{G}_{ij}^{t}$ in following subsections are equal to the number of those randomly generated rental demands, who pick up vehicles from station i at the beginning of time interval t and return vehicles to station j with the same duration.

The relocation time between each O-D pair is assumed to be the shortest travel time between them and is constant across the operation period. The movement time of dispatchers depends on the transport modes chosen. There are two optional modes for dispatchers in this study: (1) using electric bikes and (2) using regular bikes. For simplicity, we suppose that the travel time of dispatchers by regular and electric bikes is double and 1.5 times the relocation time of vehicles, respectively.

The daily fixed cost of a vehicle (i.e., VC) is estimated by assuming a total payment cost of ¥ 260,000 (including vehicle-purchase and other insurance costs) from the network of EVCARD [27] with an amortization rate of 3% per month, which leads to the daily fixed cost $VC=260$ ¥/veh-day. The relocation cost is set to be 0.6 ¥/min considering the battery wear and charge costs. Without loss of generality, we further assume that the daily payment for a dispatcher is set to be 100 ¥/per-day, i.e., $PC=100$ ¥/per-day. The dispatcher movement cost depends on the chosen transport mode. The average movement cost is set to be 0.12 and 0.06 ¥/min for the people by electric bike and regular bike, respectively. The travel time and cost are summarized in Table III. The basic maximum accumulative working time $\delta_{\max}$ and compulsory break time

$\theta_{\min}$ are set to be 150 minutes and 30 minutes, respectively.

B. Performance evaluation of the proposed algorithm

In this section, we compare the proposed method, i.e., solving the model [DN] by the optimization-solution algorithm with the MILP solver Gurobi, with two benchmark methods, a formulation-based method and a well-known metaheuristic genetic algorithm (GA). GA is developed by initially solving the VRP and DRP without the cost minimization objective to improve initial solution quality and solving efficiency.

TABLE IV

COMPARISON OF COMPUTATIONAL PERFORMANCE BETWEEN PROPOSED ALGORITHM AND BENCHMARK METHODS

$\mathbf{N}-\mathrm{T}-\mathrm{D}-\mathrm{Nv}-\mathrm{Np}-\delta_{\max}-\theta_{\min}$	Solving time(s)			Objective function value (¥)			GAP1	GAP2
	Alg	Lin	GA	Obj_Alg	Obj_Lin	Obj_GA		
10-20-100-50-10-120-15	9.6	129.4	144.8	7,830.7	7,699.6	7,862.9	1.70%	0.00%
10-40-200-100-20-150-30	8.8	1,628.8	323.8	9,207.3	9,028.6	9,142.0	2.09%	0.71%
20-40-200-100-20-180-30	24.6	92,911.5	346.5	10,229.1	10,047.1	11,217.1	1.80%	-8.81%
20-40-200-100-20-210-30	22.6	241,487.0	337.8	10,207.7	9,987.6	11,273.5	2.20%	-9.45%
20-40-400-150-40-180-30	23.2	>259,200.0	867.5	17,509.3	27,568.7	18,556.8	-36.21%	-5.64%
20-40-400-150-40-210-45	22.4	>259,200.0	872.6	17,488.6	27,568.7	18,522.8	-36.30%	-5.58%

Considering the nonlinearity of the break requirements, we use the linearization method, which is conducted by introducing a big value, denoted by M . Then, (5) can be replaced by (43)-(44), expressed as follows:

$$0 \le \delta^{tw} \le M\eta, \qquad \forall t \in \boldsymbol{T}, w \in \boldsymbol{W} \tag{43}$$

$$\left(\begin{array}{l} \delta^{(t-1)w} + \Delta \sum_{\tau=1}^{t} \sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{N} \setminus \{i\}} \sum_{\Delta\tau + \gamma_{ij}^{\tau} >= \Delta(t+1)} \tilde{z}_{ij}^{tw} \\ + \Delta \sum_{k \in \boldsymbol{K}} \sum_{\tau=1}^{t-1} \sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{N} \setminus \{i\}} \sum_{\Delta\tau + \xi_{ij}^{kt} >= \Delta t} z_{ij}^{ktw} \end{array} \right) - M(1-\eta) \le \delta^{tw}$$

$$\le \left(\begin{array}{l} \delta^{(t-1)w} + \Delta \sum_{\tau=1}^{t} \sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{N} \setminus \{i\}} \sum_{\Delta\tau + \gamma_{ij}^{\tau} >= \Delta(t+1)} \tilde{z}_{ij}^{tw} \\ + \Delta \sum_{k \in \boldsymbol{K}} \sum_{\tau=1}^{t-1} \sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{N} \setminus \{i\}} \sum_{\Delta\tau + \xi_{ij}^{kt} >= \Delta t} z_{ij}^{ktw} \end{array} \right) + M(1-\eta),$$

$$\forall t \in \boldsymbol{T}, w \in \boldsymbol{W} \tag{44}$$

A set of instances with different numbers of parking stations, time intervals, rental demands, the maximum number of available vehicles and dispatchers, maximum accumulative working time, and compulsory break time are used to test the performance of the proposed algorithm against the linearization method and GA. The maximum computational time limit is set to be 259,200 seconds (i.e., 72 hours) considering solving real-life situations.

To better illustrate the parameter settings of each instance, we use the index of instance scenario in this subsection, i.e., $\boldsymbol{N}-T-D-Nv-Np-\delta_{\max}-\theta_{\min}$, in which these letters denote the total numbers of stations, time intervals, users' demands, maximum fleet size, maximum number of dispatchers, maximum accumulative working time, and compulsory break time, respectively. Under each scenario, 5 instances are randomly generated based on the parameter setting in Subsection VI.A. For example, an instance "10-20-100-50-10-120-15" represents 10 stations, 20 time intervals, 100 randomly generated rentals, 50 maximum available vehicles, 10 maximum available dispatchers, 120-minute maximum accumulative working time, and 15-minute compulsory break time. Notably, instances in the same scenario are generated randomly based on the same stations and time intervals, which are both selected randomly from EVCARD in Subsection V.A.

Given a particular instance index, a group with five instances will be randomly generated. After the solving process, the average solving time and average objective function value are tabulated. The results of preliminary experiments illustrate that the proposed algorithm, the linearization method, and GA could obtain feasible solutions in an hour in small instances (e.g., 10-20-100-50-10-120-15 and 10-40-200-150-20-120-30) in short time. Nevertheless, if the scales of instances become bigger and beyond specific scales, optimal solutions cannot be obtained with the linearization method within the time limitation. For problem instances that are not solved to optimality within 72 h, we will present the incumbent solution from the proposed method or upper bound obtained from GUROBI within 72 h.

Two performance measures, i.e., solving time (in seconds) and objective value, are compared. Let Obj_Alg , Obj_Lin , and Obj_GA be the objective obtained from the proposed algorithm, the linearization method, and the genetic algorithm. The ratio of the gaps, i.e., GAP1=(Obj_Alg-Obj_Lin)/ Obj_Lin ×100%, GAP2=(Obj_Alg-Obj_GA)/ Obj_GA×100% are also reported for more intuitive comparisons. Specifically, GAP=0% indicates that these methods can obtain optimal solutions with the same accuracy; negative GAPs demonstrate that the proposed algorithm can obtain a better solution than other methods. Otherwise, the solutions obtained by the proposed algorithm are less accurate.

The results are tabulated in Table IV. Overall, the results demonstrate that for all scenarios, the proposed algorithm could obtain solutions within 25 seconds. It provides much better solutions within even 259,200 seconds compared to the linearization method in large-size scenarios, i.e., 20-40-400-150-40-150180-30 and 20-40-400-150-40-210-45. This outcome illustrates the superiority of the proposed algorithm when solving large-scale problems. For small-size and middle-size scenarios, the proposed solution can obtain optimal results in significantly less time. The values of GAP1 between the proposed algorithm and the linearization method are no more than 2.20%, with the former taking considerably less time than the latter. As the scenario size increases, the linearization method can no longer obtain the optimal solution within the 259,200-second time limit, while the proposed method continues to deliver better solutions. We then compare the proposed algorithm with GA. For most cases, the proposed algorithm performs better than GA, with lower objective values and less solving time. Based on the observation of the solving time of the proposed algorithm, the solving efficiency appears to be insensitive to the number of demands. For example, the demand has increased from 200 to 400 in the instances with 20-40-200-100-20-180-30 to 20-40-400-150-40-180-30, yet there is a slight change in average solving time. This can be explained by the fact that the optimization modules in the algorithm are affected by station and time interval numbers instead of the number of demands, due to aggregated requests in the origin network. In comparison to solving directly with the linearization method and GA, the computation time of the proposed algorithm is much shorter because it can obtain personnel assignment with the result from VRP and DRP, instead of solving with large-dimension individual dispatcher variables. For instance, with demand larger than 200, the average GAP1 and GAP2 are at least -36.21% and -5.56%, respectively. The solving results reveal the potential of the proposed algorithm to be implemented in a real-world transportation network. We further visualize the variations in solving time of the proposed algorithm consistently decreasing with the increase of $\delta_{\max}$ and the decrease of $\theta_{\min}$ when other parameters remain unchanged. This is consistent with our expectation that a larger $\delta_{\max}$ or a smaller $\theta_{\min}$ indicates less probability for dispatchers to require a compulsory break and thus fewer iterations from simulation to optimization module resulting from overloading work. The proposed algorithm demonstrates clear advantages over the linearization method and GA in solving the [DN] model proposed in the study.

TABLE V

IMPACT OF BREAK REQUIREMENT ON COST AND NUMBER OF DISPATCHERS DISOBEYING BREAK REQUIREMENT

#TotalRental	$\mathrm{Cost}^{\mathrm{br}}$	$\mathrm{Cost}^{\mathrm{nbr}}$	Cost_Gap (%)	$\mathrm{N}^{\mathrm{ot\_br}}$
100	4,670	4,482	4.19	0.3
200	9,967	9,730	2.43	1.1
300	14,789	14,379	2.85	2.1
400	21,532	21,171	1.71	4.3

C. Sensitivity analysis

In this subsection, we proceed to analyze the impact of the break requirement consideration of dispatchers, the payment for dispatchers, and the relocation cost on the CSS performance. We will vary the concerned parameters while keeping the other parameters the same as those introduced in Subsection V.A. To facilitate the performance comparison, we will report average results of the total daily cost (Cost), the dispatcher number (NO. Dis) calculated by $\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{w\in \boldsymbol{W}} z_{oi}^{tw}$, the total number of relocation tasks (NO. Rel) calculated by $\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\sum_{w\in \boldsymbol{W}} \tilde{z}_{ij}^{tw}$, the total number of movement tasks (NO. Move 1 and NO. Move 2) calculated by $\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\sum_{w\in \boldsymbol{W}} z_{ij}^{ktw}, k=1,2$, as well as dispatcher-related indicators, including relocation time per dispatcher (RT) calculated by $\frac{\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\gamma_{ij}^{t}\sum_{w\in \boldsymbol{W}} \tilde{z}_{ij}^{tw}}{\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{w\in \boldsymbol{W}} z_{oi}^{tw}}$, movement time per dispatcher (MT1 and MT2) calculated by $\frac{\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{j\in \boldsymbol{N}\setminus\{i\}}\xi_{ij}^{kt}\sum_{w\in \boldsymbol{W}} z_{ij}^{ktw}}{\sum_{t=1}^{T}\sum_{i\in \boldsymbol{N}}\sum_{w\in \boldsymbol{W}} z_{oi}^{tw}}, k=1,2$, and total movement time per dispatcher (TMT) calculated by $\mathrm{TMT}=\mathrm{MT1}+\mathrm{MT2}$.

1) Impact of break requirement consideration

To explore the influence of break requirement of dispatchers on the operation cost of CSSs, we formulate a model [$\mathrm{DN}^{\mathrm{nbr}}$], which is similar to the model [DN], but without the consideration of the break requirement. For ease of comparison, we rename the model considering the break requirement, i.e., [DN] as [$\mathrm{DN}^{\mathrm{br}}$] in this subsection. The parameters in this subsection are the same as those in Subsection V.A. Ten instances are randomly generated for a particular number of rental demands (#TotalRental), and the averaged results are reported. We further induce ‘ $\mathrm{Cost}^{\mathrm{br}}$ ’ and ‘ $\mathrm{Cost}^{\mathrm{nbr}}$ ’ as the operation costs obtained by solving models [$\mathrm{DN}^{\mathrm{br}}$] and [$\mathrm{DN}^{\mathrm{nbr}}$], respectively. The gap of costs indicated by ‘ Cost_Gap ’, defined as $(\mathrm{Cost}^{\mathrm{br}}-\mathrm{Cost}^{\mathrm{nbr}})/\mathrm{Cost}^{\mathrm{nbr}}$, is adopted to evaluate the impact of break requirement on the cost of CSSs. Let ‘ $\mathrm{N}^{\mathrm{ot\_br}}$ ’ be the number of dispatchers whose accumulative working time in the model [$\mathrm{DN}^{\mathrm{nbr}}$] exceed $\delta_{\max}$ (i.e., disobey break requirement) in Subsection II.B. The results are presented in Table V. It can be observed from Table V that all the gaps are greater than 1%, implying that considering break requirements results in an improvement in the total daily cost without considering driving fatigue-related loss. This can be explained by (1), which renders some cost-effective dispatcher assignments infeasible due to compulsory breaks at stations. Fortunately, the ratios generally exhibit a downward trend to 1.71 % when the number of rentals reaches 400. This may indicate that the effect on the total cost declines more significantly when the demand is higher. This can be attributed to the fact that compared to the sharp increase in investment in dispatchers and relocation in high-demand scenarios, the cost for uneconomical arrangements of dispatchers is relatively smaller. Another notable finding is that the number of dispatchers who violate (1) constantly increases from 0.3 to 4.3 as the number of rentals rises, which is consistent with our expectation that to minimize the daily cost of CSS, dispatchers will take overloading works, and cause, accordingly, driving fatigue without the constraints on maximum accumulative working time. Except for the statistic cost (i.e., the cost included in $\mathrm{DN}^{\mathrm{nbr}}$), $\mathrm{DN}^{\mathrm{br}}$ also avoid the driving-fatigue-related accidents resulting from dispatcher overloading. Thus, to further consider the loss from driving-fatigue-related accidents of dispatchers, we introduce the driving-fatigue-related loss function, i.e., $f_{accident}$, defined as $p_a c_{loss}$, where p_a and c_{loss} are the fatigue-related accident probability and the driving-fatigue-related loss of a single accident, respectively. Let $\mathrm{Cost}^{\mathrm{nbr\_accident}}$ be the sum of $\mathrm{Cost}^{\mathrm{nbr}}$ and $f_{accident}$. The average accident loss c_{loss} is set to be ¥ 6,868 [28]. For the scenario with 400 rentals, it is acceptable that if fatigue-related accident probability exceeds 0.12%, the model considering the break requirement of dispatchers will contribute to lower costs and increased safety. These findings demonstrate the necessity of incorporating the accumulative time into dispatcher number determination for service operators, which contributes to the cost decrease of CSSs with the consideration of driving-fatigue-related loss and safety of operation, and thus highlights the significance of this study.

TABLE VI

EFFECT OF PARAMETERS ABOUT BREAK REQUIREMENT ON THE PERFORMANCE OF ONE-WAY CSS

$\delta_{\max}$ (min)	$\theta_{\min}$ (min)	Cost (¥/day)	NO. Dis	NO. Rel	NO. Move 1	NO. Move 2	RT (min/per)	MT1 (min/per)	MT2 (min/per)	TMT (min/per)
120	15	21,489	13.6	53.2	20.7	16.8	70.4	24.4	49.8	74.2
120	30	21,738	16.2	53.3	18.5	14.8	59.2	17.6	35.3	52.9
150	30	21,610	14.9	53.3	17.6	16.5	64.2	18.5	43.9	62.4
150	45	21,735	15.7	53.4	16.9	13.5	60.7	17.0	33.4	50.4
180	30	21,543	14.2	53.3	19.5	16.5	67.4	21.1	46.3	67.4
180	45	21,685	15.4	53.4	18.7	15.7	61.9	19.3	40.1	59.4
210	45	21,553	14.3	53.3	20.2	15.4	66.9	21.4	44.9	66.3
210	60	21,663	15.4	53.5	19.8	16.0	62.1	20.3	40.9	61.2

We further caution that the influence of break requirement consideration may mainly depend on the parameters in the break requirement. To explore how the parameters about break requirement, i.e., the maximum accumulative working time $\delta_{\max}$ and compulsory break time $\theta_{\min}$ influence the performance of one-way CSS, we vary the value of these parameters and keep the other parameters the same value introduced in Subsection V.A. Specifically, three values of $\delta_{\max} \in \{120,150,180\}$, and three values of $\theta_{\min} \in \{15,30,45\}$, are adopted. Ten instances, each consisting of 400 randomly generated rentals, are used to conduct the experiments. The results are tabulated in Table VI. It can be seen that the dispatcher number, increases as $\theta_{\min}$ rises and $\delta_{\max}$ decreases, assuming that the other parameters stay constant. The above results can be explained that the increasing $\theta_{\min}$ and decreasing $\delta_{\max}$ imply more strict break requirements and less available working time for dispatchers. Thus, more dispatchers are required to conduct relocation tasks. For instance, for a specific $\delta_{\max}$, a larger $\theta_{\min}$ means that the dispatchers with maximum accumulative working time must take breaks at the stations for a longer time, which implies a need for additional dispatchers to handle relocation or movement tasks during the break time. A similar explanation can be provided to the situation with a smaller $\delta_{\max}$ under specific $\theta_{\min}$. Another notable result is that the total cost only experiences a slight increase, ranging from 21,489 to 21,738. Despite significant changes in break requirement parameters, there is a relatively small increase in the total cost, suggesting that there is potential to find an optimal balance between cost efficiency for service operators and the well-being of dispatchers through the appropriate setting of break requirements. By setting optimal break requirements, service operators can ensure the well-being of their dispatchers without incurring excessive costs. The rise of dispatcher numbers, in turn, results in a downward trend in dispatchers' average relocation and movement time with relatively unchanged relocations, which can be attributed to the fact that more dispatchers are available to handle the tasks due to the stricter break requirements. This allows tasks to be distributed more evenly among dispatchers, leading to a reduction in the average time spent on each task.

2) Impact of dispatcher payment

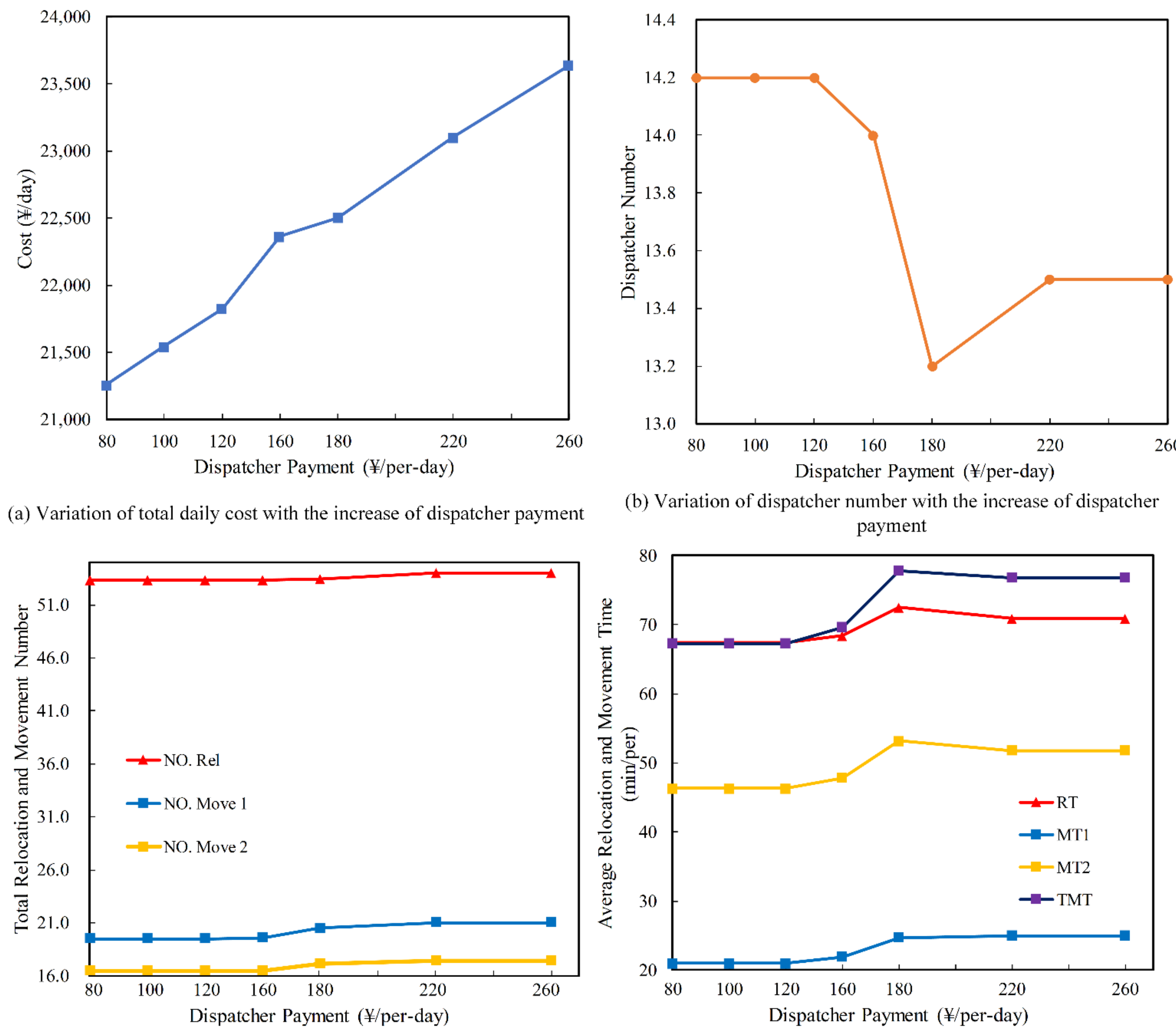
The number of dispatchers, being a major investment for CSS operators, continually poses a significant concern. Thus, this study investigates the impact of dispatcher payment on CSS performance. Unless otherwise specified, the parameters setting remains the same as those in Subsection V.A except for dispatcher payment. Ten instances, each consisting of 400 randomly generated rentals, are used to conduct the experiments.

The results of the proposed model under different values of dispatcher payment are tabulated in Table VII. The variations of total daily cost, number of dispatchers, total tasks number, and average relocation and movement time per dispatcher are visualized in Fig. 4. (a)-(d), respectively. As illustrated in Fig. 4. (a), the increase in daily payment of dispatchers contributes to a noticeable increase in the overall cost of CSSs, highlighting the dominating impact of dispatcher payment on the profitability of CSSs. Another noteworthy observation from Fig. 4. (b) is that the dispatcher number decreases with the fluctuation as dispatcher payment increases, which is attributed to the need for balancing costs. In addition, to minimize the total cost of CSSs as the payment increases, CSS operators might arrange more relocation and movement tasks for the reduced dispatchers as shown in Fig. 4. (c). Numbers of relocations and movements by different transport modes generally increase with the growth in dispatcher payment. Furthermore, Fig. 4. (d) illustrates that dispatcher-related indicators display a significant upward trend, suggesting that dispatchers with higher payments may be assigned more time-consuming tasks. This may be explained by the fact that higher payments will also potentially

TABLE VII

EFFECT OF DISPATCHER PAYMENT ON THE PERFORMANCE OF ONE-WAY CSS

PC (¥/per-day)	Cost (¥/day)	NO. Dis	NO. Rel	NO. Move 1	NO. Move 2	RT (min/per)	MT1 (min/per)	MT2 (min/per)	TMT (min/per)
80	21,259	14.2	53.3	19.5	16.5	67.4	21.0	46.3	67.3
100	21,543	14.2	53.3	19.5	16.5	67.4	21.0	46.3	67.3
120	21,827	14.2	53.3	19.5	16.5	67.4	21.0	46.3	67.3
160	22,365	14.0	53.3	19.6	16.5	68.4	21.9	47.8	69.6
180	22,505	13.2	53.4	20.5	17.1	72.5	24.7	53.2	77.8
220	23,100	13.5	54.0	21.0	17.4	70.9	25.0	51.8	76.8
260	23,640	13.5	54.0	21.0	17.4	70.9	25.0	51.8	76.8



(a) Variation of total daily cost with the increase of dispatcher payment

(b) Variation of dispatcher number with the increase of dispatcher payment

(c) Variation of total relocation number and movement number with the increase of dispatcher payment

(d) Variation of average relocation time and movement time each dispatcher with the increase of dispatcher payment

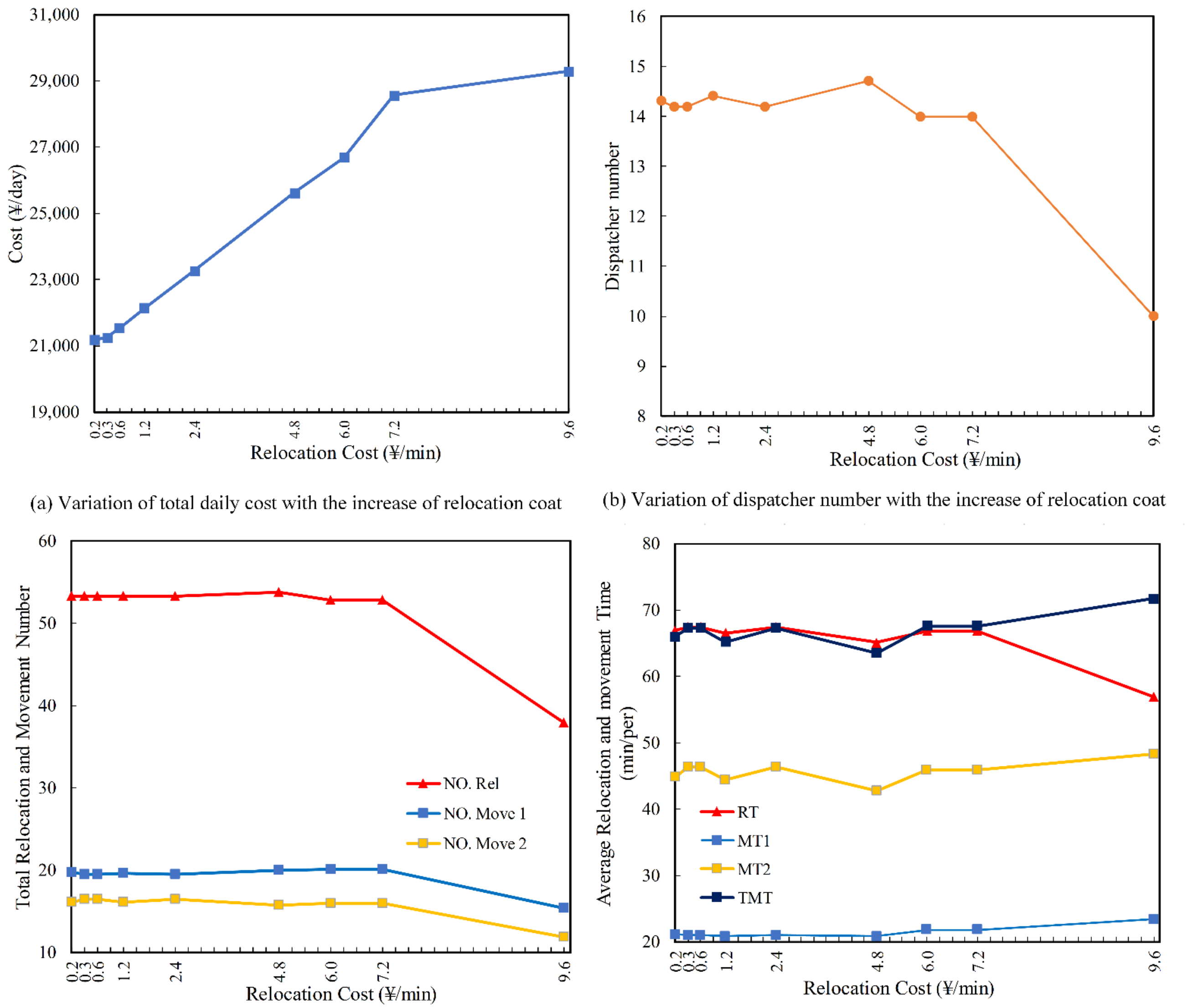
Fig. 4. Effect of dispatcher payment on the performance of one-way CSS

boost dispatcher productivity, as they are assigned to conduct more time-consuming tasks despite the break requirement, which is similar to the change in dispatcher-related indicators (i.e., RT and TMT). It is also notable to observe that dispatchers spend more time on movements than vehicle relocation tasks. This is attributed to the fact that travel times for movement by regular and electric bikes are double and 1.5 times as long as relocation time, respectively. The results further demonstrate the significance of considering movement in the definition of break requirement. In conclusion, the results demonstrate that

TABLE VIII

EFFECT OF RELOCATION COST ON THE PERFORMANCE OF ONE-WAY CSS

Relocation cost (¥/min)	Cost (¥/day)	NO. Dis	NO. Rel	NO. Move 1	NO. Move 2	RT (min/per)	MT1 (min/per)	MT2 (min/per)	TMT (min/per)
0.2	21,170	14.3	53.3	19.7	16.1	66.9	21.1	44.9	66.0
0.3	21,256	14.2	53.3	19.5	16.5	67.4	21.0	46.3	67.3
0.6	21,543	14.2	53.3	19.5	16.5	67.4	21.0	46.3	67.3
1.2	22,136	14.4	53.3	19.6	16.1	66.5	20.8	44.4	65.2
2.4	23,266	14.2	53.3	19.5	16.5	67.4	21.0	46.3	67.3
4.8	25,611	14.7	53.8	20.0	15.7	65.1	20.8	42.7	63.5
6.0	26,686	14.0	52.8	20.1	16.0	66.8	21.8	45.9	67.6
7.2	28,555	14.0	52.8	20.1	16.0	66.8	21.8	45.9	67.6
9.6	29,290	10.0	37.9	15.3	11.9	56.9	23.4	48.3	71.7



(a) Variation of total daily cost with the increase of relocation coat

(b) Variation of dispatcher number with the increase of relocation coat

(c) Variation of total relocation number and movement number with the increase of relocation coat

(d) Variation of average relocation and movement time each dispatcher with the increase of relocation coat

Fig. 5. Effect of relocation cost on the performance of one-way CSS

one-way CSSs could employ fewer dispatchers with more relocation and movement tasks while considering the break requirements of dispatchers to minimize the cost increase, even though higher salaries may be offered at the expense of increased cost (i.e., sacrificing profits).

3) Impact of relocation cost

In addition to the dispatcher payment, we also test the impact of relocation cost on the performance of CSSs. The parameter setting remains the same as Subsection V.A, except for the relocation cost. The rentals are the same as those in the analysis about dispatcher payment.

The impact of the relocation cost on the performance is illustrated in Fig. 5. The relevant data are tabulated in Table VIII. As Fig. 5. (a) shows, the daily cost of CSS operators rises as the relocation cost per minute increases, emphasizing the impact of relocation cost on the performance of CSS. The number of dispatchers (NO. Dis) follows a downward trend with fluctuations as illustrated in Fig. 5. (b), which can be explained by the fact that decreasing relocation, resulting from the increasing relocation cost, contributes to fewer dispatchers with the objective of cost minimization. In Fig. 5. (c), numbers of relocations (NO. Rel), and movements by different transport modes (NO. Move 1 and NO. Move 2) appear to fluctuate with the change in relocation cost, suggesting that CSS operators can adjust their arrangement to accommodate varying relocation costs, subsequently impacting the overall cost and operational efficiency. All indicators change significantly when the relocation cost reaches a threshold of 9.6 ¥/min. At this point, the cost-saving benefits of optimizing dispatcher arrangements are outweighed by the additional expenses incurred due to the higher relocation cost. This significant change in the number of dispatchers can be attributed to CSS operators being forced to re-evaluate their dispatching strategies and resource allocation to minimize costs. Dispatcher-related indicators, i.e., relocation time (RT), movement time (MT) with different transport modes, and total movement time (TMT) per dispatcher, show varying trends. This suggests that operators may adopt different arrangements to minimize relocation costs, such as focusing on shorter or more efficient relocations. Besides that, in some cases, the movement time may increase as dispatchers are assigned more tasks considering the higher relocation cost. The CSS operators are thus suggested to fulfill rentals by less vehicle relocation and adjusting dispatcher tasks for cost minimization under a high relocation cost.

VI. Conclusions

In this study, the DN problem considering the break requirement of dispatchers has been investigated with the objective of cost minimization for a one-way carsharing service operator. Dispatchers need to take a break to recover their attention when they feel fatigued caused by continuously conducting relocation and movement tasks for a long time. The break requirement of dispatchers was considered by introducing restrictions on maximum accumulative working time and compulsory break time. A nonlinear integer programming model was developed to minimize the cost by determining the fleet size of vehicles, the number of dispatchers, vehicle relocation, and dispatcher movement. Due to the complexity of the break requirement, obtaining an optimal solution for the proposed model using state-of-the-art solvers was challenging. After exploring the structure of the model, the nonlinear integer programming model was decomposed into two parts: the optimization module and the simulation module. The optimization model was further decomposed into the vehicle relocation problem, dispatcher rebalances problem, and dispatcher flow problem. The optimization-simulation process was iteratively updated with constraints in the related problems. At last, numerical experiments on a one-way CSS company named EVCARD were conducted. The computational results demonstrated the efficiency of the proposed model and developed algorithm and indicated the impact and necessity of considering the break requirements of dispatchers when exploring decision-making problems. The case study also evaluated the impact of the payment for dispatchers, and relocation cost on the performance of the CSS. Our study may provide valuable guidance to one-way CSS operators.

Further research can be undertaken in several aspects. First, it would be interesting to model the proposed problems considering the charging demand, and scheduling of electric vehicles. Second, since carsharing operators and dispatchers benefit differently in terms of cost minimization and working experience, more studies should be conducted to address the benefit redistribution among operators and hired dispatchers to facilitate fair carsharing systems. Third, the model is formulated under the assumption of demand known and served. It would be practically significant to consider elasticity and uncertainty of demand with several uncertain factors, e.g., users' elastic demand and rental time change.

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