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# Investigating the interrelationship between stochastic aircraft routing of airlines and maintenance staffing of maintenance providers

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**Abstract**

Stochastic aircraft routing (SAR) plays a critical role in defining the routing plans of airlines, which include assigning the aircraft to flight legs and determining the time and location of performing the maintenance to the aircraft. Based on the routing plan designed by airlines, the maintenance providers should schedule their workforce to perform maintenance operations by solving the maintenance staffing problem (MSP). MSP helps maintenance providers to build their staffing plans, which include the assignment of manpower to each aircraft, so that aircraft receive the maintenance operations as planned. Practically, to airlines, the routing plan will be interrupted (e.g. flight will be delayed) if an aircraft cannot be released from the maintenance station punctually. Similarly, for maintenance providers, if an aircraft missed the scheduled appointment at the maintenance station, this will also cause a huge interruption to their staffing plan. Therefore, there is an interrelationship between SAR and MSP. In the literature, the focus of each problem has been traditionally limited to independent scope, yet with limited consideration of their interrelationship. In this paper, we study SAR along with MSP, with an objective of investigating the interrelationship between SAR and MSP. For this purpose, we propose a coordinated configuration of SAR and MSP that is formulated as a leader-follower Stackelbery game, in which SAR acts as a leader and MSP acts as a follower. This game is enacted through a bi-level optimization model, which is solved by a bi-level nested ant colony optimization (ACO) algorithm. A case study of major airline and maintenance provider located in the Middle East is presented to demonstrate the feasibility and potential of the proposed model. The results demonstrate significant saving in the costs of both companies.

**Keywords:** Stochastic aircraft routing, Maintenance staffing, Stackelberg game, Bi-level optimization.

## 1. Introduction

In the last decade, the development of the aviation industry has shown radical economic growth because of the bloomed passenger demand, which is expected to grow by 31% from 2012 to 2017, as reported by International Air Transport Association in 2014. Despite this pleasing situation for airlines, the task of managing the aircraft routing (AR) has seen strong challenge to cope with the demand growth. Moreover, this task's complexity dramatically increased, while considering the flight delays that prohibit the generated routes to be operated as planned. For example, in 2011, it was estimated that U.S. airline industry experienced a total of 103 million minutes of delay, resulting in a $7.7 billion as an increase in the operating cost, as reported by Liang, et al. [1]. In this regard, the aircraft routing (AR), which is one of the main focuses of this study, is very significant to airlines in that it generates the maintenance feasible routes for each aircraft flown in reality.

In the literature, the focus of AR studies can be classified to three foci including; tactical (TAR), operational (OAR), and operational with flight delay consideration (OARD). The TAR generates generic

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rotations or a fixed sequence of flights for each aircraft such that some of the operational constraints are neglected [2-6]. Using these generated rotations for aircraft are not applicable due to the lack of consideration of the operational constraints. Therefore, the researchers shifted their focus from tactical to operational. The OAR studies specify each aircraft's route besides considering the operational constraints, such as limitations on the total cumulative flying time, the total number of days since last maintenance operation, and the total number of take-offs, as mandated by (FAA) [7-11]. It is noticed that several research efforts have been exerted in both TAR and OAR. However, the drawback of these studies is the ignorance of these sudden changes (e.g. flight delays), which results in generating routes that can be easily disrupted. This motivates the researchers to consider flight delay besides the operational side, to produce routes that better withstand the disruption by using re-routing mechanism [12], re-timing mechanism [13], buffer time insertion mechanism [1], and stochastic programming framework for aircraft routing (SAR) [14]. For more details about AR, we refer the reader to Eltoukhy, et al. [15].

Solving SAR provides a solution (known as the routing plan), which is a sequence of flight legs assigned to each aircraft, and that sequence includes maintenance visits to satisfy the operational requirements. Practically, each aircraft starts covering the assigned flight legs, then it goes to the maintenance provider. After finishing the maintenance operation, the aircraft should move from the maintenance station to resume covering its route. To achieve the routing plan, the airline should send the scheduled arrival and departure times for its aircraft to the maintenance provider. Based on the received scheduled times, the maintenance providers should schedule their workforce to perform maintenance operations by solving the maintenance staffing problem (MSP). MSP helps maintenance providers to build their staffing plans, which include the assignment of manpower to each aircraft, while considering the workforce capacity and the scheduled times of the aircraft [16, 17].

From this description, we can see that achieving the routing plan, is the responsibility of the maintenance provider to release the aircraft punctually from the maintenance station. Similarly, staffing plan, which as the solution of MSP, can be realized as planned, if the airlines send their aircraft on time, so that any disturbance to the staffing plan can be avoided. Therefore, SAR and MSP are closely interdependent on each other, and in turn joint decision making is deemed to be imperative.

In the literature, existing research on the SAR and MSP usually focuses on solving each problem independently, exploring the interdependence or interrelationship between these two problems has not been investigated.

In this paper, we study SAR along with MSP, with an objective of handling the inherent interdependence between SAR and MSP. To achieve this goal, we propose a coordinated configuration of SAR and MSP that is formulated as a leader-follower Stackelbery game, in which SAR acts as a leader and MSP acts as a follower. This game is enacted through a bi-level optimization model to capture the coordination between two self-interested decision makers for SAR and MSP. Furthermore, to be consistent with the bi-level optimization model, a bi-level nested ACO algorithm is developed to derive optimal or near-optimal solutions for both problems. To demonstrate the feasibility and potential of the proposed model, a case study of major airline and maintenance provider located in the Middle East is presented.

The rest of the paper is organized as follow. The decision models of SAR and MSP are presented in Sections 2 and 3, respectively. In Section 4, we elaborate the bi-level optimization model. A bi-level nested ACO algorithm is developed in Section 5. Section 6 reports a case study of major airline and maintenance provider. Conclusions of the study are given in Section 7.

## 2. SAR decision model

SAR is considered the first part of the coordinated configuration, which is handled by the airlines. The traditional task of SAR is to generate maintenance feasible routes for each aircraft flown in reality. In order to generate these routes, SAR has to deal with two main parts, as illustrated in the left part of Fig.1. First part is the aircraft that has a limited number in the airline, whereas the second part is represented by a list of flight legs. Each flight leg is characterized by six features: departure time, origin airport, arrival time, destination airport, flight duration, and non-propagated delay. The critical issue of SAR is to build the routes while satisfying all the operational maintenance requirements (e.g. restrictions on the total cumulative flying time, the total number of days between two successive maintenance operations, and the total number of take-offs). Meanwhile, it becomes more challenging to generate these routes when considering the non-

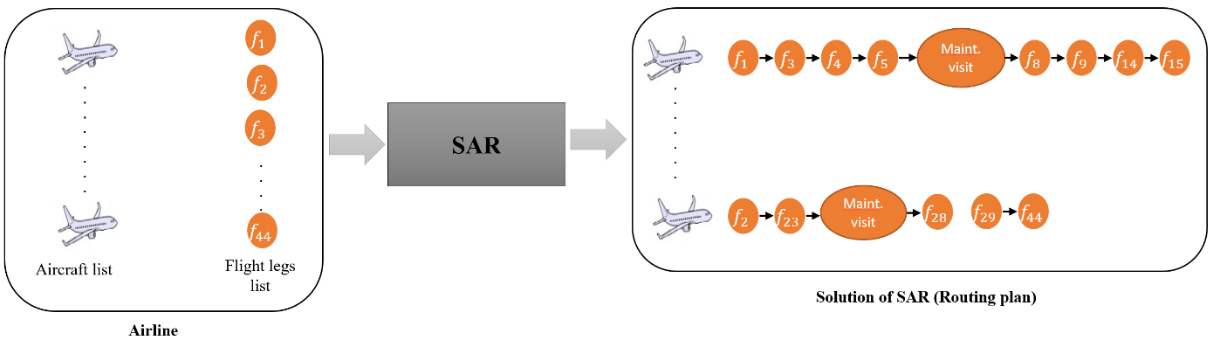


**Fig. 1: Typical representation of the routing problem and its routing plan**

propagated delay for each flight, which is handled by generating different disruption scenarios, resulting in more complexity added to the problem.

Solving SAR provides a solution (known as the routing plan), as shown in the right part of Fig. 1. The generated solution is a sequence of flight legs assigned to each aircraft, and that sequence includes maintenance visits to satisfy the operational requirements. Practically, each aircraft starts covering the assigned flight legs as planned, then it goes to the maintenance provider to be maintained. After finishing the maintenance operation, the aircraft should move from the maintenance station to resume covering their route. To achieve that, it is the responsibility of the maintenance provider to release the aircraft punctually from the maintenance station. This requires the maintenance provider to assign their workforce to perform the maintenance operations efficiently by solving the MSP.

From the previous description, it becomes clear that achieving the routing plan is not only dependent on the airline itself, but also dependent on the maintenance providers, as their operations are part of the solution. Therefore, it is imperative for SAR to be optimized in conjunction with MSP decisions.

Before presenting the SAR's formulation, it should be noted that this formulation is built based on the connection network, which is one of the commonly used network for routing problem [3, 9]. To formalize the representation of the MSP's formulation, we first define the sets, parameters and decision variables that are frequently used throughout this paper as follow:

**<u>SAR decision model</u>**

*Sets*

| | |
|---|---|
| $i, j \in NF$: | Set of flight legs. |
| $k \in K$: | Set of aircraft. |
| $m \in MT$: | Set of maintenance stations. |

| | |
|---|---|
| $a \in A$: | Set of airports. |
| $v \in \{1,2,\dots,V\}$: | Number of maintenance operations that should be performed by each aircraft |
| $\xi \in \Xi$: | Set of disruption scenarios for o non-propagated delay realizations. |
| $\{o,t\}$: | Dummy source and sink nodes of the aircraft routing network. |

*Parameters*

| | |
|---|---|
| $DT_i$: | Departure time of flight leg $i$. |
| $AT_i$: | Arrival time of flight leg $i$. |
| $TRT$: | Turn-around time. |
| $O_{ia}$: | Origin binary indicator of flight leg $i$ such that $O_{ia} = 1$ if the origin of flight leg $i$ and the airport $a$ are the same, and 0 otherwise. |
| $D_{ia}$: | Destination binary indicator of flight leg $i$ such that $D_{ia} = 1$ if the destination of flight leg $i$ and the airport $a$ are the same, and 0 otherwise. |
| $FT_i$: | Flight duration of flight leg $i$. |
| $T_{max}$: | Maximum flying time between two successive maintenance operations. |
| $NPD_{ik}^{\xi}$: | Non-propagated delay of flight leg $i$ covered by aircraft $k$, under scenario $\xi$. |
| $Mb_{ma}$: | Maintenance binary indicator of maintenance station $m$ such that $Mb_{ma} = 1$ if the maintenance station $m$ located at airport $a$, and 0 otherwise. |
| $MAT$: | Time required to perform the maintenance operation assumed by the airlines |
| $M$: | A considerable big number. |
| $p^{\xi}$: | Probability for realization of scenario $\xi$. |
| $C_{pD}$: | Per minute propagated delay cost. |
| $PD_{ijkv}^{\xi}$: | Propagated delay caused when aircraft $k$ already covered flight leg $i$ and will potentially cover flight leg $j$, before performing maintenance operation number $v$, under scenario $\xi$. |
| $PD_{ikv}^{\xi}$: | Total propagated delay of the route covered by aircraft $k$, caused from the beginning of coverage until covering flight leg $i$, before performing maintenance operation number $v$, under scenario $\xi$. |

*Decision variables*

| | |
|---|---|
| $x_{ijkv}^{\xi} \in \{0,1\}$: | =1 if flights legs $i$ and $j$ are covered by aircraft $k$, before performing maintenance operation number $v$, under scenario $\xi$ and 0 otherwise. |
| $y_{imkv}^{\xi} \in \{0,1\}$: | =1 if aircraft $k$ covers flight leg $i$ then perform maintenance operation number $v$, at maintenance station $m$, under scenario $\xi$, and 0 otherwise. |
| $z_{mjkv}^{\xi} \in \{0,1\}$: | =1 if aircraft $k$ covers flight leg $j$, after performing maintenance operation number $v$, at maintenance station $m$, under scenario $\xi$, and 0 otherwise. |
| $RTAM_{kv}^{*\xi} > 0$: | The ready time for aircraft $k$ to continue covering another flight legs after performing the maintenance operation number $v$, under scenario $\xi$. |

**MSP decision model**

*Sets*

| | |
|---|---|
| $f, b \in \{1,2,\dots,F\}$: | Set of flights that their aircraft will be maintained. |
| $s \in S$: | Set of shifts. |
| $\{o', t'\}$: | Source and sink node of the layered graph. |

*Parameters*

$SAT_{fm}^{\xi}$: Scheduled arrival time of flight $f$ that its aircraft will be maintained, at maintenance station $m$, under disruption scenario $\xi$.

$SDT_{fm}^{\xi}$: Scheduled departure time of flight $f$ that its aircraft will be maintained, at maintenance station $m$, under disruption scenario $\xi$.

$w_{sm}^{l}$: Minimal team size (number of worker) that can be formed to perform maintenance operation, during shift $s$, at maintenance station $m$.

$w_{sm}^{u}$: Maximal team size (number of worker) that can be formed to maintain aircraft, during shift $s$, at maintenance station $m$.

$Q_s^{max}$: Capacity of workforce available during shift $s$.

$l_f$: Workload (man-hours) required to maintain aircraft that covers flight $f$.

$C_{wfsm}$: Cost incurred when assigned $w$ workers to maintain aircraft that covers flight $f$, during shift $s$, at maintenance station $m$.

*Decision variables*

$wf_{fsm}^{\xi} \in \{w_{sm}^{l}, \dots, w_{sm}^{u}\}$: Number of worker (team size) assigned to maintain aircraft that covers flight $f$, during shift $s$, at maintenance station $m$, under disruption scenario $\xi$.

$RTAM_{fm}^{\xi} > 0$: Actual ready time for the aircraft that covers flight $f$ to leave the maintenance station $m$, under disruption scenario $\xi$.

Based on the above notations, the optimization model for SAR, as a leader, is formulated as below:

$$min \sum_{\xi \in \Xi} p^{\xi} \left( \sum_{v \in V} C_{pD} \left( \sum_{k \in K} \sum_{i \in NF} \sum_{j \in NF} PD_{ijkv}^{\xi} \, x_{ijkv}^{\xi} \right) \right) \tag{1}$$

s.t.

$$PD_{ijkv}^{\xi} = PD_{ikv}^{\xi} + (NPD_{ik}^{\xi} - (DT_j - AT_i - TRT))^{+} \qquad \forall\, i \in NF, \forall j \in NF, \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{2}$$

$$\sum_{k \in k} \left( \sum_{j \in NF \cup \{t\}} \sum_{v \in V} x_{ijkv}^{\xi} + \sum_{m \in MT} \sum_{v \in V} y_{imkv}^{\xi} \right) = 1 \qquad \forall\, i \in NF, \forall \xi \in \Xi \tag{3}$$

$$\sum_{j \in NF \cup \{o\}} x_{ijkv}^{\xi} + \sum_{m \in MT} z_{mikv}^{\xi} = \sum_{j \in NF \cup \{t\}} x_{ijkv}^{\xi} + \sum_{m \in MT} y_{imkv}^{\xi} \quad \forall\, i \in NF, \forall k \in k, \forall\, v \in V, \forall \xi \in \Xi \tag{4}$$

$$\sum_{j \in NF} \sum_{v \in V} y_{jmkv}^{\xi} = \sum_{j \in NF \cup \{t\}} \sum_{v \in V} z_{mjkv}^{\xi} \qquad \forall m \in MT, \forall k \in k, \forall \xi \in \Xi \tag{5}$$

$$AT_i + TRT - DT_j \leq M(1 - x_{ijkv}^{\xi}) \qquad \forall\, i \in NF, \forall j \in NF, \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{6}$$

$$\sum_{k \in k} x_{ijkv}^{\xi} \leq \sum_{a \in A} D_{ia} O_{ja} \qquad \forall\, i \in NF, \forall j \in NF, \forall v \in V, \forall \xi \in \Xi \tag{7}$$

$$\sum_{k \in k} y_{imkv}^{\xi} \leq \sum_{a \in A} D_{ia} Mb_{ma} \qquad \forall\, i \in NF, \forall m \in MT, \forall v \in V, \forall \xi \in \Xi \tag{8}$$

$$\sum_{k \in k} z_{mjkv}^{\xi} \leq \sum_{a \in A} Mb_{ma} \, O_{ja} \qquad \forall m \in MT, \forall j \in NF, \forall v \in V, \forall \xi \in \Xi \tag{9}$$

$$RTAM_{kv}^{*\xi} - DT_j \leq M\left(1 - z_{mjkv}^{\xi}\right) \qquad \forall m \in MT, \forall j \in NF, \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{10}$$

$$RTAM_{kv}^{*\xi} = \sum_{i \in NF \cup \{o\}} \sum_{m \in MT} (AT_i + MAT) z_{mjkv}^{\xi} \qquad \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{11}$$

$$RTAM_{kv}^{*\xi} = \sum_{f \in F} \sum_{i \in NF \cup \{o\}} \sum_{m \in MT} RTAM_{fm}^{\xi} \, z_{mjkv}^{\xi} \qquad \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{12}$$

$$\sum_{i \in NF \cup \{o\}} \sum_{j \in NF} FT_j \; x_{ijkv}^{\xi} \leq T_{max} \qquad \forall k \in k, \forall v = 1, \forall \xi \in \Xi \tag{13}$$

$$x_{ijkv}^{\xi} \in \{0,1\} \qquad \forall\, i \in NF, \forall j \in NF, \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{14}$$

$$y_{imkv}^{\xi} \in \{0,1\} \qquad \forall\, i \in NF, \forall m \in MT, \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{15}$$

$$z_{mjkv}^{\xi} \in \{0,1\} \qquad \forall\, m \in MT, \forall\, j \in NF, \forall k \in k, \forall v \in V, \forall \xi \in \Xi \tag{16}$$

$$RTAM_{kv}^{*\xi} > 0 \qquad \forall k \in k, \forall \in V, \forall \xi \in \Xi \tag{17}$$

The objective function (1) is the minimization of the total expected cost of propagated delay, which is calculated by using Constraints (2). Constraints (3) are cast to ensure covering each flight leg exactly by one aircraft, whereas balance constraints (4) and (5), are formulated to keep the circulation of these aircraft throughout the network. To build a connection between two successive flight legs to be covered by same aircraft, that connection should be feasible in terms of time and place considerations, as described by constraints (6) and (7). On the other hand, to prepare a maintenance visit for each aircraft, constraints (8) are formulated, so that the locations of the maintenance stations are considered. After finishing the maintenance operation, the aircraft should resume covering its route. For this purpose, constraints (9) - (12) are cast. It should be noted that constraints (12) are considered the link between SAR and MSP. Forcing the aircraft that needs maintenance to undergo maintenance operations cannot be fulfilled through coverage and balance constraints. Therefore, the operational restrictive constraints (13) are cast. Finally, constraints (14) – (17) are cast to indicate the integrality of the decision variables.

## 3. MSP decision model

The second part of the coordinated configuration is represented by MSP, which is solved by the maintenance providers. The main goal of MSP is to determine the workforce team sizes required to perform the maintenance operations to the aircraft. To do that, MSP has to manage three main parts: the received aircraft, the workforce, and the planned shifts, as illustrated in the left part of Fig. 2. Starting with the aircraft, it represents the demand for the maintenance provider, which is characterized by the scheduled arrival and departure times for each aircraft. Moving to the workforce, each maintenance provider has its own workforce capacity and limits for team sizes. Lastly, the planned shifts, each provider has its own planned shifts. For example, some providers build its plan based on two shift pattern (morning and afternoon), whereas others deal with three shift pattern (morning, afternoon, and night). The critical issue of MSP is to determine the efficient team sizes, while considering the workforce capacity and the scheduled arrival and departure times of the received aircraft.

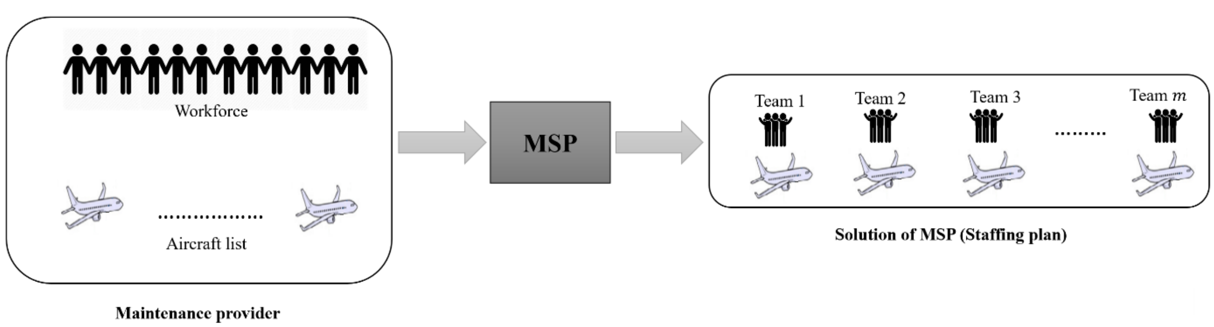


**Fig. 2: Typical representation of the staffing problem and its staffing plan**

Using the above information, MSP is solved with main objective of minimizing the worker cost. The generated solution, known as the staffing plan, indicates how many workers is assigned to maintain each aircraft, and accordingly, the actual ready time for each aircraft to leave the maintenance station can be determined. Sometimes, due to the workforce capacity restrictions, the solution in terms of ready time might be behind the scheduled departure time, resulting in a delay to the aircraft. As a result, the routing plan determined by the airline will be affected, and its plan will not be realized. Meanwhile, if the aircraft missed its the scheduled appointment at the maintenance station, this will also cause a huge interruption to maintenance provider's staffing plan. So, it is again confirmed from the MSP side that achieving the results as planned is a mutual responsibility between the airline and maintenance providers. Therefore, MSP should be solved jointly with SAR, in which the MSP decisions including the determined ready time for the aircraft are sent back to the airline. In a response to these decisions, the airline will change its decisions including the scheduled arrival and departure time, if its initial plan is affected.

MSP presented in this section is formulated based on a layered graph, which is commonly used in the literature while handling the staffing problem and worker allocation problem [18]. Based on the predefined notations, the optimization model for MSP, as a follower, is formulated as below:

$$min\ \sum_{m\in MT}\sum_{s\in S}\sum_{f\in F} C_{wfsm}\, wf_{fsm}^{\xi} \tag{18}$$

s.t.

$$RTAM_{fm}^{\xi} = \left(SDT_{fm}^{\xi} - \left(SAT_{fm}^{\xi} + TRT + l_f/wf_{fsm}^{\xi}\right)\right)^{+} \qquad \forall f \in F, \forall m \in MT, \forall \xi \in \Xi \tag{19}$$

$$\sum_{f\in F} wf_{fsm}^{\xi} \le Q_s^{max} \qquad \forall s \in S, \forall m \in MT, \forall \xi \in \Xi \tag{20}$$

$$SAT_{fm}^{\xi} = \sum_{k\in K}\sum_{i\in NF}\sum_{v\in V} AT_i\, y_{imkv}^{\xi} \qquad \forall f \in F, \forall m \in MT, \forall \xi \in \Xi \tag{21}$$

$$SDT_{fm}^{\xi} = \sum_{k\in K}\sum_{j\in NF}\sum_{v\in V} RTAM_{kv}^{*\xi}\, z_{mjkv}^{\xi} \qquad \forall f \in F, \forall m \in MT, \forall \xi \in \Xi \tag{22}$$

$$wf_{fsm}^{\xi} \in \{w_{sm}^{l}, \dots, w_{sm}^{u}\} \qquad \forall f \in F, \forall s \in S, \forall m \in MT, \forall \xi \in \Xi \tag{23}$$

$$RTAM_{fm}^{\xi} > 0 \qquad \forall f \in F, \forall m \in MT, \forall \xi \in \Xi \tag{24}$$

The objective function (18) is the minimization of the total worker cost. Constraints (19) represent the calculation of the real ready time for the aircraft when leaving the maintenance station. In order to allocate the workers properly to perform maintenance operation, the worker capacity in each visible shift should be respected, as explained by constraints (20). Acting as a follower to build efficient staffing plan requires MSP to receive some information from the airline. For this purpose, linkage constraints (21) and (22) are cast. Finally, constraints (23) and (24) define the domain of the decision variables.

## 4. The bi-level optimization model

Based on the above description, it is clear that both SAR and MSP are of inconsistent objectives. These conflicting goals as well as the interdependence between SAR and MSP make the coordinated configuration between them indispensable. One of the tools that can handle this situation is Stackelberg game, known as leader-follower game. In this game, SAR as a leader constitutes the upper-level optimization model, and the MSP as a follower forms the lower-level optimization model. Each model has its own strategies in terms of decision variables and payoffs that represented by the objective functions. Initially, SAR as a leader determines its strategy (i.e. decision variable) and send to MSP. Next, MSP responds to the SAR's decisions by choosing its strategy (i.e. decision variable) that maximizes its own payoff (i.e. objective function). The bi-level optimization model for the coordinated configuration of SAR and MSP can be summarized as follow:

$$min \sum_{\xi \in \Xi} p^{\xi} \left( \sum_{v \in V} C_{pD} \left( \sum_{k \in K} \sum_{i \in NF} \sum_{j \in NF} PD^{\xi}_{ijkv} \, x^{\xi}_{ijkv} \right) \right) \tag{25}$$

s.t. Constraints Eqs. (2) - (17) (26)

Where given decision variables ($y^{\xi}_{imkv}$, $z^{\xi}_{mjkv}$ and $RTAM^{*\xi}_{kv}$) used to solve:

$$min \sum_{m \in MT} \sum_{s \in S} \sum_{f \in F} C_{wfsm} \, wf^{\xi}_{fsm} \tag{27}$$

s.t. Constraints Eqs. (19) - (24) (28)

From the above formulation, it is observed that SAR makes decision regarding the maintenance visits for its aircraft. With these results as an input, MSP determine its staffing plan to satisfy the required maintenance operations. Next, MSP responds to the upper-level by providing its decisions. If the decisions influence the initial routing plan, SAR further adjusts its decisions by resolving the model. This process keeps iterated until reaching the Stackelberg equilibrium, in which both players are unwilling to adjust their decisions. In this way, the optimal setting for the coordinated configuration are derived.

## 5. Solution method

Before discussing the solution method used to solve the proposed model, it is very important to mention that SAR and MSP belong to the class of NP-hard problems [8, 18]. So, it is practical and reasonable to use meta-heuristic approaches, as it has been successively applied for solving different optimization problems. For example, travelling salesman problem [19], crew scheduling [20], vehicle routing problem [21], aircrew rostering problem [22], and control attitude behavior problem [23, 24].

As described in Sections (2) and (3), the upper-level and lower-level optimization problems (referred as SAR and MSP) essentially formulated based on network representation, for which ACO has proven to be advantageous for large and complex network based problems [25, 26]. This motivate us to propose a bi-level nested ACO approach, whilst each individual level is solved by one specific ACO, as shown in the next sub-sections.

### 5.1. ACO for upper-level optimization

The ACO adopted to solve SAR consists of two main parts, which are as follow:

- Route construction. This part is conducted by the help of ants (i.e. each ant simulates an aircraft $k \in K$) such that each ant constructs its route by the usage of state transition rule, stated in Eq. (29).

  $$j = \begin{cases} \arg\_max_{l \in N_i^k} \left\{ \left[ {\tau_{ij}}^{\xi} \right]^{\alpha} \left[ {\eta_{ij}}^{\xi} \right]^{\beta} \right\} & if \; q \leq q_0 \\ J & if \; q > q_0 \end{cases} \tag{29}$$

  Where $N_i^k$ is the set of potential flight legs that can be selected by the ant $k$. The terms ${\tau_{ij}}^{\xi}$ and ${\eta_{ij}}^{\xi} = 1/(C_{pD} * PD^{\xi}_{ijkv})$ are the pheromone trial and the heuristic function of the network arcs, respectively. The two parameters $\alpha$ and $\beta$ represent the relative importance of the pheromone trial and the heuristic function, respectively. $q_0$ is the exploration threshold parameter ($0 \le q_0 \le 1$) and $q$ is a uniformly distributed random number [0~1]. Typically, the ant selects the next flight leg $j$ based on the value $q$. If $q \le q_0$, then selects the flight leg $j$ according to Eq. (29). On the other side, if $q > q_0$, the ant picks the flight leg $j$ according to the following probability rule:

$$P_{ij}^{k} = \frac{\left[\tau_{ij}{}^{\xi}\right]^{\alpha}\left[\eta_{ij}{}^{\xi}\right]^{\beta}}{\sum_{l\in N_i^k}\left[\tau_{ij}{}^{\xi}\right]^{\alpha}\left[\eta_{ij}{}^{\xi}\right]^{\beta}} \qquad if\ j \in N_i^k \qquad (30)$$

- Update the pheromone trail: it can be done in accordance with the following rule:

$$\tau_{ij,new}^{\xi} = (1-\rho)\tau_{ij,old}^{\xi} + \Delta\,\tau_{ij}{}^{\xi} \qquad (31)$$

Where $\rho$ is the evaporation rate parameter $(0 < \rho < 1)$. The first term $(1-\rho)\tau_{ij,old}^{\xi}$ is used each iteration, so that a uniform reduction of the phermones can be achieved. The second term $\Delta\,\tau_{ij}{}^{\xi}$ represents the pheromone quantities, under disruption scenario $\xi$. This term is used only to update all the edges included in the best so far solution, by following the rule in Eq. (32).

$$\Delta\,\tau_{ij}{}^{\xi} = \frac{Q}{cost\left(A_{best}^{\xi}\right)} \qquad if\{i,j\} \subseteq A_{best}^{\xi} \qquad (32)$$

Where $Q$ is the control factor of laying the pheromone. The $cost\left(A_{best}^{\xi}\right)$ is the lowest propagated delay cost from the beginning until now, while handling disruption scenario $\xi$.

### 5.2. ACO for lower level optimization

ACO adopted to solve MSP is like that one proposed to solve the upper level except the heuristic information, which is as follows $\left(\eta_{bfw,worker}^{\xi} = 1/(wf_{fsm}^{\xi} * C_{wfsm})\right)$.

### 5.3. Bi-level nested ACO algorithm

Solving joint optimization of SAR and MSP in the presence of the interdependence necessitates an algorithm that capture this issue. For this purpose, we propose a bi-level nested ACO algorithm, which starts with upper-level ACO in order to solve SAR and make decisions regarding the routing plan and maintenance visits (refereed as $y_{imkv}^{\xi}$, $z_{mjkv}^{\xi}$ and $RTAM_{kv}^{*\xi}$). Upper-level solution is sent to the lower-level ACO as an input to solve MSP and make decisions regarding the staffing plan (referred as $wf_{fsm}^{\xi}$ and $RTAM_{fm}^{\xi}$) that satisfy the received solution. All the lower-level best solutions are sent back to the upper-level ACO to re-run it and adjust its solution. This process iterates until both leader and follower reach the Stackelberg game equilibrium.

For computational efficiency and reasonable problem contexts, we set the stopping criteria for both upper and lower-level ACO to be happened upon convergence (i.e. no solution improvement for successive 100 iterations), or when completing the maximum number of iterations (i.e. 500 iterations). If both stopping criteria are satisfied, then the nested algorithm is terminated, and the Stackelberg equilibrium is derived.

## 6. Case study

To reveal the importance of the proposed model, we present a case study based on data delivered from major airline and maintenance provider located in the Middle East, as shown in Table 1.

**Table 1: Data collected from airline and maintenance provider**

| Airline | Maintenance provider |
|---|---|
| - Number of flight legs=240.<br>- Fleet size=30.<br>- Maximum number of take-offs ($C_{max}$) =10.<br>- Number of airports=8.<br>- Number of maintenance stations=4.<br>- Turn-around time ($TRT$) =45 minutes.<br>- Maximum flying time ($T_{max}$) =40 hours.<br>- Time required to perform maintenance ($MAT$) =8 hours.<br>- Per minute propagated delay cost ($C_{pD}$) = 75 if propagated delay is less than or equal 15 minutes, or it is equal 125 for longer propagated delays. | - Minimal team size ($w_{Sm}^{l}$) = 10 workers.<br>- Maximal team size ($w_{sm}^{u}$) = 5 workers.<br>- Capacity of workers ($wf_{s}^{max}$) =100 workers.<br>- Potential team size ($wf_{fsm}^{\xi}$) are 5, 6, 7, 8, 9, and 10, and its corresponding cost ($C_{wfsm}$) are 670,730,800,870,950, and 1020.<br>- Per minute aircraft time delay penalty cost ($CW_d$) =100.<br>- Workload required to maintain aircraft ($l_f$) =50 hours. |

### 6.1. Results of Stackelberg model

The near optimal solutions for the joint SAR and MSP are determined by implementing the bi-level nested ACO algorithm. For computational efficiency, the number of disruption scenarios is capped at as 100 equally likely scenarios. The nested ACO algorithm adopts pheromone trail importance of 1, heuristic function importance of 2, exploration threshold of 0.95, evaporation rate of 0.05, and control factor for pheromone laying of 0.01. Regarding the ant size, the upper-level ACO adopts the size that equals the number of aircraft. On the other hand, the lower level ACO sets the size to be equal the number of flights that their aircraft will be maintained. The results obtained from nested ACO algorithm show that after 350 iterations, the upper-level ACO converges and returns its best result, whereas the lower-level ACO converges and its best result is achieved after 450 iterations. Since these results constitute the situation in which both players are unwilling to change their decisions, the Stackelberg equilibrium is achieved to be 1640.23 for the airline and 23202.53 for the maintenance provider.

### 6.2. Comparison between the proposed Stackelberg and models in the literature

To demonstrate the advantage of the leader-follower Stackelberg model (LFS) over the existing method in the literature, we conduct computational experiments in order to compare LFS performance with another traditional optimization method called non-joint optimization method (NJOP). The results show that LFS model outperforms the NJOP model by 15.61% (1640.23 vs. 1943.63), while handling SAR, whereas the outperformance of LFS model over NJOP model is 18.70% (23202.53 vs. 28539.39), while handling MSP. So, it is clear from the results that the proposed LFS model improves the results obtained by the airline and maintenance providers significantly. This echoes the importance of the coordinated configuration of SAR and MSP to be implemented in reality.

## 7. Conclusion

In this paper, we propose a joint optimization model for coordinated configuration of SAR and MSP by using leader-follower Stackelberg game. In this game, SAR, which is handled by the airlines, plays a leader's role for minimization the propagated delay cost. On the other hand, MSP, which is handled by the maintenance company, acts as a follower that responds rationally to the leader's decision regarding the departure time of the airline's aircraft from the maintenance stations. To get the Stackelberg equilibrium, a nested ACO algorithm was proposed to solve the joint optimization model. The case study of major airline and maintenance companies located in the Middle East verifies the feasibility and potential of the proposed model. The results demonstrate significant saving in the operational costs of both companies compared to those results obtained from traditional method called the non-joint optimization method.

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