A novel approach for detailed modelling and optimization to improve energy saving in multiple effect evaporator systems

Di Bo [a,1], Kunru Yang [a,1], Qingsong Xie [a], Chang He [a], Bingjian Zhang [a], Qinglin Chen [a], Zhiwen Qi [a], Jingzheng Ren [b], Ming Pan [a,*]

[a] School of Chemical Engineering and Technology, Sun Yat-Sen University, Guangzhou, 510275, China
[b] Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hong Kong SAR, China

ABSTRACT: The multiple effect evaporator (MEE) system is a typical process for liquor concentration in the energy-intensive industries. Many approaches and commercial software have been widely used for MEE simulation and optimization. However, the existing work usually assumes simplified correlations for material thermodynamic properties and evaporator operations to avoid computational complexity, which may cause a large deviation in results. This paper addresses accurate and detailed unit operations and material properties to model the MEE systems and further heat integration for optimal energy recovery. To deal with the resulting complex mixed integer non-linear programming (MINLP) problems of MEE systems, an efficient optimization strategy is developed, where the non-critical variables in the MINLP model are initialized as parameters and up-dated by solving a series of mixed integer linear programming (MILP) problems using a two-stage iterative procedure. An industrial scale problem for concentrating black liquor in a Chinese Paper Mill is carried out to demonstrate the validity and efficiency of the new approach. Based on the conditions of constant heat transfer coefficients and stream boiling point rises assumed in the well-known commercial software WinGEMS®, our method performs identically to WinGEMS® in five distinct scenarios. Moreover, our method is more capable of solving industrial problems in

* Corresponding author. Email address: panm5@mail.sysu.edu.cn (M. Pan)
[1] These authors contributed equally.

practical situations including varying stream thermal properties and evaporator heat-transfer coefficients, achieving up to 25% of energy conservation in a real-world case.

1. INTRODUCTION

The pulp and paper industry accounts for approximately 5% of total industrial energy consumption, and leads to 2% of direct carbon dioxide emissions from industries.[1] It ranks fourth in terms of energy consumption among industries.[2] The demand and production of world paper and paperboard are increasing: annual paper and paperboard production is expected to grow to between 700 Mt (low estimate) and 900 Mt (high estimate) in 2050.[3] The largest share of this growth will occur in China, India and other Asian developing countries. Due to this remarkable production increase, a corresponding significant increase of energy consumption and carbon dioxide (CO_2) emissions will emerge in the pulp and paper industry.

Evaporation, utilized in many industries to concentrate liquors, is an energy intensive process step in paper plants. Due to the high energy efficiency achieved using multiple effect evaporator (MEE) systems, they have been applied widely in large-scale and energy-consuming industries, such as paper making,[4] sugar refining,[5] and water desalination.[6] Especially, it has been reported that the energy consumption of black liquor MEE systems is around 24~30% of the overall energy consumption in a papermaking process.[7]

Three kinds of research have been reported for MME modelling and optimization, namely pinch analysis, pinch analysis combined with numerical optimization tools, and completely mathematical programming methods. In the early research stage, some researchers used pinch analysis[8] for energy conservation of MEE processes. Pinch analysis is a thermodynamically based approach and has been proposed to identify the optimal heat recovery of process. Piacentino and Cardona[9] studied the potential of energy conservation through process integration and evaluated system improvement by pinch analysis without detailed calculations.

The major limitation of pinch analysis is that it only focuses on heat transfer, rather than the changes of process pressure and detailed unit operations which are very common in the process industries.[10] Therefore, the combination of pinch analysis and numerical optimization tools has been proposed for simulating and optimizing the MEE systems[11,12]. Higa et al.[11] found that the evaporation processes presented many potentials of thermal integration in sugar plants. They applied pinch analysis in a systematic way for determining the minimum energy consumption target of the global process. Sharan and Bandyopadhyay[12] integrated a MEE system into the heat exchanger network of a corn glucose process by using a Grand Composite Curve (GCC). A mathematical theorem was also proposed to determine variation in overall energy requirement for the integration system.

Due to the modern computer technology, mathematical programming has become an efficiently automated optimization approach performing more superiorly than the pinch analysis. The mathematical programming models are categorized into simplified models and rigorous models. In the simplified models, the complex non-linearities in MEE systems are converted into linearities or simple non-linearities based on certain operational assumptions. Khanam and Mohanty[16] proposed a simple analytical linearization model for MEE problems solved by iterative methods. But this approach is not capable of finding a feasible solution in varying operating conditions. Kaya and Sarac[17] developed two mathematical models for a four-effect evaporator system with and without preheating which revealed that the MEE system with pre-heating was more energy-saving. They simplified the model by assuming that the pressure and temperature of each evaporator are known and constant. Khanam and Mohanty[18] showed that condensate can be utilized as a heating medium to pre-heat liquor. Their model was simplified by assuming equal driving force in each effect, whereas equations were solved by using the Gaussian elimination method with partial pivoting. They further used the concepts of stream analysis and temperature path to formulate model equations, and reduced capital and operating costs by adjusting the number of flash tanks and their positions in the MEE system.[19] It is easy to solve the MEE problems based on simplifying assumptions, but their solutions may deviate from actual scenarios.

To obtain more accurate results, some researchers focus on rigorous models. Verma et al.[20] utilized the Interior-Point method to solve the problem of MEE systems with seven configurations, such as configuration coupled with steam-split and pre-heaters. Their method showed less sensitivity towards initial values versus traditional algorithms, where the objective function and constraints were supposed to be continuously differentiable and meet Karush-Kuhn-Tucker conditions. Verma et al.[21] used a Genetic Algorithm to calculate the values of steam consumption, and employed mutation operations to bring random variations in the genes to maintain the genetic diversity. Chen et al.[22] and Ruan et al.[23] stated that the non-linear equations can be converted into a set of linear equations when initial variables were given. The iteration method combined with the matrix method was used to solve the linear equations effectively. Verma et al.[24] developed a generalized modelling of a heptads' effect evaporator system, where mass and energy balances were formulated to linearly independent equations represented by matrix form. Their problem can be solved iteratively utilizing different numerical techniques, such as Gauss-Jordan, Jacobi, and successive over-relaxation methods. Gautami and Khanam[25] proposed a detailed iterative procedure to obtain the optimum configuration of MEE systems among fourteen models. They developed a model for predicting fouling resistance using the experimental data. It is also noted that solving rigorous models is usually a time-consuming process because of the computational difficulties associated with the non-linearities in MEE operations.

Based on the discussion above, solving large-scale and complex non-linear mathematical problems is crucial to optimize MEE systems. These systems are usually formulated as large-scale complex mixed integer non-linear programming (MINLP) problems based on mass and energy balance, including varying material properties, flash tank locations, material pre-heating and steam mixing. Conventional approaches suffer from the complexities that demand advanced mathematical tools to be employed in various scenarios. Unlike the existing approaches, the new method proposed in this paper can avoid the computational complexity without using any simplifying assumptions and find optimal solutions effectively. To deal with the computational difficulties caused by non-linearities, an effective optimization approach is developed to convert the MINLP model to a series of mixed integer linear programming (MILP) models. First, the problems of the MEE system are formulated, including the variations of material thermodynamic properties and evaporator performance under different operating conditions, the location of the flash tank, and the strategies of material pre-heating. The non-linear terms are then linearized by initializing non-critical variables as parameters. Finally, the MINLP problems can be expressed as a complete MILP model, and a two-stage iteration algorithm is developed to solve the new MILP problem iteratively until an optimal solution is found.

The rest of the article is structured as follows: a problem statement is introduced in the second section, followed by a detailed MINLP model of a conventional multiple effect evaporator (MEE) system. The MINLP problem is then reformulated as a new MILP model and solved based on the proposed iteration approach. Finally, an industrial case is carried out to prove the validity and efficiency of the proposed approach compared with WinGEMS® results and real-world data.

2. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

An MEE is an apparatus for efficiently utilizing the heat from vapour to evaporate water, where a series of evaporators are operated by gradually decreasing pressures. Since the boiling point of water has a positive relationship with pressure, the vapour produced from the previous effect can be used as a heating medium in the next effect and thus, only the first effect (at the highest pressure) requires external steam. Based on the flow directions of black liquor and vapour, the MEE process can be operated in forward feed, backward feed, and mixed feed arrangements. The mixed feed configuration is addresses in this work. In the mixed feed operation, the feed liquid enters an intermediate effect and flows into the next effect till it reaches the last effect, and then is pumped back to the first effect. Such configuration is composed of partly forward and backward feed. The mixed feed operation not only avoids large heat consumption caused by the significant temperature difference between the fresh steam and feed liquid in the first effect under the forward feed operation, but also eliminates some of the pumps needed in backward configuration as the concentrated liquor has to be transferred from the low-pressure effect to the high-pressure effect.

This paper presents a multi-effect evaporator system for black liquor concentration in a Chinese paper mill. Although only black liquor is considered as the concentrated liquor for MEE systems, our proposed approach is generic enough to be used for accommodating other types of concentrated liquor once their properties are provided from different industries.[2,3] Figure 1 shows the schematic diagram of this MEE system with mixed feed arrangement, including five evaporators, a flash tank, and four mixers. The fresh steam enters Effect 1, and the subsequent effects utilize the latent heat of outlet steam obtained from the previous effect to heat the black liquor. The black liquor is fed to Effect 3 followed by Effects 4 and 5, and is then sent back to Effects 1 and 2. The flash tank can produce steam from part of the condensate discharged by Effect 1. This steam is mixed with the outlet steam of Effect 2 and used as a heating medium in Effect 3. The condensates from the other effects are mixed and then disposed of. The pressure difference between two adjoining effects has to be controlled in a reasonable range to guarantee that the boiling point of water is met.

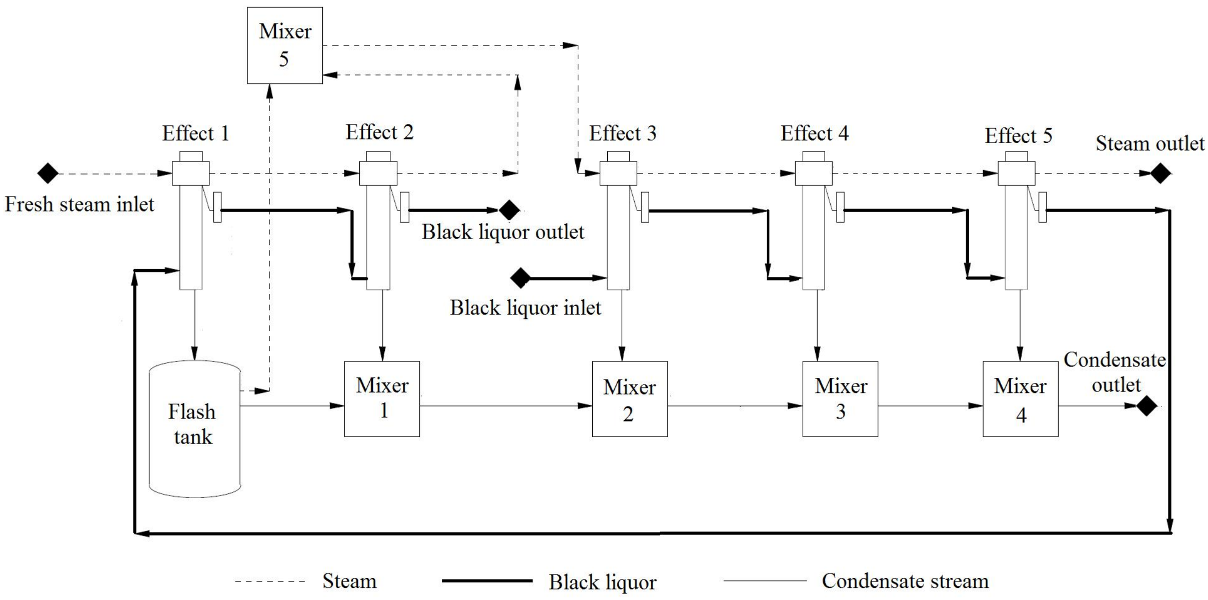


Figure 1. Schematic diagram of an MEE process with mixed feed arrangement

The black liquor MEE problem addressed in this paper is based on the following descriptions.

Given:

1. The topology of the MEE system.
2. A set of evaporators (effects) I (i=1, 2, ..., I) with constant heat transfer areas and heat losses, and the variable heat transfer coefficients relying on steam and black liquid properties, such as temperatures, flow rates, and concentrations.
3. A set of heat exchangers E (e=1, 2, ..., E) with constant heat losses and minimum temperature differences.
4. A flash tank.
5. A set of mixers M (m = 1, 2, ..., M).
6. The specific enthalpy correlations of steams and condensates at different pressures and temperatures.
7. The specific enthalpy correlations, boiling point rise (BPR) correlations, and specific heat correlations of black liquor at different temperatures and concentrations.

8. The status of fresh steam, i.e. temperature and pressure.
9. Inlet data of black liquor, i.e. flow, temperature, and concentration.

The simulation and optimization of the MEE system is to determine:

1. The operating schemes for each evaporator, such as evaporator pressures and temperatures, steam temperatures and flow rates, condensate temperatures and flow rates, and black liquor temperatures, flow rates and concentrations.
2. Operation of the flash tank, such as feeding steam to a suitable evaporator.
3. The strategy of pre-heating black liquor by selecting appropriate effect outlet streams.

The objective of simulation is to find a feasible solution based on the given temperatures and flow rates of fresh steam. While the objective of optimization is to minimize fresh steam consumption to achieve the expected outlet concentration of black liquor, or to maximize the outlet concentration of black liquor in the condition of constant fresh steam flow rate.

3. DETAILED MODELLING OF A MEE SYSTEM

As discussed in the introduction, a great number of non-linear terms associated with energy and mass balances, heat-transfer coefficients, and BPRs, have to be addressed when formulating MEE problems. These non-linear terms lead to a very complex MINLP problem including exponential equations and polynomial equations. The proposed model includes five parts, namely the calculation of material thermodynamic properties, evaporator operations, flash tank operations, condensate mixing, and heat integration, which is described in detail as follows:

3.1. Material Thermodynamic Properties

The materials related to an evaporator involve black liquors, steams and condensate. The thermodynamic properties of these materials vary with pressure, temperature and concentration, and must be taken into account in the evaporating processes.

The thermodynamic properties of black liquors include specific heat (BCP), specific enthalpy (BH), and BPR. These properties are the functions of black liquor temperature (BT), black liquor concentration (BX) and operating pressure (P), which can be expressed as:

$$BCP = f_{BCP}(BT, BX) \quad (1)$$

$$BH = f_{BH}(BT, BX) \quad (2)$$

$$BPR = f_{BH}(BH, BX, P) \quad (3)$$

The formulations of the above functions will vary with different types of black liquor (such as Kraft black liquor, and paper board black liquor), and can be regressed from practical industrial data.

Saturated steam specific enthalpy (SH) and condensate specific enthalpy (WH) are functions of temperature (T). The specific enthalpy (SH') of superheated steam is a function of pressure (P) and temperature (T). eqs 4-6 commonly formulated as non-linear equations, can be derived from the reference (20).

$$SH = f_{SH}(T) \quad (4)$$

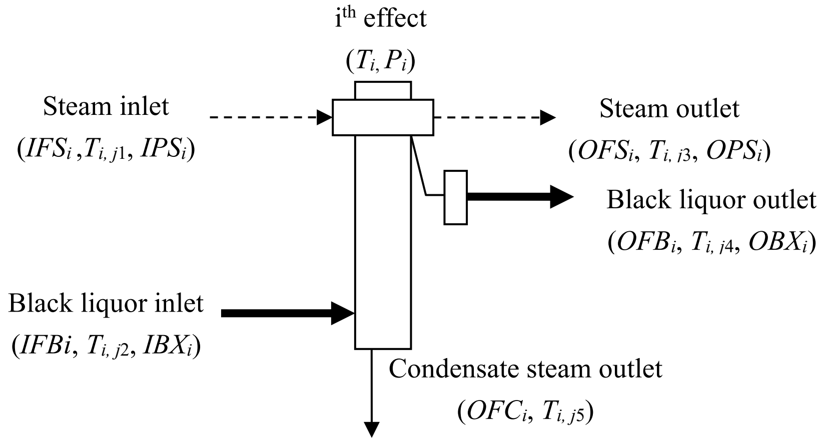
$$\mathrm{WH} = f_{WH}(T) \quad (5)$$

$$SH^{'} = f_{SH^{'}}(P, T) \quad (6)$$

3.2. Evaporator Modelling

The performance of an evaporator is determined by its operating temperature and pressure, as well as the material temperatures, concentrations and flow rates. Figure 2 is a schematic diagram of an evaporator including two input flows (namely inlet steam and black liquor), and three output flows (outlet steam, black liquor and condensate). The factors considered for evaporator operation are equipment temperature (T_i) and pressure (P_i); inlet steam flow rate

(IFS_i), temperature ($T_{i,j1}$) and pressure (IPS_i); outlet steam flow rate (OFS_i), temperature $T_{i,j3}$) and pressure (OPS_i); inlet black liquor flow (IFB_i), temperature ($T_{i,j2}$) and concentration (IBX_i); outlet black liquor flow (OFB_i), temperature (($T_{i,j4}$) and concentration



(OBX_i); and condensate flow (OFC_i) and temperature ($T_{i,j5}$).

Figure 2. Schematic diagram of an evaporator

Modelling of the evaporator includes two parts. First, the material constraints between different evaporators are considered, and then the relationship of materials are restricted in the same evaporator.

- Material constraints between different evaporators

Except in Effect 1, the outlet steam from the previous effect is fed to the next effect as the inlet steam, and thus the inlet steam (IFS_i) in Effect *i* (*i*>1) is the sum of the outlet steam (OFS_{i-1}) in the previous effect and the steam from flash tank ($FFSI_i$), as shown in eqs 7-9, where *I* is the set of all evaporators:

$$IFS_I = OFS_{i-1} + FFSI_i\,, i \in I,\ \ ord(i) \geq 2 \tag{7}$$

$$T_{i,j1} = T_{i-1,j3}\,, \qquad i \in I, \quad ord(i) \geq 2 \tag{8}$$

$$IPS_i = OPS_{i-1}\,, \quad i \in I,\ \ ord(i) \geq 2 \tag{9}$$

eqs 10-12 illustrate the process where black liquor, processed by the previous evaporator i, will enter the subsequent evaporators ($i^{'}$) for evaporation.

$$OFB_i = IFB_{i^{'}}\ ,\ i,\ i^{'} \in I \tag{10}$$

$$T_{i,j4} = T_{i^{'},j2}\ ,\ i,\ i^{'} \in I \tag{11}$$

$$OBX_i = IBX_{i^{'}},\ i,\ i^{'} \in I \tag{12}$$

- Material constraints in an evaporator

As shown in Figure 2, the constraints of material temperatures and pressures in an effect are presented as eqs 13-15.

$$T_i = T_{i,j3},\ i \in I \tag{13}$$

$$T_i = T_{i,j4},\ i \in I \tag{14}$$

$$P_i = OPS_i,\ i \in I \tag{15}$$

Eq 16 describes that if an effect is not fed with fresh steam, the impact of BPR from the previous effect (BPR_{i-1}) must be considered on the condensate temperature ($T_{i,j5}$) of the current effect.

$$T_{i,j5} = \begin{cases} T_{i,j1}\ , & if\ ord(i) = 1 \\ T_{i,j1} - BPR_{i-1}, & if\ ord(i) \geq 2 \end{cases} \quad \mathrm{i} \in \mathrm{I} \tag{16}$$

The mass balance in an evaporator includes, inlet steam and outlet condensate (eq 17), inlet black liquor, outlet black liquor, and outlet steam (eq 18), and the constant solid mass in black liquor (eq 19).

$$IFS_i = OFC_i\ ,\quad i \in I \tag{17}$$

$$IFB_i = OFB_i + OFS_i\ ,\quad i \in I \tag{18}$$

$$IFB_i \times IBX_i = OFB_i \times OBX_i\ ,\quad i \in I \tag{19}$$

The energy balance in an effect is shown in eq 20, where QL_irepresents the heat loss of the evaporator i; ISH_i and ISH_i represent the specific enthalpies of the inlet and outlet steams, respectively; IBH_i and OBH_i represent the specific enthalpies of the inlet and outlet black liquor.

$$(1 - QL_i) \times (IFS_i \times ISH_i - OFC_i \times WH_i) = OFS_i \times OSH_i + OFB_i \times OBH_i - IFB_i \times IBH_i\ , i \in I \quad (20)$$

The heat transfer coefficient (U_i) of an evaporator commonly varies with inlet steam temperature ($T_{i,j1}$) and flow rate (IFS_i), black liquor concentration (IBX_i, OBX_i) and flow rate (IFB_i, OFB_i).

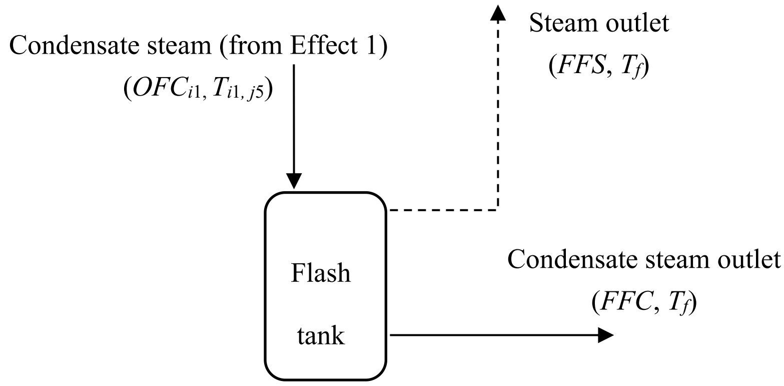
$$U_i = f_U(T_{i,j1}, IFS_i, IBX_i, OBX_i, IFB_i, OFB_i)\ , i \in I \quad (21)$$

Based on the above heat transfer coefficient (U_i), the heat transfer equation of evaporators can be formulated in eq 22. A_i is the heat transfer area of the effect (i).

$$(1 - QL_i) \times (IFS_i \times ISH_i - OFC_i \times WH_i) = U_i \times A_i \times (T_{i,j1} - T_{i,j4} - BPR_{i-1}),\ i \in I \quad (22)$$

3.3. Flash Tank

As mentioned in Section 2 and Figure 1, Effect 1 discharges the condensate with the highest pressure compared with the rest of the effects. The function of a flash tank is to flash the high-pressure condensate to a reduced-pressure steam. This reduced-pressure steam is then used to supply heat to a low-pressure effect (see Figure 3). The operation of the flash tank is described as follows:



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gure 3. Schematic diagram of a flash tank

The input stream to the flash tank is the condensate from Effect 1. The operating status of the flash tank, including the flow rates of outlet steam (FFS) and condensate (FFC), the temperatures (T_f) of outlet flashing steam and condensate, is equal to the operating status of the effect which the flashed steam will be fed to. To describe the operation of flash tank feeding its steam to a suitable evaporator, a binary variable, FI_i, is introduced. It is equal to 1, if the flashing steam is supplied to the Effect (*i*); otherwise, it is 0. eq (23) restricts that the flash tank can feed steam to only one evaporator.

$$\sum_{\substack{i \in I \\ ord(i) \geq 2}} FI_i = 1\,, \qquad (23)$$

When the flashing steam is sent to Effect *i*, the status of the flashing steam is the same as the inlet steam of the evaporator *i*, and thus the operating temperature (T_f) and pressure can be determined, as described in eqs 24 and 25, where M is a sufficiently large positive number. The flashing steam enthalpy ($FSIH_i$) is the correlation of its flow rate (*FFS*) and steam enthalpy (OSH_i), as shown in eqs 26-28. eqs 29-31 express the enthalpy of the outlet condensate ($FCIH_i$). The flow rate of outlet steam ($FFSI_i$) to the suitable Effect (*i*) is formulated in eqs 32-34.

$$T_f \geq T_{i,j1} - M \times (1 - FI_i) \quad i \in I, \quad ord(i) \geq 2 \tag{24}$$

$$T_f \leq T_{i,j1} + M \times (1 - FI_i) \quad i \in I, \quad ord(i) \geq 2 \tag{25}$$

$$FSIH_i \geq FFS \times OSH_i - M \times (1 - FI_i) \qquad i \in I, \ ord(i) \geq 2 \tag{26}$$

$$FSIH_i \leq FFS \times OSH_i + M \times (1 - FI_i) \qquad i \in I, \ ord(i) \geq 2 \tag{27}$$

$$FSIH_i \leq M \times FI_i \ i \in I, \ ord(i) \geq 2 \tag{28}$$

$$FCIH_i \geq FFC \times WH_i - M \times (1 - FI_i) \qquad i \in I, ord(i) \geq 2 \tag{29}$$

$$FCIH_i \leq FFC \times WH_i + M \times (1 - FI_i) \qquad i \in I, ord(i) \geq 2 \tag{30}$$

$$FCIH_i \leq M \times FI_i \qquad i \in I, ord(i) \geq 2 \tag{31}$$

$$FFSI_i \geq FFS - M \times (1 - FI_i) \qquad i \in I, ord(i) \geq 2 \tag{32}$$

$$FFSI_i \leq FFS + M \times (1 - FI_i) \qquad i \in I, ord(i) \geq 2 \tag{33}$$

$$FFSI_i \leq M \times FI_i \qquad i \in I, ord(i) \geq 2 \tag{34}$$

Based on the above constraints, the mass balance eq 35 and energy balance eq 36 in the flash tank can be presented as follows:

$$OFC_{i1} = FFS + FFC \tag{35}$$

$$OFC_{i1} \times WH_{i1} = \sum_{\substack{i \in I \\ ord(i) \geq 2}} (FSIH_i + FCIH_i) \tag{36}$$

3.4. Condensate Mixing

The condensates, produced by evaporators and the flash tank, are mixed and finally disposed of. The condensates pass from Mixer 1 to Mixer 4 in sequence. The mass balance and energy balance for condensate mixing can be expressed in eqs 37-40. In these equations, FM_m *and* TM_m represent the outlet condensate flow rates and temperatures, respectively.

$$FM_m = FFC + \ OFC_i \tag{37}$$

$$FM_m = FM_{m-1} + \ OFC_i, \ ord(m) = ord(i) + 1, \ \ 2 \leq ord(m) \leq 4, i \in I, m \in M \tag{38}$$

$$FM_m \times TM_m = FFC \times T_f + OFC_i \times T_{i,j5},\ ord(m) = 1, ord(i) = 2, i \in I, m \in M \quad (39)$$

$$FM_m \times TM_m = FM_{m-1} \times TM_{m-1} + \ OFC_i \times T_{i,j5}\ ,$$

$$ord(m) = ord(i) + 1,\ \ 2 \leq ord(m) \leq 4, i \in I, m \in M \quad (40)$$

3.5. Heat Recovery

Based on the MEE system studied in this paper (see Figure 1), it can be noted that the black liquor discharged from Effect 5 will enter Effect 1. Since, the largest temperature difference occurs between Effect 1 and 5 in the addressed MEE system which leads to significant heat consumption, substantial energy savings will be achieved if the temperature of black liquid fed to Effect 1 can increase by pre-heating. In the new model, some streams are selected for pre-heating the black liquor before it enters Effect 1, such as using the outlet black liquor of Effect 2, and the outlet condensate of Effect 5. Therefore, heat exchangers are utilized for heat recovery.

Binary variable, HX_{ex1}, restricts the heat transfer between the outlet black liquor from Effect 5 and the outlet condensate from Mixer 4. HX_{ex1} is equal to 1, if the black liquor from Effect 5 exchanges heat with the final condensate discharged from Mixer 4; otherwise, it is 0. eqs 41-44 constrain the outlet temperatures of cold stream (COT_{ex1}) and hot stream (HOT_{ex1}) of heat exchanger *ex1*. The minimum temperature difference (ΔT_{ex1}^{min}) in a counterflow heat exchanger is shown in eq 45 and eq 46. The heat transfer of *ex1* is formulated as eq 47, where QL_{ex1} is the heat loss, and M is a sufficiently large positive number.

$$COT_{ex1} \geq T_{i,j4} - M \times HX_{ex1}\ ,\ ord(i) = 5, i \in I \quad (41)$$

$$COT_{ex1} \leq T_{i,j4} + M \times HX_{ex1}\ ,\ ord(i) = 5, i \in I \quad (42)$$

$$HOT_{ex1} \geq TM_m - M \times HX_{ex1}\ ,\ ord(m) = 4, m \in M \quad (43)$$

$$HOT_{ex1} \leq TM_m + M \times HX_{ex1}\ ,\ ord(m) = 4, m \in M \quad (44)$$

$$TM_m - COT_{ex1} \geq \Delta T_{ex1}^{min} - M \times (1 - HX_{ex1})\ ,\ ord(m) = 4, m \in M \quad (45)$$

$$HOT_{ex1} - T_{i,j4} \geq \Delta T_{ex1}^{min} - M \times (1 - HX_{ex1}),\ ord(i) = 5, i \in I \quad (46)$$

$$OFB_i \times BCP_i \times (COT_{ex1} - T_{i,j4}) = (1 - QL_{ex1}) \times FM_m \times CP_m \times (TM_m - HOT_{ex1}),$$
$$ord(m) = 4, ord(i) = 5, i \in I, m \in M \quad (47)$$

Similar to heat exchanger *ex1*, heat exchanger *ex2* can be used to exchange the heat between the outlet black liquor from *ex1* and the outlet black liquor from Effect 2. Binary variable HX_{ex2} is equal to 1, the outlet black liquors from *ex1* exchanges heat with the outlet black liquor from Effect 2; otherwise, it is 0.

$$COT_{ex2} \geq COT_{ex1} - M \times HX_{ex2} \quad (48)$$

$$COT_{ex2} \leq COT_{ex1} + M \times HX_{ex2} \quad (49)$$

$$HOT_{ex2} \geq T_{i,j4} - M \times HX_{ex2} \quad (50)$$

$$HOT_{ex2} \leq T_{i,j4} + M \times HX_{ex2} \quad (51)$$

$$T_{i,j4} - COT_{ex2} \geq \Delta T_{ex2}^{min} - M \times (1 - HX_{ex2}), \quad ord(i) = 2, i \in I \quad (52)$$

$$HOT_{ex2} - COT_{ex1} \geq \Delta T_{ex2}^{min} - M \times (1 - HX_{ex2}) \quad (53)$$

$$OFB_{i5} \times BCP_{i5} \times (COT_{ex2} - COT_{ex1}) = (1 - QL_{ex2}) \times OFB_{i2} \times BCP_{i2} \times \left(T_{i2,j4} - HOT_{ex2}\right) \quad (54)$$

If the black liquor entering Effect 1 is pre-heated, the inlet black liquor temperature ($T_{i,j2}$) is equal to cold stream temperature (COT_{ex2}), as shown in eq 55:

$$T_{i,j2} = COT_{ex2},\ ord(i) = 1, i \in I \quad (55)$$

3.6. Objective Functions

The overall operating cost of multiple effect evaporator systems is mainly affected by the energy consumption. Therefore, energy consumption is chosen as the key issue of MEE optimization in this work. The objective of our new model is to minimize the fresh steam consumption under a certain outlet concentration of black liquor, or to maximize the final outlet concentration of black liquor with the given fresh steam supplied.

The status of black liquor entering the MEE system is shown in eqs 56-58, where $BF^{initial}, BT^{initial}, BX^{initial}$ represent the initial flow rate, temperature, and concentration of black liquor.

$$IBF_i = BF^{initial},\ ord(i) = 1, i \in I \quad (56)$$

$$T_{i,j2} = BT^{initial},\ ord(i) = 1, i \in I \quad (57)$$

$$IBX_i = BX^{initial},\ ord(i) = 1, i \in I \quad (58)$$

If the outlet concentration of black liquor ($BX^{initial}$) is known, the minimum consumption of fresh steam (IFS_{i1}) can be formulated:

$$OBX_i = BX^{final},\ ord(i) = 2, i \in I \quad (59)$$

$$Min\ IFS_{i1}$$

If the flow rate of fresh steam (IFS_{i1}) is fixed, the maximization of the concentration of outlet black liquor (OBX_{i2}) is described as follows:

$$Max\ OBX_{i2}$$

The new model of the MEE system is composed of eqs 1-59 and the objective function. This new model includes evaporation, flashing, streams mixing and heat integration, and considers variations in material thermodynamic properties and evaporator performance. As the material thermodynamic properties eqs 1-6, the heat-transfer coefficients eq 21 are nonlinear equations, the MEE problem is described as a complex and large-scale MINLP problem. It is difficult to obtain optimal solutions or even feasible solutions in a reasonable time by using conventional methods.

4. ITERATION ALGORITHMS FOR SOLVING DETAILED MEE MODELS

It is a great challenge to deal with the non-linearities in MEE problems. Non-linear correlations in the new model include material thermodynamic properties (eqs 1-6), energy balance (eqs 20-22, 47-54), and condensate mixing (eq 39 and eq 40). Recently, the MILP-based iteration approach has become an attractive method from a computational perspective versus the MINLP approach [26-29]. This paper proposes a two-stage iteration approach based on MEE operations as shown in Figure 4. In the inner loop (Loop 1), the original model is converted to a new MILP model and solved iteratively to find a feasible solution (namely simulation) under certain MEE conditions (such as given input black liquor and fresh steam). While for optimization, the fresh steam consumption (IFS_{i1}) is estimated at first and reduced gradually in the outer loop (Loop 2), the inner loop (Loop 1) is then executed repeatedly to find feasible solutions under the above updated IFS_{i1}. The details of each loop are presented as follows:

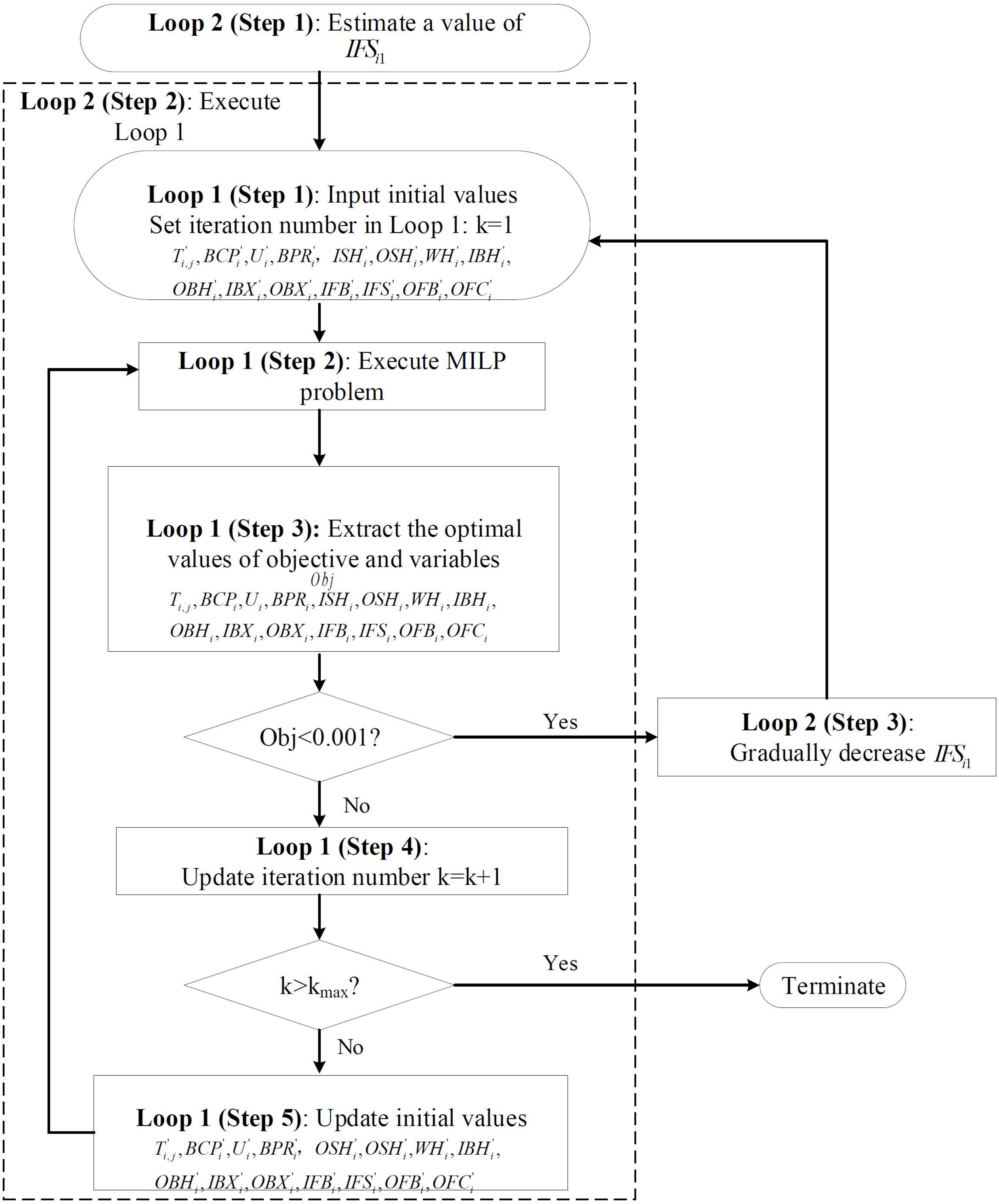


Figure 4. The procedure of the iteration optimization algorithm

4.1. Loop 1: MEE Simulation

Since heat exchanger constraints eqs 41-55 are not included in the simulation of the MEE system, the flow rate and temperature of mixed condensate can be calculated directly based on the status of each effect condensate. Therefore, the equations (eqs 37-40) for condensate mixing can be neglected. In summary, the first stage problem consists of eqs 1-36, 56-59, and the objective function. Figure 4 presents the procedure of the iteration algorithm. The detailed steps are described as follows:

Step 1: Set the initial time (*k*) for Loop 1: k = 1. It is assumed that the following information is given based on the practical MEE systems:

(1) The temperature differences between joint effects (namely, *i* and *i*+1) are 10°C.

(2) The flow rates of outlet steams ($OFS_i^{'}$) in each effect are the same, which is the mean of total steam flow rate.

(3) The consumption of fresh steam ($IFS_{i1}^{'}$).

Based on the above assumptions, the initial values of the following variables can be calculated:

(1) The flow rates and concentrations of black liquors ($IFB_i^{'}$, $OFB_i^{'}$, $IBX_i^{'}$, $OBX_i^{'}$) in eqs 10, 12, 18 and 19.

(2) The temperatures of steams and black liquors ($T_{i,j1}^{'}$, $T_{i,j2}^{'}$, $T_{i,j3}^{'}$, $T_{i,j4}^{'}$) in eqs 8, 11, 13 and 14.

(3) The flow rate of the fresh steam ($IFS_i^{'}$) in eq 7 and eq 18.

(4) The black liquor specific heats, specific enthalpies ($IBCP_i^{'}$, $OBCP_i^{'}$, $IBH_i^{'}$, $OBH_i^{'}$), and $BPR_i^{'}$ in eqs 1-3.

(5) The specific enthalpies of steams and condensates ($ISH_i^{'}$, $OSH_i^{'}$, $IBH_i^{'}$, $WH_i^{'}$) in eqs 4-6.

(6) The heat-transfer coefficients ($U_i^{'}$) in eq 21.

Thus, the heat transfer equation, eq 20 can be converted to a linear relationship, as shown in eq 60:

$$(1 - QL_i) \times (IFS_i \times ISH_i^{'} - OFC_i \times WH_i^{'}) = OFS_i \times OSH_i^{'} + OFB_i \times OBH_i^{'} - IFB_i \times IBH_i^{'}, \quad i \in I \qquad (60)$$

The heat transfer constraint eq 22 are also linearized, as presented in eq 61:

$$(1 - QL_i) \times (IFS_i \times ISH_i^{'} - OFC_i \times WH_i^{'}) = U_i^{'} \times A_i \times (T_{i,j1} - T_{i,j4} - BPR_{i-1}), \ i \in I \quad (61)$$

Based on the above descriptions, eqs 1-6 and 20 can be omitted, and thus the first stage problem containing a large number of non-linear relationships is converted into a simpler MILP problem. The MILP problems consist of eqs 7-19, 23-36, 56-61, and its objective function (eq 62) is to minimize the total differences between initial and up-dated variables.

$$\min obj = \sum differences\ between\ initial\ and\ updated\ variables \qquad (62)$$

Step 2: Solve the above MILP model.

Step 3: Obtain the results of MILP problem ($T_{i,j}$, BCP_i, BPR_i, U_i, ISH_i, OSH_i, WH_i, IBH_i, OBH_i, IBX_i, OBX_i, IFB_i, OFB_i, IFS_i, OFC_i).

Step 4: Once the objective value (obj) is less than the tolerance, a feasible solution is found; otherwise, up-date the iteration number (k) for Loop 1: k=k+1;

Step 5: If the iteration number (k) exceeds the specified maximum iteration number (namely no feasible solution is found under current fresh steam consumption (IFS_{i1})), Loop 1 will stop; otherwise, up-date the initial values of variable in the MILP problem as eq 63, and execute Steps 2-5 iteratively.

$$(T'_{i,j}, U'_i, ISH'_i, OSH'_i, WH'_i, IBH'_i, OBH'_i) = \frac{(T'_{i,j}, U'_i, ISH'_i, OSH'_i, WH'_i, IBH'_i, OBH'_i) + (T_{i,j}, U_i, ISH_i, OSH_i, WH_i, IBH_i, OBH_i,)}{2} \quad (63-1)$$

$$(BCP'_i, IBX'_i, OBX'_i) = \frac{\left(BCP'_i, IBX'_i, OBX'_i\right) + (BCP_i, IBX_i, OBX_{i,})}{2} \qquad (63-2)$$

4.2. Loop 2: MEE Optimization

The second stage problem is to find the optimal scheme for the first stage problem. This problem includes heat integration constraints (eqs 41-55) and condensate mixing constraints (eqs 37-40), where eqs 39, 40, 47 and 54 are non-linearities. Steps for Loop 2 are as follows:

Step 1: Estimate the fresh steam consumption (IFS_{i1}).

Based on Loop 1, the temperature and flow rate of the outlet condensate (TM'_{m4} , FM'_{m4}), the flow rates and specific heats of the outlet black liquors from Effect 2 and 5 ($OFB'_{i5}, BCP'_{i5}, OFB'_{i2}, BCP'_{i2}$) can be calculated, and thereby eq 47 and eq 54 are linearized to eq 64 and eq 65, where heat loss of the heat exchangers (QL_{ex1}, QL_{ex2}) and the specific heat of condensate (CP_{m4}) are constant.

$$OFB'_{i5} \times BCP'_{i5} \times \left(COT_{ex1} - T_{i5,j4}\right) = QL_{ex1} \times FM'_{m4} \times CP_{m4} \times \left(TM'_{m4} - HOT_{ex1}\right) \qquad (64)$$

$$OFB'_{i5} \times BCP'_{i5} \times (COT_{ex2} - COT_{ex1}) = QL_{ex2} \times OFB'_{i2} \times BCP'_{i2} \times \left(T_{i2,j4} - HOT_{ex2}\right) \qquad (65)$$

As the specific enthalpy of black liquor increases with the black liquor temperature when the concentration of black liquor is constant, the correlation of specific enthalpy and temperature can be expressed as a linear relationship, where A_{i1} and B_{i1} are constant parameters.

$$IBH_{i1} = A_{i1} \times T_{i,j1} + B_{i1} \qquad (66)$$

Based on the solutions of the first stage problem, eqs 7-19, 23-36, 41-46, 48-53 and 55-66 consist of the second stage problem.

Step 2: Execute Loop 1 based on the estimated fresh steam consumption (IFS_{i1}) in Step 1.

Step 3: If a feasible solution can be found in Loop 1, decrease the value of IFS'_{i1} gradually, and return to Step 2. Once no feasible solution can be obtained in Loop 1, Loop 2 can stop, and the approximate minimum fresh steam consumption is determined.

5. CASE STUDIES

An industrial process from a Chinese Paper Mill is carried out to demonstrate the validity of the new model and the efficiency of the proposed algorithm. The flow chart of the example is shown in Figure 1. The proposed iteration algorithm is performed in GAMS 24.4.6 on an Intel(R) Core ™ i7-6700 CPU @4.00 GHz and 16 GB RAM. The solver CPLEX is used to solve the MILP problems. The characterization of model complexity and computing effort in the case studies are summarized in Table 1.

Table 1. Characterization of model complexity and computing effort in case studies

		Iterations	CPU Times(s)	Continuous variables	Binary variables	Constraints
5.1. Comparisons with WinGEMS®	Scenario 1	28	12	150	0	152
	Scenario 2	33	11	150	0	152
	Scenario 3	34	12	150	0	152
	Scenario 4	35	10	150	0	152
	Scenario 5	34	11	150	0	152
5.2. Practical Applications	Base case	47	28	170	1	414
	Optimization	42	23	185	2	360

5.1. Comparisons with WinGEMS®

WinGEMS® is a well-known commercial simulation tool used in the pulp and paper industry. It can simulate MEE processes, provides reasonable assessments, and support faster and more accurate planning and decision-making. To simulate black liquor MEE systems with WinGEMS®, the heat-transfer coefficients and BPRs in all effects must be known and constant. Our model uses the same black liquid thermodynamic database embedded in WinGEMS® for fair comparison.

In this case, the information known regarding the black liquid fed to the MEE system includes 60°C of inlet temperature, 30 t/h of flow rate, and 15% of solid concentration; the temperature of saturated fresh steam is 110°C; the heat transfer area of each effect is 1000 m^2 with 10% heat loss; and BPR in each effect is 2°C. The constant fresh steam consumptions and heat-transfer coefficients of effects are given for five distinct scenarios in Table 2. WinGEMS® can calculate evaporator temperatures, the temperature, flowrate and concentration of the black liquid discharged from the MEE system, and the liquid and steam flowrates in each effect. As shown in Tables 3 and 4, the proposed approach and WinGEMS® perform identically in all cases, which demonstrates the validity of our model rather than presents a sensitivity analysis.

Table 2. Fresh steam consumptions and heat-transfer coefficients in each scenario

Scenarios	Fresh steam flow (t/h)	Heat transfer coefficient (kJ/m^2 ·h·°C)				
		Effect 1	Effect 2	Effect 3	Effect 4	Effect 5
1	4.0	1394.5	1200.0	2962.3	1200.0	1200.0
2	5.0	1392.0	1200.0	1884.1	1200.0	1200.0
3	6.0	1380.2	1200.0	1530.1	1200.0	1200.0
4	7.0	1362.5	1200.0	1362.8	1200.0	1200.0
5	7.7	1347.2	1200.0	1294.8	1200.0	1200.0

Table 3. Evaporator temperatures, and the temperatures, flowrates and concentrations of final black liquor obtained by our method and WinGEMS® in each scenario

Scenarios		Evaporator temperature (ºC)					Final black liquor		
		Effect 1	Effect 2	Effect 3	Effect 4	Effect 5	Temperature (ºC)	Flowrate (t/h)	Concentration (%)
1	Our method	104.2	97.6	93.6	90.1	86.4	97.6	21.57	20.86
	WinGEMS	104.3	97.6	93.6	90.1	86.4	97.6	21.58	20.85
2	Our method	102.8	94.9	89.0	84.0	78.9	94.9	17.72	25.39
	WinGEMS	102.8	94.9	89.0	84.0	78.9	94.9	17.73	25.38
3	Our method	101.3	92.0	84.3	77.7	71.1	92.0	13.71	32.82
	WinGEMS	101.3	92.1	84.3	77.7	71.1	92.1	13.72	32.80
4	Our method	99.7	89.0	79.4	71.1	62.9	89.0	9.53	47.22
	WinGEMS	99.7	89.1	79.4	71.2	62.9	89.1	9.51	47.32
5	Our method	98.5	86.8	75.8	66.4	57.0	86.8	6.49	69.34
	WinGEMS	98.6	86.8	75.8	66.4	57.0	86.8	6.50	69.23

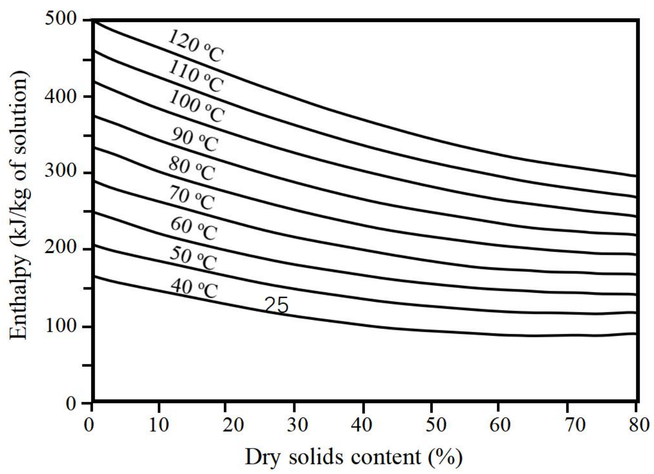
Table 4. Flowrates (t/h) of steam and black liquor between evaporators obtained by our method and WinGEMS® in each scenario

Scenarios		Effect 1→ Effect 2		Effect 2→ Effect 3	Effect 3→ Effect 4		Effect 4→ Effect 5		Effect 5→ Effect 1
		Steam	Black liquor	Steam	Steam	Black liquor	Steam	Black liquor	Black liquor
1	Our model	2.77	24.32	2.86	0.89	29.11	0.97	28.13	27.09
	WinGEMS	2.77	24.33	2.85	0.89	29.11	0.97	28.14	27.10
2	Our model	3.50	21.11	3.55	1.75	28.25	1.80	26.45	24.61
	WinGEMS	3.50	21.13	3.55	1.75	28.25	1.80	26.45	24.62
3	Our model	4.27	17.78	4.29	2.64	27.36	2.66	24.70	22.05
	WinGEMS	4.27	17.78	4.29	2.64	27.36	2.66	24.70	22.05
4	Our model	5.10	14.30	5.07	3.58	26.42	3.55	22.87	19.40
	WinGEMS	5.10	14.29	5.06	3.57	26.43	3.55	22.88	19.40
5	Our model	5.71	11.77	5.64	4.26	25.74	4.19	21.55	17.49
	WinGEMS	5.71	11.78	5.64	4.26	25.74	4.19	21.55	17.49

5.2. Practical Applications in Industry

Unlike the assumptions used in WinGEMS®, varying thermodynamic properties of black liquors, BCPs, heat-transfer coefficients, and BPRs must be considered in practical operating conditions:

1. The thermodynamic properties of black liquor are determined based on the regression of



practical process data. The specific enthalpy correlations of black liquor at different temperatures and concentrations in this case are shown in Figure 5.

Figure 5. Correlations between the specific enthalpy, temperature and concentration of black liquor

2. The specific heat relationships of black liquors (BCP) under different temperatures (BT), concentrations (BX) are shown in eq 67.

$$BCP = 4.216(1 - BX) + (1.675 + 0.00331\,BT)BX + (4.87 - 0.02BT)(1 - BX)BX^3 \qquad (67)$$

3. The heat-transfer coefficient correlations are the functions of material properties, flow rates and operating temperatures, as shown in eq 68, which is similar to the correlations reported in Reference (20).

$$U_i = 1200\left(\frac{T_{i,j1}}{100}\right)^{0.64}\left(\frac{IFS_i}{2612}\right)^{0.54}\left(\frac{IBX_i + OBX_i}{0.35}\right)^{-0.521}\left(\frac{IFB_i + OFB_i}{10^5}\right)^{0.0748}, i \in I \qquad (68)$$

4. The BPR correlations are expressed as eq 69.

$$BPR = 6.173BX - 7.48BX^{1.5} + 32.747BX^2 \qquad (69)$$

Since the above conditions cannot be addressed in WinGEMS®, the data from real world industries are used to prove the accuracy of the new approach. The information given includes: the black liquid fed to the MEE system is 80°C of inlet temperature, 50 t/h of flow rate, and 17.5% of solid concentration; fresh steam is the saturated steam at 112°C; the heat-transfer areas of each effect is 500 m^2; the heat losses of Effect 1 and 2 are 20% while the others are 15%; the heat loss of countercurrent heat exchangers is 20% with 10°C of minimum temperature differences; the outlet black liquor concentration is expected to be 23.7%. From Table 5, it is noted that the simulation results of the new model are within the range of the real-world production. Figure 6 depicts the detailed process data derived from the new method.

Table 5. Comparison between real world production and the new method

	Fresh steam flowrate (t/h)	Black liquor concentration (%)			Evaporator heating steam temperature (°C)				
		Effect 3 inlet	Effect 5 outlet	Effect 2 outlet	Effect 1	Effect 2	Effect 3	Effect 4	Effect 5
Real world	6.0~7.0	16.8~18.5	20.8~22.0	22.9~23.8	112~114	99~101	88~91	77~80	64~67
Our method	6.83	17.5	21.2	23.7	112.0	99.3	88.9	77.5	65.8

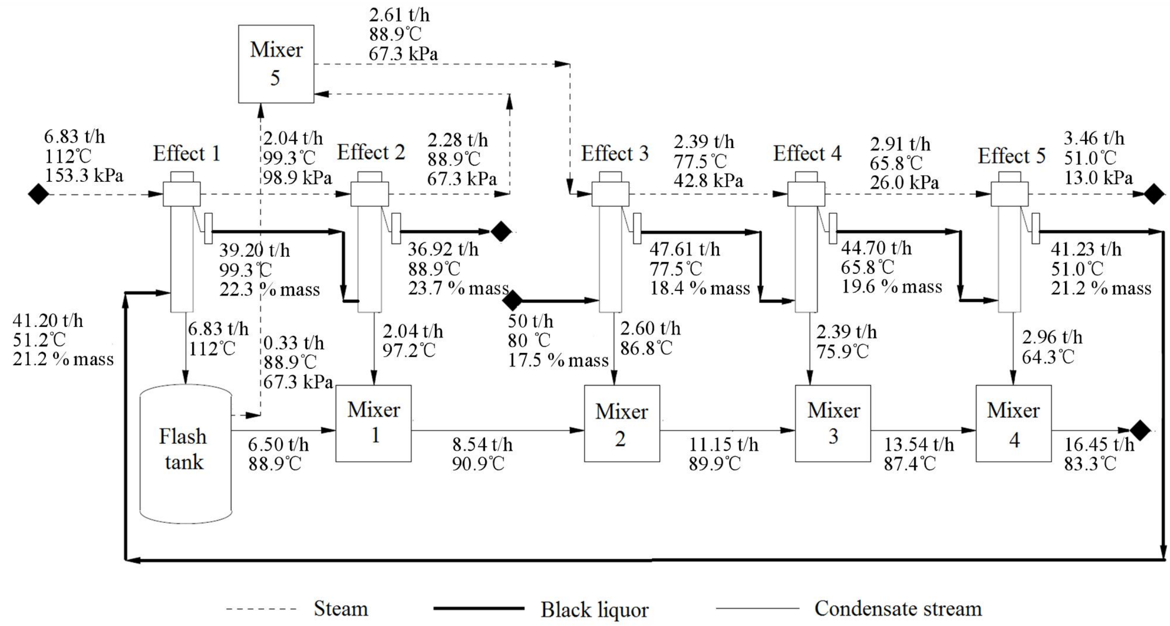


Figure 6. Detailed process simulation data obtained from the new method

As discussed in Section 3.5, there is a large temperature difference (61°C) between the black liquor entering Effect 1 (51°C) and the fresh steam (112°C) in the practical case (see Figure 6). If this temperature difference can be reduced by selecting suitable hot streams to pre-heat the black liquor entering Effect 1, substantial energy saving can be achieved. The optimal system obtained by the new approach is shown in Figure 7. In the optimal scheme, the condensate discharged from Mixer 4, and the final black liquor from Effect 2 exchange heat with the outlet black liquor from Effect 5, which increases the temperature of the black liquor entering Effect 1 from 53.4°C to 74.1°C, thereby reducing the temperature cap between the cold and hot streams in Effect 1 and minimizing fresh steam consumption. The fresh steam consumption of the optimal system is 5.12 t/h, 25% of reduction versus the original system (6.83 t/h). In addition, it is necessary to cool the mixed condensate released from the original system due to its high temperature (83.3°C). The optimal system with two heat exchangers can effectively save the cooling water as its final condensate temperature is much lower (63.4°C).

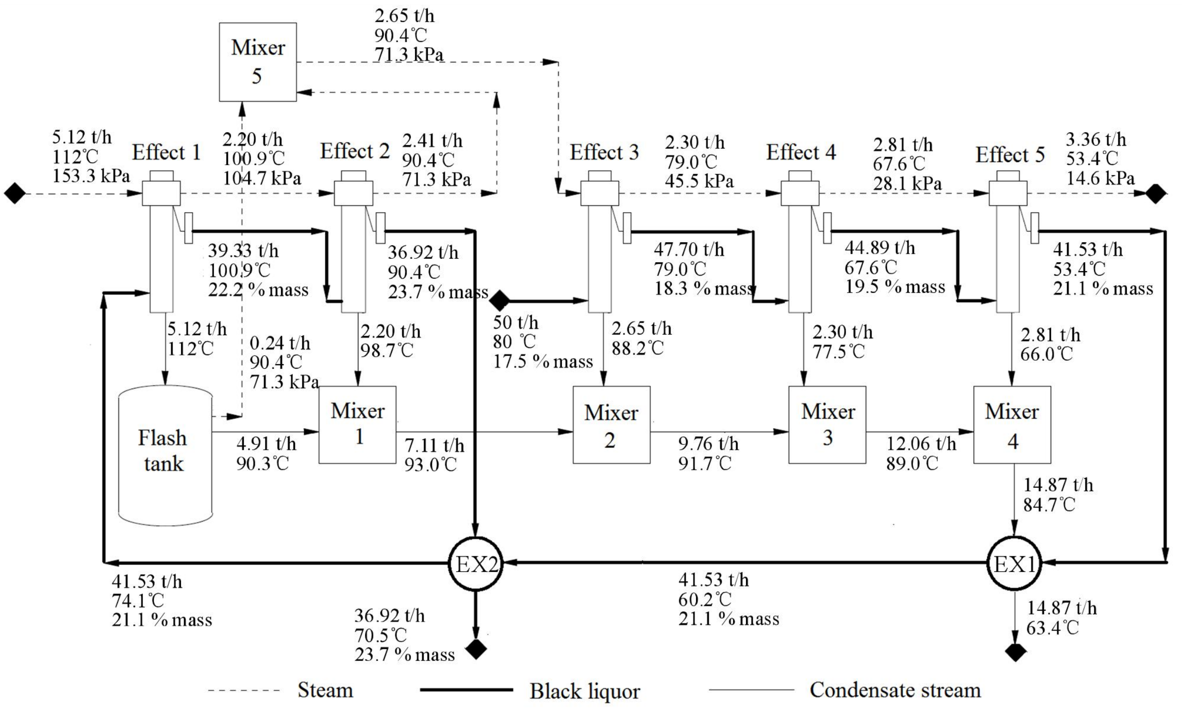


Figure 7. The optimal energy-saving solution achieved with the new model

In our optimal solution, two heat exchangers have been added into the original system for reducing the fresh steam consumption. A simple economic efficiency analysis is presented for the cost of optimal system. The capital cost of exchangers (ECC)[30] is shown in eq 70.

$$ECC = K_0 \times \mathrm{EXA}^{0.8} \qquad (70)$$

In eq 70, K_0 is the cost coefficient of exchanger, and EXA is exchanger area.

The exchanger area can be calculated in eq 71.

$$F \times CP_m \times \Delta \mathrm{T} = U_0 \times \mathrm{EXA} \times \mathrm{LMTD} \quad (71)$$

In eq 71, F is the mass flow rate of hot stream, ΔT is the temperature difference between hot stream inlet and outlet, U_0 is exchanger heat transfer coefficients, and LMTD is a logarithmic average of the temperature difference between the hot and cold streams.

Based on eqs 70 and 71, the total cost of heat exchangers is estimated as $1637. In the condition of 32 $/t of fresh steam price and 8 h/d of operating time, our optimal solution (1.7t/h of fresh steam reduction) requires a short payback period (not more than one week) and can achieve significant economic benefit after the payback period (around 130,000 $/year).

For MEE optimization, some researchers used sensitivity analysis method to find potential optimal MEE configarations.[16,18] The proposed options included the strategies with pre-heaters,[16,18] steam-split[23,25], feed-split[19,22], and condensate flashing.[20,21] The most economic operation was only chosen from the given configurations. Our method is a one-step optimization approach, and can find the optimal solution directly without listing all alternative solutions. Furthermore, the existing approach only addressed preheating the stream in MEE systems by steam-split (namely steam rearrangement), which might lead to high retrofit cost. Our approach utilized the additional heat from the condensate and final stream to preheat the stream in the MEE system, which reduced the retrofit cost and increased the energy saving efficiently.

6. CONCLUSIONS

Energy conservation for MEE systems is a challenging task for the process industries. In this paper, the MILP converted model and iteration algorithm are developed for the simulation and optimization of large-scale MEE problems. The new method features practical

MEE scenarios, such as variable material thermodynamic properties and unit operating conditions, flash tank operation, and black liquor pre-heating. In the proposed approach, a set of non-critical variables are initialized to parameters and up-dated iteratively, converting the non-linear terms into linear terms. Thus, MEE problems can be formulated as complete MILP models which reduces the computational complexities significantly.

A MEE system from a Chinese Paper Mill has been used to show that our approach can perform equally as well as the well-known simulation software WinGEMS® in the simplified operating conditions, and is more capable of practical applications as:

1. WinGEMS® only simulates the process with a specific type of black liquor. However, the new method can address different types of black liquor which are regressed from industrial collections.
2. The heat-transfer coefficients and BPRs are constant in WinGEMS®, which is not common in practice. These variables associated with material properties, flow rates and operating conditions are considered in our approach.
3. WinGEMS® cannot provide optimization for MEE processes, while the new method proposes efficient energy-saving strategies. It has been proved that our approach can find optimal solutions for practical MEE problems, and reduce the energy consumption up to 25%.

It should be noted that, only one type of concentrated liquor (black liquor) is considered for MEE simulation and optimization in this work. Other types of concentrated liquor will be further considered for ensuring the practicality of the proposed approach in more typical process industries. Furthermore, the present methodology is restricted to a given topology and cannot optimize the topology itself. The topology modifications aiming at reducing fresh steam consumption will be taken into account in our future work.

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Nomenclature

Parameters

QL_i	heat loss of an effect
QL_{ex1}	heat loss of heat exchanger 1
QL_{ex2}	heat loss of heat exchanger 2
A_i	heat-transfer area of i^{th} effect (m^2)
M	a sufficiently large number
ΔT_{ex1}^{min}	minimum temperature difference of heat exchanger 1 (°C)
ΔT_{ex2}^{min}	minimum temperature difference of heat exchanger 2 (°C)
CP_m	specific heat capacity of condensate in the m^{ath} mixer (J/kg °C)
A_{i1}	constant parameter for calculating the specific enthalpy of black liquor
B_{i1}	constant parameter for calculating the specific enthalpy of black liquor
$BF^{initial}$	initial flowrate of black liquor (kg/h)
$BT^{initial}$	initial temperature of black liquor (°C)
$BX^{initial}$	initial concentration of black liquor (%)
BX^{final}	final concentration of black liquor (%)
ISH'_i	initial value of inlet steam specific enthalpy in the i^{th} effect (kJ/kg)
WH'_i	initial value of condensate specific enthalpy in the i^{th} effect (kJ/kg)
OSH'_i	initial value of outlet steam specific enthalpy in the i^{th} effect (kJ/kg)
OBH'_i	initial value of outlet black liquor specific enthalpy in the i^{th} effect (kJ/kg)
IBH'_i	initial value of inlet black liquor specific enthalpy in the i^{th} effect (kJ/kg)

U'_i	calculated value of heat-transfer coefficient in the ith effect (W/m^2 °C)
U_0	heat-transfer coefficient in heat exchanger (W/m^2 °C)
K_0	the cost coefficient of plain exchanger
F	the mass flow rate of condensate (kg/h)
TM'_{m4}	calculated value of temperature of condensate from Mixer 4 (°C)

Variables

$T_{i,j}$	temperature of stream in the ith effect (°C)
T_i	temperature of the ith effect (°C)
T_f	temperature of a flash tank (°C)
IFS_i	inlet steam flowrate in the ith effect (kg/h)
OFS_i	outlet steam flowrate in the ith effect (kg/h)
IFB_i	flowrate of inlet black liquor in the ith effect (kg/h)
OFB_i	flowrate of outlet black liquor in the ith effect (kg/h)
OFC_i	flowrate of condensate in the ith effect (kg/h)
FFS	flowrate of flashing steam from a flash tank (kg/h)
$FFSI_i$	flowrate of flashing steam to the ith effect (kg/h)
FFC	flowrate of condensate from a flash tank (kg/h)
IBH_i	specific enthalpy of inlet black liquor in the ith effect (kJ/kg)
OBH_i	specific enthalpy of outlet black liquor in the ith effect (kJ/kg)
ISH_i	specific enthalpy of inlet black liquor in the ith effect (kJ/kg)
OSH_i	specific enthalpy of outlet black liquor in the ith effect (kJ/kg)
WH_i	specific enthalpy of condensate in the ith effect (kJ/kg)
$FSIH_i$	specific enthalpy of flashing steam to the ith effect (kJ/kg)
$FCIH_i$	specific enthalpy of condensate from a flash tank (kJ/kg)
BPR_i	boiling point rise in the ith effect (°C)
BCP_i	specific heat of black liquor in the ith effect (J/kg °C)
IBX_i	concentration of inlet black liquor in the ith effect (%)
OBX_i	concentration of outlet black liquor in the ith effect (%)
IPS_i	pressure of inlet steam in the ith effect (kPa)

OPS_i	pressure of outlet steam in the i^{th} effect (kPa)
P_i	pressure of in the i^{th} effect (kPa)
FM_m	flowrate of outlet condensate in the i^{th} mixer (kg/h)
TM_m	temperature of condensate in the i^{th} mixer (℃)
U_i	heat-transfer coefficient in the i^{th} effect (W/m^2 ℃)
COT_{ex1}	temperature of cold stream in heat exchanger 1 (℃)
HOT_{ex1}	temperature of hot stream in heat exchanger 1 (℃)
COT_{ex2}	temperature of cold stream in heat exchanger 2 (℃)
HOT_{ex2}	temperature of hot stream in heat exchanger 2 (℃)
ΔT	the temperature difference of condensate (℃)
EXA	heat-transfer area of heat exchanger (m^2)

Binary variables

$FI_i = 1$	if the flashing steam is supplied to the i^{th} effect; otherwise it is 0
$HX_{ex1} = 1$	if the black liquor from Effect 5 exchanges heat with the final condensate discharged from Mixer 4; otherwise it is 0
$HX_{ex2} = 1$	if the outlet black liquors from *ex1* exchanges heat with the outlet black liquor from Effect 2; otherwise it is 0

Subscripts

i	i^{th} effect
j	inlet and outlet
m	m^{th} mixer
ex	heat exchanger

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