Design of an optical lens for LED lighting using a hybrid Principal Components Analysis-Taguchi method

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Short title: Lens design using hybrid PCA-Taguchi method

Optical lenses are extensively used to enhance the performance of light-emitting diodes. Both uniformity and efficiency are important performance indicators in lens design, however, improving uniformity always lowers efficiency. In this study, the Taguchi method and Principal Component Analysis (PCA) are integrated to optimize the lens shape for two quality objectives, namely, uniformity and efficiency. The Taguchi method was conducted twice to establish the signal / noise ratio of the two quality characteristics for calculating the principal components in PCA. Then, the optimum parameters obtained by the Taguchi method were processed by PCA. The correlated individual responses were converted to the principal components which explained most of the dataset and were considered as the single quality characteristic for the optimization. The combined method resolved the difficulties of optimizing multiple quality characteristics without sacrificing any particular quality characteristic while the traditional Taguchi method can only be applied to the single quality characteristic. A LED source fitted with a secondary lens designed by the proposed method showed over 92% light efficiency and an improvement of uniformity .

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1. Introduction

As a light source, LEDs have the advantages of a long working life and a low energy consumption as well as being environmental friendly. For these reasons, LEDs have been introduced into lighting systems and have become one of the most important components in many lighting devices. LEDs are now replacing more conventional light sources[1]. However, LEDs are not recommended for use in lighting devices without modification because their light distribution is Lambertian. The light emitted from LED is created by the energy emission from the combination of electrons and holes in the semiconducting materials. When LED arrays are used in the practical applications, the light distribution on the target surface would be incoherent and several light rings would be appeared on the target surface[2]. Usually, the highest luminous intensity of a LED source is concentrated at 90 degrees to the epitaxial axis[3], so distinctive and bright spots are exhibited at the lit surface. Furthermore, the angular extent of light emitted by an unmodified LED is near to ±60 degrees. For the application of LEDs to particular lighting devices such as advertising boxes, road lighting luminaires and television, the light distribution and efficiency of unmodified LEDs are not sufficient to meet the requirements of current products. Therefore, LEDs have to be used with secondary lenses so that the light from the LED itself is redirected by reflective and refractive paths through secondary lenses to generate the desired light distribution with a wider light angle and higher light efficiency.

In the recent years, researchers have proposed different ways to accelerate the optical design process for LED lenses so as to raise the optical performance of lighting devices. Researchers have discussed the ways of changing light paths by refraction or reflection using secondary lenses. A specifically designed freeform lens has been used to achieve the desired light distribution using numerical computations[4]. Also, a new design of freeform reflector-array has been proposed so as to generate the required light distribution[5]. For real applications, a new technique of application specific LED packaging (ASLP) has been employed to improve the optical performance of current road lighting luminaires[6-7].New freeform lenses for extended sources were implemented in ASLP. These lenses introduced the advantages of a combination of ASLP modules and borosilicate glass freeform lenses in road lighting, the performance of the LED road lighting luminaire with ASLP modules showed excellent improvements. Later, the efficiency of a LED headlamp was improved by satisfying the requirements of the standard in which the light distribution is produced as a block shape that falls into the target area[8].

Apart from stepping up the lighting performance by the specific optical design of reflectors and the packaging method, researchers took advantage of statistical approaches to shorten design and manufacturing time for optical lenses. The logic used delivered excellent contributions to the optimization problem. A statistical computation for compensating the backlight image of a digital camera by neuro-fuzzy networks integrated with particle swarm optimization has been proposed[9]. Further, a neural network algorithm and computer simulations have been used to solve multiple variables in injection processes to produce optical lenses[10]. A neural-network model has been suggested for dealing with a multiple objective problem in a manufacturing process of a film transistor-liquid crystal display[11].

Today, the introduction of optical software makes the optical design process simpler and faster. Time for designing secondary lenses is shortened because of powerful optical software. The common practice for optical designers to obtain the best LED lens shape is to use iterative calculating steps with virtual simulations of light ray directions in optical software till the lighting requirements are fulfilled. Experienced optical designers build up the lens model usually by taking “trial and error” steps. Lens dimensions and parameters are adjusted repeatedly without clear indicators of success. When the lens model involves a complicated form, the time for resolving the optimum shape can be enormous. The long hours entailed in the design steps cause immense manufacturing costs of optical lenses. For this reason, systemizing the design process is highly beneficial to optical industries. Furthermore, the design procedures referred to can be used make further improvements as the steps are found to be robust.

The Taguchi method is a quality management method based on statistical principles to form an optimization system to strengthen product quality. The emphasis of this methodology is on the variations of factors, which could be measured in term of product performances such as time, pressure, and force[12]. One of the purposes of the experimental process in the Taguchi method is to recognize the key factors which provide the largest contribution to the variations, after that, these factors are adjusted to create the least variability. The Taguchi method has been suggested by researchers and used to contribute to lens design[13]. The lens dimensions are chosen as optimized parameters and divided into several levels based on the feedback of parameters to the quality characteristics. The ‘larger the better’ principle is applied to determine the signal to noise level in the case of optical design. However, researchers have indicated that traditional Taguchi method cannot solve multi-objective optimization problems[14-16]. Especially for optical design, the final objectives to be achieved always are multiple not single. In order to overcome this disadvantage of the Taguchi method, Principal Component Analysis (PCA) integrated with the Taguchi method can be applied. Actually, a combination of Taguchi methods and PCA has been utilized for lens designs and manufacturing applications and they have demonstrated encouraging results. Fang *et al*.[17] applied the integrated method of Taguchi methods, PCA and fuzzy theory for solving the optimization of multiple quality characteristics of non-imaging optics, the new design with the optimized parameters showed significant improvements in luminous flux and illumination uniformity. Datta *et al*.[18] successfully employed Taguchi methods and PCA to resolve the correlated multiple criteria in the optimization problem of arc welding in the manufacturing industry. Chen *et al*.[19] utilized the Taguchi method and the PCA for investigating the effects of manufacturing tolerance and joint clearance on the quality of a six bar mechanism, researchers successfully identified and optimized the key factors involved in the fabrication, the findings of the study improved the quality of the mechanism and reduced the cost. The integration of PCA and the Taguchi method attracts attention because it can be used to resolve practical optimization problems involving multiple criteria. However, the optimization problem in this study involves multi-objective characteristics while the available optimization methods are few according to literature. The available optimization methods for the optimization problem with multi-objectives with supporting research works are commonly “Regression analysis”, “DEAR approach”, “Assignment of weight”, “Fuzzy based Taguchi method” and “Taguchi method with PCA” [20]. An integrated method of Taguchi method with PCA proposed by this study is desirable for optimizing lens design which involves the optimization of two objectives - uniformity and efficiency. Also, PCA with the Taguchi method does not require complicated mathematic modelling, heavy dependence on expert knowledge and a formal definition of a decision rule, all of which are normally required by other multi-objective optimization approaches. All of the above benefits of the PCA-Taguchi method make it possible to reduce the time taken to design a lens.

PCA is a statistical tool to reduce data and extract useful information from large datasets. It makes a valuable contribution to statistics and is applied in different fields such as neuroscience, network computing, self-learning models and manufacturing[21].To start, the obtained data is normalized to avoid neglecting small or large value data, the normalized data is then used to develop the variance-covariance matrix. The sources of variation from the correlated quality characteristics are used to identify the factors causing the deterioration of objective values. The principal components (uncorrelated quality indices) with large explanatory abilities are selected and viewed as the equivalent single quality characteristic, which then is passed to the Taguchi method to implement the optimization process again.

Actually, in practical engineering optimization problems, the number of factors and functions which need to be investigated is strictly limited by cost. As the current optimization approaches normally require huge function evaluations, therefore, the multi-objective optimization for expensive cost functions becomes the main challenge for global optimization[22]. One of the methods to deal with this challenge would be find a response curve or surface to fit the data set by evaluating the objective and constraint functions within a few points[23]. Then, the response curve or surface can be employed for further visual analysis, tradeoff analysis, and optimization. For the problem addressed in this study, two quality characteristics, uniformity and efficiency, need to be optimized. For a multi-objective optimization with two dimensions, the optimization parameters can be significantly visualized by a Pareto front, and dominance can be obtained. The above provides essential information for similar studies and researches about the problems of multi-objective optimization in the future.

During the lens design for improving the performance of LEDs, uniformity and efficiency always contradict each other, so an increase in uniformity results in a decrease in efficiency and vice versa. Therefore, a balance between efficiency and uniformity is needed. In this study, a new hybrid statistical approach is used to build an optimization system for the lens shape in order to enhance both uniformity and efficiency for LED applications. This approach provides an optimization of multiple quality characteristics without sacrificing a particular quality characteristic while the traditional Taguchi method can only deal with the single quality characteristic. In this paper, the Taguchi method and PCA are integrated to develop the optimum lens shape. The experimental results illustrate that both the efficiency and uniformity achieved by the designed lens are improved, the LED with designed lens using the proposed method showed over 75% uniformity and 90% efficiency.

2. Methodology and theory

2.1 The Taguchi method

The Taguchi method focuses on minimizing the effects of variations instead of eliminating the variations themselves. By choosing a suitable orthogonal array, important factors can be adjusted to eliminate the effects of variations from original sources, this is described as robustness. The parameter design in the orthogonal array includes obtaining control factors causing fewer effects to the quality responses, therefore, factor levels and factor interval values are considered thoughtfully in the experimental design. Another use of the orthogonal array is to design effective experiments, so that all factor levels appear as the same number of occurrences in the experiments. This ensures no bias for the individual factors and allows the separation of effects from one factor to another. Furthermore, an orthogonal array does not require all combinations of factors be tested. It means that only parts of combinations of factor levels in the experiments could represent all of the combinations of factor levels. Therefore, the experimental time is reduced significantly without sacrificing necessary information.

The phrase the larger the better implies that an increase in the output values of the quality characteristics should be the final goal. In the optical design, ‘the larger the better’ is adopted as efficiency and uniformity are to be enhanced. An identification of a noise factor guarantees the robust nature of the experiments using the orthogonal array. Not every potential noise factor needs to be discovered. Instead, it is more critical to determine the potential noise factors causing the poorest situation that might appear in the reality.

2.2 Principal component analysis

Principal component analysis (PCA) is a statistical tool to reduce data and extract useful information from large datasets. It simplifies the data dimensions and obtains hidden and dynamic information. It has already been applied by many scholars on data mining analysis and it has made valuable contributions to different industries such as neuroscience, network computing and self-learning model.

PCA includes a computation process which is used to transform a set of correlated response variables into a set of uncorrelated variables named principal components (PC). Data being processed reveals the variance-covariance based on the linear combination of the original data variables. If n components are used to stand for the whole system variability, this variability is indicated by the PCs where their numbers are less than the original variables. The PC containing the largest variance is uncorrelated with another PC.

The eigenvalues of the covariance matrix and the corresponding eigenvectors of the covariance matrix are needed to calculate for determining PCs in PCA. Let Y_1, Y_2, $\cdots Y_n$ be a set of variables that need to be processed. PCs can be obtained by the transformation of the uncorrelated linear combinations as shown in equation (1):

$$Z = \alpha_1 Y_1 + \alpha_2 Y_2 + \cdots + \alpha_p Y_n$$
$$Z_2 = \beta_1 Y_1 + \beta_2 Y_2 + \cdots + \beta_n Y_n$$
$$Z_n = \gamma_1 Y_1 + \gamma_2 Y_2 + \cdots + \gamma_n Y_n \qquad (1)$$

Where α is the first eigenvector, β is the second eigenvector and γ is n-th eigenvector. Z_1 is the first PC; Z_2 is the second PC and so on. In this paper, Y_1 and Y_2 are the normalized values of uniformity and efficiency, respectively.

Each PC has some proportion of the variation of quality characteristics called the accountability portion. When the PCs are compiled and added up, the explanation of the variation of quality characteristics increases. The domination of one PC is the main indicator and stands for all multiple quality characteristics.

The values of the dataset should be normalized to create a value between 0 and 1 before they are processed using PCA. The normalization (Equation 2) prevents ignoring large and small value data.

$$X' = \frac{X - X_{min}}{X_{max} - X_{min}} \qquad (2)$$

In this paper, X' is the normalized value, X is the signal / noise (S/N) ratio of efficiency or uniformity.

3. The proposed optimization system for secondary lenses

3.1 Design steps

LED sources are not suitable to use in lighting devices without modification because they create Lambertian light distributions which diminish uniformity and efficiency of LEDs. The shape of lens strongly affects the angular distribution of the emitted light and hence the pattern of light on the illuminated surface; therefore, it influences the performance of the whole lighting system.

The steps of the proposed PCA-Taguchi optimization system for lens shape are listed below and illustrated in Figure 1.

Step 1 – Identify the effects of uniformity and efficiency on each dimensional parameter of the designed lens. Uniformity and efficiency are set as quality characteristics
Step 2 – Select the control factors and noise factors in the lens module
Step 3 – Choose the correct orthogonal array and conduct experiments in Tracepro software
Step 4 – Calculate the S/N ratio of uniformity and efficiency for every quality characteristic based on the larger the better formula. Normalize the S/N ratio.
Step 5 – Implement the PCA process. The explanatory proportion and the characteristic vector matrix should be obtained
Step 6 – Calculate the PCs by the corresponding eigenvectors and eigenvalues
Step 7 – Implement variance analysis (ANOVA) of PCs, construct the response table of factors. Select the factor levels at the highest multiple performance characteristic indices (MPCI) which are the results of optimization of the lens.

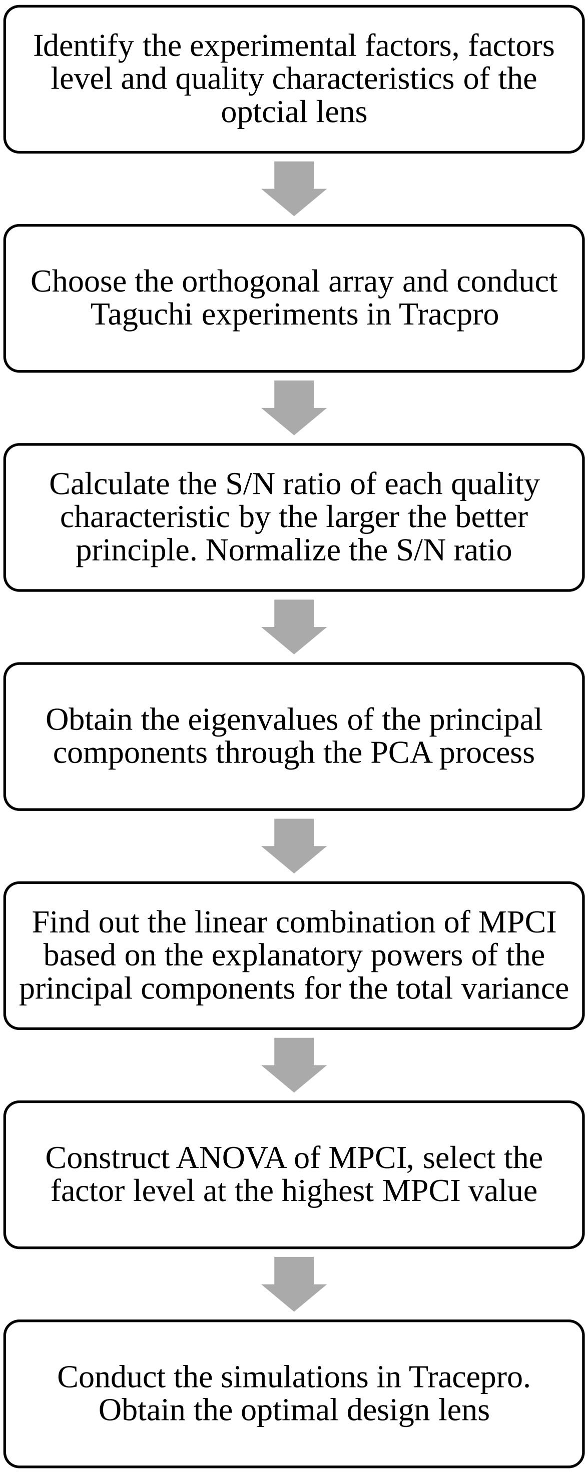

Figure 1. Flow chart of the lens design process

3.2 Optimization using the Taguchi method and PCA for lens design

As the upper part of the optical lens always has the largest effect in redirecting light rays from LED sources, therefore, the upper part of the designed optical lens is set to have the least number of adjustable factors in this study. This also reduces the possibility of manufacturing failures especially when producing lenses with different dimensions from the designed lens, providing the manufacturing tolerance in practical industries.

In this optical design for lens shape, six factors relating to its dimension are assigned; each factor being divided into five levels. The interval value of each factor level is 0.2mm. The detail of the setting range of each factor is shown in Table 1. The lens configuration of the original design is A: 17.9mm, B: 6.4mm, C: 15.5mm, D: 10.6mm, E: 10.7mm, F: 8.7mm. The degrees of freedom of 6 factors with 5 levels each is 6 X (5-1) = 24. For the case of six factors with 5 levels each and 24 degrees of freedom, a L25 orthogonal array is chosen. The assigned factors and the initial configuration of the lens in the cross section are shown in Figure 2. The six control factors are chosen based on the extent of their influence in diffusing light rays passing through particular parts of the lens. A: lens height, B: lens top width, C: lens upper width, D: lens second upper width, E: lens middle width and F: lens bottom width. The quality characteristics were uniformity and efficiency.

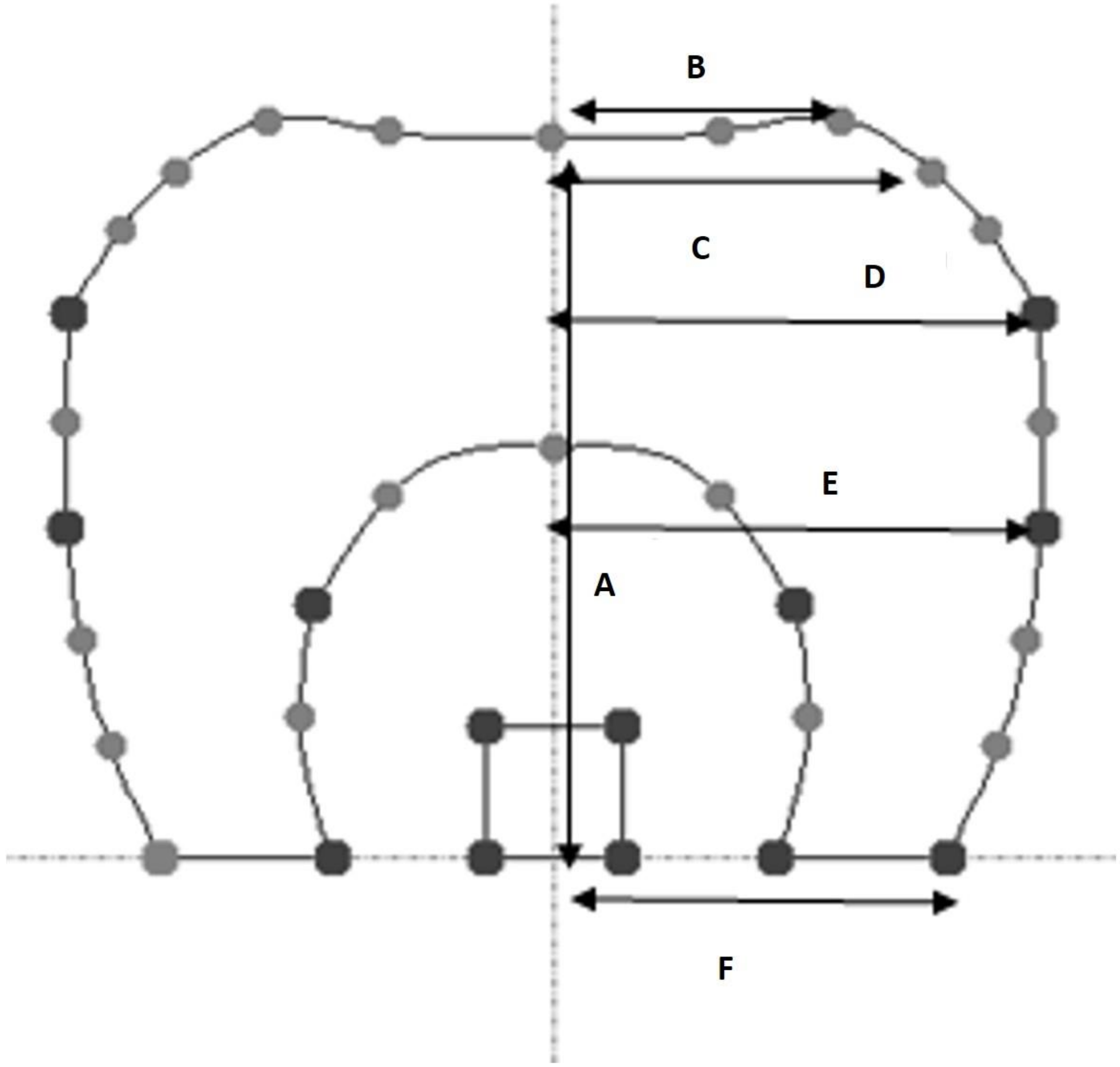

Figure 2. The proposed lens shape (cross section) with the assigned factors

The dimensions of the lens are adjusted based on the settings in the orthogonal array. The simulation result of average illuminance and efficiency are used as quality characteristics. Light efficiency and average illuminance can be read directly from the simulation results of the Tracepro software. The higher value of average illuminance suggests that the illuminated surface is receiving more and much more even incoming light rays, therefore better light uniformity is implied.

Tracepro software is used to conduct the simulations. The material of lens is PMMA. The refractive index of PMMA is 1.493. The numbers of rays used in the simulation is

50,000 in order to generate reliable results. The illuminated surface is positioned 50mm away from the bottom of the LED source.

3.3 Taguchi experiment

The simulations were conducted in Tracepro according to the orthogonal array of the Taguchi method. The orthogonal array was chosen as L25. In the Tracepro simulation, average illuminance would be the indicator of uniformity, the higher the average illuminance value the better the uniformity and vice versa. The significant lens factors affecting efficiency and average illuminance were recognized through ANOVA analysis. Those significant factors were then passed to adjust into the proper values in order to eliminate the negative impacts imposed by them. The simulation results of 25 experiments are shown in Table 2. Uniformity and efficiency were planned to increase, therefore the larger the better equation was applied to calculate the S/N ratio of two quality characteristics. ANOVAs of efficiency and uniformity were constructed and are shown in Tables 3 and 4, respectively.

ANOVA revealed that factor A and factor E contributed the most to the variation in uniformity and efficiency, the contribution percentages were 7.35% and 88.52% respectively to light efficiency and 7.96% and 87.88% respectively to average illuminance. These two factors were the key factors in the variations. Therefore, these two factors are needed to adjust the lens values in order to improve the quality characteristics. These two factors were adjusted and the second Taguchi method was conducted based on these adjusted factors. The new values of parameters in the lens dimension were assigned in the second Taguchi method, the range and interval values of factor A and factor E were adjusted to be narrow and small. The value of factor A was changed from 16.9mm – 17.1mm and the value of factor E was changed from 10.9mm – 11.1mm with the interval value 0.05 for both factors. The adjustment was made to eliminate the variations from the most contributed factors. The 25 experiments were conducted again in Tracepro. The simulation results are shown in Table 5.

According to the simulation results of the second Taguchi method, the values of efficiency and average illuminance were larger than in the first Taguchi method. The average value of light efficiency was increased from 89.5% to 91.5% and the mean value of average illuminance was increased from 9.95 to 10.15 in the second Taguchi method.

3.4 Principal component analysis

Data obtained in the second Taguchi method was input to the PCA process. The S/N ratios of efficiency and average illuminance were firstly normalized in order to ensure no extremely small or large values were neglected. Normalized data was then passed to SPSS software. The principal component vector matrix was obtained from the correlation matrix in SPSS, the value of the Pearson correlation between efficiency and uniformity being 0.891. The eigenvalues and the proportional explanatory power were extracted by SPSS and are shown in Table 6. According to Table 6, the first principal component PC_1 has 94.5% explanatory power for the total variance of the whole dataset related to efficiency and average illuminance. The second principal component PC_2 has 5.5% explanatory power which is a much lower ability to contribute to the total variation than PC_1.

The eigenvectors and eigenvalues were calculated based on their correlation. The eigenvectors with two eigenvalues of the correlation matrix are shown below as equation 3:

$$\begin{pmatrix} PC_1 \\ PC_2 \end{pmatrix} = \begin{pmatrix} 0.7072 & -0.7071 \\ 0.7072 & 0.7071 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \qquad (3)$$

Y_1 and Y_2 are the normalized values of efficiency and average illuminance, respectively. When normalized data are input into the above equation, the values of PC_1 and PC_2 for all simulations in the orthogonal array can be found. The results were shown in Table 7.

Normally, the component of explanation power is important if its eigenvalue is larger than 1. According to the Table 6, the eigenvalue of PC_1 was 1.89 while the eigenvalue of PC_2 was 0.11. Therefore, the multiple performance characteristics index (MPCI) only considered the component PC_1. PC_2 could be ignored in the calculation of MPCI. The formula of MPCI was

$$\mathrm{MPCI}=0.94538Y_1 \qquad (4)$$

The results of PC_1, PC_2 and MPCI were shown in Table 8, ANOVA and the corresponding graph of ANOVA are shown in Table 9 and Figure 3, respectively. The optimum parameter set was the factor level at the highest value of the S/N ratio of MPCI. The best level of each factor was A3, B2, C3, D4, E5 and F3 because they imposed the lowest variations of combined effects of efficiency and uniformity for the lens in LED lighting. The contribution percentages of factor D and factor E were 52% and 35% respectively while the cumulative contribution from these two factors was over 85%. The related lens parts of factor D and factor E direct the light rays from a LED source to the proper region with a wider angle so they led to an even light distribution on the target surface as a result. The larger the value of the S/N ratio in MPCI, the better the link of the factors to the overall performance of the lens for LED lighting.

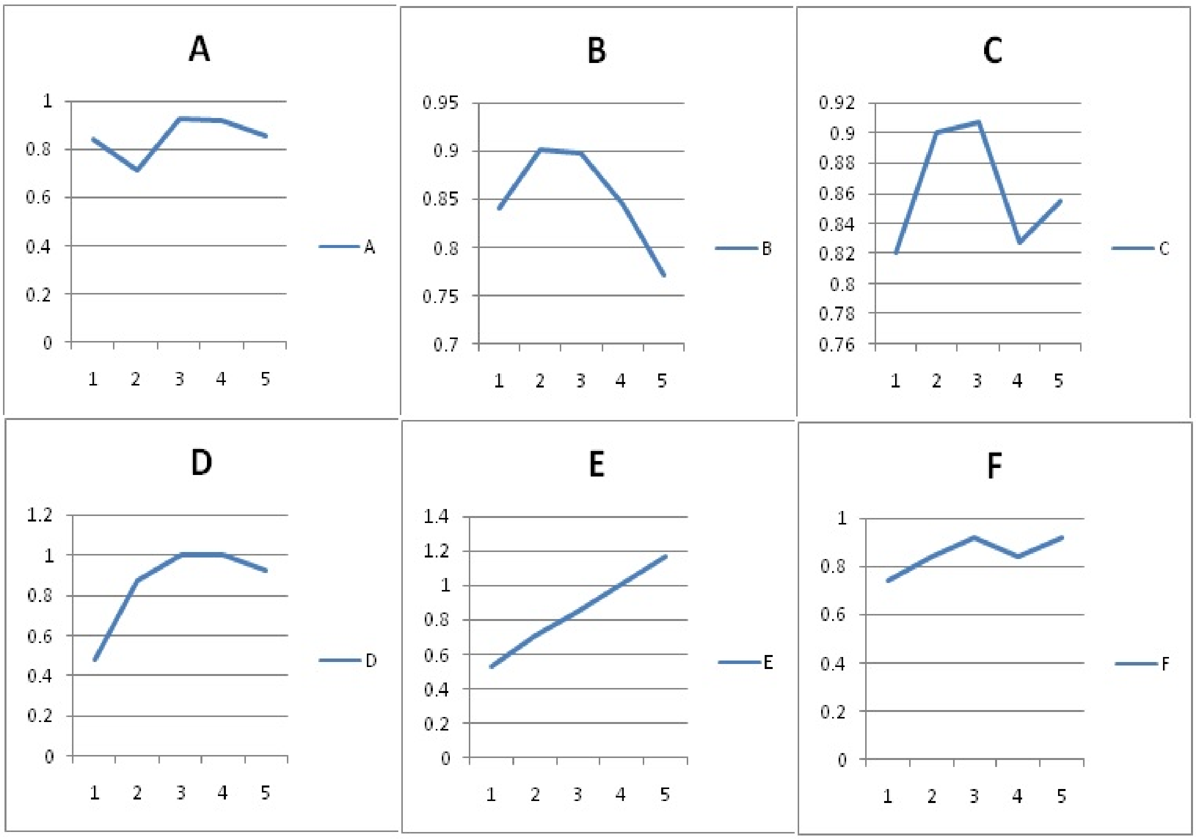

Figure 3. Results of analyses of variance of the multiple components performance index (MPCI)

3.5 Simulation Results

Simulations of the LED light source using the designed lens were conducted in order to verify the enhancement of the uniformity and efficiency produced by the lens. Figures 4 and 5 show the illuminance maps of the LED light source with the original lens and with the designed lens, respectively. The illuminance measurements regarding to the direction parallel to the width of the target surface and the direction parallel to the length of the target surface are captioned as (a) and (b) in Figures 4 and 5 respectively. The illuminance map of the LED source with the original lens exhibits several circular rings inside the light pattern, also, some dark regions appeared inside the light spot. Conversely, the illuminance map of the LED light source with the final designed lens displayed a more even light pattern covering a wider viewing angle without distinctive rings inside. By applying the designed lens using the proposed method to the LED light source, efficiency increased from 86% to 92% and uniformity is significantly improved.. Both uniformity and efficiency were enhanced without sacrificing one or the other. The simulation results indicated that the proposed optimization system using the combined Taguchi method and PCA can solve the multi-objective problem in the optical design of the lens.

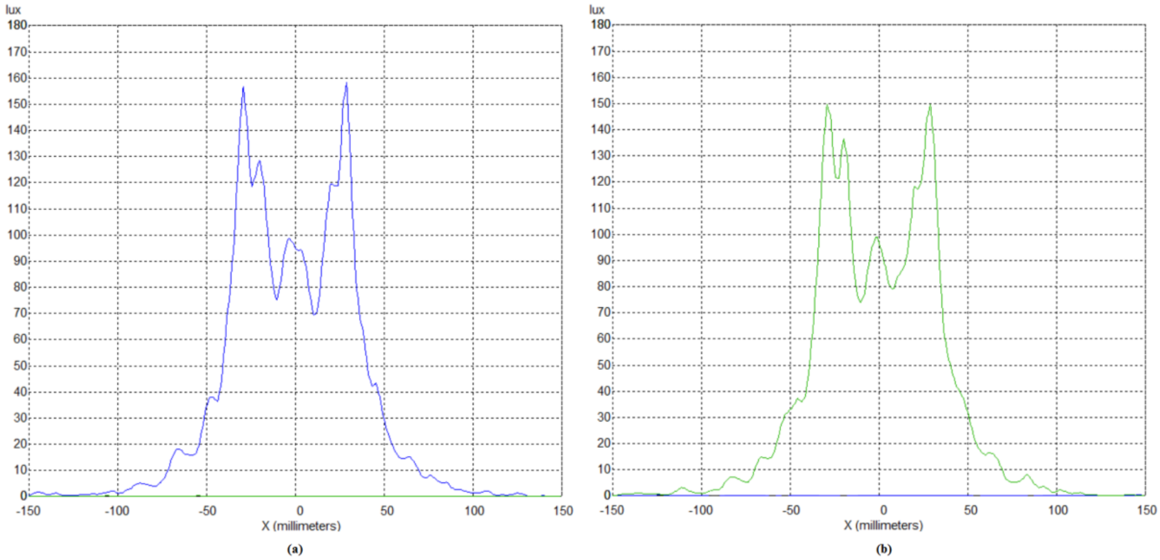

Figure 4. The illuminance map of the LED light source with the original lens. (a) The direction parallel to the width of the target surface and, (b) the direction parallel to the length of the target surface

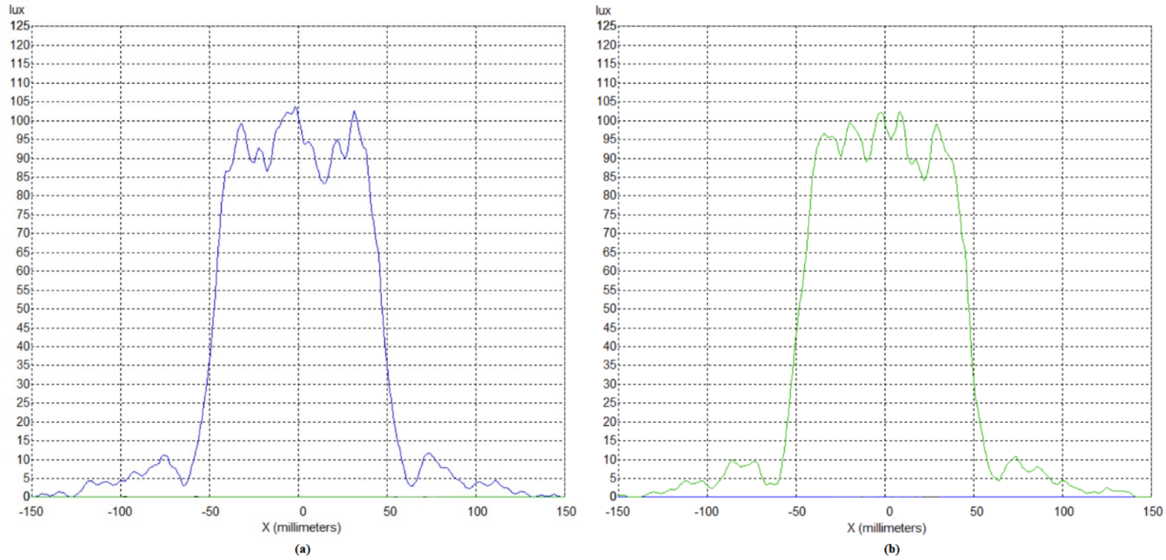

Figure 5. The illuminance map of the LED light source with the designed lens. (a) The direction parallel to the width of the target surface and, (b) the direction parallel to the length of the target surface

4. Conclusion

This paper assessed the use of a combined Taguchi method with PCA, this being a new idea for developing an optimization system in lens design by combining two statistical approaches. This new proposed optimization system effectively solved the multi-objective problem in optical design and enhanced the performance of the lens for LED lighting. Efficiency and uniformity of the LED light source using the designed lens were both enhanced, which efficiency satisfactorily reached 91.74% . The conclusions reached were as follows:

1. PCA combined with the Taguchi method can be used to extract the MPCI which shows the balance between multiple quality characteristics making it possible to solve multi-objective problems without sacrificing any important quality characteristics. The traditional Taguchi method is not suitable for this purpose.

2. From ANOVA of the Taguchi method, the significant factors contributing to the poor performance of a lens can be identified. Proper adjustments of the identified parameters can improve the performance of the quality characteristics.

3. From ANOVA of the MPCI obtained by the PCA process, the factors showing the least variations in the combined uniformity and efficiency can be identified. These factors are the most important parameters for balancing the two objectives of lens design, uniformity and efficiency.

Acknowledgements

The work described in this paper was supported by the Research Committee of The Hong Kong Polytechnic University (Project Code: RTTL).

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Figure captions

Figure 1. Flow chart of the lens design process

Figure 2. The proposed lens shape (cross section) with the assigned factors

Figure 3. Results of analyses of variance of the multiple components performance index (MPCI)

Figure 4. The illuminance map of the LED light source with the original lens.

Figure 5. The illuminance map of the LED light source with the designed lens.

Table 1. The detail of factor levels and corresponding values in the first Taguchi method.

Level	Factor(mm)					
	A	B	C	D	E	F
1	16.6	6.1	15.1	10.9	10.6	8.9
2	16.8	6.3	15.3	11.1	10.8	9.1
3	17	6.5	15.5	11.3	11	9.3
4	17.2	6.7	15.7	11.5	11.2	9.5
5	17.4	6.9	15.9	11.7	11.4	9.7

Table 2. The simulation results of signal to noise (S.N) ratio of efficiency and average illuminance of 25 experiments

Experiment no.	Factor mm						Light Efficiency (%)	Average Illuminance (lux/m^2)	Light Efficiency S/N ratio	Average illuminance S/N ratio
	A	B	C	D	E	F				
1	16.6	6.3	15.1	11.1	10.6	8.9	0.877	9.74	38.86	19.77
2	16.6	6.4	15.3	11.2	10.8	9.1	0.907	10.08	39.15	20.07
3	16.6	6.5	15.5	11.3	11	9.3	0.914	10.16	39.22	20.14
4	16.6	6.6	15.7	11.4	11.2	9.5	0.92	10.22	39.28	20.19
5	16.6	6.7	15.9	11.5	11.4	9.7	0.923	10.26	39.3	20.22
6	16.8	6.3	15.3	11.3	11.2	9.7	0.92	10.22	39.28	20.19
7	16.8	6.4	15.5	11.4	11.4	8.9	0.918	10.21	39.26	20.18
8	16.8	6.5	15.7	11.5	10.6	9.1	0.852	9.47	38.61	19.53
9	16.8	6.6	15.9	11.1	10.8	9.3	0.884	9.82	38.93	19.84
10	16.8	6.7	15.1	11.2	11	9.5	0.9	9.96	39.08	19.97
11	17	6.3	15.5	11.5	10.8	9.5	0.889	9.87	38.98	19.89
12	17	6.4	15.7	11.1	11	9.7	0.891	9.9	39	19.91
13	17	6.5	15.9	11.2	11.2	8.9	0.903	10.04	39.11	20.03
14	17	6.6	15.1	11.3	11.4	9.1	0.918	10.21	39.26	20.18
15	17	6.7	15.3	11.4	10.6	9.3	0.851	9.45	38.6	19.51
16	17.2	6.3	15.7	11.2	11.4	9.3	0.917	10.19	39.25	20.16
17	17.2	6.4	15.9	11.3	10.6	9.5	0.86	9.56	38.69	19.61
18	17.2	6.5	15.1	11.4	10.8	9.7	0.893	9.92	39.02	19.93
19	17.2	6.6	15.3	11.5	11	8.9	0.895	9.94	39.04	19.95
20	17.2	6.7	15.5	11.1	11.2	9.1	0.898	9.98	39.07	19.98
21	17.4	6.3	15.9	11.4	11	9.1	0.896	9.96	39.05	19.96
22	17.4	6.4	15.1	11.5	11.2	9.3	0.905	10.06	39.13	20.05
23	17.4	6.5	15.3	11.1	11.4	9.5	0.911	10.13	39.19	20.11
24	17.4	6.6	15.5	11.2	10.6	9.7	0.853	9.48	38.62	19.54
25	17.4	6.7	15.7	11.3	10.8	8.9	0.89	9.89	38.99	19.9

Table 3. Analysis of variance of efficiency

Light efficiency						
Level	A	B	C	D	E	F
1	39.162	39.084	39.07	39.01	38.68	39.052
2	39.032	39.046	39.05	39.04	39.01	39.028
3	38.99	39.03	39.03	39.09	39.08	39.026
4	39.014	39.026	39.03	39.04	39.17	39.044
5	38.996	39.008	39.02	39.01	39.25	39.044
Effect	0.166	0.076	0.054	0.078	0.576	0.026
Contribution	0.02756	0.0058	0.003	0.006	0.332	0.0007
Percentage Contribution	7.3525	1.5412	0.778	1.623	88.52	0.1804

Table 4. Analysis of variance of average illuminance

Average illuminance						
Level	A	B	C	D	E	F
1	20.078	19.994	19.98	19.922	19.592	19.966
2	19.942	19.964	19.966	19.954	19.926	19.944
3	19.904	19.948	19.946	20.004	19.986	19.94
4	19.926	19.94	19.938	19.954	20.088	19.954
5	19.912	19.916	19.932	19.928	20.17	19.958
Effect	0.174	0.078	0.048	0.082	0.578	0.026
Contribution	0.0303	0.0061	0.002304	0.0067	0.33408	0.000676
Percentage Contribution	7.9643	1.6004	0.60608	1.7688	87.8826	0.177825

Table 5. Experimental results of second Taguchi method.

Experiment no.	Factor (mm) A	B	C	D	E	F	Light Efficiency (%)	Average illuminance (lux/m^2)	Light efficiency S/N ratio	Average illuminance S/N ratio
1	16.9	6.1	15.1	11.1	10.9	8.9	0.90667	10.074	39.15	20.06
2	16.9	6.3	15.3	11.2	10.95	9.1	0.91086	10.121	39.19	20.1
3	16.9	6.5	15.5	11.3	11	9.3	0.91396	10.155	39.22	20.14
4	16.9	6.7	15.7	11.4	11.05	9.5	0.91678	10.186	39.25	20.16
5	16.9	6.9	15.9	11.5	11.1	9.7	0.91827	10.203	39.25	20.16
6	16.95	6.1	15.3	11.3	11.05	9.7	0.91698	10.189	39.25	20.16
7	16.95	6.3	15.5	11.4	11.1	8.9	0.91747	10.194	39.25	20.16
8	16.95	6.5	15.7	11.5	10.9	9.1	0.90897	10.1	39.17	20.09
9	16.95	6.7	15.9	11.1	10.95	9.3	0.91059	10.118	39.19	20.1
10	16.95	6.9	15.1	11.2	11	9.5	0.914	10.156	39.22	20.14
11	17	6.1	15.5	11.5	10.95	9.5	0.91203	10.134	39.21	20.12
12	17	6.3	15.7	11.1	11	9.7	0.91406	10.156	39.22	20.14
13	17	6.5	15.9	11.2	11.05	8.9	0.91443	10.16	39.22	20.14
14	17	6.7	15.1	11.3	11.1	9.1	0.91756	10.195	39.25	20.16
15	17	6.9	15.3	11.4	10.9	9.3	0.90998	10.111	39.13	20.05
16	17.05	6.1	15.7	11.2	11.1	9.3	0.91755	10.195	39.19	20.11
17	17.05	6.3	15.9	11.3	10.9	9.5	0.91054	10.117	39.11	20.03
18	17.05	6.5	15.1	11.4	10.95	9.7	0.91337	10.149	39.21	20.13
19	17.05	6.7	15.3	11.5	11	8.9	0.91251	10.139	39.21	20.12
20	17.05	6.9	15.5	11.1	11.05	9.1	0.91448	10.161	39.24	20.15
21	17.1	6.1	15.9	11.4	11	9.1	0.91309	10.145	39.21	20.13
22	17.1	6.3	15.1	11.5	11.05	9.3	0.91635	10.182	39.24	20.15
23	17.1	6.5	15.3	11.1	11.1	9.5	0.91706	10.19	39.26	20.17
24	17.1	6.7	15.5	11.2	10.9	9.7	0.91095	10.122	39.12	20.04
25	17.1	6.9	15.7	11.3	10.95	8.9	0.91052	10.117	39.23	20.15

Table 6. Eigenvalues, explanatory powers and cumulative explanatory powers of principal components (PC)

PC	Total explanatory powers	Percentage of Variance	Cumulative percentage
1	1.89	94.5	94.5
2	0.11	5.5	100

Table 7. Results of Principal Components (PC1 and PC2) and multiple components performance index (MCPI).

Experiment no.	Factor						PC1	PC2	Total PC (MPCI)
	A	B	C	D	E	F			
1	1	1	1	1	1	1	0.000	0.000	0.000
2	1	2	2	2	2	2	0.789	0.002	0.745
3	1	3	3	3	3	3	1.217	0.004	1.151
4	1	4	4	4	4	4	1.169	0.001	1.105
5	1	5	5	5	5	5	1.283	0.001	1.213
6	2	1	2	3	4	5	1.122	0.004	1.060
7	2	2	3	4	5	1	1.239	0.001	1.171
8	2	3	4	5	1	2	0.495	0.003	0.468
9	2	4	5	1	2	3	0.267	0.004	0.253
10	2	5	1	2	3	4	0.636	0.418	0.601
11	3	1	3	5	2	4	0.933	-0.002	0.882
12	3	2	4	1	3	5	0.679	-0.002	0.642
13	3	3	5	2	4	1	1.103	0.003	1.043
14	3	4	1	3	5	2	1.414	0.000	1.337
15	3	5	2	4	1	3	0.775	0.002	0.733
16	4	1	4	2	5	3	1.357	0.004	1.283
17	4	2	5	3	1	4	0.828	0.002	0.782
18	4	3	1	4	2	5	1.060	0.003	1.002
19	4	4	2	5	3	1	0.915	-0.002	0.865
20	4	5	3	1	4	2	0.709	-0.007	0.671
21	5	1	5	4	3	2	1.038	-0.001	0.981
22	5	2	1	5	4	3	1.230	-0.001	1.163
23	5	3	2	1	5	4	0.867	0.003	0.820
24	5	4	3	2	1	5	0.701	-0.007	0.662
25	5	5	4	3	2	1	0.679	-0.002	0.642

Table 8. Results of the principal components PC1 and PC2 and the multiple components performance index (MCPI)

	Factor								Total PC
Experiment no.	A	B	C	D	E	F	PC1	PC2	(MPCI)
1	1	1	1	1	1	1	0.000	0.000	0.000
2	1	2	2	2	2	2	0.789	0.002	0.745
3	1	3	3	3	3	3	1.217	0.004	1.151
4	1	4	4	4	4	4	1.169	0.001	1.105
5	1	5	5	5	5	5	1.283	0.001	1.213
6	2	1	2	3	4	5	1.122	0.004	1.060
7	2	2	3	4	5	1	1.239	0.001	1.171
8	2	3	4	5	1	2	0.495	0.003	0.468
9	2	4	5	1	2	3	0.267	0.004	0.253
10	2	5	1	2	3	4	0.636	0.418	0.601
11	3	1	3	5	2	4	0.933	-0.002	0.882
12	3	2	4	1	3	5	0.679	-0.002	0.642
13	3	3	5	2	4	1	1.103	0.003	1.043
14	3	4	1	3	5	2	1.414	0.000	1.337
15	3	5	2	4	1	3	0.775	0.002	0.733
16	4	1	4	2	5	3	1.357	0.004	1.283
17	4	2	5	3	1	4	0.828	0.002	0.782
18	4	3	1	4	2	5	1.060	0.003	1.002
19	4	4	2	5	3	1	0.915	-0.002	0.865
20	4	5	3	1	4	2	0.709	-0.007	0.671
21	5	1	5	4	3	2	1.038	-0.001	0.981
22	5	2	1	5	4	3	1.230	-0.001	1.163
23	5	3	2	1	5	4	0.867	0.003	0.820
24	5	4	3	2	1	5	0.701	-0.007	0.662
25	5	5	4	3	2	1	0.679	-0.002	0.642

Table 91. Results of the analysis of variance of the multiple components performance index (MPCI)

	A	B	C	D	E	F
Level 1	0.843	0.841	0.821	0.477	0.529	0.744
Level 2	0.711	0.901	0.901	0.867	0.705	0.840
Level 3	0.927	0.897	0.907	0.994	0.848	0.917
Level 4	0.921	0.844	0.828	0.998	1.009	0.838
Level 5	0.854	0.772	0.854	0.918	1.165	0.916
Contribution	0.047	0.017	0.008	0.272	0.404	0.030
Percentage	6.035	2.142	0.967	35.013	52.027	3.817