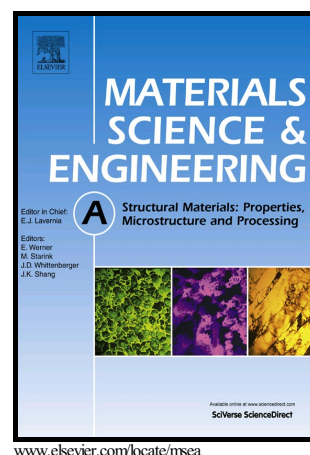


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**Micromechanics-based Damage Model for Failure  
Prediction in Cold Forming**

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**Abstract**

The purpose of this study was to develop a micromechanics-based damage (micro-damage) model that was concerned with the evolution of micro-voids for failure prediction in cold forming. Typical stainless steel SS316L was selected as the specimen material, and the nonlinear isotropic hardening rule was extended to describe the large deformation of the specimen undergoing cold forming. A micro-focus high-resolution X-ray computed tomography (CT) system was employed to trace and measure the micro-voids inside the specimen directly. Three-dimensional (3D) representative volume element (RVE) models with different sizes and spatial locations were reconstructed from the processed CT images of the specimen, and the average size and volume fraction of micro-voids (VFMV) for the specimen were determined via statistical analysis. Subsequently, the micro-damage model was compiled as a user-defined material subroutine into the finite element (FE) package ABAQUS. The stress-strain responses and damage evolutions of SS316L specimens under tensile and compressive deformations at different strain rates were predicted and further verified experimentally. It was concluded that the proposed micro-damage model is convincing for failure prediction in cold forming of the SS316L material.

**Keywords:** Micromechanics-based damage model; X-ray computed tomography; Representative volume element; Failure prediction; Large deformation; Cold forming.

**Nomenclature**

$\mathbf{F}$	Deformation gradient tensor
$\mathbf{R}$	Rotation tensor with orthogonality
$\mathbf{h}$	Hencky's logarithmic strain
$\mathbf{h}^e, \mathbf{h}^p$	Elastic and plastic strain tensors respectively
$\mathbf{D}$	Stretching tensor
$\mathbf{D}^e, \mathbf{D}^p$	Elastic and plastic stretching tensors respectively
$E$	Elastic modulus
$\nu$	Poisson's ratio
$\boldsymbol{\tau}$	Kirchhoff stress tensor
$\mathbf{D}^{p'}$	Deviatoric stress tensor
$\mathbf{D}^{p(m)}$	Hydrostatic stress tensor
$f$	Volume fraction of micro-voids (VFMV)
$\rho$	Specific density of the material
$p$	Accumulated plastic strain
$\dot{p}$	Accumulated plastic strain rate
$n, K$	Material viscosity constants
$\langle \rangle$	McCauley operator
$\mathbf{N}$	Unit normal vector of the yield surface
$F_y$	Yield surface
$s_i, i=1, 2$	Damage parameters
$Q$	Isotropic hardening deformation resistance
$R^{sat}$	Saturated stress value
$\beta$	Evolution rate of $R$

## 1. Introduction

The identification of damage initiation and evolution is always a key concern in the metal forming process [1]. Classical damage theory is based on the continuum mechanics-based thermodynamics framework initiated by Kachanov, Lemaitre and Chaboche [2-5], which is known as continuum damage mechanics (CDM). Following on the basic concept of effective stress, researchers have paid much attention to the application of the CDM approach in different material forming or loading conditions. Wang [6] developed a unified nonlinear damage evolution rule for ductile materials. By using two damage state variables, Chaboche et al. [7] proposed a modified CDM model to describe the plastic compressibility in ductile damage. Pires et al. [8] improved the original CDM model by incorporating the micro-crack closure effect to predict the damage growth and fracture initiation in ductile materials. Soyarslan and Tekkaya [9] developed a modified CDM model coupled with multiplicative finite plasticity to study the propagation of inner defects in forward extrusion. However, although this phenomenological CDM model saves computational time, it ignores the physical details, e.g., micro-defects, of the material [10-12].

Thus, another approach, the micromechanics-based damage (micro-damage) model, which considers the effects of void initiation and evolution, was developed and brought to the front as a popular research topic. This approach was originally developed by Gurson [13] and known as the GTN model [13-15]. In the GTN model, ductile fracture occurs through the void nucleation, growth, and coalescence. The void coalescence is the final stage in the fracture of ductile materials. It can be triggered by localized plastic deformation inside the inter-void ligament between neighbouring voids and can be modelled by accelerating void growth at a critical void volume fraction. Needleman and Tvergaard [14] improved the GTN model by taking into

account the void evolution and material strain hardening. In order to describe the ductile materials with irregular-shaped voids, Gologanu and Leblond [16] extended the GTN model based on an “expansion” velocity field condition. By considering the void shape, void location, strain hardening and stress triaxiality, Pardoen and Hutchinson [17] enhanced the GTN model to establish the criterion of void coalescence.

In the application of the micro-damage model for damage prediction [11, 18], the identification of micro-void initiation and growth is of great importance. For micro-voids detection, destructive methods are used commonly to measure its interior distribution. In addition, the VFMV is always evaluated by quantitative metallographic analysis or microscopic examinations [18-21]. However, such testing procedures are destructive, and it is unimaginably difficult to locate the cross-sections and obtain other useful information from numerous micro-voids accurately. In view of this, X-ray CT is an ideal alternative technique that avoids material destruction and allows for superior image quality of the inner microstructure [22]. It is not only for image capturing and generating external 3D profiles, but also for visualizing internal features of solid objects. Digital volumetric data of the tested specimens can be obtained simultaneously by the high-end data-analysis and visualization software, so as to obtain more reliable information, such as the spatial location, size, compactness, and sphericity, etc., of micro-voids. This enables more accurate calculation of the void volume fraction [23, 24]. It is a very powerful tool to realize and support the material microstructure characterization as well as high-resolution micro-interior reverse engineering for various applications such as high-precision engineering, defect detection, composite material science, dimensional metrology and other potential applications [25].

The above phenomenological CDM or micro-damage models are always implemented in finite element (FE) packages for damage prediction of metallic materials. However, the application of the X-ray CT technique for damage parameter identification in the micro-damage model has rarely been studied. Also, most of the mentioned constitutive models coupled with damage were formulated within the small deformation framework, which limited their applications in the large deformation processes of metallic materials with good plasticity [26, 27].

Thus, the main objective of this study was to develop a micro-damage model framework for failure prediction in the large deformation of metallic material undergoing cold forming. Typical stainless steel SS316L, which possesses good ductility, high strength and superior corrosion resistance [28], was selected as the specimen material for the X-ray CT scanning and uniaxial tension and compression tests. A micro-focus high-resolution X-ray CT system was employed specially to trace and measure the micro-voids of the specimen directly. In which, the micro-voids inside the material can be modelled precisely and accurately. Furthermore, 3D RVE models with different sizes and spatial locations were reconstructed and the average size and VFMV for the specimen were determined via statistical analysis. Subsequently, the aforesaid micro-damage model was compiled as a user-defined subroutine into ABAQUS. The stress-strain responses and damage evolution in cold forming of SS316L were predicted and further verified experimentally.

## **2. Micromechanics-based damage model**

In this section, a micromechanics-based damage model describing the large deformation behaviour of metallic material was presented. The specific constitutive equations of the micro-damage model based on hypo-elastic relations were derived

[17, 29]. In order to describe the elasto-plastic behaviour of metallic materials, the additive decomposition of the Hencky's logarithmic strain  $\mathbf{h}$  is defined as [30]

$$\mathbf{h} = \mathbf{h}^e + \mathbf{h}^p, \quad (1)$$

Where  $\mathbf{h}^e$  and  $\mathbf{h}^p$  are the elastic and plastic strain tensors respectively.

For large deformation conditions, the logarithmic strain  $\mathbf{h}$  can be replaced by stretching tensor  $\mathbf{D}$  as

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p, \quad (2)$$

where  $\mathbf{D}^e$  and  $\mathbf{D}^p$  are the elastic and plastic stretching tensors, respectively. Further, the elastic strain tensor  $\mathbf{h}^e$  can be assumed as a state variable to formulate the constitutive equations, and it conforms to the following expression

$$\mathbf{D}^e = \mathbf{h}^{\circ \log(e)}. \quad (3)$$

The elastic stretching tensors  $\mathbf{D}^e$  can be formulated by the hypo-elastic relationship as [30]

$$\mathbf{D}^e = \frac{1+\nu}{E} \dot{\boldsymbol{\tau}}^{\log} - \frac{\nu}{E} \text{tr}(\dot{\boldsymbol{\tau}}) \mathbf{1}, \quad (4)$$

where  $\nu$  and  $E$  are Poisson's ratio and elastic modulus, respectively, and  $\boldsymbol{\tau}$  is the Kirchhoff stress tensor.

The plastic stretching tensors  $\mathbf{D}^p$  can be decomposed into two parts, i.e., the deviatoric stress tensor  $\mathbf{D}^{p'}$  and the hydrostatic stress tensor  $\mathbf{D}^{p(m)}$ ,

$$\mathbf{D}^p = \mathbf{D}^{p'} + \mathbf{D}^{p(m)} = \mathbf{D}^{p'} + \frac{1}{3} \text{tr}(\mathbf{D}^p) \mathbf{1}. \quad (5)$$

By introducing the concept of volume fraction of micro-voids  $f = 1 - \rho$  (where  $\rho$  is the specific density of a given material), the flow rules  $\mathbf{D}^{p'}$  and  $\mathbf{D}^{p(m)}$  can be assumed as [19]



$$\mathbf{D}^p = \sqrt{\frac{3}{2}} \dot{\rho} \mathbf{N} = \sqrt{\frac{3}{2}} \left\langle \frac{F_y}{K} \right\rangle^n \mathbf{N}, \quad (6)$$

$$\mathbf{D}^{p(m)} = \frac{1}{3} f s_1 \exp\left(\frac{\text{tr}(\boldsymbol{\tau})}{3 s_2 \rho}\right) \dot{\rho} \mathbf{1}, \quad (7)$$

where  $\dot{\rho}$  is the accumulated plastic strain rate,  $n$  and  $K$  are the material viscosity

constants.  $s_1$  and  $s_2$  are damage parameters,  $\langle \cdot \rangle$  is the McCauley operator;  $\mathbf{N} = \frac{\boldsymbol{\tau}'}{\|\boldsymbol{\tau}'\|}$

is the unit normal vector of the yield surface  $F_y$ .

The yield surface of  $F_y$  can be expressed as [31]

$$F_y = \sqrt{\frac{3}{2}} \|\boldsymbol{\tau}' / \rho\| + f s_1 s_2 \exp\left(\frac{\text{tr}(\boldsymbol{\tau})}{3 s_2 \rho}\right) - Q, \quad (8)$$

where  $Q$  is the isotropic hardening deformation resistance with the following specific form

$$Q = Q_0 + R, \quad (9)$$

where  $Q_0$  and  $R$  are the initial deformation resistance and the subsequent resistance, respectively.

The evolution law of the resistance  $R$  can be formulated as

$$\dot{R} = \beta (R^{\text{sat}} - R) \dot{\rho}, \quad (10)$$

where  $R^{\text{sat}}$  is the saturated value of  $R$ ,  $\beta$  controls the evolution rate of  $R$ .

According to effective stress concept of damage mechanics [2] and mass conservation [19, 20], the damage evolution law of the material can be expressed by the evolution of VFMV  $f$  as

$$\dot{f} = \rho \text{tr}(\mathbf{D}^p) = \rho f s_1 \exp\left(\frac{\text{tr}(\boldsymbol{\tau})}{3 s_2 \rho}\right) \dot{\rho}. \quad (11)$$

The expression of the driving force is

$$F_f = \frac{s_2}{\rho_0} \ln \left( \frac{f(1-f_0)}{f_0(1-f)} \right). \quad (12)$$

### 3. Finite element implementations

The proposed micro-damage model was implemented into the ABAQUS using the subroutine UMAT which allows the strain and stress from intrinsic dissipation to be calculated at each integration point. Based on the backward Euler method and radial return [31-34], the implicit stress integration algorithm has been proposed in this study.

#### 3.1. Discretization of constitutive equations

First, an initial state of the moment  $t=0$  is considered, i.e.,

$$\boldsymbol{\tau}|_{t=0} = \mathbf{0}, \quad \mathbf{F}|_{t=0} = \mathbf{R}^{\log}|_{t=0} = \mathbf{1} \quad (13)$$

At the time interval from steps  $n$  to  $n+1$  with a time increment  $\Delta t = t_{n+1} - t_n$ , the Kirchhoff stress tensor  $\boldsymbol{\tau}_{n+1}$  can be obtained such that

$$\boldsymbol{\tau}_{n+1} = \overset{\circ}{\boldsymbol{\tau}}_{n+1}^{\log} \Delta t + \Delta \mathbf{R}_{n+1} \boldsymbol{\tau}_n \Delta \mathbf{R}_{n+1}^T. \quad (14)$$

In addition, the evolution equations of hypo-elastic relation, yield function, viscoplastic flow rule, density and VFMV can be discretized as

$$\overset{\circ}{\boldsymbol{\tau}}_{n+1}^{\log} = \rho_{n+1} \mathbb{D} : \mathbf{D}_{n+1}^e \Delta t + \frac{\Delta \rho_{n+1}}{\rho_{n+1}} \boldsymbol{\tau}_{n+1}, \quad (15)$$

Where  $\mathbb{D}$  is the fourth-order elastic tensor.

$$F_{y(n+1)} = \sqrt{\frac{3}{2}} \|\boldsymbol{\tau}'_{n+1} / \rho_{n+1}\| + f_{n+1} s_1 s_2 \exp \left( \frac{\text{tr}(\boldsymbol{\tau}_{n+1})}{3 s_2 \rho_{n+1}} \right) - Q_{n+1}. \quad (16)$$

$$(\mathbf{D}_{n+1}^p)' = \frac{\sqrt{\frac{3}{2}} \Delta p_{n+1} \mathbf{N}_{n+1}}{\Delta t}. \quad (17)$$

$$\mathbf{D}_{n+1}^{p(m)} = \frac{1}{3} f_{n+1} s_1 \exp\left(\frac{\text{tr}(\boldsymbol{\tau}_{n+1})}{3s_2 \rho}\right) \frac{\Delta p_{n+1}}{\Delta t} \mathbf{1}. \quad (18)$$

$$\Delta p_{n+1} = \left\langle \frac{F_{y(n+1)}}{K} \right\rangle^n \Delta t. \quad (19)$$

$$\Delta \rho_{n+1} = -\Delta f_{n+1}. \quad (20)$$

$$\Delta f_{n+1} = \rho_{n+1} f_{n+1} s_1 \exp\left(\frac{\text{tr}(\boldsymbol{\tau}_{n+1})}{3s_2 \rho_{n+1}}\right) \Delta p_{n+1}, \quad (21)$$

with

$$\mathbf{N}_{n+1} = \frac{\boldsymbol{\tau}_{n+1}'}{\|\boldsymbol{\tau}_{n+1}'\|} \quad \left( \mathbf{N}_{n+1} : \mathbf{N}_{n+1} = 1 \quad \text{when} \quad F_{y(n+1)} = 0 \right). \quad (22)$$

$$Q_{n+1} = Q_n + \Delta R_{n+1}, \quad (23)$$

where

$$\Delta R_{n+1} = \beta (R_{sat} - R_n) \Delta p_{n+1}. \quad (24)$$

### 3.2. Implicit stress integration algorithm

The implicit stress integration algorithm is illustrated in Fig. 1. For the given values of  $\Delta \mathbf{F}_{n+1}$  and  $\Delta t_{n+1}$ , assuming that the variables of  $\boldsymbol{\tau}_n$ ,  $\rho_n$ ,  $\mathbf{F}_n$  and  $\mathbf{R}_n^{\log}$  at time  $t_n$  have been obtained, variable  $\boldsymbol{\tau}_{n+1}$  at time  $t_{n+1}$  can be solved by adopting the approach of elastic-prediction and plastic-correction [32, 33].

Assuming that the deformation increment at time  $t_{n+1}$  is elastic, the trial variables, i.e.,

$\boldsymbol{\tau}_{n+1}^*$ ,  $\rho_{n+1}^*$ ,  $Q_{n+1}^*$ ,  $(F_y)_{n+1}^*$ , in the step n+1 can be obtained.

If

$$F_{y(n+1)}^* \leq 0, \quad (25)$$

means the yield is not reached at a given increment step. The trial stress state is the true state. Otherwise, plastic yield occurred and Eq. (14) can be rewritten as

$$\boldsymbol{\tau}_{n+1} = \frac{\rho_{n+1}}{\rho_n} \left( \frac{\rho_{n+1}}{\rho_n} \boldsymbol{\tau}_{n+1}^* - \rho_{n+1} \mathbb{D} : \mathbf{D}_{n+1}^p \Delta t - \frac{\Delta \rho_{n+1}}{\rho_n} \Delta \mathbf{R}_{n+1} \boldsymbol{\tau}_n \Delta \mathbf{R}_{n+1}^T \right). \quad (26)$$

The deviation of Eq. (26) is given as

$$\boldsymbol{\tau}'_{n+1} = \frac{\rho_{n+1}^2}{\rho_n^2} \boldsymbol{\tau}_{n+1}^* - 2\mu \frac{\rho_{n+1}}{\rho_n} \mathbf{D}_{n+1}^p \Delta t - \frac{\Delta \rho_{n+1} \rho_{n+1}}{\rho_n^2} (\Delta \mathbf{R}_{n+1} \boldsymbol{\tau}_n \Delta \mathbf{R}_{n+1}^T)'. \quad (27)$$

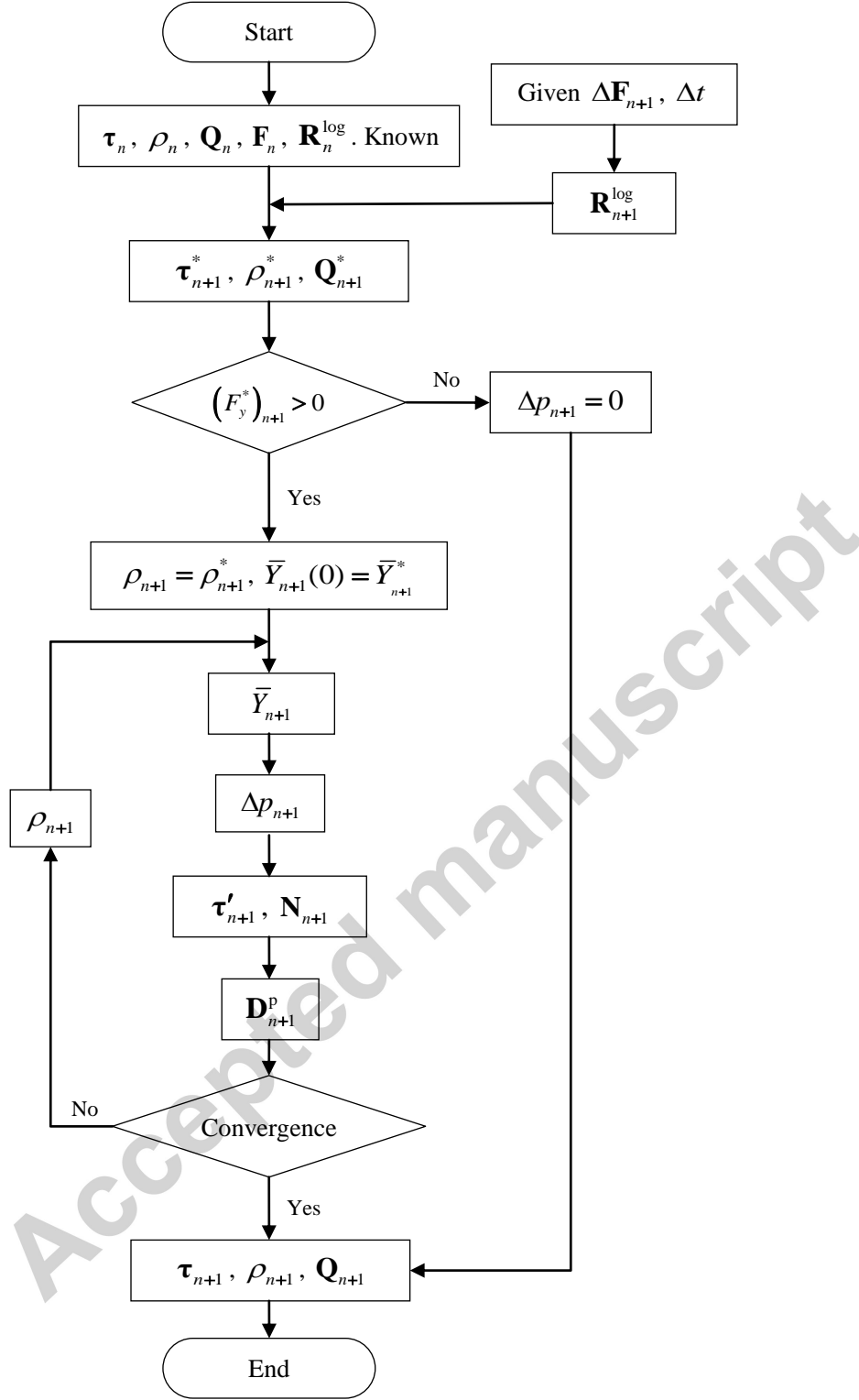
Then the yield function is

$$\begin{aligned} \|\boldsymbol{\tau}'_{n+1}\| = & \left\| \frac{\rho_{n+1}^2}{\rho_n^2} \boldsymbol{\tau}_{n+1}^* - \frac{\Delta \rho_{n+1} \rho_{n+1}}{\rho_n^2} (\Delta \mathbf{R}_{n+1} \boldsymbol{\tau}_n \Delta \mathbf{R}_{n+1}^T)' - \right. \\ & \left. 2\mu \frac{\rho_{n+1}}{\rho_n} \left( \sqrt{\frac{3}{2}} + \frac{1}{3} f_{n+1} s_1 \exp \left( \frac{\text{tr}(\boldsymbol{\tau}_{n+1})}{3s_2 \rho_{n+1}} \right) \right) \Delta p_{n+1} \right\|. \end{aligned} \quad (28)$$

Thus, the following nonlinear scalar equation can be obtained

$$\begin{aligned} \bar{Y}_{n+1} - \bar{Y}_{n+1}^* + 3\mu \frac{\rho_{n+1}}{\rho_n} \left( 1 + \sqrt{\frac{1}{6}} f_{n+1} s_1 \exp \left( \frac{\text{tr}(\boldsymbol{\tau}_{n+1})}{3s_2 \rho_{n+1}} \right) \right) \\ \left\langle \frac{\bar{Y}_{n+1}}{\rho_{n+1}} + f_{n+1} s_1 s_2 \exp \left( \frac{\text{tr}(\boldsymbol{\tau}_{n+1})}{3s_2 \rho_{n+1}} \right) - Q_{n+1} \right\rangle^n \Delta t = 0, \end{aligned} \quad (29)$$

$$\text{where } \bar{Y}_{n+1} = \|\boldsymbol{\tau}'_{n+1}\|, \quad \bar{Y}_{n+1}^* = \left\| \frac{\rho_{n+1}^2}{\rho_n^2} \boldsymbol{\tau}_{n+1}^* - \frac{\Delta \rho_{n+1} \rho_{n+1}}{\rho_n^2} (\Delta \mathbf{R}_{n+1} \boldsymbol{\tau}_n \Delta \mathbf{R}_{n+1}^T)' \right\|.$$



**Fig. 1.** Flow chart of the stress integration algorithm of UMAT subroutine

### 3.3. Identification of material parameters for failure prediction

When implementing the micro-damage model into ABAQUS, the material parameters involved need to be identified. Following is the summary of the material parameters determination.

(1) The Poisson's ratio  $\nu$  and initial elastic modulus  $E$  can be obtained through the uniaxial compression tests.

(2) The viscoplastic constants,  $K$  and  $n$ , can be calculated from the uniaxial compression tests at different strain rates.

(3) The initial yield stress  $Q_0$  can be obtained from the uniaxial compression tests; the saturated stress value  $R^{sat}$  and the evolution rate  $\beta$  are determined by fitting the curves of  $\tau$  vs.  $h$  through the following equation:

$$R = R^{sat} (1 - \exp(-\beta p)) \quad (30)$$

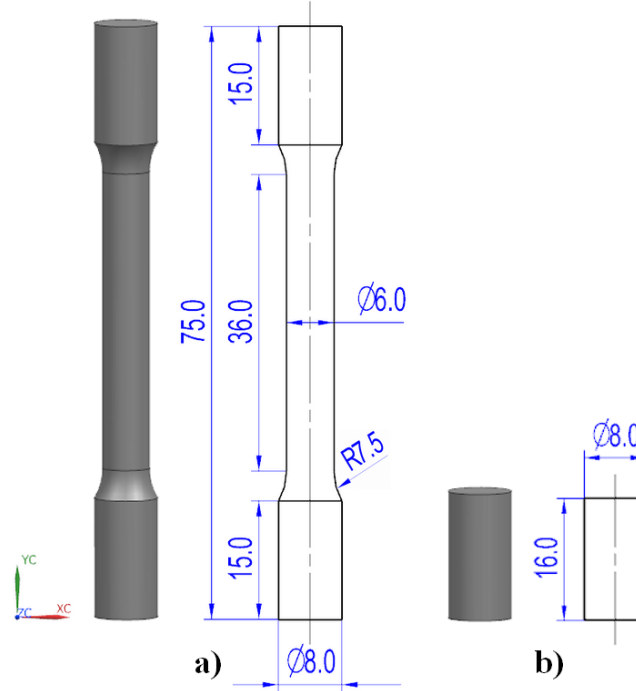
(4) The initial volume fraction of micro-voids  $f_0$  can be obtained with the aid of the X-ray CT System, i.e., the total volume of the micro-voids in a selected RVE divided by the volume of the RVE.

(5) For the damage-related parameters  $s_1$  and  $s_2$ ,  $s_1$  is considered as material-independent and the value of 2 is commonly adopted for most metallic materials [20].  $s_2$  can be calibrated by the hardening part of stress-strain curve.

#### 3.4. Finite element models for cold forming of SS316L

The geometrical models of the tensile and compressive specimens with the same size of the experimental ones are shown in Fig. 2. In the FE simulations, they were simplified as 2D axisymmetric, and y-axis was the symmetric axis of the model. The models were meshed with 2D 4-node bilinear axisymmetric quadrilateral elements (CAX4). For the boundary conditions of the FE model, fixed constraint was applied on the nodes of the bottom line and the axial loading was imposed on the nodes of the

top line. The friction coefficient of the compressive simulation was set to 0.3. The material parameters mentioned in Section 3.3 were employed in the user-defined subroutine UMAT to carry out the FE simulations.



**Fig. 2.** Specimens designed for failure prediction in cold forming: a) tensile specimen, b) compressive specimen (dimension in mm).

#### 4. Experiments

In this study, both uniaxial compression and tension tests were employed. The uniaxial compression test was carried out for deriving the material parameters for failure prediction, while the uniaxial tension test was performed for damage parameter calibration and experimental verification. A micro-focus X-ray CT system was specially employed to detect the micro-voids in the specimen. In such a situation, the configuration of RVE can be modelled and the VFMV can be obtained. In addition, experimental tensile and compressive stress-strain responses, as well as the

fractographic analysis of the fractured tensile specimens, were carried out to verify the applicability of the developed micro-damage model.

#### 4.1. Material and specimen preparation

The material used in this study was cold-rolled stainless steel SS316L bar with a diameter of 8 mm. The chemical composition of the SS316L is shown in Table 1. The specimens used for uniaxial compression and tension tests were designed according to the ASTM standard E9 – 09 and E8M – 09 respectively, i.e., cylindrical specimen with size of  $\varnothing 8\text{mm} \times 16\text{mm}$  for the compression test and dog-bone shape specimen with gauge size of  $\varnothing 6\text{mm} \times 25\text{mm}$  for the tension test. In order to detect the tiny internal micro-voids through the X-ray CT system, the overall size of the specimen should be designed and fabricated to be very small. Therefore, a cubic specimen with side length of around 1 mm was segmented inside the SS316L bar. It should be noted that all the compressive, tensile and CT scanning specimens were extracted from the same raw bar material to ensure the consistency of the material properties.

**Table 1** Chemical composition of SS316L (% in mass).

C	Mn	S	Si	Cr	Ni	Mo	Fe
0.02	1.46	0.004	0.03	17.58	11.05	1.42	balance

#### 4.2. Uniaxial compression and tension tests

The uniaxial compression and tension tests were carried out on an MTS 810 testing machine at room temperature. The strains were measured using extensometers with gauge length of 10 mm and 25 mm for compression and tension tests respectively. For



tension test, the applied load was controlled by the nominal strain and monotonic increased until the specimen fractured, while the load was gradually increased until a deformation of 30% was reached for the compression test. In order to predict the ductile damage behavior of the SS316L in cold forming under static or quasi-static state, the stress-strain curves at lower strain rates of  $0.001 \text{ s}^{-1}$ ,  $0.005 \text{ s}^{-1}$  and  $0.01 \text{ s}^{-1}$  were obtained from both compression and tension tests. For the cold forming process under dynamic state, e.g., at high strain rates ranging from  $1000 \text{ s}^{-1}$  to  $5000 \text{ s}^{-1}$  [35], the proposed micro-damage model is still effective since the elasto-plastic behaviour and hardening behaviour of the SS316L can be described by Eqs. (2) and (30), respectively.

#### *4.3. Determination of VFMV through micro-CT scanning*

The dual-tube micro-focus X-ray CT System YXLON FF35 CT was employed to capture 2D images of the specimen with defects of micro-voids before deformation, and then to reconstruct the 3D volumetric model. The installed high-resolution transmission tube enables the system to have the capability to detect the small-sized voids up to  $0.5 \text{ }\mu\text{m}$  in the material. The X-ray CT scanning was carried out at 150 kV tube voltage and  $60 \text{ }\mu\text{A}$  tube current, the integration time was set as 400 ms and 1080 projections were generated in one full rotation. Owing to the tiny size of the specimen and nano-focus of current mode adopted, the specimen was penetrated completely by X-ray and a high-resolution of  $2.7 \text{ }\mu\text{m}$  was achieved. Such resolution is high enough to detect the micro-defects in the metallic materials [36, 37].

Two-dimensional (2D) projected images with three views (front, side and top views) were obtained by rotating the specimen through  $360^\circ$ . Subsequently, 3D volume elements were reconstructed based on appropriate algorithms with the aid of the

software VG Studio MAX 2.2. By defining the relative pixel intensity values of the background and material, the surface of the material was determined. Thus, the profile and inner structure of the specimen were obtained. Furthermore, by processing the voxel data of the scanned 3D CT images, information like size, number, spatial location, compactness, and sphericity of micro-voids were obtained through the defect analysis module of VGStudio MAX 2.2, then the volume of each micro-void was calculated based on their size, compactness, and sphericity [38], thus the VFMV was obtained through dividing the total volume of micro-voids in a selected RVE by the volume of the RVE. The effectiveness of the micro-damage model is highly dependent on the accuracy and reliability of the VFMV to be calculated from the information obtained through the X-ray CT System.

#### *4.4. Fractography for fracture surfaces*

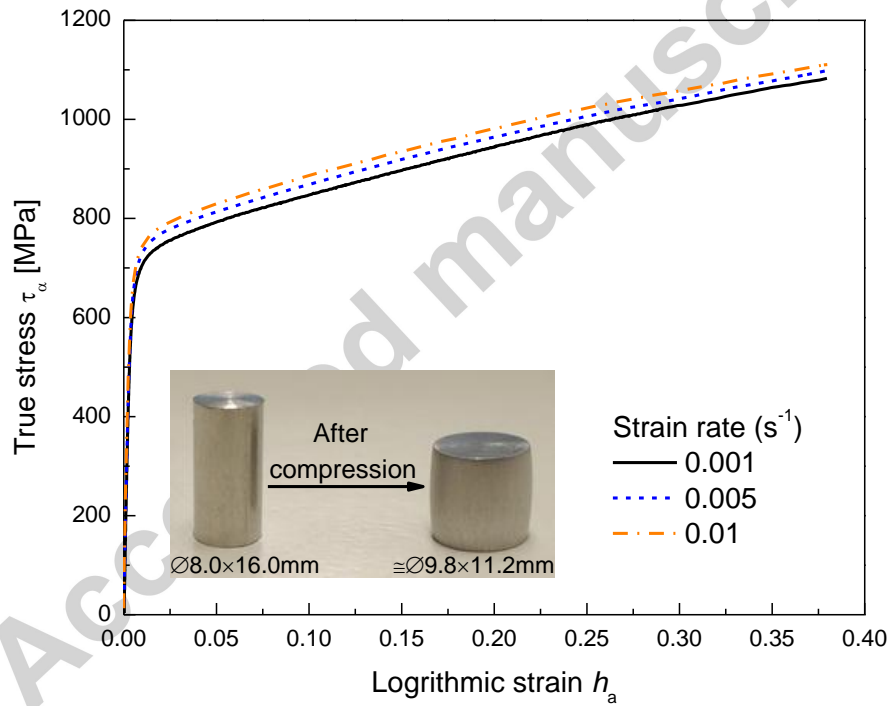
The fracture surfaces of the specimens after tension tests were analyzed by using the HITACHI S-3400N SEM. The accelerating voltage was set as 20.0 kV, the current and working distances ranged from 60–150  $\mu$ A and 10.5–16.8 mm respectively. Both low and high magnification images were captured for qualitatively analysis of the fracture mode and fractographical details such as micro-voids, dimples and porosities.

## **5. Results and discussions**

### *5.1. Experimental stress-strain curves*

The uniaxial compressive stress-strain curves of cold-rolled SS316L at three different strain rates are shown in Fig. 3. It can be observed that the strain rate has little influence on the elastic modulus of the material whereas the measured elastic modulus

is 194 GPa. However, the remarkable rate-dependence and strain-hardening effect were presented during the plastic deformation, i.e., the higher the strain rate, the greater the stress at the same measured strain. By fitting the experimental stress-strain data, the isotropic hardening parameters  $Q_0$ ,  $R^{\text{sat}}$  and  $\beta$ , as well as the viscoplastic constants  $K$  and  $n$ , were obtained. In addition, the damage parameter  $s_2$  was first calibrated by the hardening part of the compressive stress-strain curve and further calibrated by comparing the decline parts of the simulated and experimental tensile stress-strain curve. Table 2 shows the elastoplastic and damage parameters of the SS316L.



**Fig. 3.** Experimental compressive stress-strain curves of SS316L at different strain rates.

**Table 2** Elastoplastic and damage parameters of SS316L.

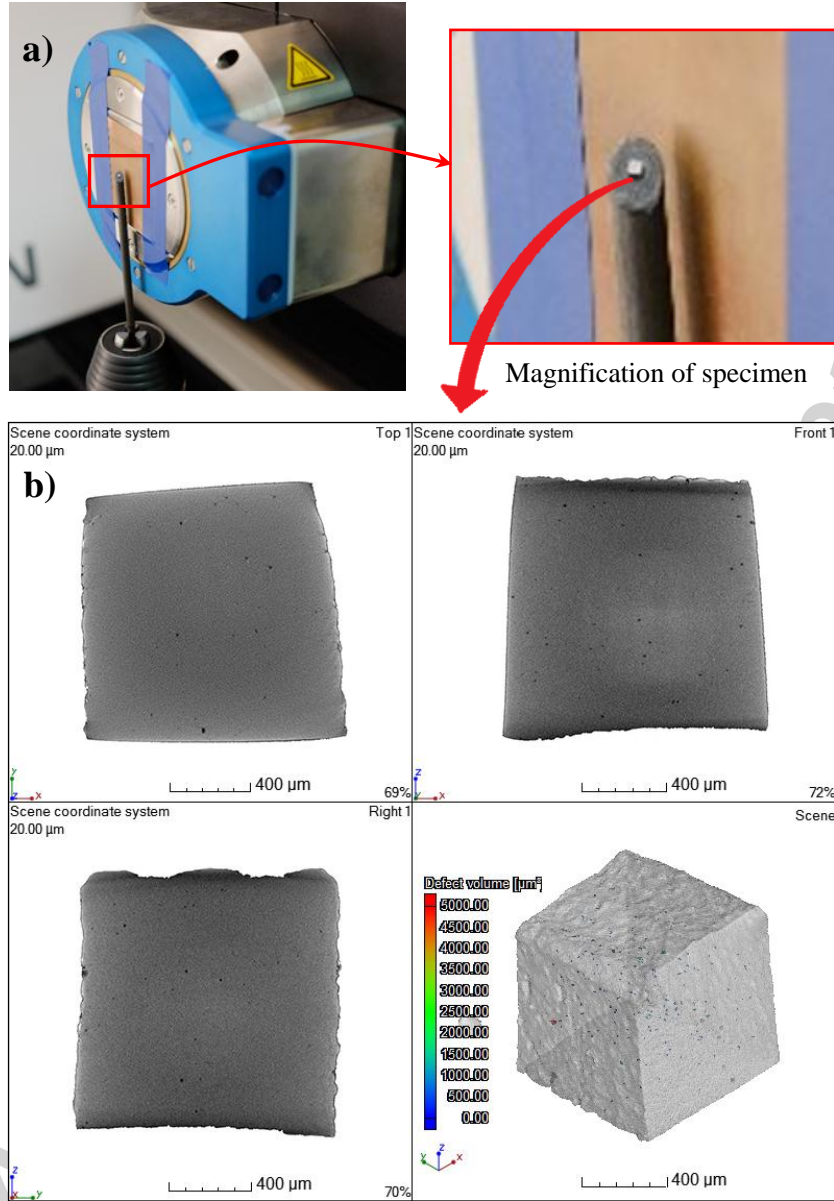
$E$	$\nu$	$Q_0$	$R^{\text{sat}}$	$\beta$	$K$	$n$	$s_1$	$s_2$
194 GPa	0.3	565 MPa	518 MPa	3.0	220	9.8	2	157

### 5.2. VFMV inside the specimen

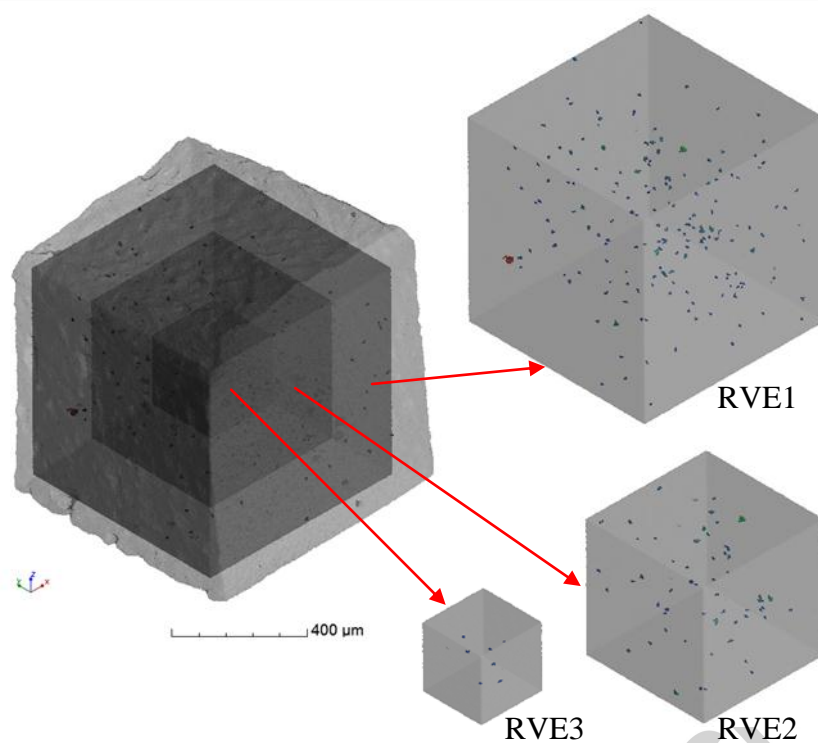
The micro-voids of the cold-rolled SS316L specimen were detected by using our newly installed YXLON FF35 CT System. Fig. 4 shows the scanned cuboid specimen and the reconstructed 2D and 3D CT images. The relative pixel intensity value was used to determine the material surfaces of the CT images, a threshold value from image histogram was adopted to separate the micro-voids and material in the images; the micro-voids inside the specimen material were analyzed further by using the defect detection module of VGStudio MAX 2.2. To better visualize the internal micro-voids of the material, the 3D CT images of the scanned specimen were displayed semi-transparently. As shown in Fig. 4 (b), the material is represented in grey and the micro-voids with sizes ranging from 5.56  $\mu\text{m}$  to 22.25  $\mu\text{m}$  are represented in other different colors.

In order to study the shape, size and spatial distribution of micro-voids inside the material, three RVE models with side lengths of 750  $\mu\text{m}$ , 500  $\mu\text{m}$  and 250  $\mu\text{m}$  were constructed from the 3D CT images, as shown in Fig. 5. The sizes distribution of micro-voids in the three RVE models are shown in Fig. 6; the results indicated that different sizes of RVE models tend to exhibit the similar trend of micro-voids distribution, i.e., following a normal distribution. The average size of micro-voids in the RVE models is around 9.8  $\mu\text{m}$ , the VFMV in the RVE model was obtained by measuring the volume fraction of the micro-voids and the RVE model, and the average value is 0.000237, as shown in Fig. 7. It was also found that the average size

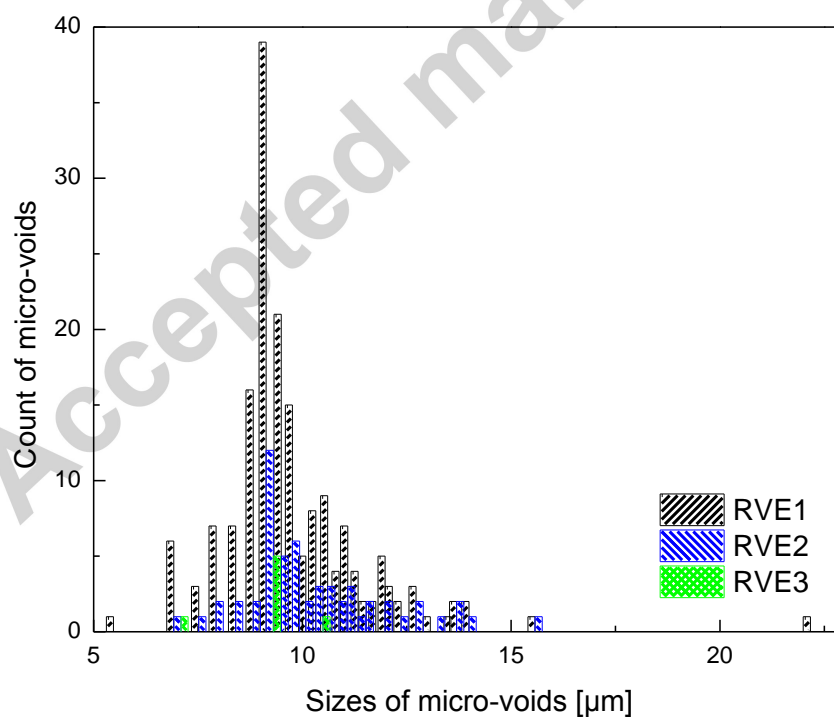
and VFMV in different RVE models were similar, indicating that the micro-voids inside the cold-rolled SS316L specimen were uniformly distributed.



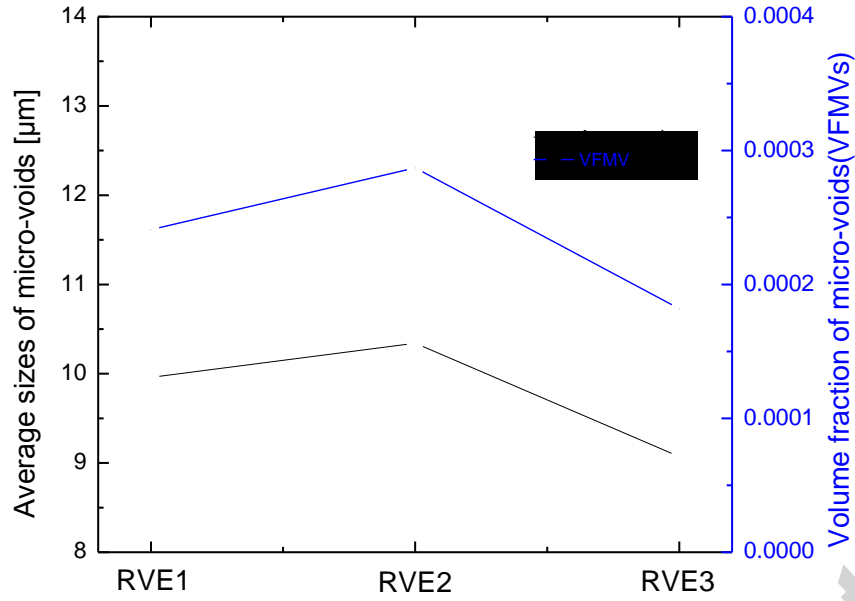
**Fig. 4.** X-ray CT scanning of the SS316L specimen: a) Cuboid specimen for CT scan, b) 2D and 3D CT images with micro-voids.



**Fig. 5.** Reconstruction of RVE models from the 3D CT images.



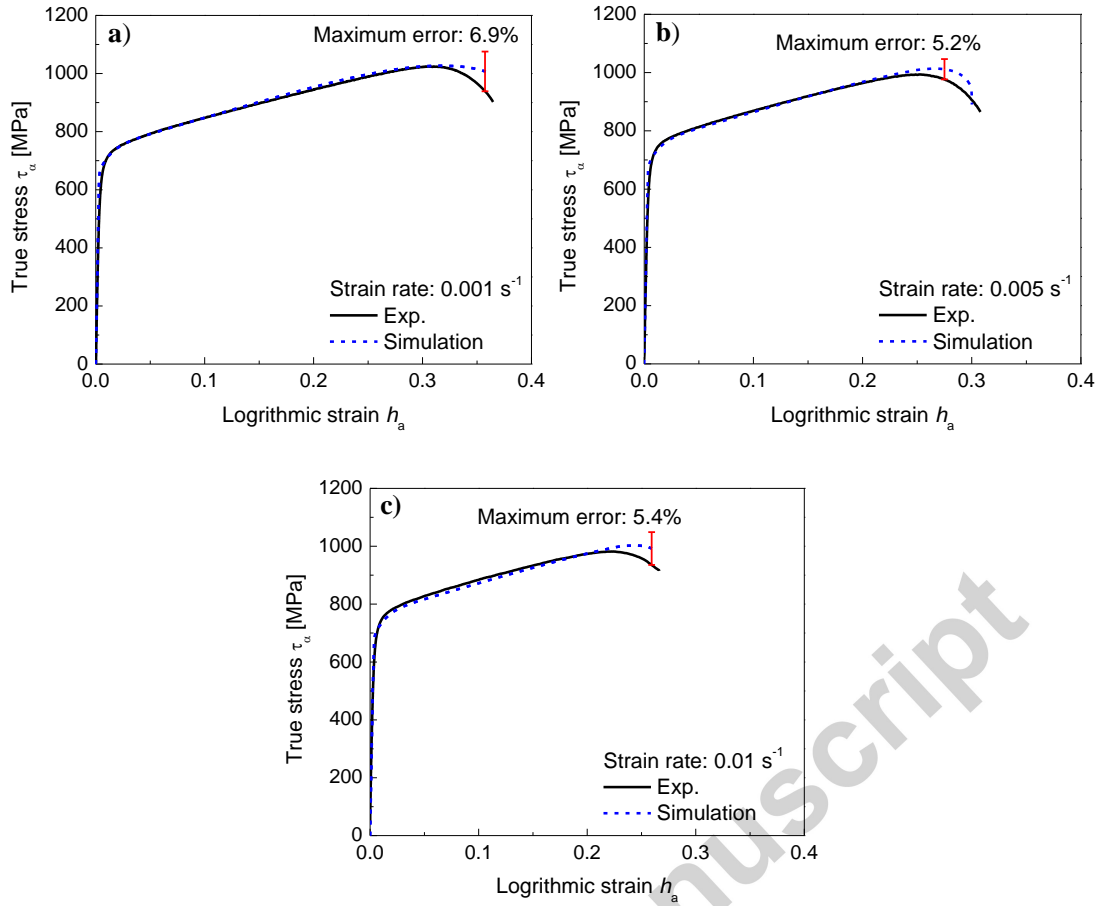
**Fig. 6.** Sizes distribution of micro-voids in RVE models.



**Fig. 7.** Average sizes and VFMVs in RVE models.

### 5.3. Failure prediction in cold forming of SS316L and the verification

In this section, the stress-strain response, damage distribution and evolution of the tensile and compressive specimens at different strain rates were predicted and verified experimentally. The experimental and predicted stress-strain curves of the SS316L specimens under uniaxial tensile deformation are shown in Fig. 8. It can be concluded that the tensile stress-strain responses of the material at different strain rates were well predicted by the micro-damage model prior to necking. Although certain discrepancies of the stresses (as shown in Fig. 8) were found after necking whereas the maximum error was around 6.9%, the onset of necking could still be predicted accurately, and the final fracture strains of the specimens under tensile tests at strain rates of  $0.001 \text{ s}^{-1}$ ,  $0.005 \text{ s}^{-1}$  and  $0.01 \text{ s}^{-1}$  were 0.365, 0.308 and 0.266, respectively. It was also observed that the maximum stresses of the SS316L specimens at different strain rates were almost the same, i.e., about 1020 MPa, however, better ductility was achieved at a lower strain rate under tensile deformation.



**Fig. 8.** Experimental and predicted stress-strain curves of SS316L specimens under uniaxial tensile deformation at strain rates of: a)  $0.001 \text{ s}^{-1}$ ; b)  $0.005 \text{ s}^{-1}$  and c)  $0.01 \text{ s}^{-1}$ .

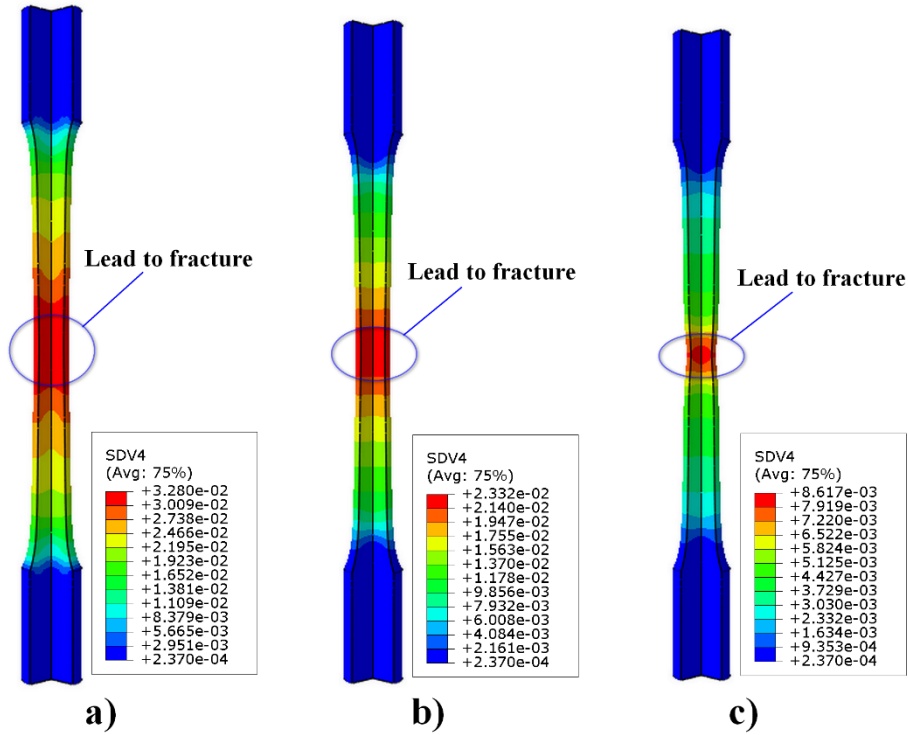
The fractured tensile test specimens and the predicted damage distributions of tensile specimens at final fracture strains are shown in Figs. 9 and 10, respectively. It is noted that all the simulated 2D geometrical models were swept  $270^\circ$  along the y axis to generate 3D models for better demonstration of the internal damage distribution. In the tensile tests, localized necking occurred in each specimen. However, obvious localized necking only occurred in specimen (c) in FE simulations. The reason for this difference is that only severe damage concentration occurred at the centre region of specimen (c). The damage concentration would result in a larger deformation of the corresponding meshes in FE simulations at which localized necking occurred. It can



be seen from Fig. 10 that higher strain rate resulted in more concentrated damage distribution, especially for specimen (c). Therefore, obvious localized necking occurred in the centre region of specimen (c), while only slight necking was found in specimens (a) and (b). Unlike the slight variation of maximum stresses, the maximum damage values of the specimen were decreased significantly with the increase of the strain rate. It should be noted that the maximum damage value indicated the allowable damage value of the specimen, i.e., the greater the allowable damage value, the stronger the ability of the material to resist fracture. Therefore, it can be concluded that the SS316L has better failure resistance at lower strain rates.



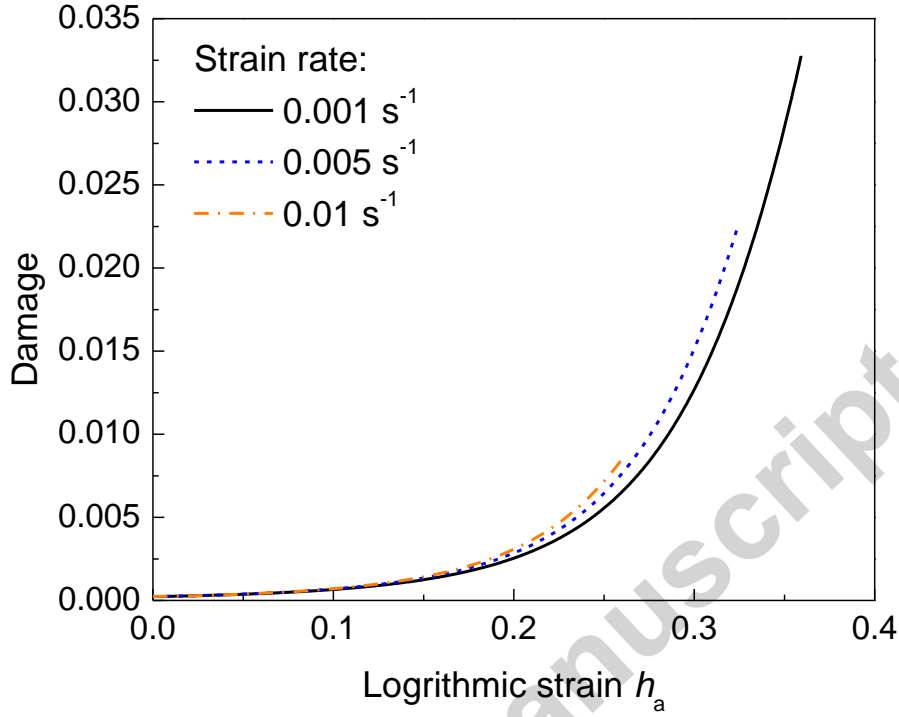
**Fig. 9.** The fractured tensile test specimens at strain rates of: a)  $0.001 \text{ s}^{-1}$ ; b)  $0.005 \text{ s}^{-1}$  and c)  $0.01 \text{ s}^{-1}$ .



**Fig. 10.** The predicted damage distributions of tensile specimens at strain rates of: a)  $0.001 \text{ s}^{-1}$ ; b)  $0.005 \text{ s}^{-1}$  and c)  $0.01 \text{ s}^{-1}$ .

Since the failure of the specimen is most likely to occur in the severely damaged location, the damage evolution in the centre regions of the specimens at different strain rates was analysed, as shown in Fig. 11. It can be seen that the evolution of damage followed a power function trend, i.e., the damage evolved slowly at the initial deformation stage, while increasing rapidly with the increased strain, and greater damage occurred in higher strain rate at the same strain level. In this study, the tensile simulations were performed up to the final fracture strain of each specimen. Therefore, the volume fraction of micro-voids corresponding to the final fracture strain was the critical value of micro-void evolution that led to fracture of the materials. According to the predicted results shown in Figs. 10 and 11, fracture of the materials would occur when the volume fraction of micro-voids of the tensile specimens reached

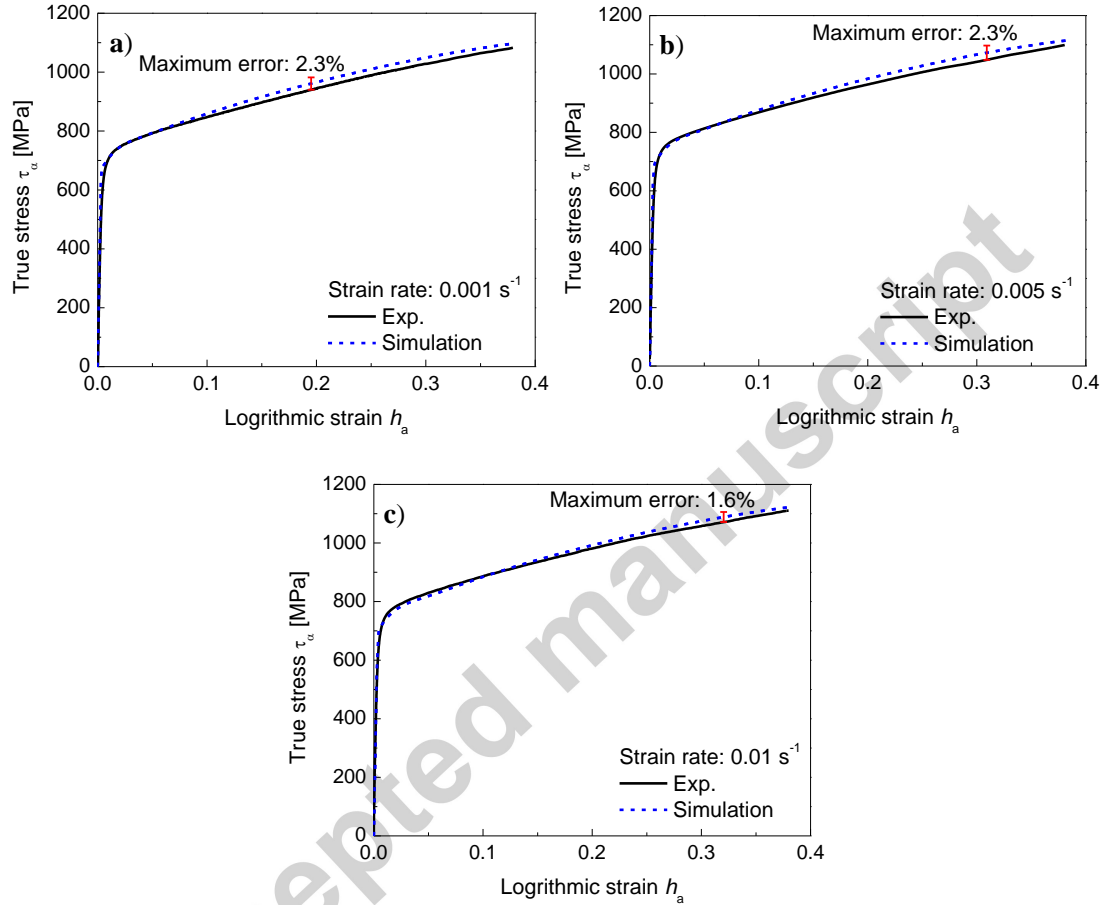
0.0328, 0.0225 and 0.00869 (these values also represent the allowable damage values) at strain rates of  $0.001 \text{ s}^{-1}$ ,  $0.005 \text{ s}^{-1}$  and  $0.01 \text{ s}^{-1}$ , respectively.



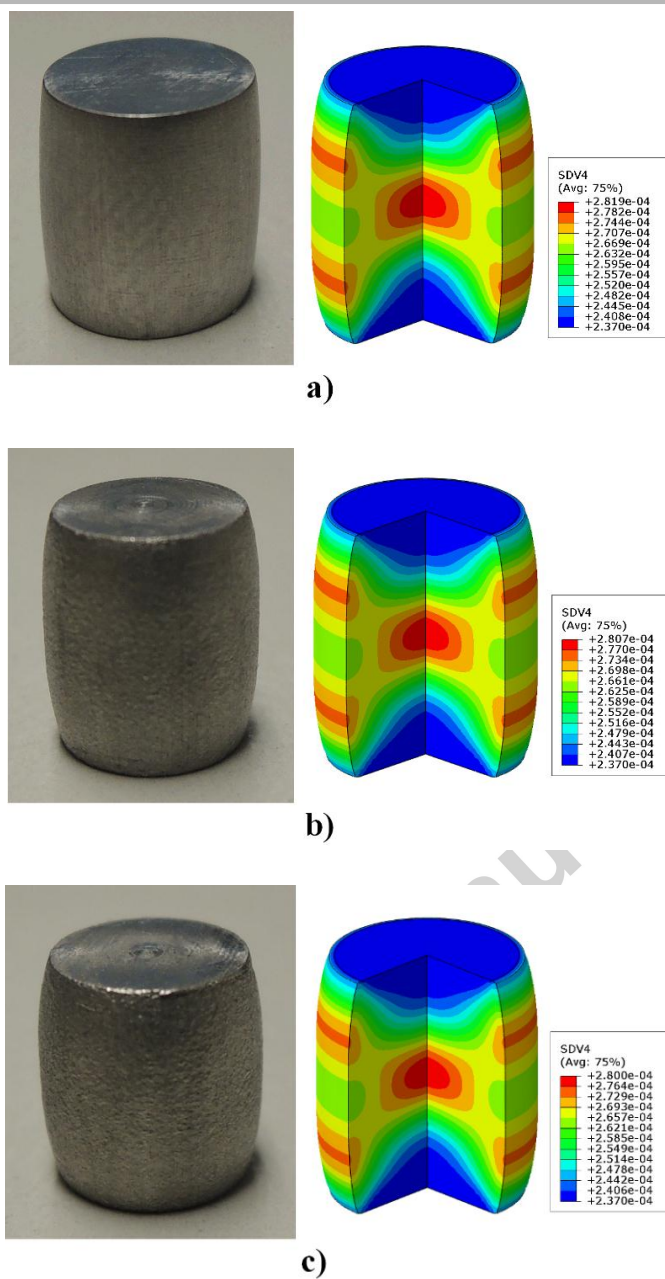
**Fig. 11.** Damage evolution at the centre regions of the tensile specimens at different strain rates.

The experimental and predicted stress-strain curves of the SS316L specimens under uniaxial compressive deformation are shown in Fig. 12. It can be seen that the predicted stress-strain curves were consistent with the experimental results at different strain rates, and the maximum error was less than 2.3%. The tested and predicted damage distributions of the compressive specimens at different strain rates are shown in Fig. 13. It can be seen that the distributions of the damage at different strain rates were basically the same, and the damage was mainly distributed in the centre and the cylindrical surface of the specimens. As shown in Fig. 14, the evolution of the damage at the centre region of the specimen was very slow under compressive

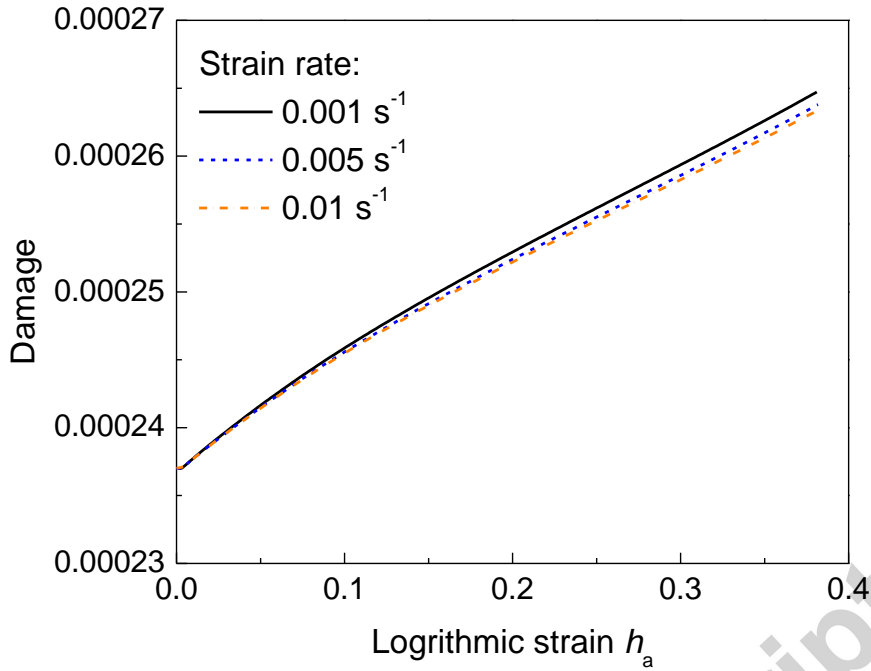
deformation, and the maximum damage value for all the specimens was about 0.00028, which was much smaller than that of the tensile deformation. It may be concluded that the failure was not likely to happen in this material due to the small damage accumulation during compressive deformation.



**Fig. 12.** Experimental and predicted stress-strain curves of SS316L specimens under uniaxial compressive deformation at strain rates of: a)  $0.001 \text{ s}^{-1}$ ; b)  $0.005 \text{ s}^{-1}$  and c)  $0.01 \text{ s}^{-1}$ .



**Fig. 13.** The tested and predicted damage distributions of the compressive specimens at strain rates of: a)  $0.001 \text{ s}^{-1}$ ; b)  $0.005 \text{ s}^{-1}$  and c)  $0.01 \text{ s}^{-1}$ .



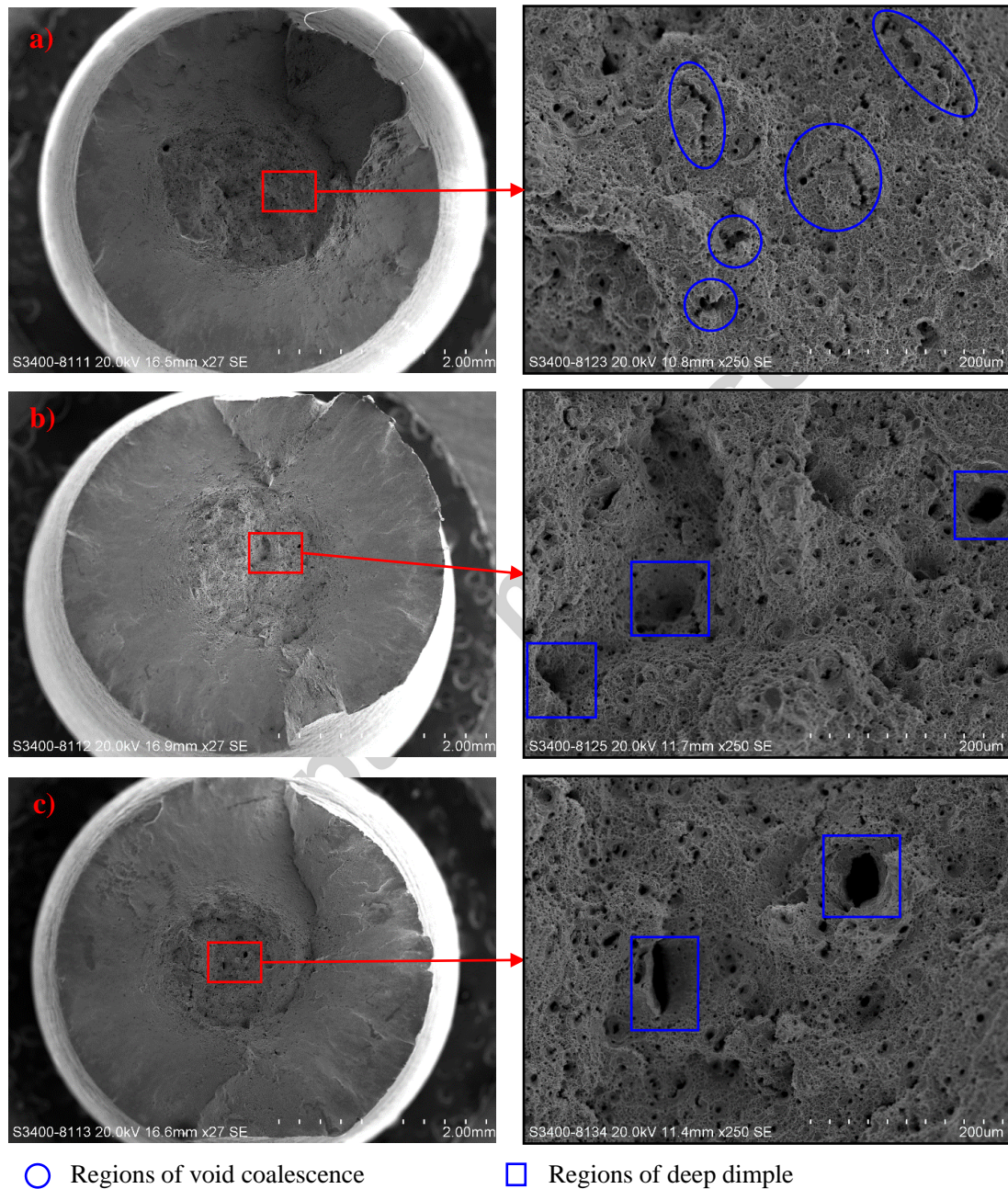
**Fig. 14.** Damage evolution at the centre regions of the compressive specimens at different strain rates.

#### 5.4. Fractographic analysis

The fractographic analysis of the fractured tensile specimens at different strain rates was performed in this study. As shown in Fig. 15, many micro-voids and dimples were found in each fracture surface, indicating that the fracture mode of the tested SS316L was ductile fracture, and the failure was largely caused by the initiation and coalescence of micro-voids in pure tension of such metallic material. It was noted that the strain rate had an obvious influence on the ductile fracture of the material. The fracture surface of the specimen, at a strain rate of  $0.001 \text{ s}^{-1}$ , was relatively flat and the size and distribution of the dimples were uniform. Several micro-void coalescence regions were shown clearly, indicating that the growth and coalescence of micro-voids can be fully formed at lower strain rates, resulting in better ductility of the material. With the increasing strain rate, the fracture surfaces of the specimens were rough and wavy (see Fig. 15 (b) and (c)), less dimples were found and their



distribution was uneven, and several large, deep dimples were observed but with no obvious coalescence of micro-voids. It may be concluded that the rapid growth of the micro-void in some local regions resulted in the remaining area becoming too small to withstand the load, eventually leading to the failure of the specimen.



**Fig. 15.** SEM fractographs of the fracture surface of tensile specimens at strain of: a)  $0.001 \text{ s}^{-1}$ , b)  $0.005 \text{ s}^{-1}$  and c)  $0.01 \text{ s}^{-1}$ .

## 6. Conclusions

In the present work, a micromechanics-based damage model for failure prediction in large deformation of SS316L undergoing cold forming was developed. A micro-focus high-resolution X-ray CT system was employed to measure the micro-voids inside the specimen. 3D RVE models were reconstructed from the processed CT images of the specimen, and the average size and VFMV for the specimen were determined via statistical analysis. Furthermore, the micro-damage model was compiled as a user-defined material subroutine into ABAQUS. The stress-strain responses and damage evolution of specimens under tensile and compressive deformations at different strain rates were predicted. Finally, the experimental tensile and compressive stress-strain response, as well as the fractographic analysis, were carried out for experimental verification. The following conclusions can be drawn:

1. A micromechanics-based damage model that was concerned with the evolution of micro-voids was developed for failure prediction in large deformation of SS316L undergoing cold forming.
2. X-ray CT scanning was performed to detect the internal micro-voids of the specimen. The results indicated that the sizes distribution of micro-voids in different regions of the specimen tended to be similar and followed a normal distribution. The average size of micro-voids in RVE models was around 9.8  $\mu\text{m}$  and the VFMV in the specimen was 0.000237.
3. For the tensile deformation, the maximum stresses of SS316L specimens at different strain rates were almost the same, i.e., about 1020 MPa. However, the allowable damage values of the specimen were decreased significantly with the increase of the strain rate, and the SS316L had better failure resistance at lower



strain rates. The evolution of damage in tensile deformation followed a power function trend, i.e., the damage evolved slowly at the initial deformation stage, while increased rapidly with the increasing of strain, and the greater damage occurred in higher strain rate at the same strain level.

4. For the compressive deformation, the predicted stress-strain responses of SS316L specimens at different strain rates were in good agreement with the corresponding experimental ones. The damage distribution and evolution of the specimens at different strain rates were basically the same. The evolution of damage in compressive deformation was very slow and the damage value was much smaller than that of tensile deformation. It may be concluded that the failure was not likely to happen in this material due to the small damage accumulation during compressive deformation.
5. Ductile fracture mode was found for the fractured tensile specimen. The growth and coalescence of micro-voids can be formed fully at the lower strain rate, which resulted in a better ductility of the material. With the increasing of strain rate, the rapid growth of the micro-void in some local regions resulted in the remaining area becoming too small to withstand the load, eventually leading to the failure of the specimen.

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