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A mathematical model and algorithms for the aircraft hangar maintenance scheduling problem

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Abstract

An aircraft hangar maintenance scheduling problem is studied, motivated by the aircraft heavy maintenance conducted in a hangar operated by an independent maintenance service company. The aircraft hangar maintenance scheduling problem in such context consists of determining a maintenance schedule with minimum penalty costs in fulfilling maintenance requests, and a series of hangar parking plans aligned with the maintenance schedule through the planning period. A mixed-integer linear programming (MILP) mathematical model, integrating the interrelations between the maintenance schedule and aircraft parking layout plans, is presented at first. In the model, the variation of parking capacity of the maintenance hangar and the blocking of the aircraft rolling in and out path are considered. Secondly, the model is enhanced by narrowing down the domain of the time-related decision variables to the possible rolling in and out operations time of each maintenance request. Thirdly, to obtain good quality feasible solutions for large scale instances, a rolling horizon approach incorporating the enhanced mathematical model is presented. The results of computational experiments are reported, showing: (i) the effectiveness of the event-based discrete time MILP model and (ii) the scalability of the rolling horizon approach that is able to provide good feasible solutions for large size instances covering a long planning period.

Keywords: Aircraft maintenance scheduling, Hangar parking layout planning, Mixed-integer linear programming, Event-based model, Rolling horizon approach

1. Introduction

The rapid development of air transport has led to significant economic growth, and the demand for commercial air transport has been increased [1, 2]. This rapid growth of air transport has imposed many challenges on planning and operations activities in the aviation industry [1, 3]. Many airline companies have been reconsidering their operations practices in conducting maintenance activities on their fleet, in order to ensure aircraft maintenance, repair and overhaul (MRO) operations continue to conform with the regulations prescribed by aviation authorities, while maintaining minimum maintenance cost [4, 5]. Instead of conducting the heavy maintenance requiring significant input in terms of the hiring of licensed engineers, holding maintenance materials and operating a maintenance hangar within the airline company, outsourcing of MRO operations to an independent service company has become an intriguing option. The cost of MRO is around 9% of the annual operating cost for airlines, and it is the third highest cost behind fuel and labour costs [6]. By outsourcing MRO activities, an airline company can concentrate on its high-value added activities and utilize the additional savings in other areas. It is estimated that the percentage of outsourcing has risen from about 25 per cent to around 70 per cent of maintenance activities between the mid-1990s and 2012. [7].

The proposed maintenance scheduling and parking layout planning problem is studied from the perspective of an aircraft maintenance service company providing heavy maintenance service. The aircraft maintenance, repair and overhaul (MRO) activities are critical for aircraft safety, and periodic maintenance checks need to be carried out on each aircraft upon meeting operating for a specified number of flying hours. Outsourcing of MRO activities from airlines to maintenance service providers continue to grow, as the air transport demand increases. According to different airlines' flight plans, many maintenance requests are initiated by their internal maintenance plans. From the perspective of the maintenance service provider, efficiently fulfilling the overwhelming maintenance requests with limited resource availability becomes challenging. One of the major tasks within the maintenance company is to develop a maintenance schedule which involves substantial operational decision making. Such a maintenance plan includes the maintenance schedule for each aircraft (roll in and roll out time) and the parking position of each aircraft in the hangar. Over the planning horizon, the roll in and roll out times of all aircraft should align with the parking plans. The development of such a plan is challenging due to the following considerations: (i) the hangar capacity that accommodates the aircraft varies according to the incoming maintenance requests at different times; (ii) blocking between aircraft occurs whenever there are many incoming maintenance requests arriving at similar times, or the planner makes improper roll in and roll out arrangements. To address these issues and provide a systematic approach to solve the problem, we propose an optimization methodology to develop maintenance plans from the perspective of the independent aircraft maintenance service company. The work described in this paper extends the mathematical model presented in a conference paper we published earlier [8], and the computational efficiency of the basic mathematical model presented in [8] is significantly enhanced by the new approaches we developed in this paper. We consider the blocking of aircraft movement operations due to improper hangar planning and overwhelming maintenance requests as a significant bottleneck in fulfilling the maintenance requests, while a such factor has not been incorporated in the other multi-period layout planning problems in the literature. In this regard, the major focus of this paper falls into the coordination between maintenance scheduling and hangar layout planning. Other practical factors, such as arranging the aircraft's position according to its maintenance type and distance to specific tooling, and manpower limitations, are not incorporated in the scope of the model although it is possible to extend the model with these practical factors.

The remainder of this paper is organized as follows. The literature review in Section 2 analyses the problem nature, research background as well as the research gaps. Afterwards, we present the solution procedures for the proposed problem in Section 3. The results of computational experiment are reported in Section 4. Finally, the conclusions and future work are discussed in Section 5.

2. Literature review

2.1 Hangar maintenance scheduling

Aircraft maintenance tasks are conducted in a set of checks periodically to ensure the aviation safety, and the frequency of various maintenance checks is prescribed by the combinations of flying hours as well as the number of take-off and landing cycles [9]. There are four major types of checks (Type A, B, C and D checks) that are regulated by the Federal Aviation Administration according to the maintenance scope, duration as well as the frequency [10]. Aircraft maintenance is high cost activity regarding the equipment, inventory and manpower. Samaranayake et al. [11] studied the complexity of conducting aircraft maintenance checks involving extensive equipment, tools and materials, then developed an engineering structure to efficiently manage the scheduling of aircraft maintenance. While classifying the maintenance checks according to their work places, the maintenance checks can be categorized into line maintenance and hangar maintenance [12]. Line maintenance refers to "on line" maintenance that is conducted within the turnaround time between two flights, as the aircraft is parked at the gate or the apron, to guarantee a reliable aircraft dispatch [13], and Type A check is usually classified into line maintenance. For the other check types (B, C and D checks), they usually refer to "hangar" maintenance, as they require intensive maintenance inputs and long maintenance lead-times compared with the line maintenance. Some studies covered workforce scheduling problems from the aircraft maintenance company's perspective [14, 15]. De Bruecker et al. [16] considered an aircraft maintenance personnel rosters problem from an independent aircraft line maintenance company serving several airline companies. Liang et al. [17] considered an aircraft maintenance routing problem incorporating propagated delays in optimization, and Gavranis and Kozanidis [18] proposed an exact algorithm to solve a maintenance scheduling problem that maximized the fleet availability of a military aircraft unit. Chen et al. [19] considered the licensed technicians assignment optimization problem in the context of an aircraft maintenance hangar operated within single airline company, assuming constant hangar capacity. Recently, Qin et al. [20] proposed an aircraft parking stand allocation model for a maintenance company serving different size aircraft in batches, considering the variation of hangar capacity.

2.2 Layout planning problem

The problem studied in this paper involves a dynamic layout planning problem. In the literature, some optimization problems share some similarities in the problem nature and assumptions. The extension of the traditional Vehicle Routing Problem (VRP) incorporating simultaneous picks-up and deliveries, and two-dimensional loading constraints (2L-SPD) belongs to the class of the composite routing-packing optimization problem [21]. In Vehicle Routing Problem with Two-dimensional Loading and picks-up/deliveries constraints, one has to determine the route of a vehicle that satisfies customers at different demand and delivery points and consider a two-dimensional packing problem for the placing the goods in the vehicle for different customers [22], requiring that the routing of the vehicle must satisfy the Last-In-First-Out (LIFO) loading and unloading constraint. In addition, in the literature, the items to be arranged in the vehicle are all rectangle [21-25]. Moreover, the Facility Layout Problem (FLP) is another classic layout planning problem, which aims to determine the locations of rectangular facilities at different sites, minimizing the material handling costs between the facilities [26-29]. Dynamic Facility Layout Problems consider arranging the facilities over a planning period instead of one-time planning [30, 31]. Though layout planning problems have been extensively studied in the literature from various perspectives, such as the manufacturing industry [32-36], the relevant approaches cannot be directly applied in our problem due to the following considerations: (i) the shape of an aircraft is irregular. (ii) The Last-In-First-Out constraint can be relaxed as a soft constraint in the maintenance scheduling problem. (iii) Blocking during the aircraft roll in/out operations significantly affects the efficiency and needs to be characterized.

2.3 Non-overlapping constraints for irregular items

The aircraft parking stand allocation problem embedded in the maintenance scheduling problem can be modelled as a cutting and packing problem in a two-dimensional fixed dimension container. The most widely used tool for checking whether two irregular polygons overlap in the cutting and packing problem is the No-Fit Polygon (NFP). Bennell and Oliveira [37] and Bennell and Oliveira [38] provided a detailed tutorial on how to generate NFP between two non-convex irregular polygons. Alvarez-Valdes et al. [39] introduced a horizontal slices formulation approach to enhance the formulation of Fischetti and Luzzi [40]. Martinez-Sykora et al. [41] adopted horizontal slices in their MIP formulation to solve the irregular pieces packing problem with guillotine cuts. Cherri et al. [42] proposed two robust mixed-integer formulations for the irregular polygon packing problem that decompose the non-convex polygons into several convex pieces to generate NFP.

3. Problem statement and mathematical formulation

3.1 Problem statement

Aircraft heavy maintenance has to be conducted in the aircraft hangar after meeting the flying hours prescribed by the aviation authorities[12]. The aircraft is taken out of service and sent to a maintenance service company for heavy maintenance. The maintenance service company receives the maintenance requests initiated by the respective airlines according to their internal flying plans. To fulfill these maintenance requests from clients, the maintenance service company has to determine 1) a maintenance schedule serving the requests, consisting of the timing of movement operations for each aircraft and 2) hangar parking layouts at different times whenever there are any movement of aircraft inducing the changes of the hangar layout along the planning period, subject to the capacity of hangar space. The main goal is to minimize the penalty cost induced in fulfilling the maintenance requests. Figure 1 demonstrates a solution of hangar maintenance problem. In particular, Figure 1 demonstrates the transitions of the hangar layout plan from time 1 to 10, and it specifies the position assigned for each aircraft and the respective roll in and roll out timings along the planning period. It is note that the hangar layout at Time 2 is omitted as there is not any movement operation conducted at that time, and therefore the layout at Time 2 is kept unchanged and it is the same as Time 1. Specifically, the downward arrow represents that the respective aircraft is rolled into the hangar at current time, and the upward arrow means that the aircraft has finished the maintenance task and is rolled out from the hangar at respective time. The hangar layout of all times are coherent with respective preceding and subsequent layout plans to ensure the continuity. Moreover, if there are both rolling in and rolling out operation taken place at same point of time, the roll in operation commences after all rolling out operations finish. Take the hangar layout at Time 3 as an example, there is one large aircraft rolling out from the hangar, then two newly arrival small aircraft rolling into the hangar to take up the space vacant from the large departing aircraft.

The contributions of the studied problem can be summarized as two fold: 1) from the perspective of the MRO industry, many researches focused on airline-operated MRO activities' optimizations, which makes the existing approaches inapplicable for the maintenance service company in actual situations. Given the situation that the service company carries out the hangar maintenance schedule manually, in current practice, the developed mathematical model is tailored for the hangar maintenance service company, which significantly increases their planning efficiency and accuracy. 2) from the perspective of academia, it fills the gaps in the literature regarding the aircraft maintenance problem in the context of the MRO service company that have not been addressed yet. Moreover, the problem studied in this paper extends the multi-period layout planning problem, as the blocking during the facility movements during planning was not regarded as a main bottleneck in the other studies.

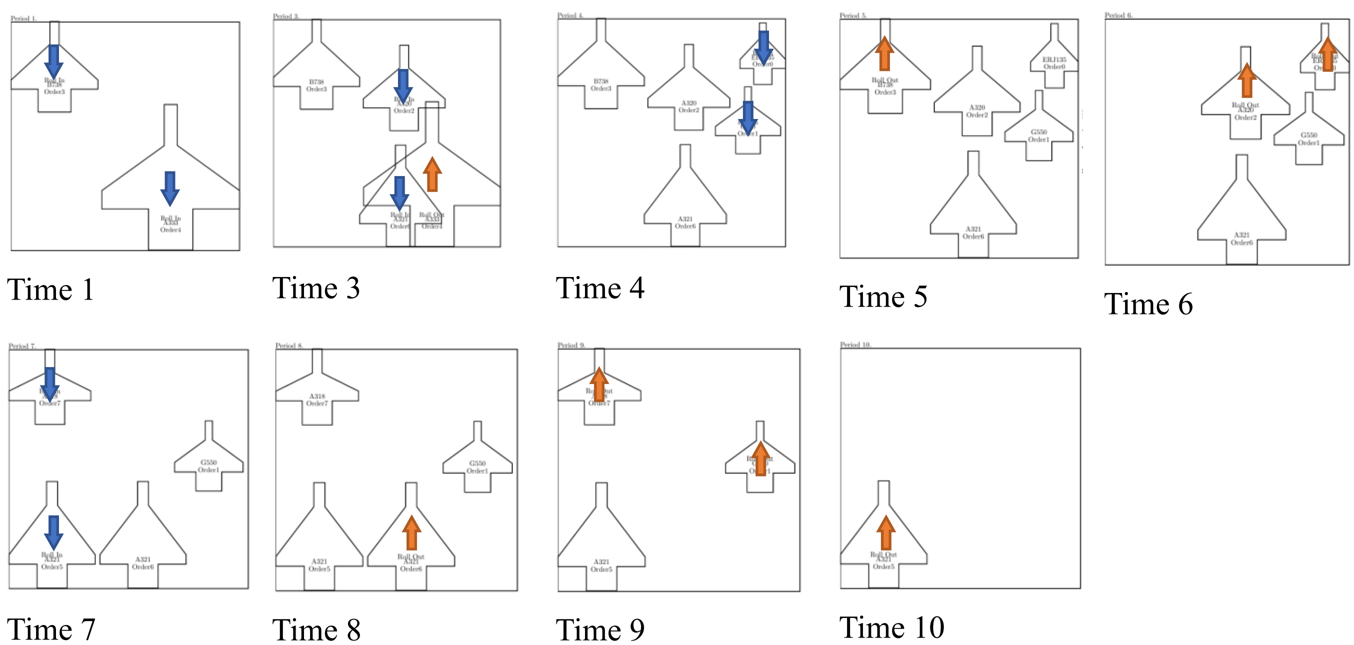


Figure 1 Hangar maintenance problem

3.2 Aircraft non-overlapping approach and three-dimensional parking

As we consider the physical shape of an aircraft in undertaking the parking planning, appropriate modelling of aircraft is fundamental to fully utilize the hangar space. The non-overlapping approach discussed in this section is incorporated in the mathematical model. Given the geometric shape of an aircraft, it can be characterized as a non-convex polygon (Figure 2). We denote the reference point of each aircraft to be the middle point at the bottom of the aircraft, and the coordinates of the reference point of aircraft p_i in two-dimensional space are denoted as (x_i, y_i). For a pair of aircraft p_i and p_j, the No-fit polygon NFP_{ij} is the region in which the reference point of aircraft p_j cannot be placed if aircraft p_i remains stationary since it would overlap aircraft p_i. A feasible zone for placing aircraft p_j without overlap with p_i is the region outside NFP_{ij}. Given these two polygons, the NFP_{ij} is generated by tracing the path of the reference point on p_j as p_j slides around the boundary of p_i, such that two polygons always touch but never overlap (Figure 3). Therefore, if the reference point of *j* moves into the NFP_{ij} then the two polygons overlap, and the interior of the NFP_{ij} represents all overlapping positions.

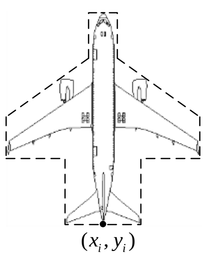


Figure 2 Geometric Representation and reference point of aircraft

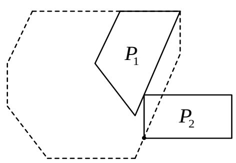
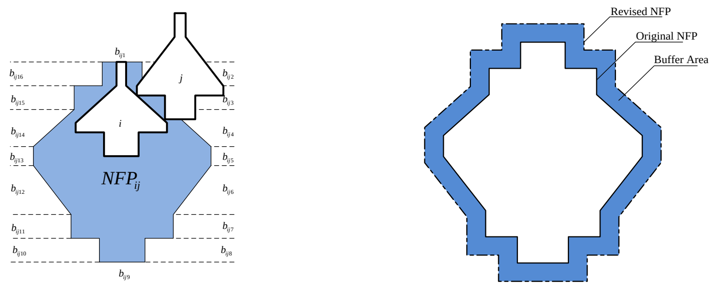


Figure 3 No Fit Polygon of P_1 and P_2



(a) **(b)**

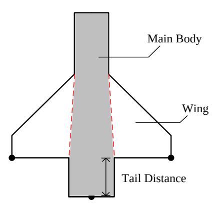
Figure 4 Horizontal slices outside NFP

According to Alvarez-Valdes et al. [39], each horizontal slice is defined by drawing one or two horizontal line(s) outwards from each vertex of the NFP, and they are then characterized by one or two horizontal edge(s) as well as the part of boundary of the NFP (Figure 4 (a)). A set of variables b_{ijk} is associated with each horizontal slice and the reference point of p_j is placed in the slice *k* if $b_{ijk} = 1$. Therefore, a general form of the constraint preventing overlap is

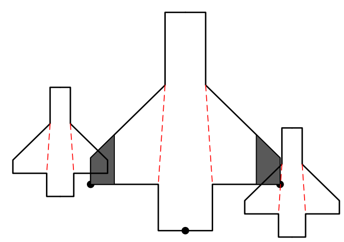
$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) \le q_{ijk} + M \cdot (1 - b_{ijk}),\ \forall i, j \in P, i \ne j, k = 1, 2, ..., m_{ij}$$

where $\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) = q_{ijk}$ is the equation of the line of the *f*th edge of the *k*th slice in NFP_{ij} and m_{ij} is the number of slices outside the NFP_{ij}. In a real situation, we cannot allow two aircraft to touch each other during the movement operation. Therefore, a safety margin between aircraft needs to be imposed in NFPs. Imposing a safety margin for an aircraft is equivalent to adding a buffer area outside each aircraft. Moving the edges of NFP for a pair of aircraft outward is equivalent to enlarging the boundary of the non-allowable area for the reference point of the relative movable aircraft in that pair. Each edge of the original NFPs is moved outwards by distance *n* (Figure 4 (b)), and the minimum safety distance between two aircraft is prescribed as one meter.

To make the most of the hangar space, the wing of a smaller aircraft can be placed under the wing of a larger aircraft. Such "overlap" of aircraft wings between two aircraft is permissible as the two aircraft's wings are of different heights, within a safety distance (Figure 5), while keeping main bodies of the aircraft separate.



Parts of aircraft



"Three-dimensional" parking with wings "overlap" in two-dimensional view

Figure 5 Three-dimensional parking arrangement

After decomposing the aircraft components into the main body and the aircraft, another set of non-overlapping constraints can be derived by using two sets of *NFPs* that separate each pair of aircraft from three-dimensional space, as shown in Figure 5: The *Main Body NFP* is used to separate the main body of the two aircraft, and the *Revised Wing NFP* is used to separate the wings of the aircraft with an allowance for "overlap" within a safety margin. For those pairs of aircraft of different wing heights with safety margins between the wings, the non-overlapping constraints (4) derived from original *NFP* are replaced by that derived from the *Main Body NFP* and the *Revised Wing NFP*. In this connection, two sets of binary variables that act as a similar function to b_{ijkt} are introduced and are used to separate the main body and the wings of aircraft, respectively.

3.3 Mathematical formulation

3.3.1 Assumptions

The basic assumptions that describe the proposed problem are as follow:

- the estimated time of arrival, estimated time of departure, and required maintenance time are assumed to be deterministic, and the time spent on movement is incorporated in the required maintenance time;
- once the aircraft is rolled into the hangar, its parking position cannot be adjusted until the maintenance task is finished and the aircraft leaves the hangar;
- once the aircraft is rolled into the hangar, the maintenance task must be finished before leaving the hangar. If the planning period ends before finishing the maintenance task (due to the delays of rolling in), such maintenance request is deemed as failed to deliver
- if the arriving aircraft (or the departing aircraft) is blocked by any parked aircraft in the hangar, its movement operations cannot be conducted until its pathway is cleared;
- the moving path of an aircraft is a straight line and turning is not allowed due to safety consideration.
- the aircraft cannot revisit the maintenance hangar after leaving, i.e. the rolling in and rolling out operations can be conducted only once.
- the time spent on roll in and roll out operations are incorporated in the required maintenance time.
- the model applies to planning for regular maintenance. Unexpected events or demands are not considered
- the manpower is assumed to be sufficient to complete the maintenance tasks.

3.3.2 Parameters and decision variables

The given information (parameters) of the problem consists of:

- The specification of each maintenance request, including the aircraft type, the required maintenance services (maintenance check), estimated time of arrival (ETA) to the hangar, and estimated time of departure (ETD), also known as the expected delivery time. The weightiness of each maintenance request.
- The geometric information of the different aircraft types, including the size of the aircraft type and the No-Fit Polygons for each pair of aircraft.
- Different penalty costs induced while fulfilling the maintenance requests.

- The dimensions of the maintenance hangar.

The list of notations for parameters mentioned above are as follows:

Notations

a_t	Set of scheduled arrival maintenance request at time *t*
d_t	Set of schedule departure aircraft in hangar at time *t*
A_t	Set of cumulative scheduled arrival aircraft in hangar from beginning to time *t*. $A_t \in \bigcup_{i=0}^{t} a_t$
D_t	Set of cumulative scheduled departure aircraft in hangar from beginning to time *t*. $D_t \in \bigcup_{i=0}^{t} d_t$
A_T	Set of maintenance requests received during planning horizon
t	Index of time, where *T* is the length of planning horizon
ETA_i	Estimated time of arrival of maintenance request associated with aircraft *i*
ETD_i	Estimated time of departure of maintenance request associated with aircraft *i*
$MTime_i$	Required maintenance time of maintenance request associated with aircraft *i*
w'_{ij}	Adjusted aircraft width *i* when aircraft *j* placed next to it
TD_i	Tail distance of aircraft *i*
$penalty1$	Penalty of not serving aircraft *i* during planning period (per request)
$penalty2$	penalty of late delivery of aircraft *i* during planning period (per minute)
$penalty3$	Penalty of failure to deliver aircraft *i* during planning period (per request)
$Weightness_i$	Weightiness of maintenance request *i*
W	width of hangar
H	length of hangar
w_i	width of aircraft *i*
h_i	length of aircraft *i*
NFP_{ij}	*NFP* of aircraft *i* and *j* with minimal safety distance
s_{ij}^k	*k*th slice of the region outside the NFP_{ij}
$\alpha_{ij}^{kf}, \beta_{ij}^{kf}, q_{ij}^{kf}$	parameters used to define the *f*th linear equation of the slice s_{ij}^k outside the NFP_{ij}
m_{ij}	number of slices outside NFP_{ij}
t_{ij}^k	number of linear equations used to define the slice s_{ij}^k
M	a large number

To determine a maintenance schedule to fulfill the maintenance requests as well as hangar layouts at different times, the following decision variables are introduced, and the uses of auxiliary decision variables in developing specific constraints are discussed in Section 3.3.3.

Decision Variables

(x_i, y_i)	position of reference point of aircraft *i* in the hangar
out_{it}	binary decision variable that takes the value 1 if aircraft *i* is rolled out at time *t*, and 0 otherwise
in_{it}	binary decision variable that takes the value 1 if aircraft *i* is rolled in at time *t*, and 0 otherwise
out_{iT^*}	binary decision variable that takes the value 1 if fail to deliver aircraft *i* at the end of planning horizon, and 0 otherwise
p_{it}	binary decision variable that takes the value 1 if aircraft *i* is parked in hangar at time *t*, and 0 otherwise
h_{ijt}	binary decision variable that takes the value 1 if aircraft *j* blocks aircraft *i* from rolling in or out at time *t*, and 0 otherwise
L_{ij}	binary decision variable that takes the value 1 if aircraft *i* is on the left side of aircraft *j* without overlap, and 0 otherwise
R_{ij}	binary decision variable that takes the value 1 if aircraft *i* is on the right side of aircraft *j* without overlap, and 0 otherwise
U_{ij}	binary decision variable that takes the value 1 if aircraft *i* is above aircraft *j* without overlap, and 0 otherwise
b_{ijkt}	binary decision variable that takes the value 1 if the reference point of aircraft *j* is placed into the slice s_{ij}^k of the region outside NFP_{ij} at time *t*, and 0 otherwise

3.3.3 Objective and constraints

$$Minimize \sum_{\forall i \in A_T} Weightness_i \cdot \left[(1 - \sum_{t \geq ETA_i} in_{it}) \cdot penalty1_i + \sum_{t \geq ETD_i} out_{it}(t - ETD_i) \cdot penalty2_i + out_{iT^*} \cdot penalty3_i \right]$$

The objective function minimizes the overall penalty costs while fulfilling the maintenance request. It includes the penalty costs of 1) lateness in fulfilling the maintenance requests along the planning horizon; 2) failure to complete the maintenance requests by the end of the planning period and 3) the lost cost in failing to receive the maintenance request.

s.t.

As mentioned earlier, the maintenance hangar operates in a multiperiod context, and the total planning horizon is represented by discrete times along the entire period (Figure 6). Each point on the timeline is used to represent the decision and status of the maintenance hangar at time *t*. The integrated decision at time *t* involves 1) determining if there are any movement operations conducted at time *t* (out_{it} and in_{it}) and 2) assigning the position of aircraft (x_i, y_i) parking in the hangar. As the position of an aircraft cannot be changed once it is moved into the hangar, the coordinates of the aircraft are not indexed with time *t*. The other auxiliary decision variables, i.e. out_{iT^*}, p_{it}, h_{ijt}, L_{ij}, R_{ij}, U_{ij} and b_{ijkt}, ensure the outcome of a solution is a logical and rational one. In this regard, it is possible that there may not have any movement operations for a consecutive period, as all maintenance requests are being processed or there are no newly arrival maintenance requests at that time. By combining the integrated decision for each discrete time along the planning horizon, the mathematical model allows us to determine the maintenance schedule and respective aircraft parking arrangement.

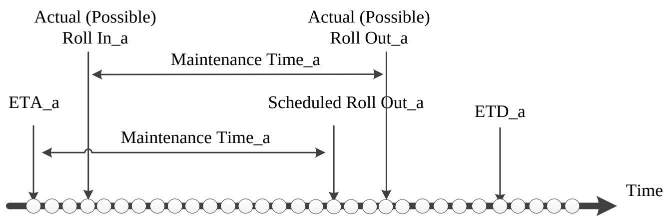


Figure 6 Basic discrete-time model

The constraints in the mathematical model can be divided into several functions:

1) Non-overlapping constraint

The aircraft received by the maintenance service company should be served within the boundary of hangar, and the aircraft should be separated with the minimum safety margin while parked in the hangar, using the No-Fit Polygons given in Section 3.2.

$$x_i + w_i / 2 \le W, \ \forall i \in A_t \tag{1}$$

$$x_i \ge w_i / 2, \ \forall i \in A_t \tag{2}$$

$$y_i + h_i \le H, \ \forall i \in A_t, \forall t \ge 0 \tag{3}$$

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(x_j - x_i) \le q_{ij}^{kf} + M \cdot (1 - b_{ijkt}), \ \forall i, j \in A_t, \ \forall k = 1, 2, ..., m_{ij}, \ \forall f = 1, 2, ..., t_{ij}^{k},, \forall t \ge 0 \tag{4}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{it}, \ \forall i, j \in A_t, \forall t \ge 0 \tag{5}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{jt}, \ \forall i, j \in A_t, \forall t \ge 0 \tag{6}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le 1 - out_{it}, \ \forall i \in D_t, \forall t \ge 0 \tag{7}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le 1 - out_{jt}, \ \forall j \in D_t, \forall t \ge 0 \quad (8)$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - 1, \ \forall i, j \in A_t \backslash D_t, \forall t \ge 0 \quad (9)$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - (out_{it} + out_{jt}) - 1, \ \forall i, j \in D_t, \forall t \ge 0 \quad (10)$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - out_{it} - 1, \ \forall i \in D_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \quad (11)$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - out_{jt} - 1, \ \forall i \in A_t \setminus D_t, \forall j \in D_t, \forall t \ge 0 \quad (12)$$

Constraints (1) – (3) ensure that the aircraft are placed within the boundary of the maintenance hangar. No-Fit Polygons between two aircraft are expressed in Constraint (4). Constraints (4) – (12) are entire non-overlapping constraints set for a pair of aircraft parking at time *t*. In particular, the non-overlapping constraint is activated when two aircraft are parked in the hangar simultaneously at time *t* (constraints (9) – (12)), and the non-overlapping is deactivated if any one of them is not arranged to be parked at time *t* or one of them is rolled out altogether at that time (constraints (5) – (8)). The auxiliary decision variable p_{it} indicates if aircraft *i* is placed in the hangar at time *t*, activating the non-overlapping constraints. The set of binary variables b_{ijkt} associated with the horizontal slice *k* outside the NFP between aircraft *i* and *j* in constraint (4).

2) Movement blocking constraints

During the movement operations of aircraft, there shall not have any obstacles blocking its path of movement. If an aircraft is about to leave or enter the hangar, the other aircraft parking in the hangar should not become the obstacle, blocking the moving aircraft. In this regard, the position between two aircraft need to be determined by the auxiliary decision variables h_{ijt}, L_{ij}, R_{ij}, U_{ij}. If the aircraft about to move at time *t* is blocked by any other aircraft, its movement operation has to be cancelled at this time.

$$(x_i + w'_{ij}/2) - (x_j - w'_{ji}/2) \le M \cdot (1 - L_{ij}) \ \ \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \quad (13)$$

$$(x_i - w'_{ij}/2) - (x_j + w'_{ji}/2) \geq -M \cdot (1 - R_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \geq 0 \tag{14}$$

$$(y_i + TD_i) - (y_j + TD_j) \geq -M \cdot (1 - U_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \geq 0 \tag{15}$$

$$(1 - h_{ijt}) \geq \frac{1}{6} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in D_t, \forall t \geq 0 \tag{16}$$

$$(1 - h_{ijt}) \leq L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in D_t, \forall t \geq 0 \tag{17}$$

$$(1 - h_{ijt}) \geq \frac{1}{5} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \geq 0 \tag{18}$$

$$(1 - h_{ijt}) \leq L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \geq 0 \tag{19}$$

Constraints (13) – (19) indicate and prescribe the correlation between the parking position of the aircraft and the blocking in aircraft movement operations. In particular, binary variables L_{ij}, R_{ij} and U_{ij} prescribe that if they take value 1, then aircraft *i* is placed on the left-hand side, right-hand side and upper position of aircraft *j*, respectively, so that aircraft *j* does not block the movement operations of aircraft *i*.

The binary variable h_{ijt} reflecting whether aircraft *i* is blocked by aircraft *j* is controlled by constraints (16) – (19). Specifically, aircraft *j* does not block the movement of aircraft *i* under the following conditions: 1) aircraft *j* undertakes the movement operations at the same time as aircraft *i*; 2) aircraft *j* is not placed in the hangar at time *t*; 3) aircraft *i* is on the the left-hand side, right-hand side or the upper position of aircraft *j*, as indicated by binary variables L_{ij}, R_{ij} and U_{ij}, respectively.

3) Movement Operations and aircraft blocking:

The constraints in this section prescribe that if the movement path of the aircraft rolling in and rolling out is blocked by other aircraft parked in the hangar, the movement actions cannot be conducted. In particular, for an aircraft pending leaving the hangar, the rolling out operation has to wait until the aircraft blocking the path leaves first (or concurrently). For the arrival aircraft, its parking position can be adjusted so that the aircraft can be timely moved in, or the movement operation has to be postponed until the aircraft blocking the pathway leaves the hangar.

$$out_{it} \leq 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in D_t, \forall t \geq 0 \tag{20}$$

$$in_{it} \le 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in A_t, \forall t \ge 0 \qquad (21)$$

Constraints (20) and (21) state that the rolling out and rolling in operations of aircraft i cannot be conducted if it is blocked by any parked aircraft in the hangar at time t. The auxiliary decision variable h_{ijt} indicates the relations between each pair of aircraft at time *t* acting as the mediator between the movement operations decision variable (out_{it} , in_{it}) and the movement blocking constraints (Constraints (13)-(19)).

4) Staying time requirements:

The duration that each aircraft stays in the hangar should sufficient for conducting the maintenance task. The constrains set in this section ensure the staying time of an aircraft served by the company equals or is longer than its required maintenance. Moreover, the rolling in and rolling out operations for each aircraft can be conducted only once, as the aircraft cannot revisit the hangar during the planning period. The auxiliary decision variable p_{it} acts as a mediator, establishing the relation between the non-overlapping constraint in constraint set 1) and the staying time requirement in this section.

$$(\sum_{t \ge ETD_i} out_{it} \cdot t - \sum_{t \ge ETA_i} in_{it} \cdot t) + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}) + M \cdot (1 - \sum_{t \ge ETD_i} out_{it}) \ge MTime_i, \forall i \in A_T \qquad (22)$$

$$p_{it} = \sum_{ETA_i \le m \le t} in_{im}, \forall i \in A_T, \forall ETA_i \le t \le ETD_i \qquad (23)$$

$$p_{it} = \sum_{ETA_i \le m \le t} in_{im} - \sum_{ETD_i \le m \le t-1} out_{im}, \forall i \in A_T, \forall t \ge ETD_i + 1 \qquad (24)$$

$$\sum_{t \ge ETA_i} in_{it} \le 1, \forall i \in A_T \qquad (25)$$

$$\sum_{t \ge ETD_i} out_{it} \le 1, \forall i \in A_T \qquad (26)$$

$$out_{it} \le \sum_{ETA_i \le m < t} in_{im}, \forall i \in A_T, \forall t \ge ETD_i \qquad (27)$$

$$(1 - out_{iT*}) \le \sum_{t \ge ETD_i} out_{it} + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}), \forall i \in A_T \qquad (28)$$

$$(1 - out_{iT*}) \le \sum_{t \ge ETD_i} out_{it} + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}), \forall i \in A_T \qquad (29)$$

Constraint (22) determines the duration of stay for each aircraft, prescribing that if such aircraft is accepted by the service company then its parking time must equal or be longer than its required maintenance time.

Constraints (23) and (24) prescribe that p_{it} indicates whether the aircraft is parked in the hangar takes value 1 by the time it rolls into hangar until it rolls out. If the value of p_{it} equals to one, the respective non-overlapping constraints are activated accordingly.

Constraints (25) – (27) ensure that the rolling in operations happens after the arrival time of the maintenance request (ETA), and rolling out operations are conducted only after the aircraft has been rolled in. Constraints (28) – (29) imposes that out_{iT^*} equals to one if the aircraft is still parked in the hangar at the end of the planning horizon.

5) Variable domination constraints

$$x_i, y_i \geq 0 \quad \forall i \in A_T \tag{30}$$

$$b_{ijkt} \in \{0,1\} \ \forall i, j \in A_t, k = 1, 2, ..., m_{ij}, \forall t \geq 0 \tag{31}$$

$$p_{it} \in \{0,1\} \quad \forall i \in A_t, \forall t \geq 0 \tag{32}$$

$$in_{it} \in \{0,1\}, \forall i \in A_t, \forall t \geq 0 \tag{33}$$

$$out_{it} \in \{0,1\}, \forall i \in D_t, \forall t \geq 0 \tag{34}$$

$$h_{ijt}, L_{ij}, R_{ij}, U_{ij} \in \{0,1\} \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \geq 0 \tag{35}$$

Constraint (30) ensures that the coordinates of the aircraft are positive, and constraints (31) – (35) indicate the binary variables in the mathematical model.

6) Tightening the model

To further tighten the mathematical model, we propose the following constraints:

$$L_{ij} + L_{ji} \leq 1, \forall i, j \in A_T, j \neq i \tag{36}$$

$$R_{ij} + R_{ji} \leq 1, \forall i, j \in A_T, j \neq i \tag{37}$$

$$L_{ij} \leq R_{ji}, \forall i, j \in A_T, j \neq i \tag{38}$$

$$R_{ij} \le L_{ji}, \forall i, j \in A_T, j \ne i \quad (39)$$

The feasibility of the tentative solution is examined by firstly determining a feasible maintenance schedule, then fixing the position-related binary variables. After branching on all the position-related variables, the geometry constraints are imposed to examine if such a parking plan is feasible. In this regard, the LP relaxation of the model is not tight, and the updates of the lower bound do not progress well to tighten the optimality gap. Constraints (36-39) impose a side-by-side relation between a pair of aircraft

4. Solution approaches

The original model presented in Section 3 is inefficient as it relies on a basic discrete time model. In this section, two newly developed solution approaches are discussed to enhance the efficiency in solving the problem.

4.1 Event-based discrete time formulation for the problem

Generally, the decision variables in the MILP formulation are indexed by the discrete time to cover the planning horizon (Figure 6), such as the basic discrete-time formulation (DT) [43] and the disaggregated discrete-time formulation (DDT) [44] in the resource-constrained project scheduling problem, while the number of variables indexed by time increase proportionally with the length of the scheduling horizon *T* [45]. Moreover, the setting of the time interval of two consecutive time points along the horizon, e.g. one-minute, 5-minute, 10-minute based, also has an impact on the scale and the accuracy of the problem. The Basic Discrete Time (BDT) model is inefficient and may visit lots of unpromising time points along the horizon. Inspired by the work related to the project scheduling problem[45], an event-based discrete time model is developed to identify the possible time point that may trigger roll in and roll out operations along the planning period.

The main idea of reducing the domain of the time-related decision variables along the planning period is to exclude all the points in the timeline that cannot trigger any movement operations so as to leave only the promising time point along the planning period, as many time points in Figure 6 cannot trigger movement operations while involving a great number of decision variables in those time points.

Ideally, a maintenance check should commence upon arrival of a new aircraft and the expected roll out time for each aircraft equals $ETA_i + MTime_i$. When blocking occurs due to the insufficient hangar space or improper parking planning, some events (roll in and roll out operations) cannot be triggered at their ideal time, e.g. ETA_i or $ETA_i + MTime_i$ respectively. Under such circumstances, the roll in (roll out) operation can be performed once the blocking is cleared or the hangar has enough space to accommodate the aircraft. The possible roll in / roll out event time can be determined by recursively calculating the possible roll in / roll out time. If the aircraft arrives during the middle of the planning period, it is possible that the hangar capacity has been used up by some earlier arrival aircraft, and the only possible way to accommodate the later arrival aircraft is to wait until the earlier arrival aircraft complete their maintenance tasks and leave the hangar. As a result, the possible roll in time for the arrival aircraft includes its own *ETA*, or the actual maintenance completion time of the other aircraft parked in the hangar. Similarly, an aircraft arriving during the middle of a planning period can also have a blocking effect on the consequent arrival aircraft. In addition, a set of aircraft with the same *ETA* can also have roll in blocking effects on each other. With regard to the possible roll out time, the later arrival aircraft have effect on the earlier arrival aircraft. It is possible that the earlier arrival aircraft have finished the maintenance, but the later arrival aircraft blocks the roll out path due to the limited hangar space or an improper pakring arrangement. Therefore, the only possible way to move out the aircraft after finishing the maintenance task is to wait until the later arrival aircraft finish their maintenance task. As a result, the possible roll out time for an aircraft includes its own *ETA+MTime*, its actual roll in time plus its *MTime*, or the actual roll out time of the aircraft blocking the movement path.

The detailed procedures of calculating the possible roll in and roll out time for the development of the Event-based Discrete Time model are shown in Algorithm 1.

Algorithm 1

Calculation of promising event times

Notations	Meanings
M	Set of maintenance requests
$Possible\_RollIn_i$	Set of Possible Roll In time of maintenance request i
$Possible\_RollOut_i$	Set of Possible Roll Out time of maintenance request i
$Rank_i$	The rank of maintenance request i in sorting list
Set_Rank_n	Set of maintenance request at position n . $H = \{1, 2, 3, ..., n_H\}$ be the index set of maintenance request in respective
1:	Sort all maintenance requests from M in increasing order according to ETA_i, then derive the position of maintenance request $Rank_i$ according to the result of sorting list.
2:	Input the maintenance requests into respective Set_Rank_k according to the $Rank_i$ (Computation of possible roll in time)

for $n = 1, 2, 3, ..., n_H$ do
For k in Set_Rank_n
Include ETA_k into set $Possible\_RollIn_k$
(same position blocking)
Calculate the combination d of maintenance request in $Set\_Rank_n \setminus k$ to determine the possible blocking in same position
for the combination of maintenance requests may block k
Possible roll in time of k = possible roll in time of the request in combination for k + respective required maintenance time
If $n \neq 0$
for $n^{'} = 0, 1, 2, ..., n-1$
for m in $Set\_Rank_{n^{'}}$
If the possible roll in time of the previous request + required maintenance time >= ETA_k
Include possible roll in time of the previous request + required maintenance time into $Possible\_RollIn_k$
(Computation of possible roll out time)
for $n = 1, 2, 3, ..., n_H$ do
For k in Set_Rank_n
Include $ETA_k + MTime_k$ into $Possible\_RollOut_k$
Include all entries in $Possible\_RollIn_k$ plus $MTime_k$ into $Possible\_RollOut_k$
while $n^{'} = n+1, n+2, ..., n_H$
For m in $Set\_Rank_{n^{'}}$
For all entries in $Possible\_RollIn_m$
If $Possible\_RollIn_m + MTime_m \geq ETA_k + MTime_k$
Include $Possible\_RollIn_m + MTime_m$ into $Possible\_RollOut_k$

4.2 Rolling horizon approach

Though the Event-based Discrete Time (EDT) model presented in Section 4.1 significantly reduces the model size and the solution time, it sometimes still requires a long solution time or may be incapable for solving some instances that contain a large number of maintenance requests. One can use the rolling horizon approach to speed up the overall solution process. Inspired by the idea of the rolling horizon approach from [46], we develop a tailored rolling horizon approach suitable for this problem. The horizon is divided into several sub-problems with n maintenance requests in each sub-problem (except the last sub-problem that may include fewer requests), and sub-problem k overlaps with the next one $(k+1)$ if the operation time for some maintenance requests determined in the current sub-problem k exceed the planning start time of the next sub-problem $(k+1)$, i.e. the earliest event time of the next subproblem. Let $K = \{1, 2, ..., n_k\}$ be the index set of those subproblems and

$W_k = [B_k, E_k]$ ($B_k = \min_{i \in W_k} ETA_i$, $E_k = \min_{i \in W_{k+1}} ETA_i$, $i \in A_T$). The rolling horizon sequentially solves the subproblems. The pseudo-code can be found in Algorithm 2. At each iteration k, a subproblem restricted to the current time domain W_k is solved using the MILP model presented in Section 4.1. To ensure continuity in the overall solution, the subsequent subproblem $(k+1)$ includes the initial condition derived from the last sub-problem k, i.e. for the maintenance requests planned to finish after E_k. The initial condition for $(k+1)$ stipulates that the position of the aircraft and the determined roll out time remain unchanged so as to ensure the connectivity of the solution between the previous and current subproblems.

Two strategies for dividing maintenance requests into subproblems are considered:

- First Come First Served (FCFS): Sorting the maintenance requests in increasing order according to ETA, then dividing the maintenance requests into respective subproblems according to the predetermined maximum number of requests to be included in one sub-problem (n).
- Mixed mode: Sorting the maintenance requests in increasing order according to *ETA*, then selecting a set of maintenance requests according to limit r, and determining $W_k = [B_k, E_k]$. Examine the rest of the maintenance requests not included in the subproblem k. If there is any maintenance job with the ETD_i within $W_k = [B_k, E_k]$, include such maintenance job into the present subproblem k.

Optimizing more maintenance requests within one sub-problem usually leads to an overall solution of better quality, as the maintenance scheduling is optimized with a wider local view of the problem [46]Following a series of preliminary tests to achieve a tradeoff between efficiency and quality, the values of r were set from 5 to 7 in performing the experiments described in Section 5.2.2.

Algorithm 2 Rolling horizon approach

Notations	Meanings
M	Set of maintenance requests
r	Number of maintenance requests to be included in one sub-problem
n_k	Number of subproblems
K	Set of subproblem. $K = \{1, 2, ..., n_k\}$

B_k	Beginning time of subproblem k
E_k	Ending time of subproblem k
$earliest(k)$	Earliest ETA_i in subproblem k .
ATD_i	Actual departure time of maintenance request i
$Coordinate_i$	Determined position of maintenance request i

1:	Set the number of subproblem as $n_k = \left\lceil \frac{M}{r} \right\rceil$, and divide the request in M into n_k subset according to respective subproblem dividing strategies (FCFS and Mixed)
2:	for $k = 1, 2, ..., n_k$ do
3:	Take kth subset of maintenance requests containing r (if $k < n_k$) or $M - r*(n_k - 1)$ (if $k = n_k$) maintenance request to establish subproblem k . $B_k = \min_{i \in W_k} ETA_i$, $E_k = \min_{i \in W_{k+1}} ETA_i$, $i \in A_T$
4:	for $k = 1, 2, ..., n_k$ do
5:	Solve the subproblem k by MILP model in Section 3.2.2 or Section 4.1
6:	For the maintenance request i with $ATD_i \geq E_k$
7:	Pass the aircraft i with ATD_i and $Coordinate_i$ to subproblem $k+1$ as initial constraints
	For the maintenance request i with $ATA_i \geq E_k$
	Pass the aircraft i with ATA_i, ATD_i and $Coordinate_i$ to subproblem $k+1$ as initial constraints
	For the maintenance request i has not scheduled to rolled into in subproblem k
	Pass the aircraft i with ETA_i, ETD_i to subproblem $k+1$ as ordinary maintenance request
8:	Integrate the solution from $k = 1, 2, ..., n_k$ to produce complete solution along planning horizon

4.2.1 Enhancement of rolling horizon approach

While solving the subproblems by the conventional rolling horizon approach mentioned above, the impact of the current parking layout on the subsequent subproblems is not considered. In particular, the subproblem solely focuses on obtaining a feasible aircraft parking layout with an optimal local objective value. It is possible that the earlier arrival aircraft might park at the anterior space of the hangar near the entrance (Figure 7 (a)), even if there is a lot of available empty space in the inner area, making the later arrival aircraft in a subsequent subproblem unable to find the parking place due to the blocking at the anterior area. In this regard, we propose an approach for this problem so as to further enhance the algorithm stability. After finding the optimal solution of each subproblem, we supplement a *layout compaction* stage to further tighten the parking layout to spare more space for the subsequent subproblems, without violating the position relation between aircraft. In particular, we compact the aircraft's positions, with the position relations unchanged, by constructing a MILP model with predetermined variable values, i.e. the coordinates in the x-axis x_i and the binary variables, L_{ij}, R_{ij} and U_{ij}, then minimize the coordinates y_i in the *layout compaction* stage (Figure 7 (b)) to spare space in the y-axis.

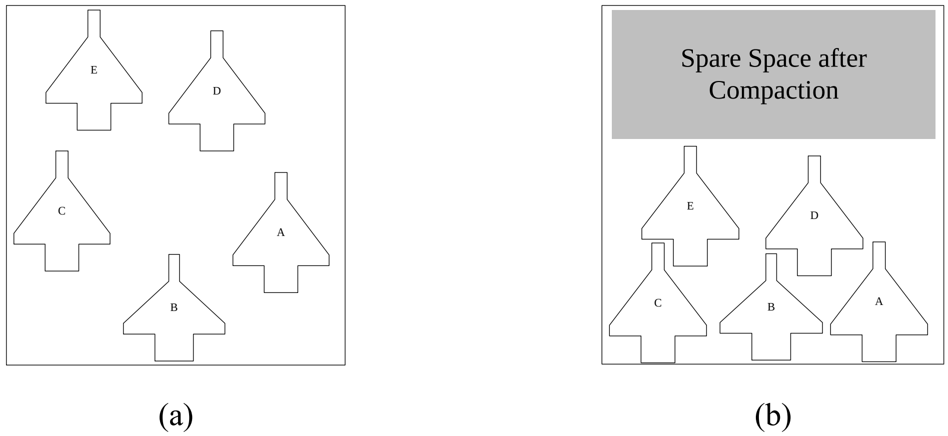


Figure 7 Before and after layout compaction in solving the subproblem

5. Computational experiments

This section presents the results of different computational experiments. All the procedures described in the previous sections are coded in C# in Visual Studio 2010 and run on a computer with an Intel Core i7 processor, at 3.6 GHz with 32 Gb of RAM. The Mixed-Integer Linear Programming is solved by the CPLEX 12.7 serial model.

5.1 Description of test instances

For our tests, we considered maintenance request data derived from an aircraft hangar maintenance service provider in Hong Kong, serving over 50 clients, including airlines, business jet companies and utility aircraft companies, as a case study. We obtained the information of the estimated arrival time (*ETA*), departure time (*ETD*), aircraft type and check type of each maintenance requests from clients over 157 days from January to May in 2015 to create instances, which were used in our previous study [20]. The characteristics of the historical data is presented in Table 1, which briefly classify the number of aircraft in three categories, small-, medium- and large-sized, as shown in Table 2. The shortest distance between two aircraft is prescribed as one meter in conducting the computational experiments given in Section 5.2.

Table 1 Characteristics of receiving maintenance requests

Month	Number of maintenance requests	Small-sized Aircraft	Medium-size aircraft	Large-size aircraft
January 2015	24	11	12	1
February 2015	19	8	11	0
March 2015	33	12	18	3
April 2015	39	17	27	0
May 2015	33	9	22	2

Table 2 Classification of aircraft size

Classification	Aircraft models
Small-sized	G200 CL600 CL605 F900LX F2000EX F2000LX ERJ135 F7X G450 GIV
Medium-sized	GL5T G550 G5000 G6000 G650 A318 ERJ190 A319 A320 B738 A321
Large-sized	A332 A333

5.2 Computational experiment

We performed three series of computational experiments, with the design of the numerical experiments as follows: the first series in Section 5.2.1 compares the computational efficiency of the Basic Discrete Time (BDT) model with different time intervals, the Event-based Discrete Time (EDT) model by solving small- and medium-size instances. After illustrating the superiority of the EDT model, the large instances were solved by the rolling horizon approach incorporating the EDT model, and its performance is reported in Section 5.2.2. The weightiness of each maintenance request was regarded as equal, and the unit penalty cost was set as (80, 1, 30) for penalties 1-3 respectively.

5.2.1 Model's evaluation

In this section, we compared the effectiveness of the proposed model formulations by presenting the results of solving small- and medium-size aircraft. Table 3 shows the results for 8 instances solved by the two models. Since our preliminary experiment in using the BDT model suggested that setting the time interval of the model as less than 20 minutes involves large numbers of binary variables, making the instances intractable. In this regard, we prescribe the time interval in the BDT model as 30 minutes, 45 minutes and 60 minutes. The first column of Table 3 stands for the instance name. Data of maintenance requests collected from the maintenance company are organized on a monthly basis, and the monthly maintenance requests are further divided into several sub-sections to create small- and medium size instances. The name of the instance is presented in "month_division_number of requests (number of planning days)" form, e.g. 1_1_3 (7) stands for the instance covering the first subsection of January with 3 maintenance requests covering 7 days. Each instance was solved by the BDT model with three different time interval settings as well as the EDT model. The third column denotes the preprocessing time before CPLEX solves the instance. The preprocessing time includes the initialization of the mathematical model, i.e. defining the decision variables and initializing the constraints, as well as the time spent on calculating the event time, as discussed in Section 4.1 for the EDT model. The number of binary variables involved in each model in solving the instance is reported in the fourth column. The best-known solution, lower bound, optimality gap and the CPU time elapsed when the termination criterion was met are recorded from the fifth to eighth columns, respectively. The time limit for each instance was 3600 seconds.

The overall results in Table 3 demonstrate the superiority of the Event-based Discrete Time (EDT) model formulation in terms of the number of instances optimally solved, optimality gap and CPU running time. It is noted that the Basic Discrete Time (BDT) model formulation with different time intervals involves a significant number of binary variables in each instance, which grows significantly as the planning period increases. Moreover, setting a large time interval (minute) in the BDT model may eliminate the true optimal solution at the model initialization stage, e.g. for instance 1_1_3 (7), the BDT (45minutes) and the BDT (60 minutes) models found that the optimal objective value was 90. In the BDT model with a large time interval, the discrete event time in the model may be later than the ETA and the ETD of the maintenance requests, which caused lateness in rolling in and rolling out operations. We further analyzed the performance of the EDT model by solving the medium-size instances and the results are presented in

Table 4, where it is noted that the number of maintenance requests in each instance is one of determinants of the model scale. However, it is worth to point out that the complexity of the instances does not solely depend on the number of maintenance requests, but also the distribution of arrival time along the planning period and also correlates with the aircraft type in each maintenance request and its arrival time. For example, while solving the medium-size instance set, it is recorded in Table 4 that the Instance 3_2_9(9) cannot be solved to optimality within the time limit with optimality gap 100%, while the other instances were solved optimally within an hour. The optimality gap of Instance 3_2_9(9) implies that the event-based discrete time model can only found a feasible solution for this instance without updating the bounds. After further investigating on Instance 3_2_9(9), it is found that the maintenance requests' arrival time concentrate on a short period of time though the length of planning period is moderate, which induces great number of possible rolling in and rolling out time compared with other instances along the planning period. As a result, this instance involves the greatest number of binary variables (406756) among all instances, making it challenging for the event-based discrete time model to tackle within the time limit. When maintenance requests are mainly for medium-size and large-size aircraft, the hangar space becomes limited in accommodating all aircraft, and may induce lateness. In addition, when the number of maintenance requests within an instance approaches 9, the number of possible event times involved in the EDT model grows significantly.

Table 3 Comparisons among basic discrete time and event-based discrete time models

Instance	Models	Prepossessing Time (seconds)	Binary Variables	Best-known solution	Lower bound	Gap	CPU (seconds)
1_1_3 (7)	BDT (30)	1.07	26940	0	0	0	9.66
	BDT (45)	0.67	17891	90	90	0	68.55
	BDT (60)	0.51	13513	90	90	0	34.23
	EDT	0.09	431	0	0	0	**0.06**
1_1_4 (8)	BDT (30)	3.18	79265	60	60	0	428.38
	BDT (45)	1.88	52877	120	120	0	964.74
	BDT (60)	1.54	39685	220	120	45.45	3600
	EDT	0.12	1528	0	0	0	**0.09**
1_1_5 (13)	BDT (30)	17.51	350766	160	30	81.25	3600
	BDT (45)	10.40	233927	170	120	29.41	3600
	BDT (60)	6.83	175448	150	150	0	3600
	EDT	0.15	2864	0	0	0	**0.28**
1_2_7 (10)	BDT (30)	39.81	860161	N/A	0	N/A	3600
	BDT (45)	24.36	573353	1485	82.0256	94.48	3600
	BDT (60)	16.23	430221	N/A	230	N/A	3600
	EDT	1.31	31896	0	0	0	**5.40**
2_1_7 (13)	BDT (30)	15.14	323657	160	160	0	1273.03
	BDT (45)	8.68	215615	295	295	0	291.10
	BDT (60)	6.16	161772	310	310	0	209.99
	EDT	0.51	12001	0	0	0	**1.28**
1_1_8 (15)	BDT (30)	68.21	1357537	N/A	0	N/A	3600
	BDT (45)	42.81	904561	N/A	0	N/A	3600
	BDT (60)	28.62	678545	N/A	240	N/A	3600
	EDT	1.43	37689	0	0	0	**5.13**

Table 4 Experiments on event-based discrete time model

Instance	Prepossessing Time	Binary Variables	Best-known solution (Upper bound)	Lower Bound	Gap	CPU
1_1_9 (15)	3.63	90688	0	0	0	16.57
1_2_9 (12)	6.73	190168	0	0	0	31.29
1_2_10 (13)	16.77	454004	0	0	0	95.05
2_1_8 (13)	1.51	29358	0	0	0	2.84
2_1_9 (14)	2.34	66384	0	0	0	9.63
2_1_10 (15)	4.64	129259	0	0	0	23.99
2_2_8 (13)	3.51	99135	0	0	0	14.29
3_1_8 (12)	1.82	33438	0	0	0	2.67
3_2_9 (9)	15.06	406756	6010	0	100	3600
3_3_9 (9)	6.96	174871	0	0	0	30.82
4_1_8 (10)	0.65	18245	0	0	0	2.00
4_2_8 (19)	0.55	14799	0	0	0	1.51
4_3_9 (18)	5.06	110869	0	0	0	457.86
4_4_9 (8)	12.42	287555	0	0	0	45.12
5_1_7 (9)	1.42	40192	0	0	0	5.12
5_2_8 (14)	5.66	119019	0	0	0	23.14
5_3_9 (15)	9.08	238335	0	0	0	30.28
5_4_9 (28)	0.28	6271	0	0	0	0.41

5.2.2 Rolling horizon approach

Though the EDT model has illustrated its superiority in solving small- and medium-size instances as reported in Section 5.2.1, the monthly maintenance requests received by the service provider ranges from 19 to 39 at present, which are intractable solely using the EDT model. Specifically, the preprocessing time is quite long in solving instances with more than 15 maintenance requests. In this section, we did not have the comparison with the solutions obtained from CPLEX as it was unable to initiate instances with more than 20 maintenance requests. In this regard, CPLEX itself cannot solve the monthly instances tested this section, as the minimum number of maintenance requests is 19 in the problem set. Referring to the computational comparison approach adopted in [9] while tackling large-scale instance, we compared the performance among different rolling horizon strategies in tackling large-scale instances, and demonstrate the advantages of enhanced rolling horizon strategy discussed in Section 4.2.1. We examine the performance of the rolling horizon approaches with different strategies in solving large-scale instances with three job dividings. The job limit in each subproblem is described as 5,6 and 7 to examine the differences in computational efficiency and solution quality. The EDT model is embedded in solving each subproblem and the time limit for solving each subproblem is 3600 seconds.

Table 5 reports the computational results of adopting different subproblem dividing strategies in the rolling horizon approach, and we compare the performance among the different strategies and the job limits in the subproblem. The computational time and objective function values are two indicators in comparing the performance of different strategies in this section [9] in solving large-scale problem, so as to demonstrate the advantages of heuristic in tackling challenging problems. A total of ten replications for each instance were conducted to evaluate the average performance of the rolling horizon approach with different strategies and job limits. The rolling horizon approaches with different strategies are able to obtain feasible solutions while prescribing the job limits in the subproblem as 5 and 6. It is noted that all rolling horizon approaches with job limits as 7 cannot tackle Instance 4, as many maintenance requests in the preceding subproblems were passed to the subsequent subproblems that exceed the EDT model, and feasible solution cannot be obtained within the time limits. The advantages of the rolling horizon approach embedding the Mixed strategy with layout compaction method was manifested while solving the instances with high maintenance demands, i.e. Instances 3, 4 & 5. It is recorded that the Mixed strategy with layout compaction method outperforms the FCFS and Mixed strategies while examining the computational efficiency and solution quality in solving complex instances. In particular, the Mixed strategy with compaction method was able to reach the same or better solution measured by objective value (as highlighted in <u>**bold underline**</u>) while require less computational time, which means that it was able to identify a solution with less tardiness in fulfilling maintenance demands. Moreover, the average, Max. and Min. CPU time of Mixed strategy with compaction also showed that the stability of solving time was better than the other two strategies. Therefore, the importance of compacting hangar layout before passing the partial preceding solution to the next subproblem has emerged, as the hangar layout can be tightened to spare more space for the subsequent subproblem's planning so as to improve the solution quality. Through conducting ten replications for each instance, it is observed that the average computational performance of the Mixed strategy with the layout compaction approach is more stable when in investigating the Average, Maximum and Minimum CPU times of ten replications. In particular, the Maximum and Minimum CPUs of the FCFS and Mixed strategies differ quite a lot when solving Instance 5 with the job limit set as 6, and the layout compaction approach spent significantly less time to obtain the same objective value as the FCFS and Mixed strategies.

Table 5 Comparison among rolling horizon approaches

Instance	No. of Maintenance Request	Job Limit	FCFS				Mixed				Mixed w/ Compaction			
			Avg. Obj.	Avg. CPU	Max. CPU	Min. CPU	Avg. Obj.	Avg. CPU	Max. CPU	Min. CPU	Avg. Obj.	Avg. CPU	Max. CPU	Min. CPU
1	24	5	240	18.99	21.77	17.65	240	17.90	20.99	16.75	240	22.84	23.68	22.12
		6	160	39.22	43.69	38.16	160	36.55	37.57	35.94	160	45.93	47.85	45.21
		7	320	373.37	376.64	371.95	320	381.84	399.73	371.43	320	444.82	468.08	440.01
2	19	5	160	8.63	9.28	8.21	160	7.88	8.17	7.61	160	10.89	11.38	10.72
		6	80	20.35	22.26	19.48	80	18.85	19.79	18.53	80	26.42	27.58	25.74
		7	80	127.51	129.15	126.39	80	127.53	128.87	126.37	80	130.93	132.48	129.59
3	33	5	400	280.56	287.07	277.99	400	265.91	270.09	260.46	**320**	**192.49**	196.07	190.48
		6	510	2012.76	2052.96	1990.72	510	2044.74	2101.88	1979.64	**430**	**1893.54**	1909.34	1879.91
		7	320	127.51	129.15	126.39	320	1330.69	1353.38	1301.08	320	1550.78	1525.29	1473.31
4	39	5	640	1264.04	1411.91	1166.15	640	1141.92	1181.14	1117.15	640	**1010.75**	1026.52	985.68
		6	560	1269.84	1288.42	1248.98	560	1358.85	1401.86	1323.09	**480**	**709.49**	718.07	701.37
		7	-	-	-	-	-	-	-	-	-	-	-	-
5	33	5	400	719.72	750.22	687.11	400	685.82	711.85	675.17	**240**	**68.56**	74.48	66.62
		6	220	623.39	4561.03	138.97	220	804.03	3540.54	141.22	220	**108.39**	112.61	107.15
		7	240	203.41	218.54	195.93	240	197.38	204.77	188.70	240	244.29	264.11	233.55

- the subproblem cannot obtain feasible solution within 3600s.

5.3 Sensitivity analysis

In this section, a sensitivity analysis is performed to display the impact of the changes on the weightiness value to the objective function using Instance 1 as a case study. The computational experiment in Section 5.2 prescribes that the weightiness of all maintenance requests is the same (value 1). The weightiness of each maintenance request was reviewed and evaluated by the maintenance company, with values that fluctuate under different situations. Moreover, the weightiness of each maintenance request given by the maintenance company is kept confidential in actual operations. Nevertheless, we sought the maintenance company's suggestion in assigning the weightiness to design the sensitivity analysis. As suggested by the practitioners, the weightiness of each maintenance request is proportional to the size of the aircraft type in general cases, as larger aircraft usually requires more inputs in maintenance work. We examined the impact of the variations in weightiness by using 6 different settings in the sensitivity analysis, and the difference of the weightiness among small-, medium- and large-size aircraft were adjusted accordingly so as to reflect the preferences and priorities of the different requests. The unit penalty cost was set as (4000, 1, 1000) for penalties 1-3 respectively in this section, so as to amplify the effect of variation of the weightiness for each maintenance request.

We deployed Instances 3 & 5, both involving many maintenance requests, to conduct the sensitivity analysis. Table 6 and Table 7 reflect the effect of the weightiness changes on the objective value and the optimal solutions. It is found that: 1) the penalty costs have an increasing trend when the weightiness increases; 2) widening the difference of weightiness among three groups of requests or increasing the weightiness do not necessarily increase cumulative delays, and the impact of such changes is reflected on the optimal solutions. 3) it is noted that the optimal solutions under different settings may have the same schedule, and the optimal decisions under different settings prone to reject some same requests, which implies that the demands of maintenance (aircraft type, arrival time, deadline and maintenance) may have larger impact on decision-making compared with the weightiness settings.

Table 6 Sensitivity Analysis on Instance 3

Settings	Weightiness			Objective Value	Cumulative Delays (Minutes)	Rejected Requests (Aircraft Type)
	Small-sized	Medium-sized	Large-sized			
1	1	2	4	36540	7050	2 (GL5T, A333)
2	2	4	5	61080	7050	2 (GL5T, A333)
3	3	4	8	50380	4565	2 (GL5T, G550)
4	3	6	8	71550	4565	2 (GL5T, G550)
5	4	7	9	84115	4565	2 (GL5T, G550)
6	4	8	10	122160	7050	2 (GL5T, A333)

Table 7 Sensitivity Analysis on Instance 5

Settings	Weightiness			Objective Value	Cumulative Delay (Minutes)	Rejected Requests (Aircraft Type)
	Small-sized	Medium-sized	Large-sized			
1	1	2	4	28740	3000	2 (A332, G5000)
2	2	4	5	45480	3000	2 (A332, G5000)
3	3	4	8	67780	1260	3 (A332, G5000, GL5T)
4	3	6	8	70220	3000	2 (A332, G5000)
5	4	7	9	50260	3180	1 (G5000)
6	4	8	10	90960	3000	2 (A332, G5000)

6. Conclusions

Outsourcing of aircraft Maintenance, Repair, and Overhaul (MRO) activities has been continuously increasing as airline companies begin to change their MRO practices, which has brought great challenges to aircraft maintenance service companies in fulfilling maintenance requests. For the maintenance company, hangar space is a bottleneck in planning the maintenance schedule, as the movement of aircraft causing blocking and geometric factor for aircraft parking are unique features in hangar maintenance scheduling A hangar maintenance problem is described in this paper, integrating the scheduling and parking layout planning problems. We develop a mathematical model to accommodate the scheduling practice for hangar maintenance activities. To enhance the efficiency, an event-based discrete time model for reducing the solution space is introduced, which outperforms the basic discrete time model in solving small- and medium-size instances. Moreover, the rolling horizon approach for this problem is developed to provide good quality feasible solutions for large-scale instances. All the developed approaches are tested on a large set of instances, based on real data collected from an aircraft maintenance service company. We assessed the effectiveness of the proposed approaches, then conducted a sensitivity analysis to study the impacts made by the variations of the weightiness of the maintenance requests. Given the difficulties in evaluating the hangar capacity due to unique geometric features involved in the problem, the parking and service capacity varies according to the incoming maintenance requests' demand and the specification of aircraft from time to time. Under different variation of weightiness settings for maintenance requests, the computational results on solving challenging instances have revealed that the congestion of arrival maintenance requests create peak periods requiring much hangar space demand for aircraft parking, but results in rejection of some maintenance requests or lateness in fulfillment since the blocking of movement and insufficient space occurs. In this regard, the negative effects of lateness and rejecting maintenance requests should not be underestimated with the rising maintenance demands, which induces clients' dissatisfaction and adverse profit lost in the company. To enhance the service level of maintenance service provider in serving the increasing demands and fulfill the maintenance requirement of different airlines' fleets, it is recommended that the independent service company and airlines work jointly ahead of time to arrange the maintenance plan, which avoids overwhelming maintenance requests arrive at similar time in a proactive manner. Such tactics enables the service company to have enough time to review their service capacity and carry out the maintenance service plan. When congestions of maintenance request occur, moderate time buffer still allow maintenance company and airlines to negotiate and adjust the maintenance plan in a flexible manner. Further avenues of this research topic include: (1) consideration of additional practical constraints for practitioners in industry, such as the inclusion of repositioning decisions while undergoing the maintenance task; assigning the position according to the maintenance type and distance to the tooling/material stores. (2) the development of exact algorithms, heuristic algorithms and improvement of the existing rolling horizon approaches in solving the more challenging large-scale instances so as to be able to make a theoretical impact on the problem. (3) stochastic modelling incorporating the uncertainties due to unscheduled maintenance requests and material as well as man power shortage constraints.

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