https://doi.org/10.1016/j.precisioneng.2019.12.003

Any-degrees-of-freedom (anyDOF) registration for characterization of freeform surfaces

M. Y. Liu[a,b], C. F. Cheung[a*], X. Feng[b,c], C. J. Wang[a], Z. C. Caoa[a,d]

[a]State Key Laboratory of Ultra-precision Machining Technology, Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong

[b]Manufacturing Metrology Team, Faculty of Engineering, University of Nottingham, Nottingham, United Kingdom

[c]School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai, China

[d]Key Laboratory of Mechanism Theory and Equipment Design of Ministry of Education, Tianjin University, Tianjin, China

[*]corresponding author: benny.cheung@polyu.edu.hk

Abstract

This paper presents an any-degrees-of-freedom (anyDOF) registration method for characterization of freeform surfaces. The method attempts to fill the research gap for traditional surface registration methods which are normally dedicated to solve the global optimization problem with all DOF but they are lack of flexibilities. The proposed anyDOF method is capable of registering surfaces with any specified and combination of DOF. This is particularly useful when some of the DOF are known to be unchanged according to the *a priori* knowledge. The anyDOF surface registration method is regarded as a typical optimization problem of finding the minimum distance from target surface to the reference surface, with constraints of the unwanted DOF. The problem is solved by the Levenberg-Marquardt method. Simulated experiments for a two-dimensional (2D) profile and a three-dimensional (3D) surface were undertaken, together with two measurement experiments including a fluid-jet polished surface and a bonnet polished surface. Experimental results show that the anyDOF registration method is highly flexible for characterization of freeform surfaces.

Keywords: any-degrees-of-freedom (anyDOF); registration; characterization, freeform surfaces; ultra-precision machining; precision surface measurement

1. Introduction

Freeform surfaces [1] have attracted a lot of research attentions in the last decades. Technologies for manufacturing [2], measurement [3], and characterization [4] of freeform surfaces have been rapidly developed to meet the stringent requirements of freeform surfaces [5]. Characterization plays an important role in the development of precision freeform surfaces since one can never know whether the machined surfaces meet the design requirements without the characterization information. During characterization, the measured surface is compared with the design surface to determine the difference between them, which is a sum of machining errors, measurement errors and characterization errors. Measurement and characterization errors must be relatively small in order to accurately determine machining errors and enable improvement to the accuracy of machined surfaces through error compensation.

Since the design surface and the measured surface are obtained from different coordinate systems, i.e. the design surface is in the coordinates of design space and the measured surface is in the coordinates of the measurement instrument, alignment of the two surfaces is necessary for comparison. Alignment is typically achieved by a surface registration process that transforms the measured surface to the design surface. Research on the registration of 3D surfaces has a long history and the topic has been intensively investigated. The most widely used is probably the iterative closest points (ICP) method developed by Besl and McKay [6] and Zhang [7], from which several variants have been developed [8]. Other frequently used methods include the least square method [9], the intrinsic feature-based method [10], and methods based on image registration [11]. Most of these methods perform surface transform in all six degrees of freedom (DOF), which in most cases provides accurate alignment of the measured and designed surfaces. Take the ICP for example, the core algorithm is a singular value decomposition (SVD) method which determine the rotation matrix in all DOF and it is not easy to separate specified DOF during registration.

However, the complete freedom of transformation sometimes results in false alignment due to similarities in the surface form and measurement error. Such false alignment occurs more often in freeform surfaces that are asymmetric or contain periodic features. For example, when evaluating fluid jet polished [12] and bonnet polished [13] surfaces where large inhomogeneous errors are present, registration with full six DOF may result in unwanted tilting or shifting, even though the overall deviation from the designed surface is minimized. Such unwanted registration error can be avoided using *a priori* knowledge of the surface, i.e. utilizing pre-existing highly accurate reference features on the surface such as flat or spherical features [14] to assist the registration process. In such cases, the registration process is performed in two steps. The first step is registration of the reference features, which will constrain some of the DOF; and the second step is to register the entire surface using the remaining DOF. In this paper, a two-step any-degrees-of-freedom (anyDOF) registration method is proposed to enable the alignment of surfaces using less than six DOF.

In the first step, the target surface is pre-aligned using reference features, resulting in some of the DOF to be constrained. In the second step, the remaining unconstrained DOFs are used to minimize the overall difference between the target and reference surfaces. With the proposed method, the unwanted misalignment can be avoided and therefore more accurate characterization of the surface can be achieved. The rest of the article is structured as follows: Section 2 describes the alignment algorithm used in the anyDOF method; in Section 3, two simulated examples are used to demonstrate the limitations of the traditional ICP method and the potential improvement to be achieved by the proposed anyDOF method, the effectiveness of which is further verified using measurement data of two real machined surfaces. The results demonstrated that the proposed method is highly robust and suitable for the characterization of freeform surfaces with inhomogeneous errors. Section 4 summarizes the findings and contributes of this work.

2. Any-degrees-of-freedom (anyDOF) registration method

The schematic diagram of the any-degrees-of-freedom (anyDOF) method is shown in Fig. 1. The target surface is first subject to outlier removal to remove spurious points due to measurement noise, dirt on the surface and/or measurement artefacts often present in optical measurement instruments [15-17]. The surface is subsequently transformed using *a priori* knowledge, such as removing tilt or pre-alignment using reference features. Depends on the type of features used during pre-alignment, some of the DOF will be constrained. For example, pre-alignment using a planar reference feature will constrain two rotations and one translation, while pre-alignment using a spherical feature will constrain all three translations. The pre-aligned surface is compared to the reference surface and the root-mean-squared (RMS) distance from each point on the transformed surface to the reference surface is determined and used as the cost function. To find the solution of the specified DOF is to minimize the cost function, which is a typical nonlinear optimization problem. The nonlinear minimization problem is solved by the Levenberg-Marquardt algorithm (LMA) [18], which is iteratively executed until the minimum tolerance is found or the maximum number of iterations is reached. As a result, the unknown variables are determined numerically and the final transformation matrix for the anyDOF registration is obtained.

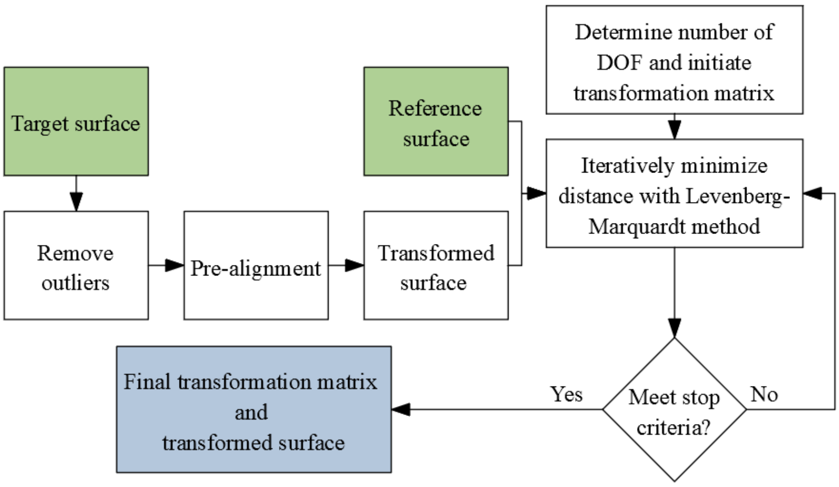


Figure 1. Diagram of the anyDOF registration method

In the case of 2D profiles, at most three DOF (i.e. two translations and one rotation) are available for optimization during registration. In the case of 3D surfaces, all or part of six DOF could be available for optimization. In this section, the discussion is focused on the registration of 3D surfaces, which is a superset of the case for 2D profiles.

The rigid-body transformation for a 3D surface has 6 DOF, i.e. translation along *x, y, z* axes and rotation about *x, y, z* axes, representing the yaw, pitch, and roll angles. The translation matrices can be determined by

$$T_x(v_x) = \begin{bmatrix} 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$T_y(v_y)=\begin{bmatrix}1&0&0&0\\0&1&0&v_y\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{2}$$

$$T_z(v_z)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&v_z\\0&0&0&1\end{bmatrix} \tag{3}$$

where v_x, v_y and v_z are the translation variables along *x, y, z* axes. The rotation matrices can be determined by

$$R_x(\alpha)=\begin{bmatrix}1&0&0&0\\0&\cos(\alpha)&-\sin(\alpha)&0\\0&\sin(\alpha)&\cos(\alpha)&0\\0&0&0&1\end{bmatrix} \tag{4}$$

$$R_y(\beta)=\begin{bmatrix}\cos(\beta)&0&\sin(\beta)&0\\0&1&0&0\\-\sin(\beta)&0&\cos(\beta)&0\\0&0&0&1\end{bmatrix} \tag{5}$$

$$R_z(\gamma)=\begin{bmatrix}\cos(\gamma)&-\sin(\gamma)&0&0\\\sin(\gamma)&\cos(\gamma)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{6}$$

where α, β, and γ are the rotation variables about *x, y, z* axes.

Assume the transformation is determined by first rotating about the *x, y, z* axes and then translating along the *x, y, z* axes, the final transformation matrix with all 6 DOF can be determined by

$$M = T_z(v_z)T_y(v_y)T_x(v_x)R_z(\gamma)R_y(\beta)R_x(\alpha) \tag{7}$$

In the anyDOF method, any DOF previously used for pre-alignment becomes a unit matrix, and the final transformation matrix is the product of the remaining translation and/or rotation matrices. For the point set P of the target surface, the transformed point set P' can be determined by

$$P' = MP \tag{8}$$

where P and P' are both n×4 matrix, n is the number of points, 4 elements are the coordinates of *x, y, z* and a unit value padded for the matrix calculation. Assume point set X represents the reference surface, the cost function is determined by

$$F = RMS(D) \tag{9}$$

where D is the vector of the distances from every point in X to P'.

In this study, the anyDOF method is implemented in Matlab. The LMA iterative procedure is started with the initial values of the unknown variables set to zero.

3. Experiments

In order to verify the effectiveness of the proposed anyDOF method, a series of experiments were conducted including a simulated 2D profile, a simulated 3D surface and two measurement experiments including a fluid-jet polished surface and a bonnet polished surface.

3.1 Simulations

The simulations include a 2D sinusoidal profile and a 3D sinusoidal surface. The target profile and surface were modified to create an inhomogeneous deviation. The modified profile and surface were then registered with the reference profile and surface to determine the errors. The widely used ICP method was applied to obtain full DOF registration, which was compared to the proposed anyDOF method. Simulation is an effective way to compare the two methods, as the determined registration error is not affected by other errors (e.g. measurement noise) that would have occurred in experiments.

(a) 2D profile

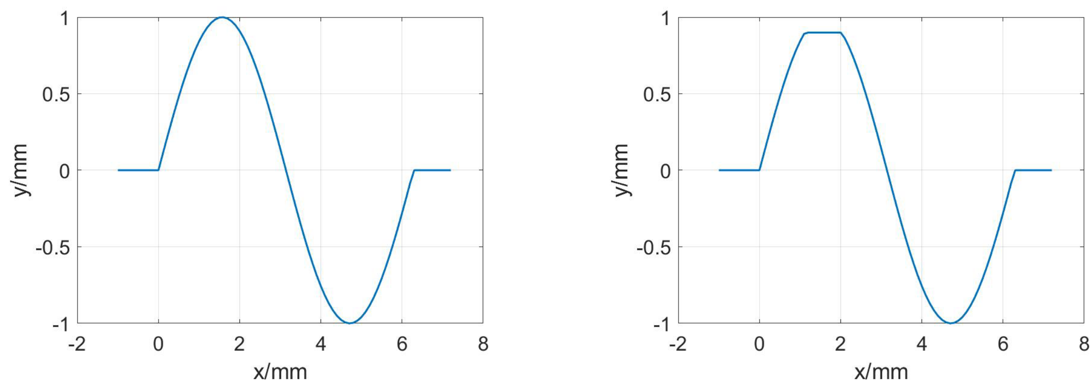
A sinusoidal profile was designed as shown in Fig. 2. The reference profile shown in Fig. 2(a) can be determined by

$$y_{ref} = \begin{cases} \sin x, & \text{if } 0 < x < 2\pi \\ 0, & \text{others} \end{cases} \tag{10}$$

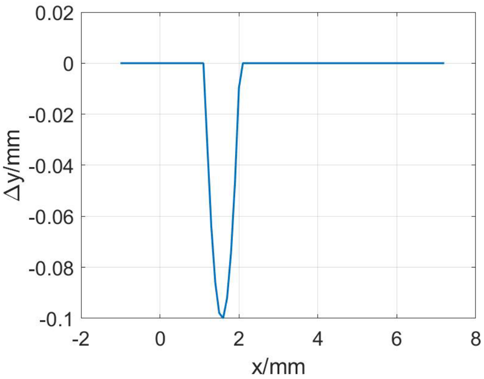
where $x \in [-1, 2\pi + 1]$ mm. The sampling distance was 0.1 mm. To simulate an inhomogeneous error in the target profile, thresholding by profile height was applied to the reference profile and replacing the missing points with new values. The modified profile, as shown in Fig. 2(b), can be determined by

$$y_{tar} = \begin{cases} y_{ref}, & \text{if } y_{ref} < 0.9 \\ 0.9, & \text{others} \end{cases} \tag{11}$$

The error profile is determined by subtracting the reference profile from the target profile, as shown in Fig. 2(c). The RMS and peak-to-valley (PV) value of the error profile are 24.1 μm and 100 μm, respectively. The error profile in Fig. 2(c) is the ideal result to be aimed for.



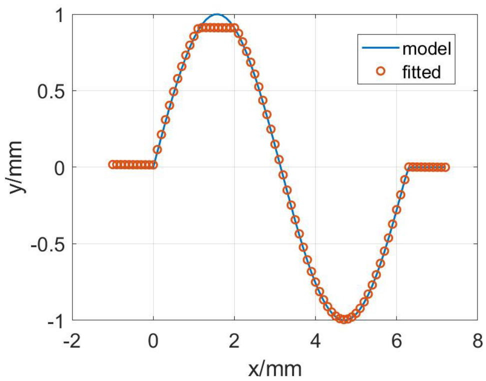
(a) Reference profile (b) Target profile



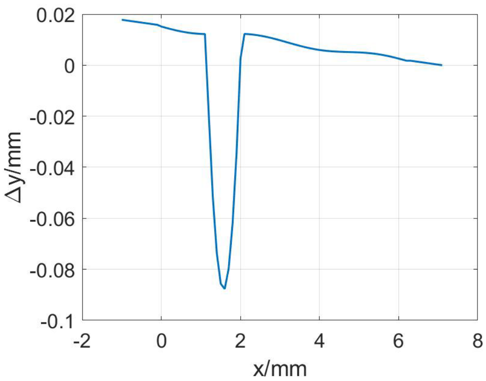
(c) Error profile

Figure 2. Simulated sinusoidal reference profile, target profile and error profile

The registration result using the ICP method is shown in Fig. 3. The error profile shown in Fig. 3(b) indicates that a tilting error of approximately 20 µm along the 8 mm long profile was present as a result of the simulated inhomogeneous error. The tilting error was introduced in the registration process, where the ICP method aimed to find the minimum RMS distance for the entire profile. The RMS value of the error profile is 22.7 µm and the PV value of the error profile is 105.5 µm. With the ICP method, registration was performed in 3 DOF including two translations in the *x* and *y* direction and one rotation in the *x-y* plane. The translation distances in the *x* and *y* directions are 0.4518 µm and 15.6 µm, respectively. The rotation angle is -0.1259 °.



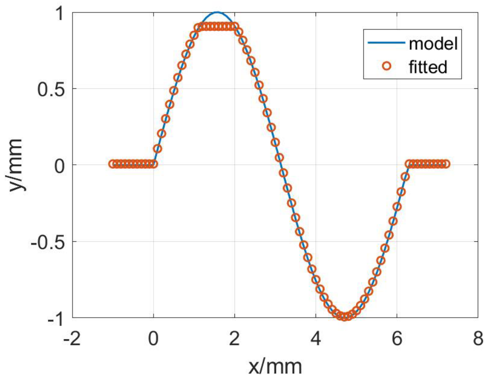
(a) Registered profile



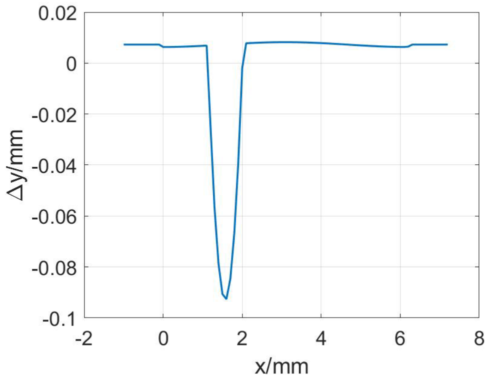
(b) Error profile

Figure 3. Registration results with the ICP method

With the anyDOF method, *a priori* knowledge was utilized to pre-align the target profile using the flat portion in the outer area, which constrained the rotation in the *x-y* plane. Hence, only translations in the *x* and *y* directions were part of the optimization problem. Figure 4 shows the registration result using the anyDOF method. As a result of restricting rotation of the profile, registration error shown in Fig. 4(b) was much closer to the ideal registration. The RMS value of the error profile is 23.0 µm and the PV value of the error profile is 100.8 µm, which are also closer to the ideal registration errors than those obtained from the ICP method. The translation distances in the *x* and *y* directions are 0.96 µm and 7.30 µm, respectively. The results for ICP method and anyDOF method are summarised in Table 1.



(a) Registered profile



(b) Error profile

Figure 4. Registration results with the anyDOF method

Table 1. Summarized results for ICP method and anyDOF method

	ICP	anyDOF
DOF	Full DOF: T_x, T_y , R_z	2 DOF: T_x, T_y
RMS error (µm)	22.7	23.0
PV error (µm)	105.5	100.8
Translation in *x* direction (µm)	0.4518	0.9568
Translation in *y* direction (µm)	15.60	7.3
Rotation (°)	-0.1259	-

The RMS error obtained using the anyDOF method (23.0 µm) was slightly larger than that obtained using the ICP method (22.7 µm), which is expected as the ICP method is a global optimization method to find the minimum distance from the target surface to the reference surface, while the anyDOF method has limited one DOF, i.e. the DOF of rotation. Nevertheless, the anyDOF method has been shown to produce a registration result closer to what is deemed by the authors to be ideal.

(b) 3D surface

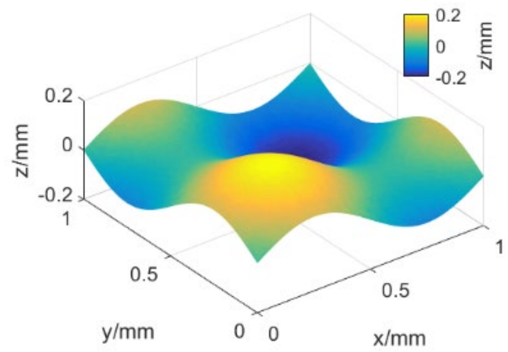
A simulated sinusoidal surface is shown in Fig. 5(a), and can be determined by

$$z_{ref} = 0.1\left[\sin\left(2\pi x\right) + \sin\left(2\pi y\right)\right] \tag{12}$$

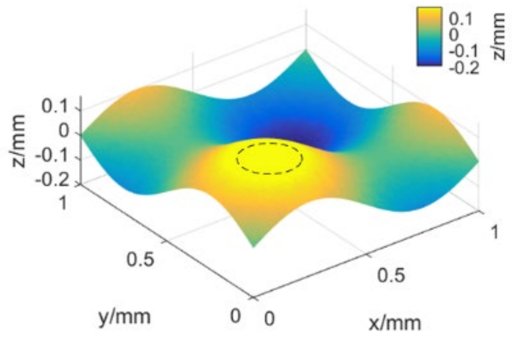
where $x, y \in [0,1]$ mm. The sampling distance is 0.01 mm. Thresholding by surface height was applied to the reference surface to create the target surface, which is shown in Fig. 5(b) and can be determined by

$$z_{tar} = \begin{cases} z_{ref}, & \text{if } z_{ref} < 0.16 \\ 0.16, & \text{others} \end{cases} \quad (13)$$

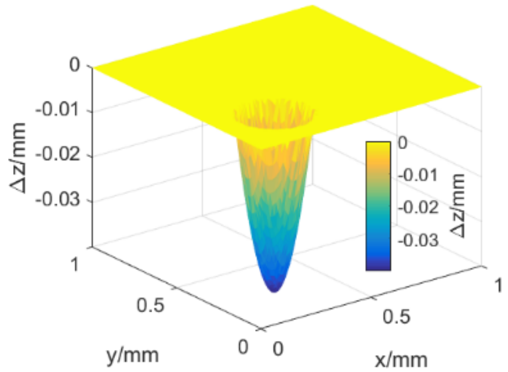
Hence, the error map of the target surface compared to the reference surface can be determined and it is shown in Fig. 5(c). The RMS and PV values of the error map are 5.8 μm and 40.0 μm, respectively.



(a) Reference surface



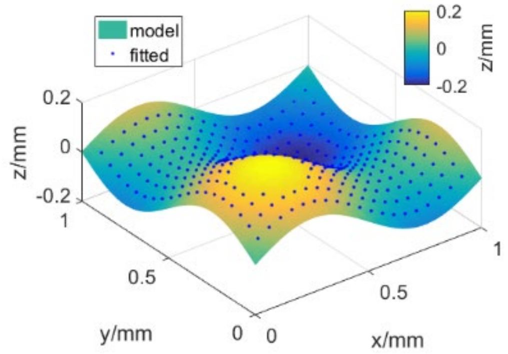
(b) Target surface



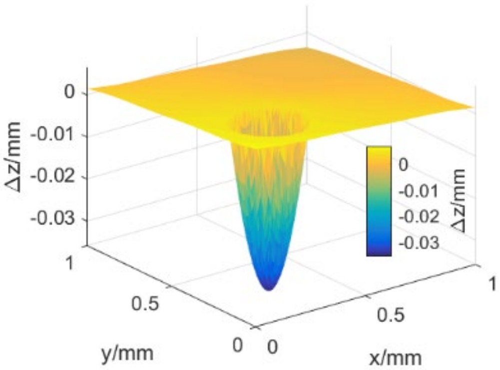
(c) Error map

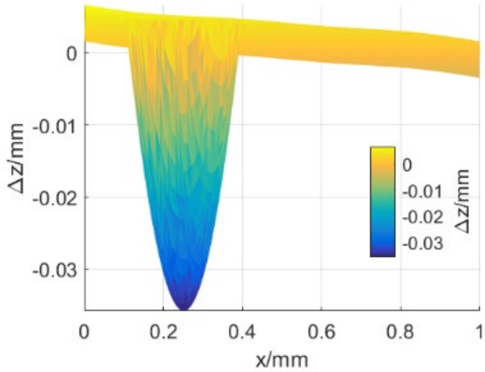
Figure 5. Simulated sinusoidal reference surface, target surface and error map

Registration result obtained using the ICP method is shown in Fig. 6. The registered surface was downsampled to have better visualization. The error map is shown in Fig. 6(b) and shown in Fig. 6(c) in a view angle from which the tilting error is better visualized. Translations in *x, y and z* directions are 0.0908 μm, 0.0938 μm, and 6.7 μm, respectively. Rotations about *x, y, z* axes are -0.2908°, 0.2907°, and -0.0008°, respectively.



(a) Registration result

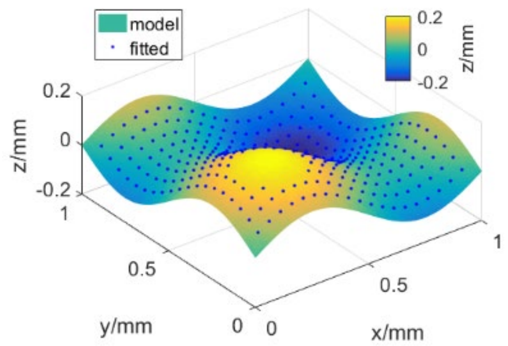




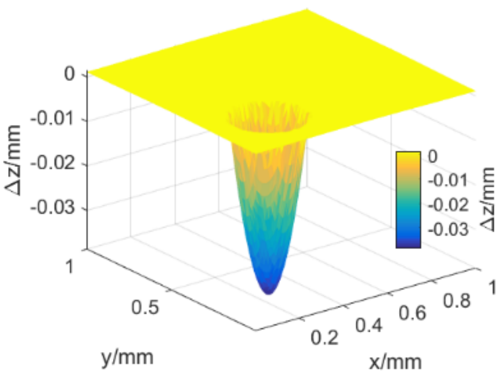
(b) Error map (c) Error map showing tilting

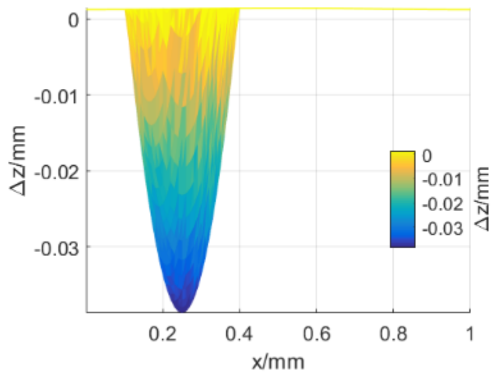
Figure 6. Registration results with the ICP method

The target surface was then registered to the reference surface using the anyDOF algorithm and the result is shown in Fig. 7. The DOF used in the anyDOF were 3 translations in *x, y* and *z* directions, according to the *a priori* knowledge. Fig. 7(a) shows the registered surface with the reference surface. The registered surface was down sampled to have a better visualization. The error map is shown in Fig. 7(b) and in Fig. 7(c) viewed along the *y* axis to demonstrate the lack of tilting in the error map, which is expected since the DOF of rotations were excluded. Translations in *x, y and z* directions are 0.1597 µm, 0.1597 µm, and 1.3 µm, respectively. The results for ICP method and anyDOF method are summarised in Table 2.



(a) Registration result





(b) Error map (c) Error map in another angle

Figure 7. Registration results with the anyDOF method

Table 2. Summarized results for ICP method and anyDOF method

	ICP	anyDOF
DOF	Full DOF: T_x, T_y, T_z, R_x, R_y, R_z	3 DOF: T_x, T_y, T_z
RMS error (µm)	5.5	5.7
PV error (µm)	42.4	40.2
Translation in *x* direction (µm)	0.0908	0.1597
Translation in *y* direction (µm)	0.0938	0.1597
Translation in *z* direction (µm)	6.7	1.3
Rotation about *x* axis (°)	-0.2908	-
Rotation about *y* axis (°)	0.2907	-
Rotation about *z* axis (°)	-0.0008	-

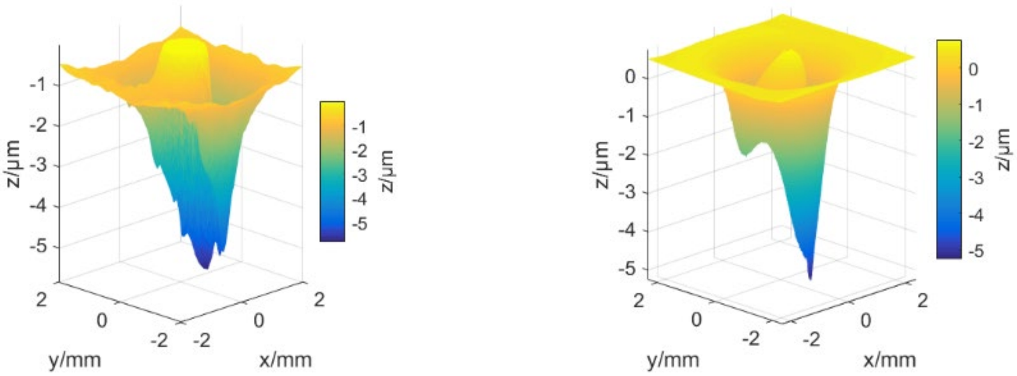
The RMS error obtained using the anyDOF method (5.7 µm) was slightly larger than that obtained using the ICP method (5.5 µm). However, the error map obtained using the anyDOF can better represent the error simulated in the ideal registration. Limiting the DOF according to the *a priori* knowledge using the anyDOF method shows high flexibility and its uniqueness compared to the ICP method, which can specify any DOF for the registration process.

3.2 Measurement experiments

Two measurement experiments were conducted to verify the effectiveness of the proposed anyDOF method: a fluid jet polished sample and a bonnet polished sample. Due to the nature of the polishing processes [13, 19], the polished workpieces have significant inhomogeneous deviation to the predicted models. Characterization of the polished workpieces using traditional full-DOF registration methods such as the ICP method results in unwanted rotational error [12], while using the proposed anyDOF method can avoid this problem.

(a) Fluid jet polished sample

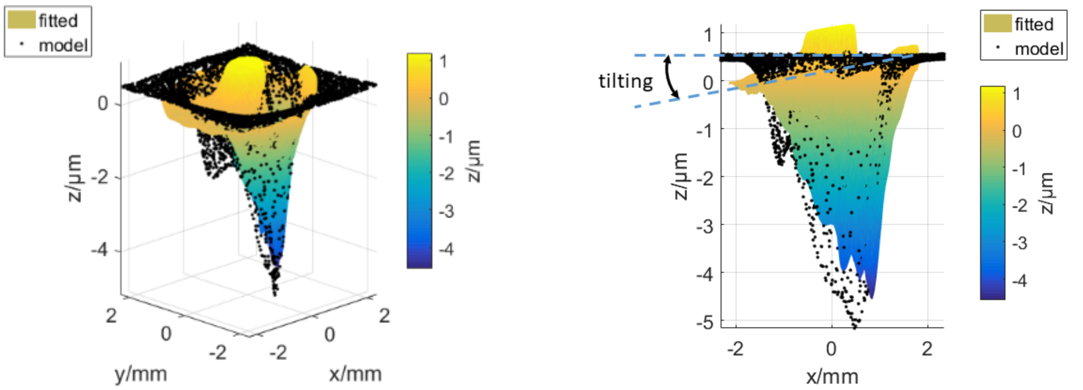
Fluid jet polishing is one of the most promising polishing processes, especially for freeform surface finishing, depending on its unique advantages, such as high adaptability to the freeform surface, no temperature increase of the workpiece, etc. [12]. Modelling of the tool influence function is critical for the modelling of the surface generation during the fluid jet polishing process, to predict the polished surface form. In this experiment, one footprint of fluid jet polishing on a BK7 optical glass surface was conducted using the 5 wt.% silicon carbide polishing slurry. The diameter of the nozzle was 1.4 mm. The impinging angle was 75° and the dwell time was 3 minutes. The surface after polishing predicted using a process model [12] is shown in Fig. 8(a), and the actual polished surface, measured using a coherence scanning interferometer (CSI) Zygo Nexview, is shown in Fig. 8(b). It should be noted that the surface was levelled in advance using the unpolished flat surface of the sample.



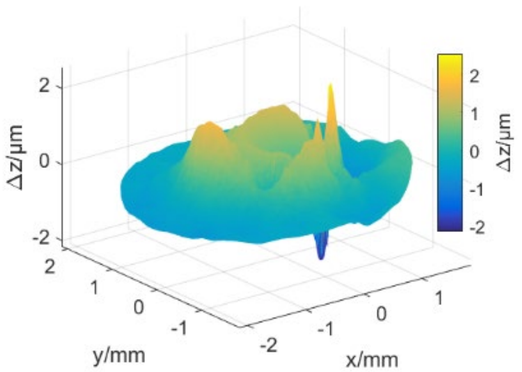
(a) Predicted surface after polishing (b) Measured surface

Figure 8. A fluid jet polishing experiment

Registration was first performed using the ICP method and the result is shown in Fig. 9. A tilting angle between the target surface and the reference surface was observed in Fig. 9(b). The RMS and PV values of the error map is 0.5550 μm and 4.7013 μm, respectively. Translations in *x, y and z* directions are -153.5782 μm, 137.0135 μm, and 1.1019 μm, respectively. Rotations about *x, y, z* axes are 0.0062°, -0.0079°, and -31.3300°, respectively. Tilting errors about the *x* and *y* axes are small but they exist visually, although the surface has already been levelled in advance. The large rotation angle about the *z* axis is due to the fact that there was no pre-alignment process in this axis.



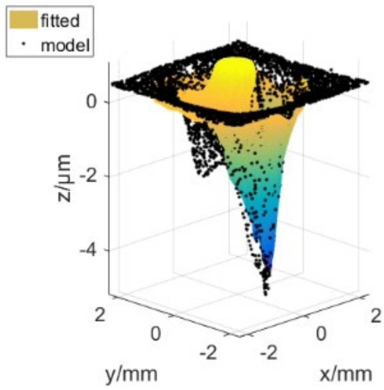
(a) Registration result (b) Registration result showing tilting



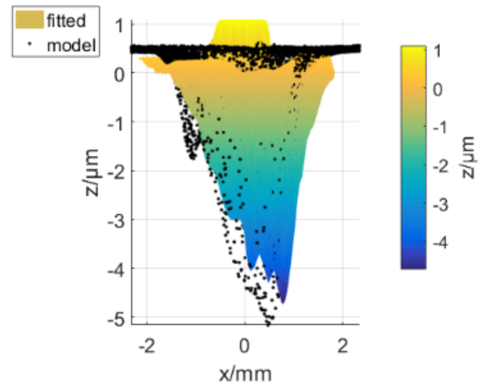
(c) Error map

Figure 9. Registration result using the ICP method

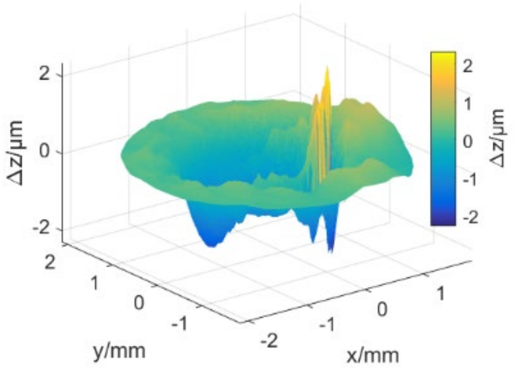
The target surface was also registered using the anyDOF method and the result is shown in Fig. 10. Three translations (in the *x, y* and *z* directions) and one rotation about the *z* axis were enabled, as the other two DOF (rotations about the *x* and *y* axes) were removed in advance during pre-alignment, using the unpolished flat surface. The tilting error introduced during ICP registration was successfully avoided, as shown in Fig. 10(b). The error map is shown in Fig. 10(c). Translations in *x, y and z* directions are -160.9646 µm, 139.6536 µm, and 1.1020 µm, respectively. Rotation about *z* axis is -37.7597°. The results for ICP method and anyDOF method are summarised in Table 3.



(a) Registration result



(b) Registration result showing no tilting



(c) Error map

Figure 10. Registration result using anyDOF method

Table 3. Summarized results for ICP method and anyDOF method

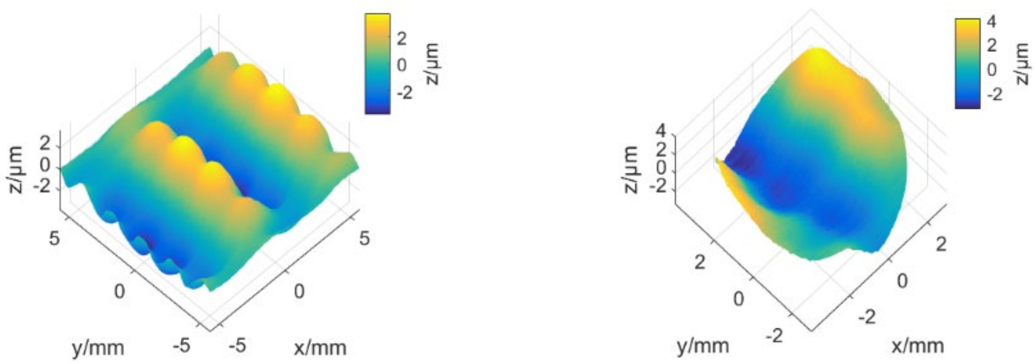
	ICP	anyDOF
DOF	Full DOF: T_x, T_y, T_z, R_x, R_y, R_z	4 DOF: T_x, T_y, T_z, R_z
RMS error (µm)	0.5550	0.5636
PV error (µm)	4.7013	4.6340
Translation in *x* direction (µm)	-153.5782	-160.9646

Translation in y direction (μm)	137.0135	139.6536
Translation in z direction (μm)	1.1019	1.1020
Rotation about x axis (°)	0.0062	-
Rotation about y axis (°)	-0.0079	-
Rotation about z axis (°)	-31.3300	-37.7597

The results obtained with both the ICP method and the anyDOF method showed similar RMS error, PV error, translation distances in all x, y and z directions and even rotation angle about the z axis, which demonstrated the effectiveness of the proposed anyDOF method. While the ICP method produced slightly lower RMS error, it was achieved at the expense of unwanted tilting error, which can not correctly represent the characterization result. The result demonstrates the advantage of the proposed anyDOF method.

(b) Bonnet polishing sample

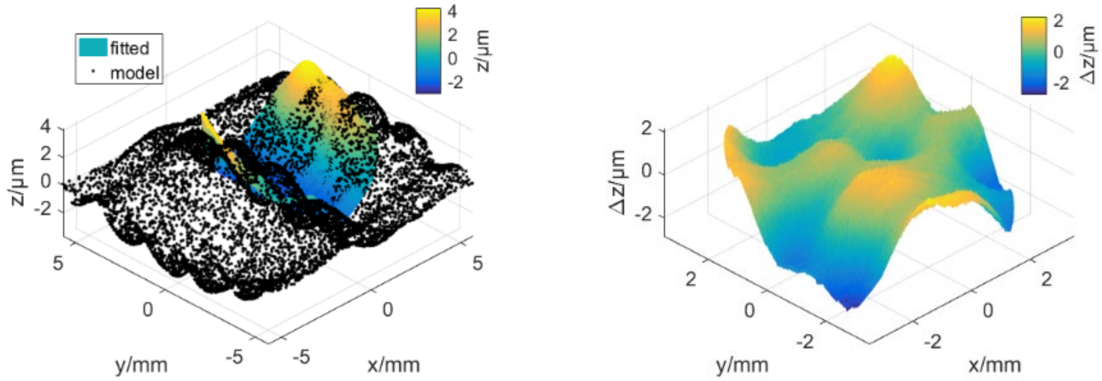
Bonnet polishing is another promising method to achieve ultra-fine surface finishing. The mechanic of the polishing process is studied based on the contact mechanics, kinematics theory, abrasive wear mechanism, as well as the relative and cumulative removal process of surface generation [13]. The polished pattern is compared to the simulated model through registration. In this section, a polishing experiment with bonnet polishing method was conducted to evaluate the effectiveness of the proposed anyDOF method. The evaluated surface is a bonnet polished surface with the following machining parameters: the tool pressure was 1.2 bar, the spindle speed was 1500 rpm, the precess angle was 15°, the tool offset was 0.28 mm, the feed rate was 50 mm/min, the tool spacing was 0.6 mm, and the vertical swing speed was 250 degree per minute. The surface after polishing was predicted using the process model [13] and it is shown in Fig. 11(a). The measured surface using CSI is shown in Fig. 11(b). It is noted that the tilting was removed in the measurement result in advance, using the unpolished flat surface. In this experiment, the predicted surface was regarded as the reference surface.



(a) Predicted surface after polishing (b) Measured surface

Figure 11. A bonnet polishing experiment

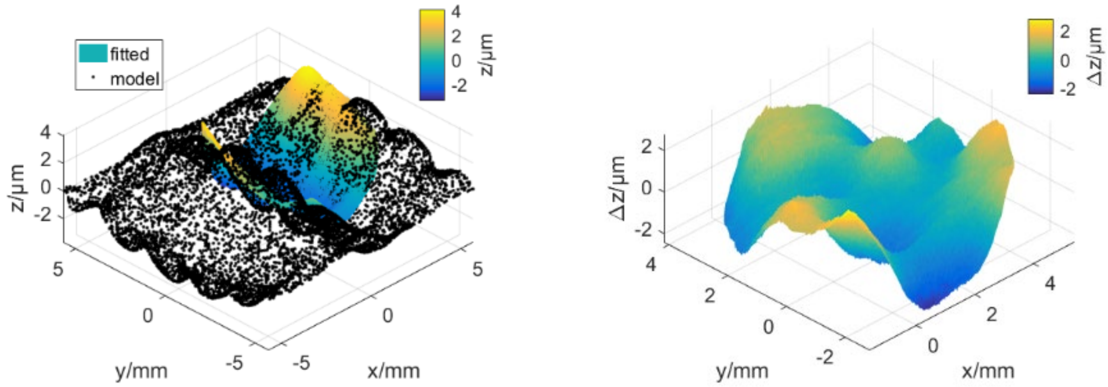
Registration result using the ICP method is shown in Fig. 12, which showed significant deviation from the reference surface due to the complexity of the polishing process. The error map is shown in Fig. 12(b). Translations in *x, y and z* directions are 2.2763×10^{3} µm, 83.9525 µm, and -0.08158 µm, respectively. Rotations about *x, y, z* axes are 0.0044°, 0.0029°, and -8.3323°, respectively. The result shows that there are still tilting errors in rotation about the *x* and *y* axes, although the tilting was removed in advance using the unpolished flat surface of the workpiece.



(a) Registration result (b) Error map

Figure 12. Registration result using ICP method

The target surface was also registered to the reference using the anyDOF method. Since the tilting of the target surface was removed in advance, the DOF considered in the registration were 4 DOF including three translations in the *x*, *y* and *z* directions and one rotation about the z axis. The result is shown in Fig. 13. Fig. 13(b) shows the error map. Translations in *x, y and z* directions are 2.3089×10^{3} µm, 98.4012 µm, and -0.01754 µm, respectively. Rotation about *z* axis is -7.8028°. The results for ICP method and anyDOF method are summarised in Table 4.



(a) Registration result (b) Error map

Figure 13. Registration result using anyDOF method

Table 4. Summarized results for ICP method and anyDOF method

	ICP	anyDOF
DOF	Full DOF: T_x, T_y, T_z, R_x, R_y, R_z	4 DOF: T_x, T_y, T_z, R_z

RMS error (μm)	0.8374	0.8507
PV error (μm)	5.0332	5.2029
Translation in *x* direction (μm)	2.2763×10^3	2.3089×10^3
Translation in *y* direction (μm)	83.9525	98.4012
Translation in *z* direction (μm)	-0.08158	-0.01754
Rotation about *x* axis (°)	0.0044	-
Rotation about *y* axis (°)	0.0029	-
Rotation about *z* axis (°)	-8.3323	-7.8028

The results showed similarity to those obtained from the fluid jet polishing experiment. The RMS error for the anyDOF method is larger than that from ICP method. However, the result from the anyDOF gain more confidence for characterization of the polished surface, according to the *a priori* knowledge of the registration for the specified DOF, i.e., removing tilting using unpolished flat surface of the workpiece.

All experiments including simulated 2D profile, 3D surface and actual measurements demonstrated that the proposed anyDOF method can successfully register the target surface with the reference surface and hence further characterize the target profile/surfaces by calculate the error map. The advantage of the anyDOF method is that it can specify any combination of all or part of six DOF. For a 2D profile, it can be any of the three DOF including two translations and one rotation. For a 3D surface, it can be any of the six DOF. The anyDOF method is particularly useful when the target profile/surface have inhomogeneous deviations which introduce unwanted tilting if registration is performed using all DOF. With the flexibility of the anyDOF method and *a priori* knowledge of the surface, pre-alignment of reference features such as planes or spheres can be performed in advance, and characterization of the complex freeform surfaces can be more accurate. Furthermore, the anyDOF method can also utilize all DOF in the registration process when it is deemed needed. Hence, it is a more generalized method which is expected to be able to have wide application in the field of characterization of 2D profiles and 3D surfaces.

4. Conclusion

In this paper, an any-degrees-of-freedom (anyDOF) registration method is presented to provide a more flexible solution for characterization of freeform surfaces. Unlike the traditional full DOF methods such as the ICP method, the enabled and disabled DOF can be specified in the anyDOF method with any combination of all available DOF. Solving the anyDOF problem is achieved using the Levenberg-Marquardt method, which is a classical optimization procedure. A number of experiments including simulations and actual measurement were conducted and the results demonstrated that the method is effective to provide accurate characterization results when limiting some unwanted DOF. This method is particularly useful when *a priori* knowledge of the surface is utilized, e.g. the surface is pre-aligned with reference features such as reference planes or spheres, or the surface is pre-processed such as by removing tilting.

Acknowledgments

The work described in this paper was mainly supported by a grant from the Ministry of Science and Technology of China (Project Code: K-BBXY) and a grant from the Research Grants Council of the Government of the Hong Kong Special Administrative Region, China (Project No. 15202717). The authors would also like to express their thanks to the Research Office of The Hong Kong Polytechnic University (Project code: BBX7).

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