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Seaport Investments in Capacity and Natural Disaster Prevention

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Abstract: Most seaports face constraints in financial resources to some degree, and thus need to balance the investments in capacity and natural disaster prevention. On the one hand, due to budget constraint, limited resources need to be allocated between the two tasks. On the other hand, the benefit of natural disaster prevention investment is likely to be higher for ports with larger capacity. This study builds a stylized analytical model to examine the managerial and policy implications of such interactions between the two counteracting mechanisms. We find that the port managers would always prioritize capacity investment over natural disaster prevention investment. Social welfare maximizing ports invest more in both capacity and disaster prevention than those chosen by profit maximizing operators. However, compared with profit maximizing ports, welfare maximizing ports also require a larger budget to justify the investment in disaster prevention. Moreover, with increasing intensity of natural disaster, a port's capacity investment decreases and its disaster prevention investment increases, irrespective of its objective. The magnitudes of both changes are larger for welfare maximizing ports than for profit maximizing ports.

Key words: Seaport; Capacity; Natural disaster prevention; Uncertainty; Ownership

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1. Introduction

Seaport management is a very challenging task, partly due to the complications created by various factors, such as the competition with other ports, the vertical relationship with shipping lines as well as the coordination with other stake holders in the maritime supply chain. In recent years, global climate change and the resultant occurrence of natural disasters have placed another layer of complexity to seaport management. Being one of the businesses that are the most susceptible to risks imposed by natural disasters such as hurricanes and flooding, most seaports are nowadays facing a crucial decision on whether and how much to invest in natural disaster prevention. This decision is closely connected to capacity investment of the seaports due to at least two reasons. First, as any other businesses, seaport operators are subject to budget constraint, meaning that the limited resources need to be allocated to different tasks. In this context, whenever a budget constraint exists, the more the resources allocated to port capacity expansion, the fewer the resources left for natural disaster prevention. Second, the benefit of natural disaster prevention of a port may well depend on its capacity level. In particular, if there is no prevention in place, when a natural disaster hits, a port with larger operation is likely to suffer larger loss due to disruption. In other words, natural disaster prevention may yield a higher return of investment if it is implemented in a larger port. Clearly, these two mechanisms impose counteracting effects on a seaport's resource allocation between the investments in capacity vis-à-vis natural disaster prevention. However, existing literature has treated these two investment decisions independently without modelling their interactions.

The dilemma faced by seaports in allocating resources between capacity expansion and natural disaster prevention is evident. In many cases, budget earmarked for natural disaster prevention ends up in capacity investment. For example, in 2005 hurricane Katrina caused serious damage to the port facilities at Gulfport, Mississippi. In order to help Gulfport to recover, US$570 million was allocated to the restoration process by the Federal Community Development and Block Grant. The original plan was to increase the height of the West Pier by 25 feet as a preventative measure for future disaster. However, one day after New York City was hit by hurricane Sandy, Gulfport released the announcement that it would instead use the money to double its capacity with a new inland port complex and a deeper channel. In other words, the focus was shifted from future disaster prevention to capacity expansion (Xiao et al., 2015). Another example is the Port of Seattle. It's betting that significant sea-level rise will not occur until about 50 years from now. The port managers argue that 50 years is the longest design life assigned to any of their assets, and they will be replacing those assets before sea-level rise inundates them (McClearn, 2018). It might appear that the decisions made by Gulfport and the Port of Seattle were irresponsible and short-sighted. Yet, the decisions made by experienced port operators may also be based on solid economic rationale, as the need for natural disaster prevention is "derived" upon the capacity of port facilities.

Uncertainty is another aspect that further complicates the related decision-making. More importantly, uncertainty exists not in one of the investment decisions but in both. Port managers need to consider demand uncertainty when they make capacity investment decision, while they also need to take into account the uncertainty of natural disaster occurrence when the decision regarding disaster prevention investment is made. The role of these two types of uncertainty in a port manager's resource allocation decision is very important but nevertheless under-investigated. Furthermore, seaports can be either publicly owned or privately owned, and the decision makers' objectives under these two scenarios will be quite different. A private port tends to pay more attention to operation profit attributable to share-holders, while a public port, often under municipal and local governments, tends to care more of social welfare instead. This difference in ownership structure and resultant operation objectives are expected to alter port managers' allocation of budget between capacity expansion and disaster prevention. In this study, we will benchmark the case of a profit-maximizing port vs. a welfare maximizing port. In reality, a port's objective may be mixed/balanced subject to the influences of regulation, ownership structures, competition and development stage. The two scenarios we considered nevertheless serve as good benchmark cases so that general conclusions can be obtained with our models.

In this paper, we build a stylized analytical model to look into a port's budget allocation decisions when both capacity expansion and natural disaster prevention are taken into account. We find that in the case of a tight budget, the port puts priority on capacity investment. However, as the budget increases, the focus of the port shifts to natural disaster prevention. We further consider the impacts of different operation objectives, and find that welfare-maximizing ports invest more in both capacity expansion and natural disaster prevention. However, they also require larger budgets to justify the investment in disaster prevention. Finally, we also investigate the effects of increasingly intense natural disaster. We find that the capacity investments of the ports decrease and their disaster prevention investments increase, while the magnitudes of both changes are larger for welfare-maximizing ports than for profit-maximizing ports.

The contributions of this paper are two-fold. First, as far as we know, this is the first theoretical study to explicitly investigate the resource allocation of a seaport between capacity expansion investment and disaster prevention investment when it faces uncertainties on both demand and disaster occurrence. Although there are many studies on the port investment and disaster prevention or adaptation (please see the following section for the detailed literature review), none of them explores these two issues simultaneously. On the one hand, capacity expansion and disaster prevention compete for the constrained investment budget. On the other hand, these two investments have different but related implications on port development, with the former promoting the port production and the latter rescuing the port from loss. That is, the latter is "derived" from the former. Our investigation on the complicated trade-offs between these two issues is expected to provide useful guides on port investment, especially when ports face multiple uncertainty sources (on market and disaster). Second, we compare the resource allocation rules between capacity expansion and disaster prevention for both profit maximizing ports and social welfare maximizing ports. Some clear differences are identified, which can provide practical policy implications on the governance of ports.

The paper is organized as follows. Section 2 provides literature review while Section 3 sets up the model. Section 4 focuses on analytical results and Section 5 presents further discussion and sensitivity analysis. Section 6 offers concluding remarks.

2. Literature Review

There are three streams of literature related to our study, namely, port investment, port adaptation and disaster management. The port investment literature is well developed. Most existing studies discuss this issue under certainty, albeit under different settings so that the effects of various issues can be evaluated, such as inter-port competition, regional development, port specialization, port pricing, facilitating maritime supply chain. For some recent studies, see Koh (2001), Musso et al. (2006), Luo et al. (2012), Xiao et al. (2012), Wang et al. (2012), zhuang et al. (2014), Zheng and Negenborn (2014), Zheng et al. (2017) and Zhu et al. (2019). The others investigate this issue under uncertainty, e.g., Meersman (2005), Allahviranloo and Afandizadeh (2008), Zheng and Negenborn (2017) and Balliauw et al. (2019). Some studies discuss port investment on disaster prevention or adaptation (e.g., Xiao et al., 2015; Wang and Zhang, 2018; Randrianarisoa and Zhang, 2019; Wang et al., 2020), which can also be included in the second stream of literature of port adaptation.

Relatively fewer studies have been carried out on port adaptation. Port adaptation refers to the investment that can help prevent or alleviate the damages caused by coastal and marine natural disasters. According to Xiao et al. (2015), such investments "may include: building storm-surge barriers and promoting beach nourishments, raising the height of roads (causeways), improving groins, dykes, levees and seawalls, strengthening a port's storm water system, and improving potable and wastewater emergency response and maintenance for more common and more extensive coastal flooding in vulnerable areas." Most of the existing studies in this area examine port adaptation using empirical analysis (e.g., Ng et al., 2013, Yang et al., 2018) or theoretical economic models (e.g., Xiao et al., 2015; Wang and Zhang, 2018; Randrianarisoa and Zhang, 2019; Wang et al., 2020). Specifically, both Wang and Zhang (2018) and Randrianarisoa and Zhang (2019) discuss the port adaptation investment under ambiguity, a quite general way of considering uncertainty. Nevertheless, none of these studies have explicitly modelled the inter-dependence of port adaptation and capacity investments.

The third stream related to our study is on disaster management. The comprehensive disaster management involves four steps: mitigation, preparedness, response and recovery (Coppola, 2015). In the broader topics of disaster management, there are three groups of literature which are directly related to our research: (1) coordination of different players (including the PPP) in disaster management; (2) resource allocation in disaster management and (3) disaster Insurance. Next, we review the related literature based on these groups.

In disaster management, private sectors play important roles in the implementation of the disaster relief tasks, e.g., they supply foods, services and technical supports. Therefore, how to align their behavior and encourage them to realize the government's disaster management objectives becomes one of the central topics in this domain. Specifically, the public-private-partnership (PPP) agreements are commonly used to determine the obligations of all parties in the disaster relief. Some studies (e.g., Guan et al. 2015, 2018) discuss the key issues in the PPP agreements in disaster management, e.g., the private sectors' risk attitudes, the centralization or decentralization management modes.

The resource allocation is a crucial decision for the governments in disaster management. The current studies focus on the resource allocations in critical infrastructure protection (Golalikhani and Zhuang, 2011), in the various phases of disaster life cycle (Coppola, 2015), on the distribution networks for relief (Anaya-Arenas et al. 2014), and on subsidizing citizens (Gruber and Levitt, 2011). Furthermore, some studies discuss the resource allocation on simultaneous disaster (Doan and Shaw, 2019) and mutual aid (Su et al., 2016).

The effective and sustainable insurance schemes are important arrangements against losses from natural disasters. There are some empirical studies on the disaster insurance. For example, Paleari (2019) reviews the insurance schemes in 28 EU countries from the perspectives of insurance penetration rate, compulsory or voluntary nature, push factors, risk reduction measures and the state roles. McAneney et al. (2016) overview the insurance pools for natural disasters sponsored by the government using the samples of US, New Zealand, UK, France and Netherland. Moreover, they discuss some related problems, e.g., pricing of risk, deficit and mitigation. The welfare gain from the insurance schemes against the natural disasters is another important research topic. For example, Borensztein et al. (2017) uses a dynamic optimization model to estimate the welfare gains from the insurance with catastrophe (CAT) bonds. Besides the traditional government-sponsored insurance schemes, there are some new forms of insurance against natural disasters, e.g., self-insurance (Mol et al., forthcoming) and volunteer crowdsourcing (Kankanamge et al., 2019).

Although there is plenty of literature discussing topics related to our paper, none of them simultaneously investigates the port investment on capacity expansion and disaster prevention under uncertainty and common budget constraint. Moreover, they do not explore the impacts of the port objective to its investment on disaster prevention, especially when the disaster occurrence is uncertain.

3. Model Setting

We consider one seaport, whose total budget is B. The port manager needs to make two investment decisions, namely K on capacity expansion and I on disaster prevention. The unit costs of the two investments are r_1 and r_2, respectively. Corresponding to the two investments, there is uncertainty in both market demand and marine natural disaster occurrence. The inverse demand function is given by $p = x - bq$, where p is the port charge, q is the port operating volume and x is the market demand potential, which is assumed to be random. For modelling tractability, it is assumed that x follows a Bernoulli distribution, with $P(x = \underline{x}) = n$ and $P(x = \overline{x}) = 1 - n$, where $0 < \underline{x} < \overline{x}$ and $0 < n < 1$.[1] Meanwhile, there is a chance for marine natural disaster to occur, and we assume that the associated intensity of the disaster is y, which also follows a Bernoulli distribution, with $P(y = 0) = m$ and $P(y = \overline{y}) = 1 - m$, where $\overline{y} > 0$ and $0 < m < 1$. Furthermore, when a natural disaster occurs, the cost it imposes on the port is proportional to the capacity of the port. We use θ to represent the per unit cost the disaster brings to the port proportional to capacity. Intuitively, a larger port suffers more loss in the case of natural

[1] There exist many similar modelling practices in the transportation literature (see, for example, Xiao et al., 2012, 2013, 2015, 2016).

disaster, although the relationship may not always be linear. A more general specification may be examined in future studies. In the case when disaster prevention investment has been made, the cost associated with the disaster would be mitigated in proportion to the disaster prevention investment level. In particular, we use ρ to denote the per unit cost saving from such investment, where $\rho > r_2$. In other words, the actual damage to the port is given by $max\{0, y\theta K - \rho I\}$.

Two benchmark cases of different objective functions are considered for the port, namely profit maximization and social welfare maximization. The port's decision process is modeled as sequential. In particular, there are two stages, and the port manager makes decision in each stage as follows:

Stage 1: The port manager determines the capacity expansion investment K and disaster prevention investment I of the port based on the distribution of x and y so as to maximize the expected profit/social welfare.

Stage 2: The port manager determines the operating volume q after observing the realization of x.

This two-stage setting is common in economics. Intuitively, the first stage describes long-term strategic decisions while the second stage depicts short-term operational decisions. The long-term decisions, once made, are fixed for a certain period of time. The short-term decisions, on the other hand, only have short-lived impacts and are made in the context of the long-term decisions.

We first look at the case of profit maximizing port. We start with the decision making in the second stage, which is characterized by the following optimization problem:

$$\begin{aligned} &Max\ \pi = (x - bq)q \\ &s.t. \quad 0 \le q \le K \end{aligned} \tag{1}$$

In stage 1 the port manager maximizes the expected profit, which can be specified as:

$$\begin{aligned} &Max\ E(\pi) = E((x - bq)q - r_1K - r_2I - max\{0, y\theta K - \rho I\}) \\ &s.t. \quad r_1K + r_2I \le B \\ &K \ge 0 \end{aligned} \tag{2}$$

$$I \geq 0$$

In the case of social welfare maximizing port, the optimization problem of the port manager in the second stage is:

$$\begin{aligned} & Max\ SW = xq - \frac{1}{2}bq^2 \\ & s.t. \quad 0 \leq q \leq K \end{aligned} \tag{3}$$

In stage 1 the port manager maximizes the expected social welfare, which can be specified as:

$$\begin{aligned} & Max\ E(SW) = E(xq - \frac{1}{2}bq^2 - r_1K - r_2I - max\{0, y\theta K - \rho I\}) \\ & s.t. \quad r_1K + r_2I \leq B \\ & K \geq 0 \\ & I \geq 0 \end{aligned} \tag{4}$$

4. Main Results

With backward induction for both optimization problems, the above two scenarios can be analytically solved under the common model set up as described in the above section. The derivation process is a bit tedious and a sketched proof can be found in the Appendix 1. The equilibrium levels of both *K* and *I* depend on two factors: the upper bound of the demand $\overline{x}$ and the budget level *B*. The detailed expressions of *K* and *I* and the corresponding conditions of $\overline{x}$ and *B* have been summarized in Tables 1-3 for the case of profit maximization and in Table 4-6 for the case of social welfare maximization. When $\overline{x}$ is very small (in our specification, when smaller than $\frac{r_1+\overline{y}\theta(1-m)}{1-n}$), *K* and *I* will both be zero irrespective of *B*. This corresponds to the trivial case when market demand is so small that sustained port operation is not justified. It is easy to see from the tables that both investment levels increase along with larger values of $\overline{x}$ and *B*. This is intuitive, as the larger the market potential, the greater the needs for both investments. Meanwhile, with larger budget, the port is able to invest more resources.

<Insert Tables 1-6 about here>

To compare the case of profit maximization and the case of social welfare maximization, we also present the results in three figures, which lead to the following propositions.

<Insert Figures 1-3 about here>

Proposition 1: As the budget increases, the port invests first in capacity and then later in natural disaster prevention.

Proof: As shown in Figures 1 to 3, it can be observed that $A1 > 0, A1' > 0$ in Figure 1, $C2 > C1, C2' > C1'$ in Figure 2, and $E2 > E1, E2' > E1'$ in Figure 3. Therefore, in both profit maximizing and social welfare maximizing cases, we can observe $K > I$.

Q.E.D

Proposition 1 suggests that a port prioritizes capacity investment over natural disaster prevention investment. This is intuitive, because due to the uncertainty of natural disaster, the expected monetary value of loss prevention is only a proportion of that of the gain from capacity expansion, especially when the capacity level is quite low (smaller than the lower bound of the uncertain demand). Therefore, it makes sense for ports to first ensure capacity expansion and only start to invest in disaster prevention when they have sufficient resources. However, this conclusion has important policy implication. It explains why Gulfport utilized the federal fund in capacity investment instead of natural disaster prevention, when it attempted to first recover from the devastating disaster. Whereas it may be argued that fund allocated for natural disaster prevention should be earmarked, the above results suggest that this may not always be optimal even from a social welfare maximization perspective, especially when port capacity investment/level is also considered.

Proposition 2: Social welfare maximizing ports invest more in both capacity and natural disaster prevention than profit maximizing ports.

Proof: From figure 1 to figure 3, it can easily be identified that $\mathrm{A1}' > A1, 0 = 0$ in figure1, $\mathrm{C2}' > C2, C1' > C1$ in figure2, and $\mathrm{E2}' > E2, E1' > E1$ in figure3. As a result, $K_{SW} > K_{\pi}, I_{SW} \geq I_{\pi}$.

Q.E.D

Proposition 2 is also easy to understand. Economic theories suggest that the equilibrium traffic in social welfare maximizing ports is larger than that in profit maximizing ports, which requires social welfare maximizing ports to invest more in capacity and natural disaster prevention than their profit maximizing counterparts.

Proposition 3: Social welfare maximizing ports require a larger budget to justify the investment for natural disaster prevention.

Proof: As is shown in figure 2, when $B > D1'$, social welfare maximizing ports start to invest in natural disaster prevention. When $B > D1$, profit maximizing ports start to invest in natural disaster prevention. Since $\mathrm{D}1' > D1$, we can observe that social welfare maximizing ports require a larger budget to justify the investment for natural disaster prevention.

Q.E.D

Proposition 3 is somehow counterintuitive and surprising, as it suggests that when the budget level is low, social welfare maximizing ports are less likely to invest in natural disaster prevention compared with profit maximizing ports. This is in fact due to the combined effect of Propositions 1 and 2. In particular, ports prioritize capacity expansion over natural disaster prevention irrespective of their objectives, while the optimal capacity investment level is higher for welfare maximizing ports than for profit maximizing ports. As a result, welfare maximizing ports require a larger budget before they get involved in natural disaster prevention compared with profit maximizing counterparts (recall disaster prevention is a "derived" need dependent on capacity. This result can be useful for regulators, as it suggests that a larger budget may need to be in place for public / government owned ports if proper disaster prevention measures are desired.

Proposition 4: Higher natural disaster intensity decreases a port's capacity investment and increases its disaster prevention investment. The magnitudes of such effects are larger for welfare maximizing ports than for profit maximizing ports.

Proof: Because figure 2 and figure 3 are similar to figure 1, we use figure 1 as an example. If $\overline{y}$ increases, we can observe that B1 ↘, A2↘ (i.e. K↘), and A1↗ (i.e. I↗) for profit as well as social welfare. However, if $\overline{y}$ increases by the same amount $\Delta\overline{y}$, the magnitudes of changes suggest that $\Delta K_{SW} > \Delta K_{\pi}$, $\Delta I_{SW} > \Delta I_{\pi}$.

Q.E.D

Proposition 4 also has important policy implications. As the impacts of climate change are more evident and significant over time, many natural disasters including those coastal disasters have become a lot more common. Under such a situation, it is reasonable to advise investors, either private or public (profit-maximizing or welfare-maximizing), to be more cautious about increasing port capacities. Meanwhile, the necessity for natural disaster prevention has also been enhanced. It may even be mandatory in the near future. Indeed, the Taskforce for Climate-related Financial Disclosures argued that major investments have to disclose if they are subject to significant impacts of climate change. Such requirements have been followed by hundreds of major investment projects world-wide.[2] Compared with their private counterparts (more similar to the benchmark case of profit-maximization), these changes should be more imperative for the public ports (closer to the benchmark case of welfare maximization), as they are responsible not only for the asset owners, but also the general public.

5. Further discussion and sensitivity analysis

In our model, the port is assumed to have monopoly power in the market. However, in recent years, port competition has become more and more severe. Meanwhile, port cooperation and integration are also gaining momentum. Qualitatively, a higher level of competition should lead to a lower output for any individual port, leading to less investments in both capacity expansion and disaster prevention, as well as a lower expected profit for the port. The only exception is that when the budget is intermediate, more competition may lead to higher disaster prevention investment. This is because more competition leads to lower port outputs and thereby lower the capacity expansion requirement. When the budget is intermediate, the two types of investment compete for the use of the budget. As the port capacity expansion requires a smaller budget, there may

[2] For more information, visit https://www.fsb-tcfd.org/.

be more resource left for the disaster prevention, which may promote the relevant investment. Port cooperation and integration, on the other hand, alleviate the port competition with each other. In other words, more port cooperation or a higher degree of port integration should have the opposite effects as those due to intensified port competition.

It is clear from our previous analysis that the intensity ($\overline{y}$) and the probability (θ) of the disaster are critical to the results. We implement the sensitivity analysis on these two parameters, and find that they have very similar impacts on the investment decisions of the port.[3] In summary, an increasing disaster intensity/probability always leads to lower capacity expansion and lower expected profit for the port. For the disaster prevention investment, when the budget is small, an increasing disaster intensity/probability either leads to lower disaster prevention investment, or has no impacts on the disaster prevention investment. When the budget is intermediate, an increasing disaster intensity/probability leads to higher disaster prevention investment. When the budget is large, the impacts of the disaster intensity/probability depend on its own magnitude, i.e., it leads to higher (or lower, respectively) disaster prevention investment if the intensity is small (or large, respectively). The reasons are as follows. An increasing disaster intensity/probability increases the cost of potential disasters, which in turns lower the port's incentive to invest in capacity expansion. When the budget is small, all the budget is still allocated port capacity expansion, with no resource left for disaster prevention. When the budget is intermediate, the port capacity expansion and the disaster prevention compete for the use of the budget. As the incentive for capacity expansion is decreased, there is more resource left for the disaster prevention investment. When the budget is large, the two types of investment can both be fully satisfied. If the disaster intensity/probability is relatively small (or large respectively), the marginal cost of the disaster prevention investment is lower (or higher, respectively) than its marginal contribution to the disaster remedy, the increasing intensity promotes (or prohibits, respectively) the disaster prevention investment.[4]

[3] The detailed analysis is illustrated in Appendix 2.

[4] Recall the objective function (2). The the marginal cost of the disaster prevention investment is r_2. Its marginal contribution to the disaster remedy can be either 0 (when the disaster is low, i.e., $y\theta K \leq \rho I$) or

6. Concluding remarks

In this paper, we build a stylized analytical model to study the decision-making process of a seaport in allocating budget between two types of investments: capacity expansion and natural disaster prevention. The effects of demand uncertainty and the uncertainty of natural disaster occurrence are explicitly modelled, as they both affect resource allocation of the port. Furthermore, we have also considered two benchmark cases, namely profit-maximizing ports and welfare-maximizing ports. We find that under both cases, port managers would always prioritize capacity investment over disaster prevention investment. Welfare-maximizing ports invest more in both capacity and natural disaster prevention than their profit-maximizing counterparts. However, compared with the latter, welfare-maximizing ports also require a larger budget to justify the investment in disaster prevention. Moreover, with increasing intensity of natural disaster, a port's capacity investment decreases and its disaster prevention investment increases, irrespective of port objectives. The magnitudes of both changes are larger for welfare-maximizing ports.

There are a few takeaways from this study. First, as we have observed in the cases of Gulfport and Seattle, it seems that some ports have the tendency to prioritize capacity investment over natural disaster prevention. Our analysis suggests that this could in fact reflect economic rationales considerations. In other words, regulators may not be overly concerned when ports first invest in capacity expansion instead of disaster prevention. Second, although welfare-maximizing ports should invest more in both capacity and disaster prevention than profit-maximizing ports, they also require a larger budget to justify the investment in disaster prevention. In other words, compared with profit-maximizing ports, welfare-maximizing ports may need to wait a longer time for the accumulation of capital in order to invest in disaster prevention. Third, with an increase in the intensity of natural disasters, we should expect to see that all ports, especially welfare-maximizing ports, would shift their focus from capacity investment to disaster prevention investment. These findings could be valuable in design practical government

positive (when the disaster is high, i.e., $y\theta K > \rho I$). Therefore, when y or θ is small (or large respectively), $y\theta K$ is less (or larger, respectively) than ρI.

policies. Because public ports are usually better characterised with the welfare-maximizing case whereas private ports are better characterized with the profit-maximizing case, practical policies may be adjusted/adapted based on ports' ownership forms. For example, *ceteris paribus*, our analytical results suggest that private ports invest more in disaster prevention than public ports when they are under similar budget constraints. Meanwhile, with increased risks of natural disasters, public ports will invest more on disaster prevention. Therefore, with increased risks of natural disasters caused by climate change, governments should set aside more resources for ports to boost natural disaster prevention, more so for the public ones.

There are certain policy implications from these results. First, policy makers need to understand that it is a sensible economic decision for the ports to prioritize capacity expansion over natural disaster prevention. With climate change becoming one of the most pressing challenges for the whole society, in order to ensure the ports to invest in disaster prevention, the government should either allocate a sufficient amount of funding for the ports or strictly earmark the funding for disaster prevention purposes. Second, currently most of the ports in the world are public ports, but privatization and public-private-partnership (PPP) are also starting to gain momentum. Policy makers should be made aware of the fact that public ports require larger budgets to justify for disaster prevention investment. Therefore, when public funding is less than sufficient, privatization or PPP might be realistic ways to encourage the ports to invest in natural disaster prevention. It gives the regulator another reason to consider these alternative forms of port ownership seriously.

Moreover, our model can potentially be empirically applied and tested with real-world cases, such as Australian ports (Nursey-Bray and Miller, 2012; Ng el al., 2013), the ports of Gulfport and Providence (Becker et al., 2015), and the port of Rotterdam (Vellinga and De Jong, 2012).

Whereas our analytical results are obtained under fairly generic assumptions, further studies are needed before the above-mentioned policy recommendations can be adopted in large scale. There are certain limitations with this study. First, we have only considered risk neutral port managers. Although this implicit assumption is commonly used in the literature, in reality, many ports especially the public ones are likely to be risk adverse. Second, we have only considered the investment decisions of one single port. In reality, ports rarely make decision independently. Competition and complementarity among ports are common in the maritime industry. Even within one port, multiple terminals and various stakeholders would all interact with and possibly change the decisions of each other. A game theoretical structure with multiple decision makers should better reflect industry reality. Third, our setting may have underscored the impacts of natural disasters. The model has only considered the costs of natural disasters on the port, but these costs may go well beyond (i.e. spill-over effects). This is expected to cause an underestimation of socially optimal disaster prevention investment. A more sophisticated modelling on the impacts of natural disaster has the potential to expand the discussion to involve more stakeholders and engage in a more holistic planning process, although it is beyond the scope of the current study.

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Table 1: Equilibrium K and I under profit maximization when $\frac{r_1+\bar{y}\theta(1-m)}{1-n} \leq \bar{x} \leq \frac{r_1+\bar{y}\theta(1-m)}{1-n} + \underline{x}$

B	K	I
$0 < B \leq \frac{r_1}{2b}\left(\bar{x}(1-n) + \underline{x}n - r_1 - \bar{y}\theta(1-m)\right)$	$\frac{B}{r_1}$	0
$B > \frac{r_1}{2b}\left(\bar{x}(1-n) + \underline{x}n - r_1 - \bar{y}\theta(1-m)\right)$	$\frac{1}{2b}\left(\bar{x}(1-n) + \underline{x}n - r_1 - \bar{y}\theta(1-m)\right)$	0

Table 2: Equilibrium K and I under profit maximization when $\frac{r_1+\overline{y}\theta(1-m)}{1-n}+\underline{x}<\overline{x}\leq\frac{(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{1-n}+\underline{x}$

B	K	I
$0<B\leq\frac{r_1}{2b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$	$\frac{B}{r_1}$	0
$\frac{r_1}{2b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)<B\leq(r_1+\frac{r_2\overline{y}\theta}{\rho})\frac{1}{2b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$	$\frac{\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)}{2b}$	$\frac{B-r_1K}{r_2}$
$B>(r_1+\frac{r_2\overline{y}\theta}{\rho})\frac{\overline{x}(1-n)+\underline{x}n-(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{2b}$	$\frac{\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)}{2b}$	$\frac{\overline{y}\theta}{\rho}K$

Table 3: Equilibrium *K* and *I* under profit maximization when $\overline{x} > \frac{(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{1-n} + \underline{x}$

B	K	I
$0 < B < r_1 \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{2b(1-n)}$	$\frac{B}{r_1}$	0
$r_1 \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{2b(1-n)} < B \leq (r_1 + \frac{r_2\overline{y}\theta}{\rho}) \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{2b(1-n)}$	$\frac{\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)}{2b(1-n)}$	$\frac{B - r_1 K}{r_2}$
$B > (r_1 + \frac{r_2\overline{y}\theta}{\rho}) \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{2b(1-n)}$	$\frac{\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)}{2b(1-n)}$	$\frac{\overline{y}\theta}{\rho}K$

Table 4: Equilibrium *K* and *I* under social welfare maximization when $\frac{r_1+\overline{y}\theta(1-m)}{1-n} \leq \overline{x} \leq \frac{r_1+\overline{y}\theta(1-m)}{1-n} + \underline{x}$

B	K	I
$0 < B \leq \frac{r_1}{b}\left(\overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m)\right)$	$\frac{B}{r_1}$	0
$B > \frac{r_1}{b}\left(\overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m)\right)$	$\frac{1}{b}\left(\overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m)\right)$	0

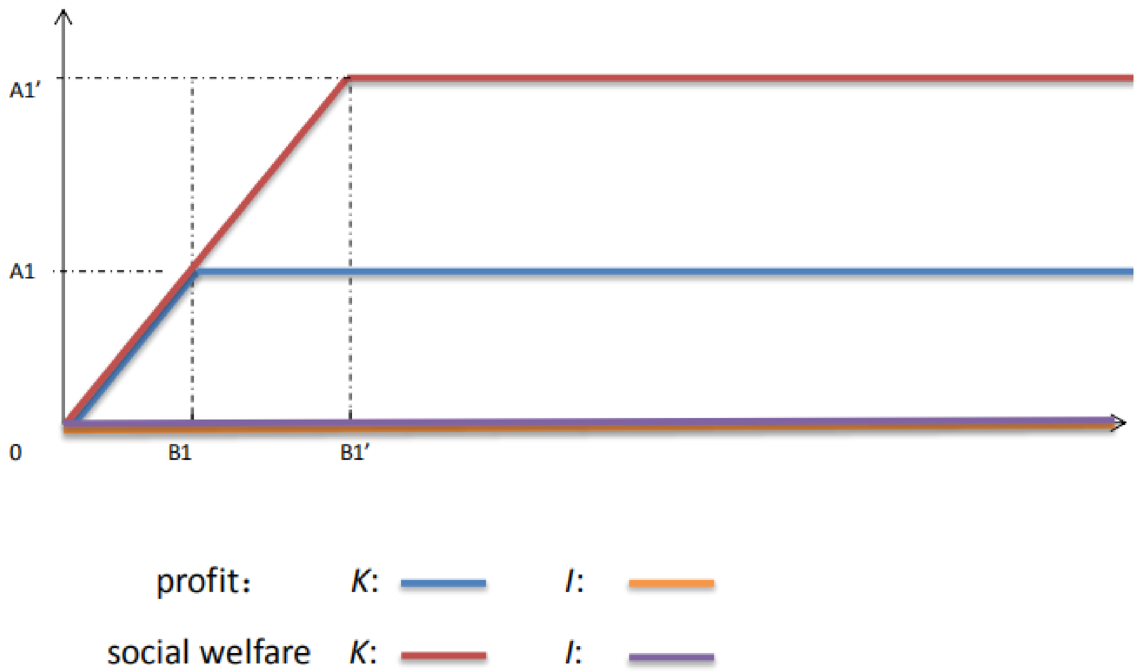
Table 5: Equilibrium *K* and *I* under social welfare maximization when $\frac{r_1+\overline{y}\theta(1-m)}{1-n}+\underline{x}<\overline{x}\leq\frac{(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{1-n}+\underline{x}$

B	K	I
$0<B\leq\frac{r_1}{b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$	$\frac{B}{r_1}$	0
$\frac{r_1}{2b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)<B\leq(r_1+\frac{r_2\overline{y}\theta}{\rho})\frac{1}{2b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$	$\frac{\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)}{b}$	$\frac{B-r_1K}{r_2}$
$B>(r_1+\frac{r_2\overline{y}\theta}{\rho})\frac{\overline{x}(1-n)+\underline{x}n-(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{b}$	$\frac{\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)}{b}$	$\frac{\overline{y}\theta}{\rho}K$

Table 6: Equilibrium K and I under social welfare maximization when $\overline{x} > \frac{(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{1-n} + \underline{x}$

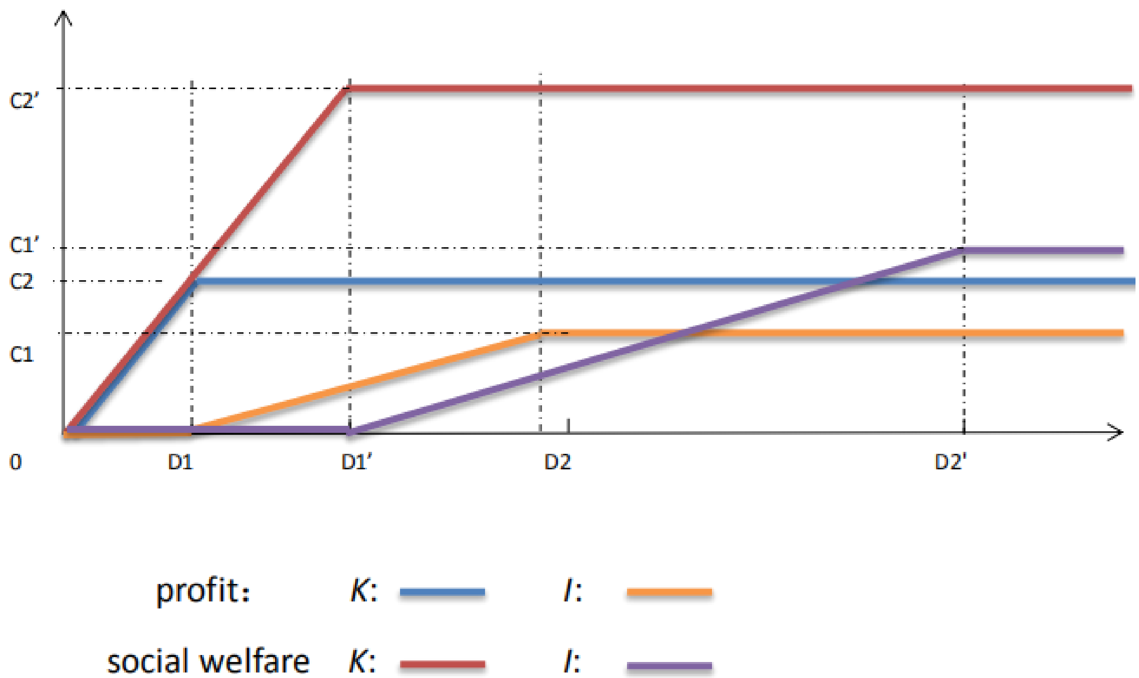
B	K	I
$0 < B < r_1 \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{b(1-n)}$	$\frac{B}{r_1}$	0
$r_1 \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{b(1-n)} < B \le (r_1 + \frac{r_2\overline{y}\theta}{\rho}) \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{b(1-n)}$	$\frac{\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)}{b(1-n)}$	$\frac{B - r_1 K}{r_2}$
$B > (r_1 + \frac{r_2\overline{y}\theta}{\rho}) \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2} + \overline{y}\theta)(1-m)}{b(1-n)}$	$\frac{\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)}{b(1-n)}$	$\frac{\overline{y}\theta}{\rho}K$

Figure 1: The equilibrium K and I when $\frac{r_1+\bar{y}\theta(1-m)}{1-n} \leq \bar{x} \leq \frac{r_1+\bar{y}\theta(1-m)}{1-n} + \underline{x}$



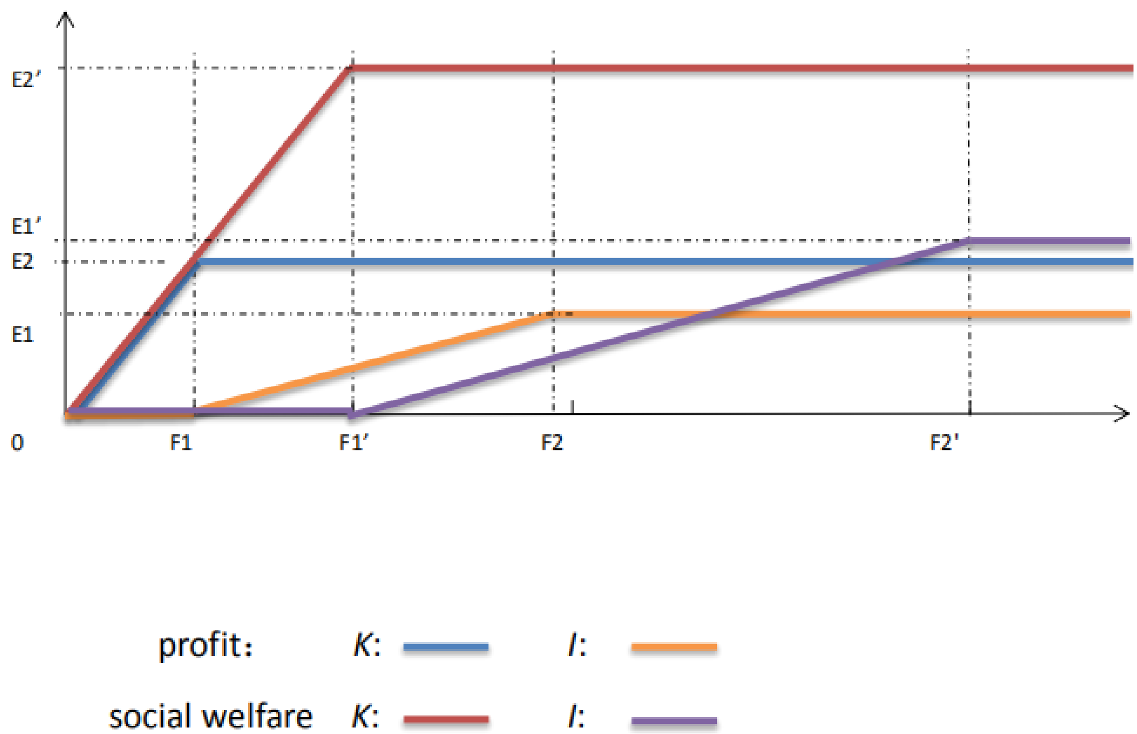
$$B1 = \frac{r_1}{2b}\left(\bar{x}(1-n) + \underline{x}n - r_1 - \bar{y}\theta(1-m)\right)$$
$$A1 = \frac{1}{2b}\left(\bar{x}(1-n) + \underline{x}n - r_1 - \bar{y}\theta(1-m)\right)$$
$$B1' = \frac{r_1}{b}\left(\bar{x}(1-n) + \underline{x}n - r_1 - \bar{y}\theta(1-m)\right)$$
$$A1' = \frac{1}{b}\left(\bar{x}(1-n) + \underline{x}n - r_1 - \bar{y}\theta(1-m)\right)$$

Figure 2: The equilibrium K and I when $\frac{r_1+\overline{y}\theta(1-m)}{1-n}+\underline{x}<\overline{x}\leq\frac{(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{1-n}+\underline{x}$



$$D1=\frac{r_1}{2b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$
$$D2=\frac{1}{2b}(r_1+\frac{r_2\overline{y}\theta}{\rho})\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$
$$C1=\frac{\overline{y}\theta}{2b\rho}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$
$$C2=\frac{1}{2b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$
$$D1'=\frac{r_1}{b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$
$$D2'=\frac{1}{b}(r_1+\frac{r_2\overline{y}\theta}{\rho})\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$
$$C1'=\frac{\overline{y}\theta}{b\rho}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$
$$C2'=\frac{1}{b}\left(\overline{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$

Figure 3: The equilibrium K and I when $\overline{x} > \frac{(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{1-n} + \underline{x}$



$$F1 = r_1 \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{2b(1-n)}$$

$$F2 = (r_1 + \frac{r_2\overline{y}\theta}{\rho}) \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{2b(1-n)}$$

$$E1 = \frac{\overline{y}\theta}{2b\rho(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$

$$E2 = \frac{1}{2b(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$

$$F1' = r_1 \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{b(1-n)}$$

$$F2' = (r_1 + \frac{r_2\overline{y}\theta}{\rho}) \frac{\overline{x}(1-n) - (\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{b(1-n)}$$

$$E1' = \frac{\overline{y}\theta}{b\rho(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$

$$E2' = \frac{1}{b(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)\right)$$

Appendix 1: Detailed derivation process

We construct the corresponding Lagrangian function for equation (1):

$$L(q,\mu,\omega) = (x - bq)q - \mu(q - K) + \omega q$$

where μ, ω are Lagrangian multipliers. Then we differentiate it with respect to q, and the first-order condition (FOC) requires $\frac{\partial L}{\partial q} = x - 2bq - \mu + \omega = 0$. Let $g_1(q) = q - K$, $g_2(q) = q$, we need to discuss the following four cases:

Case 1: If $g_1(q)$ and $g_2(q)$ are not binding, then we know $q < K$, $q > 0$ and $\mu = \omega = 0$. Solving the FOC of the Lagrangian function, we can imply $q = \frac{x}{2b} < K$.

Case 2: If $g_1(q)$ is binding and $g_2(q)$ is not binding, it shows that $q = K, q > 0$ and $\mu \geq 0, \omega = 0$. Solving the FOC of the Lagrangian function yields $q = K \leq \frac{x}{2b}$.

Case 3: If $g_1(q)$ is not binding and $g_2(q)$ is binding, it means $q < K, q = 0$ and $\mu = 0, \omega \geq 0$. Solving the corresponding FOC, it can be shown that $\mathrm{q} = 0, \mathrm{x} \leq 0$.

Case 4: If $g_1(q)$ and $g_2(q)$ are binding, it is obviously that $q = K = 0$.

In summary, when the market demand is sufficiently large, i.e., $x \geq 2bK$, then the optimal operating volume is $q = K$ and maximal profit is $\pi_{K,I} = -bK^2 + xK - r_1K - r_2I - max\{0, y\theta K - \rho I\}$. Otherwise if the market demand is not very large, i.e., $0 < x < 2bK$, the optimal operating volume is $q = \frac{x}{2b}$ and maximal profit is $\pi_{K,I} = \frac{x^2}{4b} - r_1K - r_2I - max\{0, y\theta K - \rho I\}$. Clearly, when $x = 0$, we have $q = 0$.

Since the uncertainty of *x* and *y*, we determine $E(\pi_{K,I})$ by discussing the value of $\frac{\rho I}{\theta K}$ and *K*. It can be specified for the following nine cases.

Case I. If $\frac{\rho I}{\theta K} \leq \underline{y} = 0$ and $K \leq \frac{\underline{x}}{2b}$, the expected profit is $E(\pi) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1K - \overline{y}\theta K(1-m)$. The condition $\frac{\rho I}{\theta K} \leq \underline{y} = 0$ can imply $I = 0$. Stage1 can be specified as

$$E(\pi) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1K - \overline{y}\theta K(1-m)$$

$$s.t.\ r_1K \leq B$$

$$K \geq 0$$

Let $g_3 = r_1K - B$ and $g_4 = K$. We construct the corresponding Lagrangian function

$$L(K,\mu,\omega) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1K - \overline{y}\theta K(1-m) - \mu(r_1K - B) + \omega K$$

where μ, ω are Lagrangian multipliers. We differentiate it with respect to *K,* then the first-order condition (FOC) requires $\frac{\partial L}{\partial K} = -2bK + \overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m) - \mu r_1 + \omega = 0$. Similar to above, if g_3 and g_4 are not binding, solving the corresponding FOC implies $K = \frac{1}{2b}\left(\overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m)\right)$, $B > \frac{r_1}{2b}\left(\overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m)\right)$. According to the condition $K \le \frac{\underline{x}}{2b}$, we obtain $\overline{x} - \underline{x} \le \frac{r_1 + \overline{y}\theta(1-m)}{1-n}$. If g_3 is not binding and g_4 is binding, solving the corresponding FOC implies $K = 0$, $B > 0$ and $\overline{x}(1-n) + \underline{x}n < r_1 + \overline{y}\theta(1-m)$. If g_3 is binding and g_4 is not binding, solving the corresponding FOC implies $K = \frac{B}{r_1}$. When $\overline{x} - \underline{x} \le \frac{r_1 + \overline{y}\theta(1-m)}{1-n}$, we observe $B \le \frac{r_1}{2b}\left(\overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m)\right)$. Otherwise, when $\overline{x} - \underline{x} > \frac{r_1 + \overline{y}\theta(1-m)}{1-n}$, we observe $B \le \frac{r_1}{2b}\underline{x}$.

Case II. If $\frac{\rho I}{\theta K} \le \underline{y} = 0$ and $\frac{\underline{x}}{2b} < K \le \frac{\overline{x}}{2b}$, the expected profit is $E(\pi) = (\overline{x} - bK)K(1-n) + \frac{\underline{x}^2}{4b}n - r_1K - \overline{y}\theta K(1-m)$. Stage1 can be specified as

$$E(\pi) = (\overline{x} - bK)K(1-n) + \frac{\underline{x}^2}{4b}n - r_1K - \overline{y}\theta K(1-m)$$

$$s.t.\ r_1K \le B$$

$$K \ge 0$$

We construct the corresponding Lagrangian function:

$$L(K,\mu,\omega) = (\overline{x} - bK)K(1-n) + \frac{\underline{x}^2}{4b}n - r_1K - \overline{y}\theta K(1-m) - \mu(r_1K - B) + \omega K$$

where μ, ω are Lagrangian multipliers. We differentiate it with respect to *K,* then the first-order condition (FOC) requires $\frac{\partial L}{\partial K} = (\overline{x} - 2bK)(1-n) - r_1 - \overline{y}\theta(1-m) - \mu r_1 + \omega = 0$. If g_3 and g_4 are not binding, solving the corresponding FOC implies $\mathrm{K} = \frac{1}{2b}\left(\overline{x} - \frac{r_1 + \theta(1-m)}{1-n}\right)$, $\mathrm{B} > \frac{r_1}{2b}\left(\overline{x} - \frac{r_1 + \overline{y}\theta(1-m)}{1-n}\right)$. According to the condition $\frac{\underline{x}}{2b} < K \le \frac{\overline{x}}{2b}$, we observe $\overline{x} - \underline{x} > \frac{r_1 + \overline{y}\theta(1-m)}{1-n}$. If g_3 is not binding and g_4 is binding, solving the corresponding FOC implies $K = 0$, $B > 0$ and $\overline{x} \le \frac{r_1 + \overline{y}\theta(1-m)}{1-n}$. If g_3 is binding and g_4 is not binding, solving the corresponding FOC implies $K = \frac{B}{r_1}$, $\frac{r_1}{2b}\underline{x} < B \leq \frac{r_1}{2b}\left(\overline{x} - \frac{r_1+\overline{y}\theta(1-m)}{1-n}\right)$ and $\overline{x} - \underline{x} > \frac{r_1+\overline{y}\theta(1-m)}{1-n}$.

Case III. If $\frac{\rho I}{\theta K} \leq \underline{y} = 0$ and $K > \frac{\overline{x}}{2b}$, the expected profit is $E(\pi) = \frac{1}{4b}\left(\overline{x}^2(1-n) + \underline{x}^2 n\right) - r_1 K - \overline{y}\theta K(1-m)$. The condition $K > \frac{\overline{x}}{2b}$ can imply $K = 0$. Stage1 can be specified as

$$E(\pi) = \frac{1}{4b}\left(\overline{x}^2(1-n) + \underline{x}^2 n\right) - r_1 K - \overline{y}\theta K(1-m)$$

$$s.t.\ r_1 K \leq B$$

We construct the corresponding Lagrangian function

$$L(K,\mu,\omega) = \frac{1}{4b}\left(\overline{x}^2(1-n) + \underline{x}^2 n\right) - r_1 K - \overline{y}\theta K(1-m) - \mu(r_1 K - B)$$

As $\frac{\partial L}{\partial K} = -r_1 - \overline{y}\theta(1-m) - \mu r_1 < 0$, there is no optimal solution.

Case IV. If $0 < \frac{\rho I}{\theta K} \leq \overline{y}$ and $K \leq \frac{\underline{x}}{2b}$, the expected profit is $E(\pi) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1 K - r_2 I - (\overline{y}\theta K - \rho I)(1-m)$. The condition $0 < \frac{\rho I}{\theta K} \leq \overline{y}$ can imply $K > 0$ and $I > 0$. Stage1 can be specified as

$$E(\pi) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1 K - r_2 I - (\overline{y}\theta K - \rho I)(1-m)$$

$$s.t.\ r_1 K + r_2 I \leq B$$

Let $g_5 = r_1 K + r_2 I - B$. We construct the corresponding Lagrangian function

$$L(K,I,\mu,\omega) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1 K - r_2 I - (\overline{y}\theta K - \rho I)(1-m) - \mu(r_1 K + r_2 I - B)$$

where μ is Lagrangian multipliers. We differentiate it with respect to *K* and *I,* then the first-order condition(FOC) requires $\frac{\partial L}{\partial K} = -2bK + \overline{x}(1-n) + \underline{x}n - r_1 - \overline{y}\theta(1-m) - \mu r_1 = 0$ and $\frac{\partial L}{\partial I} = -r_2 + \rho(1-m) - \mu r_2 = 0$. Similar to above, if g_5 is not binding, solving the corresponding FOC implies $K = \frac{1}{2b}\Bigg(\overline{x}(1-n) + \underline{x}n - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-$

$m)\Big)$, $I = \frac{\bar{y}\theta}{\rho}K$, $1 - \mathrm{m} = \frac{r_2}{\rho}$. According to the conditions $0 < \frac{\rho I}{\theta K} \le \bar{y}$ and $K \le \frac{\underline{x}}{2b}$, we observe $\bar{x} - \underline{x} \le \frac{(\rho\frac{r_1}{r_2}+\bar{y}\theta)(1-m)}{1-n}$ and $B > (r_1 + \frac{r_2\bar{y}\theta}{\rho})\frac{\bar{x}(1-n)+\underline{x}n-(\rho\frac{r_1}{r_2}+\bar{y}\theta)(1-m)}{2b}$. If g_5 is binding, solving the corresponding FOC implies $K = \frac{1}{2b}\Big(\bar{x}(1-n) + \underline{x}n - \left(\rho\frac{r_1}{r_2} + \bar{y}\theta\right)(1-m)\Big)$, $\mathrm{I} = \frac{B-r_1K}{r_2}$, $1 - \mathrm{m} \ge \frac{r_2}{\rho}$. According to the conditions $0 < \frac{\rho I}{\theta K} \le \bar{y}$ and $K \le \frac{\underline{x}}{2b}$, we observe $\bar{x} - \underline{x} \le \frac{(\rho\frac{r_1}{r_2}+\bar{y}\theta)(1-m)}{1-n}$ and $r_1\frac{\bar{x}(1-n)+\underline{x}n-(\rho\frac{r_1}{r_2}+\bar{y}\theta)(1-m)}{2b} < B \le (r_1 + \frac{r_2\bar{y}\theta}{\rho})\frac{\bar{x}(1-n)+\underline{x}n-(\rho\frac{r_1}{r_2}+\bar{y}\theta)(1-m)}{2b}$.

Case V. If $0 < \frac{\rho I}{\theta K} \le \bar{y}$ and $\frac{\underline{x}}{2b} < K \le \frac{\bar{x}}{2b}$, the expected profit is $E(\pi) = (\bar{x} - bK)K(1 - n) + \frac{\underline{x}^2}{4b}n - r_1K - r_2I - (\bar{y}\theta K - \rho I)(1 - m)$. Stage1 can be specified as

$$E(\pi) = (\bar{x} - bK)K(1 - n) + \frac{\underline{x}^2}{4b}n - r_1K - r_2I - (\bar{y}\theta K - \rho I)(1 - m)$$

$$s.t.\ r_1K + r_2I \le B$$

We construct the corresponding Lagrangian function

$$L(K, I, \mu, \omega) = (\bar{x} - bK)K(1 - n) + \frac{\underline{x}^2}{4b}n - r_1K - r_2I - (\bar{y}\theta K - \rho I)(1 - m) - \mu(r_1K + r_2I - B)$$

where μ is Lagrangian multipliers. We differentiate it with respect to *K* and *I,* then the first-order condition(FOC) requires $\frac{\partial L}{\partial K} = (\bar{x} - 2bK)(1 - n) - r_1 - \bar{y}\theta(1 - m) - \mu r_1 = 0$ and $\frac{\partial L}{\partial I} = -r_2 + \rho(1 - m) - \mu r_2 = 0$. If g_5 is not binding, solving the corresponding FOC implies $K = \frac{\bar{x}(1-n)-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)}{2b(1-n)}$, $I = \frac{\bar{y}\theta}{\rho}K$, $1 - \mathrm{m} = \frac{r_2}{\rho}$. According to the conditions $0 < \frac{\rho I}{\theta K} \le \bar{y}$ and $\frac{\underline{x}}{2b} < K \le \frac{\bar{x}}{2b}$, we observe $\bar{x} - \underline{x} > \frac{(\rho\frac{r_1}{r_2}+\bar{y}\theta)(1-m)}{1-n}$ and $B > (r_1 + \frac{r_2\bar{y}\theta}{\rho})\frac{\bar{x}(1-n)-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)}{2b(1-n)}$If g_5 is binding, solving the corresponding FOC implies

$K = \frac{\overline{x}(1-n)-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)}{2b(1-n)}$, $\mathrm{I} = \frac{B-r_1K}{r_2}$, $1-\mathrm{m} \geq \frac{r_2}{\rho}$. According to the conditions $0 < \frac{\rho I}{\theta K} \leq \overline{y}$ and $\frac{\underline{x}}{2b} < K \leq \frac{\overline{x}}{2b}$, we observe $\overline{x} - \underline{x} > \frac{(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{1-n}$ and $r_1\frac{\overline{x}(1-n)-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)}{2b(1-n)} < B \leq (r_1 + \frac{r_2\overline{y}\theta}{\rho})\frac{\overline{x}(1-n)-\left(\rho\frac{r_1}{r_2}+\overline{y}\theta\right)(1-m)}{2b(1-n)}$.

Case VI. If $0 < \frac{\rho I}{\theta K} \leq \overline{y}$ and $K > \frac{\overline{x}}{2b}$, the expected profit is $E(\pi) = \frac{1}{4b}\left(\overline{x}^2(1-n) + \underline{x}^2 n\right) - r_1K - r_2I - (\overline{y}\theta K - \rho I)(1-m)$. The conditions $0 < \frac{\rho I}{\theta K} \leq \overline{y}$ and $K > \frac{\overline{x}}{2b}$ can imply $K > 0$ and $I > 0$. Stage1 can be specified as

$$E(\pi) = \frac{1}{4b}\left(\overline{x}^2(1-n) + \underline{x}^2 n\right) - r_1K - r_2I - (\overline{y}\theta K - \rho I)(1-m)$$

$$s.t.\ r_1K + r_2I \leq B$$

We construct the corresponding Lagrangian function

$$L(K,I,\mu,\omega) = \frac{1}{4b}\left(\overline{x}^2(1-n) + \underline{x}^2 n\right) - r_1K - r_2I - (\overline{y}\theta K - \rho I)(1-m) - \mu(r_1K + r_2I - B)$$

As $\frac{\partial L}{\partial K} = -r_1 - \overline{y}\theta(1-m) - \mu r_1 < 0$, there is no optimal solution.

Case VII. If $\frac{\rho I}{\theta K} > \overline{y}$ and $K \leq \frac{\underline{x}}{2b}$, the expected profit is $E(\pi) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1K - r_2I$, The conditions $\frac{\rho I}{\theta K} > \overline{y}$ can imply $I > 0$. Stage1 can be specified as

$$E(\pi) = \frac{1}{4b}\left(\overline{x}^2(1-n) + \underline{x}^2 n\right) - r_1K - r_2I - (\overline{y}\theta K - \rho I)(1-m)$$

$$s.t.\ r_1K + r_2I \leq B$$

$$K \geq 0$$

We construct the corresponding Lagrangian function

$$L(K,I,\mu,\omega) = \left(\overline{x}(1-n) + \underline{x}n\right)K - bK^2 - r_1K - r_2I - \mu(r_1K + r_2I - B) + \omega K$$

As $\frac{\partial L}{\partial I} = -r_1 - \mu r_1 < 0$, there is no optimal solution.

Case VIII. If $\frac{\rho I}{\theta K} > \overline{y}$ and $\frac{\underline{x}}{2b} < K \leq \frac{\overline{x}}{2b}$, the expected profit is $E(\pi) = (\overline{x} - bK)K(1 - n) + \frac{\underline{x}^2}{4b}n - r_1K - r_2I$. The conditions $\frac{\rho I}{\theta K} > \overline{y}$ and $\frac{\underline{x}}{2b} < K \leq \frac{\overline{x}}{2b}$ can imply $K > 0$ and $I > 0$. Stage1 can be specified as

$$E(\pi) = (\overline{x} - bK)K(1 - n) + \frac{\underline{x}^2}{4b}n - r_1K - r_2I$$

$$s.t.\ r_1K + r_2I \leq B$$

We construct the corresponding Lagrangian function

$$L(K, I, \mu, \omega) = (\overline{x} - bK)K(1 - n) + \frac{\underline{x}^2}{4b}n - r_1K - r_2I - \mu(r_1K + r_2I - B)$$

As $\frac{\partial L}{\partial I} = -r_1 - \mu r_1 < 0$, there is no optimal solution

Case IX. If $\frac{\rho I}{\theta K} > \overline{y}$ and $K > \frac{\overline{x}}{2b}$, the expected profit is $E(\pi) = \frac{1}{4b}\left(\overline{x}^2(1 - n) + \underline{x}^2 n\right) - r_1K - r_2I$. The conditions $\frac{\rho I}{\theta K} > \overline{y}$ and $\frac{\underline{x}}{2b} < K \leq \frac{\overline{x}}{2b}$ can imply $K > 0$ and $I > 0$. Stage1 can be specified as

$$E(\pi) = \frac{1}{4b}\left(\overline{x}^2(1 - n) + \underline{x}^2 n\right) - r_1K - r_2I$$

$$s.t.\ r_1K + r_2I \leq B$$

We construct the corresponding Lagrangian function

$$L(K, I, \mu, \omega) = \frac{1}{4b}\left(\overline{x}^2(1 - n) + \underline{x}^2 n\right) - r_1K - r_2I - \mu(r_1K + r_2I - B)$$

As $\frac{\partial L}{\partial I} = -r_1 - \mu r_1 < 0$, there is no optimal solution

Appendix 2: Detailed sensitivity analysis

As the sensitivity analysis is similar for the cases of profit maximizing and social welfare maximizing ports, here we just present the sensitivity analysis for the parameters θ and $\overline{y}$ under profit maximization by taking partial derivatives for the parameters.

Case 1: Under $\boldsymbol{\frac{r_1 + \overline{y}\theta(1-m)}{1-n} \leq \overline{x} \leq \frac{r_1 + \overline{y}\theta(1-m)}{1-n} + \underline{x}}$

I. When $0 < B \leq \frac{r_1}{2b}\left(\overline{x}(1 - n) + \underline{x}n - r_1 - \overline{y}\theta(1 - m)\right)$

By taking partial derivatives, we have $\frac{\partial K}{\partial \theta}=\frac{\partial K}{\partial \bar{y}}=0$, $\frac{\partial I}{\partial \theta}=\frac{\partial I}{\partial \bar{y}}=0$ and $\frac{\partial E(\pi)}{\partial \theta}=-\frac{\bar{y}(1-m)}{r_1}B<0$, $\frac{\partial E(\pi)}{\partial \bar{y}}=-\frac{\theta(1-m)}{r_1}B<0$. Obviously, *K* and *I* will not be affected no matter how the parameters change, however, $E(\pi)$ will decrease with the increasing of the parameters.

II. When $B>\frac{r_1}{2b}\left(\bar{x}(1-n)+\underline{x}n-r_1-\bar{y}\theta(1-m)\right)$

By taking partial derivatives, we imply that $\frac{\partial I}{\partial \theta}=\frac{\partial I}{\partial \bar{y}}=0$, $\frac{\partial K}{\partial \theta}=-\frac{1}{2b}\bar{y}(1-m)<0$, $\frac{\partial K}{\partial \bar{y}}=-\frac{1}{2b}\theta(1-m)<0$ and $\frac{\partial E(\pi)}{\partial \theta}=-\frac{1}{2b}\bar{y}(1-m)\left(\bar{x}(1-n)+\underline{x}n-r_1-\bar{y}\theta(1-m)\right)<0$, $\frac{\partial E(\pi)}{\partial \bar{y}}=-\frac{1}{2b}\theta(1-m)\left(\bar{x}(1-n)+\underline{x}n-r_1-\bar{y}\theta(1-m)\right)<0$. Clearly, *I* will not be affected, but *K* and $E(\pi)$will decrease with the increasing of the parameters.

Case 2: Under $\frac{r_1+\bar{y}\theta(1-m)}{1-n}+\underline{x}<\bar{x}\le\frac{\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)}{1-n}+\underline{x}$

I. When $0<B\le\frac{r_1}{2b}\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)$

The analysis is the same as **Case 1,** part *I*.

II. When $\frac{r_1}{2b}\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)<B\le(r_1+\frac{r_2\bar{y}\theta}{\rho})\frac{1}{2b}\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)$

By taking partial derivatives, we have $\frac{\partial K}{\partial \theta}=-\frac{1}{2b}\bar{y}(1-m)<0$, $\frac{\partial K}{\partial \bar{y}}=-\frac{1}{2b}\theta(1-m)<0$, $\frac{\partial I}{\partial \theta}=\frac{\bar{y}r_1}{2br_2}(1-m)>0$, $\frac{\partial I}{\partial \bar{y}}=\frac{\theta r_1}{2br_2}(1-m)>0$, $\frac{\partial E(\pi)}{\partial \theta}=-\frac{1}{2b}\bar{y}(1-m)\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)<0$, $\frac{\partial E(\pi)}{\partial \bar{y}}=-\frac{1}{2b}\theta(1-m)\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)<0$. As a consequence, when the parameters increase, *K* and $E(\pi)$ will decrease but *I* will increase.

III. When $B>(r_1+\frac{r_2\bar{y}\theta}{\rho})\frac{1}{2b}\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)$

By taking partial derivatives, we have $\frac{\partial K}{\partial \theta}=-\frac{1}{2b}\bar{y}(1-m)<0$, $\frac{\partial K}{\partial \bar{y}}=-\frac{1}{2b}\theta(1-m)<0$, $\frac{\partial E(\pi)}{\partial \theta}=-\frac{1}{2b}\bar{y}(1-m)\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)<0$, $\frac{\partial E(\pi)}{\partial \bar{y}}=-\frac{1}{2b}\theta(1-m)\left(\bar{x}(1-n)+\underline{x}n-\left(\rho\frac{r_1}{r_2}+\bar{y}\theta\right)(1-m)\right)<0$ which imply that *K* and

$E(\pi)$ will decrease with the increase of the two parameters. Meanwhile, if $\theta < \frac{\overline{x}(1-n)+\underline{x}n-\rho\frac{r_1}{r_2}(1-m)}{2\overline{y}(1-m)}$ then $\frac{\partial I}{\partial\theta} > 0$. If $\theta > \frac{\overline{x}(1-n)+\underline{x}n-\rho\frac{r_1}{r_2}(1-m)}{2\overline{y}(1-m)}$ then $\frac{\partial I}{\partial\theta} < 0$. Similarly, if $\overline{y} < \frac{\overline{x}(1-n)+\underline{x}n-\rho\frac{r_1}{r_2}(1-m)}{2\theta(1-m)}$ then $\frac{\partial I}{\partial\overline{y}} > 0$. If $\overline{y} > \frac{\overline{x}(1-n)+\underline{x}n-\rho\frac{r_1}{r_2}(1-m)}{2\theta(1-m)}$ then $\frac{\partial I}{\partial\overline{y}} < 0$.

Case 3: Under $\overline{\boldsymbol{x}} > \frac{\left(\boldsymbol{\rho}\frac{r_1}{r_2}+\overline{y}\boldsymbol{\theta}\right)(1-\boldsymbol{m})}{1-\boldsymbol{n}} + \underline{\boldsymbol{x}}$

I. When $0 < B < r_1\frac{\overline{x}(1-n)-(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{2b(1-n)}$

The analysis is the same as **Case 1,** part *I*.

II. When $r_1\frac{\overline{x}(1-n)-(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{2b(1-n)} < B \leq (r_1 + \frac{r_2\overline{y}\theta}{\rho})\frac{\overline{x}(1-n)-(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{2b(1-n)}$

By taking partial derivatives, we have $\frac{\partial K}{\partial\theta} = -\frac{\overline{y}(1-m)}{2b(1-n)} < 0$, $\frac{\partial K}{\partial\overline{y}} = -\frac{\theta(1-m)}{2b(1-n)} < 0$, $\frac{\partial I}{\partial\theta} = \frac{\overline{y}r_1(1-m)}{2br_2(1-n)} > 0$, $\frac{\partial I}{\partial\overline{y}} = \frac{\theta r_1(1-m)}{2br_2(1-n)} > 0$, $\frac{\partial E(\pi)}{\partial\theta} = -\frac{\overline{y}(1-m)}{2b(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)\right) < 0$, $\frac{\partial E(\pi)}{\partial\overline{y}} = -\frac{\theta(1-m)}{2b(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)\right) < 0$. The result is the same as **Case 2**, part *II*.

III. When $B > (r_1 + \frac{r_2\overline{y}\theta}{\rho})\frac{\overline{x}(1-n)-(\rho\frac{r_1}{r_2}+\overline{y}\theta)(1-m)}{2b(1-n)}$

By taking partial derivatives, we have $\frac{\partial K}{\partial\theta} = -\frac{\overline{y}(1-m)}{2b(1-n)} < 0$, $\frac{\partial K}{\partial\overline{y}} = -\frac{\theta(1-m)}{2b(1-n)} < 0$, $\frac{\partial E(\pi)}{\partial\theta} = -\frac{\overline{y}(1-m)}{2b(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)\right) < 0$, $\frac{\partial E(\pi)}{\partial\overline{y}} = -\frac{\theta(1-m)}{2b(1-n)}\left(\overline{x}(1-n) - \left(\rho\frac{r_1}{r_2} + \overline{y}\theta\right)(1-m)\right) < 0$, if $\theta < \frac{\overline{x}(1-n)-\rho\frac{r_1}{r_2}(1-m)}{2\overline{y}(1-m)}$ then $\frac{\partial I}{\partial\theta} > 0$, if $\theta > \frac{\overline{x}(1-n)-\rho\frac{r_1}{r_2}(1-m)}{2\overline{y}(1-m)}$ then $\frac{\partial I}{\partial\theta} < 0$, if $\overline{y} < \frac{\overline{x}(1-n)-\rho\frac{r_1}{r_2}(1-m)}{2\theta(1-m)}$ then $\frac{\partial I}{\partial\overline{y}} > 0$, if $\overline{y} > \frac{\overline{x}(1-n)-\rho\frac{r_1}{r_2}(1-m)}{2\theta(1-m)}$ then $\frac{\partial I}{\partial\overline{y}} < 0$. The result is the same as **Case 2**, part *II*.