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Combating Lead-Time Uncertainty in Shipment-Assignment: Is It Wise to be Risk-Averse?

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Abstract

In global supply chains, high uncertainty coming from liner shipping forces manufacturers who commit to on-time delivery to face great losses from tardiness or earliness. To find a reliable operational solution for shipment assignment, realizing the trade-off between the operations cost and reliability is highly essential. In this paper, a risk-hedging policy for shipment assignment is proposed. We incorporate risk aversion into the stochastic optimization model where the objective is to minimize the total deterministic operations cost and the weighted value-at-risk. The closed-form expressions of value-at-risk are derived and some structural properties of the optimization problem are revealed. Linearization techniques are utilized to make the proposed model tractable and solved by an exact algorithm. Our computational studies further demonstrate the cost efficiency on systems reliability improvement through the risk hedging approach with job combination. It is interesting to uncover that being moderately risk-averse is wise, but possessing a very high risk-averse attitude is doing more harm than good as the increase of deterministic operations cost is much more significant than the decrease of value-at-risk. Being risk neutral may also be unwise as the chance of achieving the optimal expected total cost may be very low. In the extended models, we relax the assumptions and further consider scenarios with correlated shipping lead-times and stochastic exchange rate.

Keywords:
Risk management; stochastic shipping lead-time; production and distribution scheduling.

1. Introduction

1.1. Background and Motivation

Consider a supply chain with a domestic manufacturer who produces and supplies to its overseas retailers who span across two or more countries/regions, product delivery is supported by maritime transportation. This forms a typical global supply chain as 90% of the international trades are realized through maritime transportation (Yang et al., 2019). A set of orders coming from different buyers are available at the beginning. Each order has its due date for delivery. Both early arrival and late delivery lead to penalties. The finished goods are shipped by vessels which have their specific schedules announced in advance. The

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critical feature of our model is that the shipping lead-times are random and in general deviated from the expected ones. The objective of this paper is to find reliable solutions for shipment assignment so as to make a trade-off between cost and reliability. The main focus of this study is to derive the optimal risk-hedging policy for shipment assignment, obtain reliable integrated solutions for the manufacturer who faces high liner shipping lead-time uncertainty, and highlight the importance of being risk-averse in decision making.

Our work is primarily motivated by the challenging situations faced by manufacturers in global supply chains involving liner shipping. On one hand, they face pressure coming from their buyers. With the trend of products diversification and customization, life cycles of products become shorter and shorter. The situation involves an increasing number of industries, including retail (Wal-mart, Target, Ikea), electronics (Apple, Samsung, HP), fashion (H&M, Nike, Adidas), etc. Fierce market competition reduces the profitability of retail operations. Hence, cutting costs down is one of the key aspects to stay competitive. Reported by Bloomberg, Wal-mart has implemented the "On-time, In-full (OTIF)" program to squeeze its vast network of suppliers since 2017. Late delivery without 100% in full will be fined 3% of their value. Early shipment gets dinged as well because they create overstocks. OTIF is also used to evaluate the performance of the suppliers. Wal-mart is not the first big retailer to implement policies of this kind. Obviously, this is a big trend. Fast-fashion retailers such as H&M and Zara, who hire lots of manufacturers in China, require their manufacturers to provide flexible and just-in-time production so as to reduce their inventory. The pressure is pushing back to the upstream. How to achieve a high service level while keeping the cost low is critical for the manufacturer to survive in the fast-tuning market, and logistics service providers such as liner shipping companies play a crucial role.

On the other side, unreliability is a big "headache" for the supply chain. When it comes to the global supply chain with liner shipping, the manufacturer faces high uncertainty for on-time deliveries due to the characteristics of maritime transportation. For example, if we ship from Hong Kong to the United States, it usually takes a 30-days trip along with multiple stops. In practical settings, container liner shipping lead-times always fluctuate due to external factors, e.g., port congestions, inefficient port operations, bad weather, etc., which are not controllable by the shipping company. At the same time, the container liner shipping's "schedule unreliability" is well-documented. As reported by Notteboom (2006), the actual vessel schedule reliability can be as low as 50% for many shipping routes compared with their expected ones. In a recent report by Drewry (2016), the universal average on-time performance of the shipping schedule reliability in February 2016 was 62.7%, and for Asia-Europe trade, it was less than 60%. The situation has not improved much even with current advances in technologies. As shown in Fig. 1a, by reviewing 20 ocean carriers and 13 trade lanes, it is reported that the carriers achieved 64.8% and 62% on-time delivery in 2017 and 2018, respectively. The variations of on-time delivery percentages by trade are as shown in Fig. 1b.

According to the Drewry and the European shippers' council (ESC) shipper satisfaction survey 2019, the top three least favorable aspects of the carriers are clarity of prices and surcharges, shipping lead-times, and reliability of booking/cargo shipped as booked. Obviously, it is a big challenge for the commit-to-delivery manufacturers to keep a high service level and a low cost at the same time so as to compete with other competitors in such uncertain situations.

In the field of operational risk analysis, it is common to consider risk-aversion for manufacturers in supply chain systems (Shen et al., 2011, 2013). Particularly, in a make-to-order supply chain, even without shipping uncertainty, the manufacturer is usually risk-averse due to the financial risk, which is incurred by the volatility of the orders from buyers (Xie et al., 2011). However, in international trade, high uncertainty from the long shipping lead-times creates risk for both the upstream and downstream members in the supply chain, and the manufacturer bears even higher risk as its buyers would push back the financial pressure (e.g., the Wal-Mart's OTIF policy). However, the impact of container liner shipping on supply chain performance remains under-studied compared to the impact of other aspects such as inventory management or road transportation. How to take the travel time variability into account and establish the reliability-cost trade-off in the analytical model is a challenging but crucial issue (Fransoo and Lee, 2013). As a result, in this highly stochastic and challenging environment, the traditional approach of being "risk neutral" with an objective of minimizing expected cost may be insufficient and unwise. In this paper, we try to reveal if being risk-averse is indeed wise if we consider the systems reliability which relates to the likelihood of achieving the respective low expected total cost.

a. Overall monthly schedule reliability on average.

b. Monthly schedule reliability by trade.

Source: https://www.cargosmart.ai/en/blog/2018-schedule-reliability-review/
Figure 1: Liner shipping schedule reliability (measured by the percentage of on-time delivery).

Motivated by the importance of the topic, we first propose a new risk-averse model for an integrated jobs allocation, production scheduling and shipment assignment problem in a decentralized production network, in which the stochastic lead-times of the container liner shipping are involved. The objective function is to minimize the total deterministic operations cost and the weighted value-at-risk so as to make trade-off between cost and reliability. The explicit expressions of the value-at-risk under different shipment assignment cases are formulated and their structures are discussed in order to develop the optimal risk-hedging shipment assignment policy. Corresponding to different shipment assignment policies, two risk-averse models by adopting linearization techniques are developed and solved by Cplex. The traditional risk-neutral model, i.e., expected cost minimization model, is used as a benchmark for studying the proposed models.

1.2. *Main Contributions and Results*

In this study, firstly we contribute to a deeper understanding of the impact of stochastic shipping lead-times on the shipment assignment for commit-to-delivery manufacturers with a goal to improve the scheduling reliability. Secondly, we explore the risk-hedging policy in terms of the shipment assignment for ocean container transportation. Thirdly, we prove the significance of job combination. To the best of our knowledge, this is the first study on the optimal risk-hedging shipment assignment policy for the commit-to-delivery manufacturers. We also highlight the importance of being risk-averse as it is critical to achieve a reasonably low cost with a sufficiently high systems reliability level.

Firstly, the proposed risk-hedging policy is consistent with the common risk management concept in financial industry: "do not put all eggs into the same basket". In other words, one of the functions of the risk-hedging policy is to diversify the shipment selection when enough reliable shipments are available. However, when shipping with low-reliability shipments is inevitable, diversifying the jobs being assigned to the same shipment can help mitigate risk. For instance, when facing the peak season of the shipping market, the port congestion is getting worse. Under such a case, the shipping uncertainty level will get increased, the jobs with distinct due dates and cost ratios should be shipped together for risk hedging, especially for the case when jobs with early due dates have a high unit storage cost, and jobs with late due dates have relatively high unit penalty costs[1]. However, for those jobs with close or same due dates, they should be shipped separately unless there is no other reliable shipment available.

Computational studies are carried out to demonstrate the performance of the proposed model based on the risk-hedging shipment assignment policy. Compared with the total cost (including the deterministic operations cost, and the value-at-risk from earliness and tardiness) needed by the risk-neutral model, only a small increase in the total cost is needed by the proposed risk-averse model to yield a substantial increase of the reliability level. It indicates that a small increase on the total cost will yield a substantially higher reliability level in return by the proposed risk-averse model. The Pareto front under the proposed risk-averse model indicates that, the consideration of value-at-risk is highly critical as long as the shipping

[1] As for the jobs with early due dates, they have a high chance of delay; and the jobs with late due dates have a high chance to arrive early. Therefore, such combinations help to keep the value-at-risk at the lowest level for most uncertain cases.

uncertainty exists. Being risk neutral may be unwise as the chance of achieving the expected minimum total cost is very low. However, an extremely high risk-averse attitude induces the decision maker to pay a high deterministic operations cost whereas further reduction in the value-at-risk becomes very limited. So, a moderate risk-averse attitude makes the reliability improvement "cost effective".

Moreover, in the extended models, impacts of the shipping lead-times correlation and exchange rate fluctuation are explored. Firstly, the results demonstrate that high correlation among shipments amplifies the negative impact of shipping lead-times uncertainty. However, its negative impact on the reliability of the whole system is significantly reduced by the proposed risk-averse model without sacrificing much on the total cost. In terms of the exchange rate scenario, it is demonstrated that the basic model corresponds to a specific case when exchange rate takes its expectation and keeps valid for other cases. It can achieve the coordination with the risk coming from exchange rate fluctuation.

1.3. ***Structure of this Study***

In Section 2, we review some recent related studies. The model description and its formulation are provided in Section 3. Section 4 analyzes the structures of the value-at-risk under different shipment assignment situations, i.e., single-job shipment and multiple-jobs shipment, and the risk-hedging shipment assignment policy is also proposed. In addition to the benchmark model, i.e., the risk-neutral method, the individual value-at-risk model is proposed for further comparisons with the risk-hedging policy based approach. The computational analyses are reported in Section 5. Section 6 explores the extended models for more general cases, i.e., correlated shipments and exchange rate scenarios. Section 7 discusses the conclusion and future research. All proofs are placed in the supplementary appendix.

2. Literature Review

Our work relates to three streams of literature, which include ocean container transportation, risk management in supply chains, and approximation methods for stochastic programming.

2.1. ***Ocean Container Transportation***

Ocean container transportation plays a critical role in global supply chain management. It involves multiple parties, mainly including shippers, liner shipping companies, port operators, and hinterland container operators (Yu et al., 2018). The liner shipping company is the core member in the ocean container transportation and hence lots of studies are conducted around liner shipping operations. For instance, Du et al. (2019) studied the influence of speed and trim on ship fuel efficiency by a novel two-phase optimization method. It helps achieve a higher fuel efficiency for the liner shipping company. Karsten et al. (2018) studied an integrated optimization model for the liner shipping company to determine the optimal sailing speed, needed fleet and cargo routing under transit time restrictions. Zheng et al. (2017) explored the pricing strategy for two risk-averse liner shipping companies with different capacity. The authors showed that risk-aversion makes the optimal prices of both companies decrease. Yang et al. (2017) studied how a novel floating price contract can help avoid contract default induced by low spot market price. In addition to liner shipping companies, container seaports play a critical role in ocean transportation, as their throughputs directly affect the on-time delivery of the containers. As the starting point of port operations, berth planning has drawn great attention. Recently, Zhen et al. (2017) solved an integrated berth allocation and quay crane problem for tidal ports with an additional consideration of time constraint from the navigation channel. Moreover, the port alliance problems have also been investigated as a mechanism to improve the level of service (Guo et al., 2018). Sun et al. (2018) investigated an integrated production and distribution problem in which the impacts of uncertain liner shipping on production scheduling were discussed from the perspective of the shippers. Different from the above reviewed studies, we focus on the discussion of the shipment assignment policy for the risk-averse shippers who may encounter late delivery due to uncertain arrival time of the containers.

2.2. ***Risk Management in Supply Chains***

For risks in supply chains, we know that they can mainly be classified into supply risk, operational risk and demand risk. In the literature, most attention focuses on risk mitigation for demand uncertainty or supply disruption in inventory ordering policies (He et al., 2017; Lim and Wang, 2017). Facing operational risk, decision makers have different risk preferences/attitudes and reactions. In general, risk aversion is the commonly assumed risk preference for manufacturers in supply chain systems (Shen et al., 2011, 2013). Particularly, in a make-to-order supply chain, the manufacturer is usually risk-averse towards the volatility of the orders from the buyers (Xie et al., 2011). In the recent literature, Kouvelis et al. (2018) analyzed a joint replenishment and financial hedging problem for a manufacturer who faces volatile price and uncertain demand simultaneously. A "mean-variance" objective was proposed to derive the optimal solution. Chiu et al. (2018) studied optimal "advertising budget allocation" problem for uncertain inter-dependent customers' groups in luxury fashion market. A "mean-variance" framework was utilized to investigate the optimal "customer portfolios and budget allocation" problem. In the literature, how the degree of risk aversion affects the optimal decisions has been well-explored (Choi et al., 2018). The Pareto non-inferior curve can be constructed and the company can choose the most desirable solution with a tradeoff between "risk and return". However, in terms of the degree of risk aversion, there is no optimal solution which is "best" for every company because it is a matter of the company's own preference. Readers may refer to Choi et al. (2019) for a comprehensive review of the mean-risk theory for logistics and transportation problems. In this study, we assume the manufacturer is risk-averse (see Appendix (A6) for the discussion on the risk seeking case). Stimulated by the mean-risk theory, a novel cost-reliability trade-off objective is proposed.

As for the operational risk, it is derived from internal and external factors. Internal factors refer to the uncertainties associated with manufacturing and processing, such as breakdown of operations (Briskorn et al., 2011; Jabbarzadeh et al., 2016). Jabbarzadeh et al. (2016) studied a resilient supply chain network design problem, in which both disruption risk of facility shutdown and demand/supply uncertainties were taken into account. External factors refer to the uncertainties in logistics operations. The uncertain logistics operations affect the reliability of supply chains significantly especially in the global supply chain environment. As retailers are much closer to the end users and more sensitive to the volatility of market demands, most studies discussed the impact of stochastic shipping lead-times from the retailer's perspective for inventory control (Kouvelis and Li, 2012; Kouvelis and Tang, 2012; Song et al., 2017). Another mainstream discussion of stochastic shipping lead-times is on transport planning, routing design or fleet deployment for shipping service providers (Xiao et al., 2017; Lee et al., 2015; Adulyasak and Jaillet, 2015). In recent years, many researchers focused on investigating shipping uncertainties in the liner shipping operations. In fact, the delivery reliability is affected by many factors, e.g., disruption events (extreme weather conditions, labor strike, machine breakdown, etc.) and regular uncertainties (port congestion, inefficient port operations). The disruption events usually refer to one-off events, while the regular uncertainties refer to the recurring probabilistic events in shipping operations (Lee and Song, 2017). Many studies were conducted on the vessel schedule recovery problems focusing on handling disruption events in liner shipping (Brouer et al., 2013; Li et al., 2015). Except for disruptive events, the regular uncertainties were also discussed for route scheduling and fleet deployment problems (Qi and Song, 2012; Wang and Meng, 2012; Li et al., 2016). For example, Qi and Song (2012) modelled the port times to be normally and uniformly distributed random variables. Wang and Meng (2012) considered random port times following the uniform distribution. Li et al. (2016) assumed the sea times and port times follow uniform and normal distributions, respectively. In this study, to make the model tractable, following the literature, we assume the shipping lead-times which include sea transportation times and port operations times follow the known continuous probability distributions. Although some studies considered the customer service levels as the constraints in the fleet deployment and route scheduling problems, their priority was the profitability of the shipping companies. In this study, different from the existing literature, we discuss the uncertainty in the liner shipping operations not from the perspective of the shipping companies, but from the shipper's (i.e., the manufacturer's), who hire the shipping company. Therefore, the impact of liner shipping uncertainty on the shipment assignment is discussed and the optimal shipment assignment policy with risk considerations is further proposed.

2.3. *Approximation Methods for Stochastic Programming Problems*

There are mainly two approaches to tackle stochasticity in optimization problems, namely the sampling/scenario based approach (Pantuso et al., 2015) and the analytical based approach. For the analytical based approach, the key point is to convert the stochastic optimization problem into its deterministic counterpart so as to obtain the solutions. When the random variables are involved in the constraints, the common way to turn the stochastic constraints into deterministic counterparts is to add the respective probabilistic constraints. Then the stochastic optimization problem becomes a chance constrained programming (CCP) problem. For instance, Zhen et al. (2019) developed a chance constrained non-linear optimization model for solving a stochastic fleet deployment problem. Different linearization techniques were utilized to convert the model into a mixed integer second-order cone program which can be solved by the solver Cplex. Shen (2014) converted the stochastic program into a mixed-integer program by giving each stochastic constraint a probabilistic constraint with the consideration of risk. The deterministic counterparts were approximated by the assumption of realization sets of the random variables. Wang et al. (2013) proposed a sample average approximation method to tackle the joint chance constraint so as to obtain the deterministic counterpart for the stochastic optimization model. To simplify the CCP problems, the linearization of nonlinear deterministic equivalents is inevitable. Based on the proposed linear transformation theorem, Bilsel and Ravindran (2011) solved a multi-objective CCP for the manufacturer selection problem under uncertainties by using goal programming. Sun et al. (2013) developed an inexact joint probabilistic left-hand-side chance constrained model and solved it by a non-equivalent but sufficient linearization form. By using discretization and linearization, Wang and Meng (2012) converted the mixed-integer nonlinear convex program into mixed-integer program to solve the problem by using a cutting-plane method based exact algorithm. When the random variables are involved in the objective function, the common way to tackle it is to take its expected value by discretization (Ghilas et al., 2016; Wang and Disney, 2017). In this study, the random variables are involved in the objective function. Instead of taking the expected value, the stochastic objective function is converted into a value-at-risk based function as a way to corporate risk-aversion attitude into the stochastic optimization problem.

3. Basic Model

In this section, the basic model is described in details, followed by a discussion of modeling assumptions, and the problem formulation.

3.1. *Model Description and Notation*

We consider a global supply chain in which a single upstream firm, i.e., a manufacturer, produces customized goods and delivers the goods to its overseas buyers, e.g., retailers. We focus our study on a finite planning horizon (usually referring to a selling cycle), e.g., a season. A set of jobs J ordered by different buyers are available at the beginning of the planning horizon. Each job j has its specific due date d_j. Earliness or tardiness costs will be induced for each job due to early arrival or late delivery. Earliness cost refers to the storage cost c_j^{DC} at the overseas distribution center (DC) after the job's arrival at the destination port. Tardiness cost refers to the penalty c_j^{TA} of each job for its late delivery which is paid to its buyer. The manufacturer owns several factories F located in different cities for cheap labor and raw materials as well as production flexibility. As illustrated in Fig. 2, in each factory, parallel production lines l are ready to produce multiple products with the same quality. Setup times for changeovers between two jobs are combined into the total production processing time as the setup time is specified by its respective job. Due to different levels of labor and facilities, the standard daily production capacity can be distinct in different factories. Thus, the processing time $t_{j,l}$ and cost $c_{j,l}^{PR}$ for each job j, which is operated on production line $l \in L$, can be different in different factories. The finished jobs are shipped and stored in the attaching warehouse of the respective factory $f \in F$ waiting for overseas delivery. The transportation time between the factory and its attaching warehouse is negligible compared to the production processing time. This is a rather common case in which both the factory and warehouse are located in the same city (e.g., in many industrial cities of China).

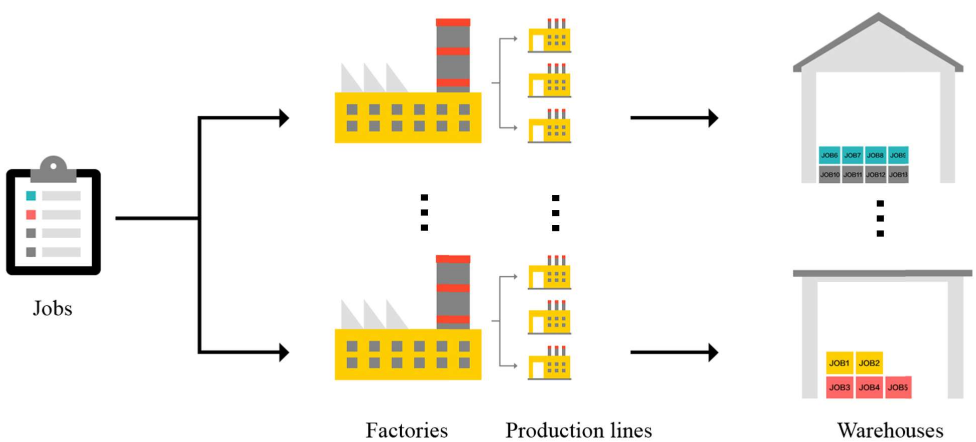
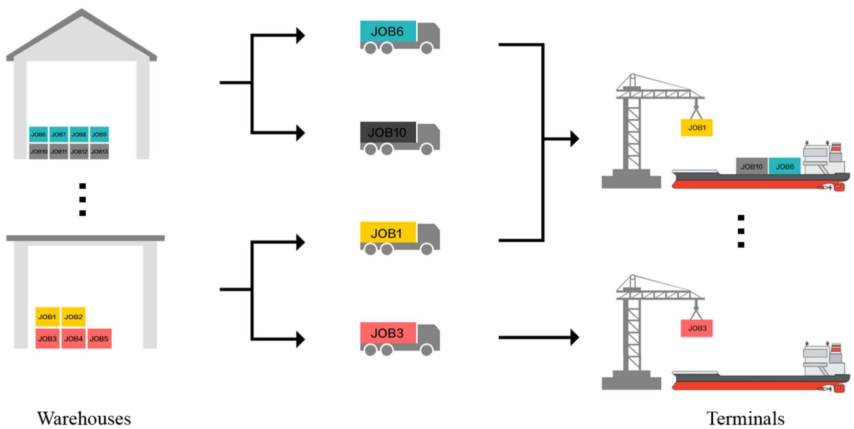
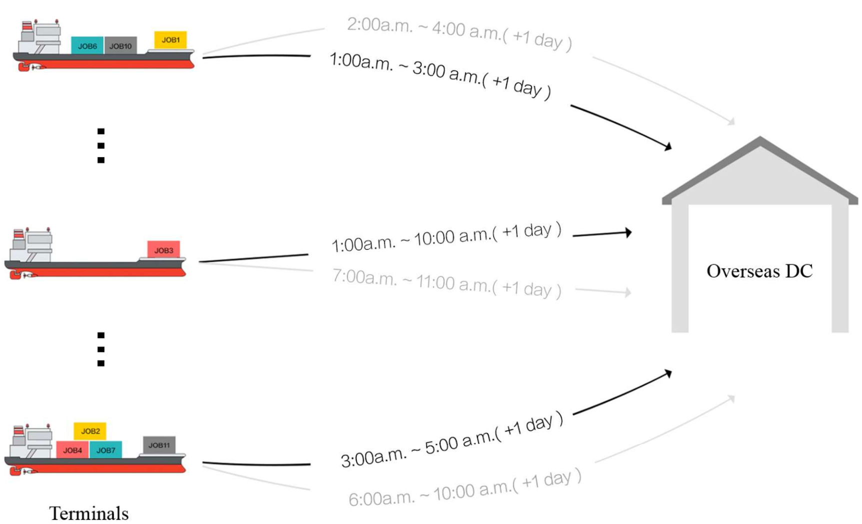


Figure 2: Illustration of the production part of the manufacturer.

As for the stochastic maritime transportation (see Fig. 3), the expected shipping schedules of a set of shipments S are announced by the shipping companies (e.g., OOCL, Cosco, Maersk Line) in advance, which provide the specific departure time a_s at the specific port of departure $h \in H$ and respective expected shipping lead-time μ_s to the port of destination for each shipment $s \in S$. In addition, there is a fixed unit freight rate c_s for each shipment $s \in S$, which has a negative relationship with the length of shipping lead-time. The shipment schedules limit the latest time to get arrival at the port. Here, each shipment corresponds to a container vessel. So, one shipment can carry multiple jobs, whereas each job cannot be split into suborders and delivered by multiple shipments. On the other hand, the inland transportation, which is outsourced to the third-party logistics company, is available all the time. However, the inland transportation time $t_{f,h}$ is a variable which depends on locations of the factories and the ports of departure. The inland transportation cost $c^{TR}_{j,f,h}$ further depends on the quantity q_j of each job. Thus, the inland transportation further shrinks the processing time for production and affects the jobs allocation and shipment assignment as well.



a. Multi-distance inland transportation by the third-party logistics company



b. Stochastic maritime transportation by the liner shipping company

Figure 3: Illustration of the two types of transportation modes with stochastic shipping lead-times.

The summary of the main notation is listed as follows:

Decision variables:

$x_{j,k,l}$	$= 1$, if job j is served immediately preceding job k on production line l; $= 0$, otherwise;
$y_{j,l,s}$	$= 1$, if job j is produced on production line l and delivered by shipment s; $= 0$, otherwise.

Dependent variables:

$t_{j,b}$	the starting time for the production of job j;
$t_{j,o}$	the completion time for the production of job j;
$t_{j,f}$	the duration for job j to be stored in the warehouse.

Random variables:

t_s	the shipping lead-time of shipment s;
$G_s(t_s)$	the economic loss from the earliness or tardiness costs due to shipment s.

Sets:

F	set of factories;
L	set of total production lines in all factories;
J	set of jobs;
J_s	set of jobs assigned to shipment s;
S	set of shipments;
H	set of terminals.

Parameters:

$t_{j,l}$	the processing time of job j on production line l;
$t_{f,h}$	the transportation time from factory f to terminal h;
q_j	the net weight of job j in tonnage;
d_j	the due date of job j;
a_s	the departure date of shipment s from the terminal;
$c_{j,l}^{PR}$	unit production cost of job j on production line l;
$c_{j,f}^{WH}$	unit storage cost of job j in the warehouse (attached to factory f);
$c_{j,f,h}^{TR}$	the inland transportation cost from the warehouse (attached to factory f) to terminal h of job j in TEU;
c_s	the liner shipping cost of shipment s in TEU;
c_j^{DC}	unit storage cost of job j in the overseas DC per day (i.e., unit earliness penalty);
c_j^{TA}	unit penalty cost for tardiness of job j per day;
c_s^r	a threshold for the loss $G_s(t_s)$ which refers to the total earliness and tardiness costs given the jobs assignment on shipment s;
L^{max}	maximum cargo payload of 20-foot ISO container (i.e., TEU);
$I_{s,h}$	= 1, if shipment s will depart from terminal h; = 0, otherwise;
$I_{l,f}$	= 1, if production line l belongs to factory f; = 0, otherwise;
μ_s, σ_s	the mean and standard deviation of shipping lead-time of shipment s;
$o(s), o(e)$	dummy jobs for the starting and ending points of a job sequence;
i, j, k	index for jobs;
l	index for production lines;
f	index for factories;
s	index for shipments;
h	index for terminals;
β_s	the reliability level assigned to each shipment s, which is determined by the operations manager, according to his or her risk preference;
ω	the risk-averse factor reflecting the risk preference of the operations manager.

3.2. Assumptions

Our model is constructed based on the following assumptions. These assumptions either reflect real-life situations, or help sharpen our findings by focusing on the core issues, while the problem remains analytically tractable.

Make-to-order supply chain: a set of jobs are given by customers and the required raw materials are available at the beginning of the planning horizon. These are common assumptions in the literature on make-to-order supply chain operations (Chen and Pundoor, 2006; Gharaei and Jolai, 2018; Ullrich, 2013) which are also realistic in practice. For instance, those garment manufacturers located in developing countries (e.g., China, Southeast Asia) usually get orders from different overseas customers for the next season in advance.

Perfect manufacturing environment: the production lines in each factory are available at the beginning of the planning horizon with no breakdowns. Besides, each job only needs to be processed by one production line once. These assumptions are consistent with the literature on make-to-order supply chains (Behnamian and Ghomi, 2013). Our model could also be extended to the case when "ready times" (or maintenance times) are required for each production line.

Standard inland container transportation: the inland container transportation provided by the third-party logistics company is available with the standard delivery mode. This assumption reflects real-life settings of inland container transportation. For instance, the freight forwarder can always offer its shipper (i.e., the manufacturer) the international logistics services, including inland transportation from the shipper's warehouse to the departure port and port related services.

Independent stochastic shipping lead-times: the shipping lead-times t_s of the shipments s ($\forall s \in S$) are independent random variables following arbitrary continuous probability distributions, i.e. $F_s(\cdot)$. To make our study more comprehensive, we have extended the analysis to the correlated shipping lead-times scenario in Section 6.

Delivery form: the delivery of each order is non-splittable. This is consistent with the literature of integrated production and outbound distribution scheduling (Chen and Pundoor, 2006; Zhong et al., 2010). In reality, the buyers prefer to receive the orders in full, just like the "OTIF" program implemented by Walmart. Hence, following this industrial practice, partial delivery [2] is not considered in our model, and each job can only be shipped by one shipment.

Fixed overseas inland transportation: for each job, the time and cost of transportation from the destination port to the overseas DC are the same and fixed, which does not affect the total cost. This assumption refers to the case that the shipping from domestic sea port to the overseas DC is a direct shipment, which is a common assumption in the literature on production and outbound distribution scheduling (Chen, 2010). Clearly, multiple transportation modes after arriving at the destination port maybe applied to enhance the delivery efficiency. Since our focus is on studying the impacts of uncertain maritime transportation time and the corresponding shipment assignment policy, we do not consider the availability of multiple transportation modes after the job has arrived at its destination port. We postpone the respective

[2] In practice, in the terms and conditions of purchase contracts, partial delivery of goods is usually disallowed unless prior agreements in writing by the buyers have been signed. This practice is implemented in many industries, such as semiconductor manufacturing (ZEISS), braking system manufacturing (KNORR-BREMSE), ICT infrastructure (SIZWE),etc. Please find some examples in the online supplementary appendix 6.

exploration to future research.

3.3. *Formulation*

Stimulated by the mean-risk theory (Gan et al., 2004; Xiao and Choi, 2009), a new objective function is proposed here, which is composed of the deterministic operations cost and the weighted risk measure. Its purpose is to study the impact of inherent uncertainties in shipping operations, and obtain the solutions to realize trade-off between cost and reliability of the whole system.

3.3.1. *Deterministic Operations Cost*

The deterministic operations cost includes total production cost c_j^{PR}, warehouse storage cost c_j^{WH}, inland transportation cost c_j^{TR} and liner shipping cost c_j^{LS}. Their definitions are shown in Eqs. (1)–(5).

(i). Production cost:

$$c_j^{PR} = \sum_{l \in L} \sum_{k \in J \cup o(e), k \neq j} c_{j,l}^{PR} x_{j,k,l}. \tag{1}$$

(ii). Warehouse storage cost:

$$c_j^{WH} = \sum_{f \in F} c_{j,f}^{WH} t_{j,f}, \tag{2}$$

$$t_{j,f} = \max\left(\sum_{l \in L} \sum_{s \in S} a_s I_{l,f} y_{j,l,s} - \sum_{l \in L} \sum_{s \in S} \sum_{h \in H} I_{s,h} I_{l,f} t_{f,h} y_{j,l,s} - t_{j,o}, 0\right). \tag{3}$$

(iii). Inland transportation cost:

$$c_j^{TR} = \sum_{f \in F} \sum_{l \in L} \sum_{h \in H} \sum_{s \in S} I_{s,h} I_{l,f} c_{j,f,h}^{TR} y_{j,l,s}. \tag{4}$$

(iv). Liner shipping cost:

$$c_j^{LS} = \sum_{l \in L} \sum_{s \in S} c_s y_{j,l,s}. \tag{5}$$

Eq. (1) states that the unit production cost of job j depends on production line l to which the job is assigned. The unit storage cost of job j (see Eq. (2)) in a warehouse depends on factory f that job j is assigned to and the duration for which job j is stored in the warehouse. The duration not only depends on the completion time of production $t_{j,o}$ and the departure time of the assigned shipment a_s, but also depends on its corresponding inland transportation time $t_{f,h}$ (see Eq. (3)). In addition, Eq. (4) indicates that c_j^{TR} depends on the distance between the warehouse and the terminal where job j will be served. Lastly, the unit liner shipping cost of job j depends on the unit shipping cost of shipment s to which the job is assigned.

3.3.2. *Value-at-Risk*

Value-at-Risk (VaR) is a widely used risk assessment measure in the financial industry (Alexander and Sarabia, 2012; Lwin et al., 2017). Nowadays, even non-financial companies use VaR as a measure for their supply chain risk management (Amorim et al., 2016; Park et al., 2017). In a very recent study, Kouvelis and Li (2019) propose an integrated risk management problem for newsvendors with VaR constraints. The authors point out that VaR is usually preferred by regulators as it is easy to implement and intuitive. It is useful to achieve integrated risk management in many regulated industrial settings.

Following the literature (Qi and Song, 2012; Wang and Meng, 2012; Li et al., 2016), for naturally present "recurring probabilistic events"(Lee and Song, 2017), the shipping lead-times are modeled as random variables which follow known continuous probability distributions. Observing that, we utilize a probabilistic risk measure, called VaR at a confidence/reliability level β_s to measure the risk incurred by the assigned shipment s. To be specific, VaR at a confidence level β_s, denoted by VaR_{β_s}, refers to the lowest amount such that, with probability β_s, the loss will not exceed. For instance, $VaR_{0.95}$ refers to the lowest amount that, with probability 95%, the loss will not exceed.

For our problem, VaR_{β_s}[3] is defined as follows:

$$VaR_{\beta_s} = inf\{c_s^r | Pr(G_s(t_s) \leq c_s^r) \geq \beta_s\}. \tag{6}$$

where the loss function $G_s(t_s)$ refers to the total earliness and tardiness costs incurred by shipment s, given the jobs assignment, i.e., for fixed $y_{j,l,s}$ ($\forall l \in L$, $j \in J$). Eq.(7) presents the details on its formulation.

$$G_s(t_s) = \sum_{l \in L} \sum_{j \in J} q_j c_j^{DC} (t_s + a_s - d_j)^- y_{j,l,s} + \sum_{l \in L} \sum_{j \in J} q_j c_j^{TA} (t_s + a_s - d_j)^+ y_{j,l,s} \tag{7}$$

Here, $x^+ = \max(x, 0)$, $x^- = -\min(x, 0)$. According to Rockafellar and Uryasev (2000), the probability $P_r(G_s(t_s) \leq c_s^r)$ is non-decreasing with respect to c_s^r, but not necessarily continuous from the left as jumps may exist. However, in our model, the probability of the loss function $G_s(t_s)$ is continuous from both right and left sides. Therefore, we can convert Eq. (6) into the following

$$VaR_{\beta_s} = (c_s^r | P_r(G_s(t_s) \leq c_s^r) = \beta_s). \tag{8}$$

In other words, VaR_{β_s} is the β_s-quantile of $G_s(t_s)$, which helps analyze the closed forms of VaR_{β_s} under different cases. For the case that $y_{j,l,s} = 0$ ($\forall i \in I, j \in J$), namely, shipment s is not assigned to any job, $VaR_{\beta_s} = 0$.

[3] Note that the conditional value-at-risk ($CVaR$), the alternative common risk measure, is equivalent/approximately equivalent to VaR by adjusting the confidence/relibility level under normal/any arbitrary continuous distribution assumptions (Mínguez et al., 2011),i.e., $VaR_{\beta_s} = CVaR_{\beta_s^*}$, where $\beta_s^* = 1 - \frac{\int_{VaR_{\beta_s}}^{\infty} G_s(t_s) f_s(t_s) dt_s}{VaR_{\beta_s}}$, and $f_s(t_s)$ is the probability density function of shipping lead-time t_s.

3.3.3. *Objective Function*

Therefore, the objective function to make trade-off between cost and reliability is formulated as follows.

$$\min Z_1 = \sum_{j \in J} q_j (c_j^{PR} + c_j^{WH}) + \sum_{j \in J} \left\lceil \frac{q_j}{L^{max}} \right\rceil (c_j^{TR} + c_j^{LS}) + \omega \sum_{s \in S} VaR_{\beta_s}, \tag{9}$$

where $\left\lceil \frac{q_j}{L^{max}} \right\rceil$ refers to the minimum integer value greater than $\frac{q_j}{L^{max}}$. Observe that $\omega \geq 0$[4] is a risk-averse factor which reflects the risk preference of the decision maker. The larger ω is, the more risk-averse the decision maker is. (As a remark, $\omega = 0$ refers to the case when the decision maker is risk-neutral).

For the novel risk-aversion objective function (9), two parameters ω and β_s are involved, which reflect the risk preference of the operational risk manager/operator from different levels. β_s reflects the risk-aversion of the operator from the shipment level. In other words, it indicates how reliable the operator expects for each shipment. The commonly used range for β_s is [0.8,1). However, parameter ω refers to the risk-aversion attitude in terms of the possible loss from a whole system's level. It reflects the operator's trade-off between the deterministic operations cost and the possible loss. A high reliability level setting for each shipment is better to work with a moderate setting from the system's level. Our findings also verify that a moderate setting on ω contributes to a more cost-effective solution. However, a low reliability level (e.g., 0.5) for each shipment may have to cooperate with a higher ω to guarantee a reliable solution that a risk-averse operator expects.

The following details present the details on the constraints from both production and shipping sides.

3.3.4. *Production and Shipping Constraints*

(i). Production constraints:

$$\sum_{l \in L} \sum_{k \in J \cup o(e), k \neq j} x_{j,k,l} = 1, \forall j \in J. \tag{10}$$

$$\sum_{l \in L} \sum_{j \in J \cup o(s), j \neq k} x_{j,k,l} = 1, \forall k \in J. \tag{11}$$

$$\sum_{j \in J \cup o(s)} \sum_{n \in J \cup o(e)} (x_{j,k,l} - x_{k,n,l}) = 0, \forall k \in J; l \in L. \tag{12}$$

$$\sum_{k \in J \cup o(e)} x_{o(s),k,l} = 1, \forall k \in J; l \in L. \tag{13}$$

$$\sum_{j \in J \cup o(s)} x_{j,o(e),l} = 1, \forall k \in J; l \in L. \tag{14}$$

$$x_{j,k,l} + x_{i,k,j} \leq 1, \forall i \in I; j \in J; k \in J, j \neq k. \tag{15}$$

[4]Note that $\omega < 0$ refers to the case when the manufacturer is risk-seeking. For detailed discussion, please refer to Appendix (A6).

$$t_{j,o} = t_{j,b} + \sum_{l \in L} \sum_{k \in J \cup o(e), k \neq j} t_{j,l} x_{j,k,l}, \forall j \in J. \tag{16}$$

$$t_{k,b} - t_{j,b} \geq \sum_{l \in L} t_{j,l} x_{j,k,l} - M(1 - \sum_{l \in L} x_{j,k,l}), \forall j \in J; k \in J, j \neq k. \tag{17}$$

$$t_{k,b} - t_{j,b} \leq \sum_{l \in L} t_{j,l} x_{j,k,l} + M(1 - \sum_{l \in L} x_{j,k,l}), \forall j \in J; k \in J, j \neq k. \tag{18}$$

(ii). Shipment constraints:

$$\sum_{l \in L} \sum_{s \in S} y_{j,l,s} = 1, \forall j \in J. \tag{19}$$

(iii). Transportation constraints:

$$\sum_{l \in L} \sum_{s \in S} a_s y_{j,l,s} - \sum_{f \in F} \sum_{l \in L} \sum_{s \in S} \sum_{h \in H} I_{s,h} I_{l,f} t_{f,h} y_{j,l,s} \geq t_{j,o}, \forall j \in J. \tag{20}$$

(iv). Connection constraints:

$$\sum_{k \in J \cup o(e)} x_{j,k,l} - \sum_{s \in S} y_{j,l,s} = 0, \forall l \in L; j \in J. \tag{21}$$

(vi). Non-negativity constraints:

$$x_{j,k,l}, y_{j,l,s} \in \{0,1\}; t_{j,b}, t_{j,o} \in \mathbb{Z}^+; VaR_{\beta_s} \in \mathbb{R}^+. \tag{22}$$

Constraints (10)—(15) control the production sequence on each production line in different factories. All the production lines start producing to maximize the utilization of the facility, and each job can be assigned to only one production line. In addition, each job can have only one predecessor and one successor. $x_{o(s),j,l}$ and $x_{k,o(e),l}$ indicate that jobs j and k are the first and last jobs served on production line l. Constraints (16)—(18) state the relationship of the production starting times between two successive jobs and the calculation of the completion time of each job. Here, M is the "big M" which represents a huge constant. Note that, here the unit time of production is per day. Constraints (19) limit each job to be assigned to just one shipment. Constraints (20) state that the departure time of the assigned shipment cannot be earlier than the sum of the production completion time and inland transportation time of the job. Constraints (21) link the production scheduling with the shipment assignment for each job.

4. Analysis

For the shipment assignment, two possible situations will arise: (i) the assigned shipment s is only responsible for one job j, and no other job will be shipped by it, namely, $|J_s| = 1$; (ii) the assigned shipment s is responsible for multiple jobs, namely, $|J_s| = m \leq |J|$. Here $|J|$ is the total number of jobs in the planning horizon. In this section, the explicit expressions of the value-at-risk VaR_{β_s} are derived under different shipment assignment cases, i.e., $VaR_{\beta_s,j}$ of single-job shipment and VaR_{β_s,J_s} of multiple-jobs shipment. The structures of VaR_{β_s} under different cases are discussed. The risk-hedging shipment assignment policy is then established.

4.1. *Special Case: Single-job Shipment*

For the case when $|J_s| = 1$, such as job j assigned to shipment s, we have the following definition:

Definition 1 (Individual value-at-risk). *For the case of one-job-shipment assignment, i.e., there is only one job j assigned to the specific shipment s which will ship the specific goods to the designated overseas port, we call the value-at-risk incurred by the specific shipment as individual value-at-risk $VaR_{\beta_s,j}$.*

In fact, the individual value-at-risk can be viewed from the job side as well. Namely, it can be referred to as the value-at-risk faced by the job j when assigned to shipment s under a given reliability level β_s. According to Eq. (8), we have the individual value-at-risk satisfying the following probability:

$$P_r(G_s(t_s) \leq VaR_{\beta_s,j}) = \beta_s. \tag{23}$$

Fig. 4 depicts the individual value-at-risk $VaR_{\beta_s,j}$, which is the minimum height of the line intersected with the function $G_s(t_s)$, where the probability[5] within the points of intersection equals β_s. Proposition 1 gives the closed-form expression of individual value-at-risk $VaR_{\beta_s,j}$ for the situation of single-job shipment.

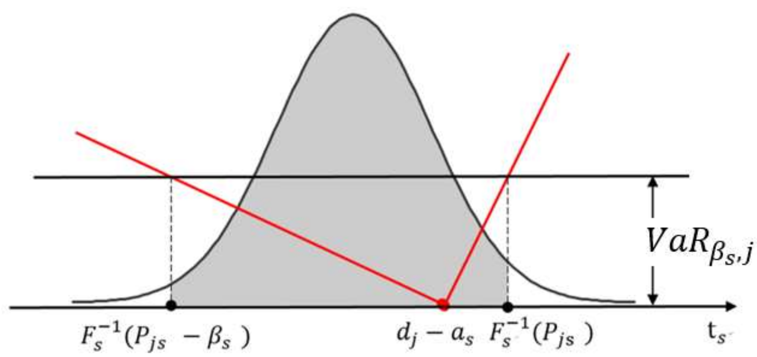


Figure 4: Illustration of the individual value-at-risk.

Proposition 1 (Closed form of individual value-at-risk). *When job j is assigned to shipment s, the individual value-at-risk VaR_{β_s} under a specific shipment reliability level β_s (i.e., determined by the operational risk manager) is:*

$$VaR_{\beta_s,j} = c_j^{TA} q_j[F_s^{-1}(P_{j,s}) + a_s - d_j], \tag{24}$$

where the probability $P_{j,s}$ is the root of

$$\begin{cases} c_j^{TA}[F_s^{-1}(P_{j,s}) + a_s - d_j] = c_j^{DC}[d_j - a_s - F_s^{-1}(P'_{j,s})] \\ P_{j,s} - P'_{j,s} = \beta_s. \end{cases} \tag{25}$$

Here, $F_s^{-1}(\cdot)$ represents the inverse cumulative distribution function of the shipping lead-time t_s.

$P_{j,s}$ and $P'_{j,s}$ are the probabilities of t_s corresponding to the two points of intersection (from right to left) that the horizontal line has (with the loss function $G_s(t_s)$). In addition, the probability of t_s within

[5] Note that the normal distribution is used here for an easy-to-understand illustration.

these two points of intersection is β_s (i.e., the shaded area[6]). Proposition 1 reflects the impact of shipping uncertainty on the individual value-at-risk. When the random shipping lead-time follows the normal distribution $\mathcal{N}(\mu_s, \sigma_s^2)$, we have: $F_s^{-1}(P_{j,s}) = \sigma_s \Phi^{-1}(P_{j,s}) + \mu_s (\forall s \in S)$. Note that a highly uncertain shipment, reflected by its high σ_s, will bring a large value-at-risk due to the amplification effect between penalty cost and shipping uncertainty. So, the selection of shipment directly determines the scale of the value-at-risk. It inevitably needs more coordination with jobs allocation among the factories to make the completion time "not too late". The cost ratio of the earliness (i.e., early arrival at the overseas DC) and tardiness penalties will further affect the individual value-at-risk. The non-negligible storage cost at DC indicates that pure selection of early departure and short lead-time shipment cannot guarantee a lower value-at-risk.

The individual value-at-risk focuses on the effects the shipping uncertainty has on the single job. The next subsection will give the generalized closed-form expression for the multiple-jobs shipment. The relationship among the jobs assigned to the same shipment as well as the relationship between the jobs and their responsible shipments will be discovered.

4.2. ***General Case: Multiple-jobs Shipment***

With more than one job being assigned to the same shipment, the situation becomes much more complicated. The pure consideration of individual value-at-risk cannot reflect the relationship among the jobs assigned to the same shipment. In this subsection, the generalized deterministic expression of "joint" value-at-risk is solved. Similarly, we have the following definition:

Definition 2 (Joint value-at-risk). *For the multiple-jobs-shipment case when a specific shipment s which will ship all the goods from multiple jobs to the designated overseas port, we call the value-at-risk incurred by the shipment s as the joint value-at-risk VaR_{β_s}.*

Assume that two jobs, $j(1)$ and $j(2)$, sequenced in the non-decreasing order of their due dates, are assigned to shipment s. Then we have the following relationship:

$$G_s(d_{j(1)} - a_s) - G_s(d_{j(2)} - a_s) = (d_{j(2)} - d_{j(1)})(c_{j(2)}^{DC} q_{j(2)} - c_{j(1)}^{TA} q_{j(1)}). \tag{26}$$

Thus, $d_{j(1)} - a_s$ is the minimum point of the function $G_s(t_s)$ as long as the daily storage cost of $j(2)$ is no more than the daily penalty cost of $j(1)$[7]. In addition, the job is only allowed to select the shipments which can guarantee at least $1 - \beta_s$ chance of on-time delivery. For instance, if $\beta_s = 0.95$, then those shipments of less than 5% chance of on-time delivery for job j will not be considered for shipping job j at

[6] In our study, without loss of the key insights, two extreme cases are excluded to make the analysis less complex. They are: i. $c_j^{TA}(U_s + a_s - d_j) \leq c_j^{DC}(d_j - a_s - F_s^{-1}(1 - \beta_s))$; ii. $c_j^{TA}(F_s^{-1}(\beta_s) + a_s - d_j) \geq c_j^{DC}(d_j - a_s - L_s)$, where (L_s, U_s) are the lower and upper bounds of the shipping lead-times $t_s (s \in S)$.

[7] Note that, we only consider the cases when daily penalty for tardiness is larger than the daily storage cost for earliness, which is reasonable in reality.

the beginning of planning. Under this condition, Lemma 1 gives the joint value-at-risk for the case when two jobs are assigned to the same shipment.

Lemma 1 (Closed form of two-jobs-shipment joint value-at-risk). *When two jobs $j(1)$ and $j(2)$, which are sequenced in a non-decreasing order of their due dates, are assigned to the same shipment s, the joint value-at-risk of these two jobs $VaR_{\beta_s,j(1)j(2)}$ under a specific shipment reliability level β_s (i.e., determined by the operational risk manager) is:*

$$VaR_{\beta_s,j(1)j(2)} = \begin{cases} c^{TA}_{j(1)}q_j[F_s^{-1}(P_1)+a_s-d_{j(1)}] & \\ +c^{DC}_{j(2)}q_j[d_{j(2)}-a_s-F_s^{-1}(P_1)] & d_{j(1)}-a_s < F_s^{-1}(P_1) \leq d_{j(2)}-a_s \\ c^{TA}_{j(1)}q_j[F_s^{-1}(P_2)+a_s-d_{j(1)}] & \\ +c^{TA}_{j(2)}q_j[F_s^{-1}(P_2)+a_s-d_{j(2)}] & F_s^{-1}(P_2) > d_{j(2)}-a_s \end{cases} \tag{27}$$

where the probabilities P_1 and P_2, respectively, are the roots of

$$c^{TA}_{j(1)}q_{j(1)}[F_s^{-1}(P_1)+a_s-d_{j(1)}]+c^{DC}_{j(2)}q_{j(2)}[d_{j(2)}-a_s-F_s^{-1}(P_1)] = \sum_{k=1}^{2} c^{DC}_{j(k)}q_{j(k)}[d_{j(k)}-a_s-F_s^{-1}(P_1-\beta_s)]$$

and

$$c^{TA}_{j(1)}q_{j(1)}[F_s^{-1}(P_2)+a_s-d_{j(1)}]+c^{TA}_{j(2)}q_{j(2)}[F_s^{-1}(P_2)+a_s-d_{j(2)}] = \sum_{k=1}^{2} c^{DC}_{j(k)}q_{j(k)}[d_{j(k)}-a_s-F_s^{-1}(P_2-\beta_s)].$$

The first situation in Lemma 1 is illustrated by Fig. 5. The combined function (represented by red line) is the loss function $G_s(t_s)$ of jobs $j(1)$ and $j(2)$. The horizontal line has two points of intersection with $G_s(t_s)$. When the probability of the shipping lead-time t_s within the two points of intersection (i.e., ψ_1 and ψ_2 in Fig. 5) is equal to β_s (i.e., the shaded area), the corresponding height of the horizontal line corresponds to joint value-at-risk $VaR_{\beta_s,j(1)j(2)}$. P_1 is a probability measure equal to $F_s(\psi_2)$ (P.S.: $\psi_1 = F_s^{-1}(P_1-\beta_s)$). Here, for a given shipment s, we denote two mononotic functions, i.e., $G_s^+(t_s)$ and $G_s^-(t_s)$, based on the loss function $G_s(t_s)$. To be specific, $G_s^+(t_s) = \{G_s(t_s)|t_s \geq \arg\min_{t_s}(G_s(t_s))\}$ and $G_s^-(t_s) = \{G_s(t_s)|t_s < \arg\min_{t_s}(G_s(t_s))\}$. Hence, we have the following probability:

$$P_{(d_j-a_s)} = Pr(t^* < t_s < d_j - a_s), \tag{28}$$

where t^* satisfies $G_s^+(d_j-a_s) = G_s^-(t^*)$. Given the shipment s, if the due date of $j(2)$ is shifted to an earlier date, and the probability $P_{(d_{j(2)}-a_s)} < \beta_s$ (which is equivalent to $d_{j(2)}-a_s < \psi_2$), then the second case in Lemma 1 will be true. It implies that the calculation of joint value-at-risk may correspond to at most $|J_s|$ cases. The number of possible cases is determined by the number of different due dates of those jobs assigned together, i.e., $m' \leq |J_s|$. Thus, we have m' check points, i.e., $t_s = d_{j(m)} - a_s (m = 1, 2, \ldots, |J_s|)$ to determine the range of ψ_2. Accordingly, we have Lemma 2.

Lemma 2 (Closed form of multiple-jobs-shipment joint value-at-risk). *Given m jobs $j(i)$ $(i = 1, 2, ..., m)$ being assigned to the same shipment s, which are sequenced in a non-decreasing order of their due dates, the joint value-at-risk of m jobs under a specific shipment reliability level β_s (i.e., determined by the*

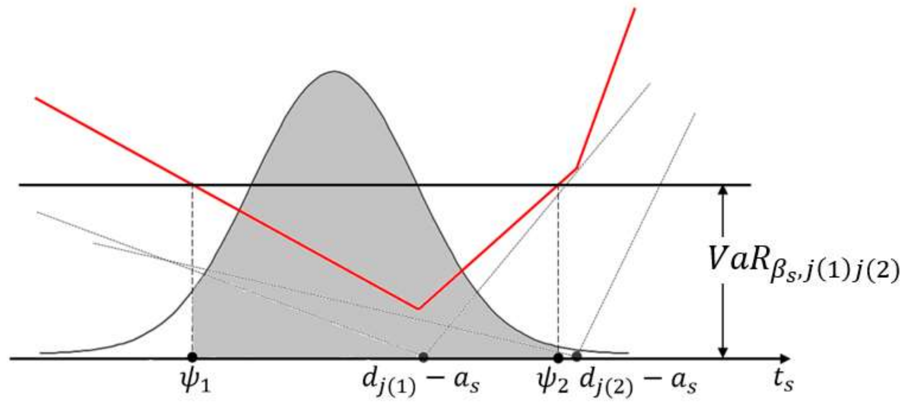


Figure 5: Illustration of the joint value-at-risk.

operational risk manager) is

$$VaR_{\beta_s,J_s} = \begin{cases} \sum_{k=1}^{i} c_{j(k)}^{TA} q_{j(k)}[F_s^{-1}(P_i) + a_s - d_{j(k)}] & \\ + \sum_{k=i+1}^{m} c_{j(k)}^{DC} q_{j(k)}[d_{j(k)} - a_s - F_s^{-1}(P_i)], & d_{j(i)} - a_s < F_s^{-1}(P_i) \le d_{j(i+1)} - a_s \\ \sum_{k=1}^{m} c_{j(k)}^{TA} q_{j(k)}[F_s^{-1}(P_m) + a_s - d_{j(k)}], & F_s^{-1}(P_m) > d_{j(i+1)} - a_s \end{cases} \tag{29}$$

where $|J_s| = m$*, the probabilities* P_i $(i = 1, 2, ...m-1)$ *and* P_m*, respectively, are the roots of*

$$\sum_{k=1}^{i} c_{j(k)}^{TA} q_{j(k)}[F_s^{-1}(P_i)+a_s-d_{j(k)}]+\sum_{k=i+1}^{m} c_{j(k)}^{DC} q_{j(k)}[d_{j(k)}-a_s-F_s^{-1}(P_i)] = \sum_{k=1}^{m} c_{j(k)}^{DC} q_{j(k)}[d_{j(k)}-a_s-F_s^{-1}(P_i-\beta_s)]$$

and

$$\sum_{k=1}^{m} c_j^{TA} q_{j(k)}[F_s^{-1}(P_m) + a_s - d_{j(k)}] = \sum_{k=1}^{m} c_{j(k)}^{DC} q_{j(k)}[d_{j(k)} - a_s - F_s^{-1}(P_m - \beta_s)].$$

Accordingly, the generalization of joint value-at-risk can be expressed in a simple and clear format as shown in Proposition 2 which is equivalent to Lemma 2.

Proposition 2 (Closed form of value-at-risk). *The value-at-risk induced by shipment* s *under given shipment reliability level* β_s *(i.e., determined by the operational risk manager) is*

$$VaR_{\beta_s} = \sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{TA} q_j[F_s^{-1}(P_s^*) + a_s - d_j]^+ + \sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{DC} q_j[F_s^{-1}(P_s^*) + a_s - d_j]^-, \tag{30}$$

where the probability P_s^* *is the root of*

$$\begin{aligned} &\sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{TA} q_j[F_s^{-1}(P_s^*) + a_s - d_j]^+ + \sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{DC} q_j[F_s^{-1}(P_s^*) + a_s - d_j]^- \\ &= \sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{DC} q_j[d_j - a_s - F_s^{-1}(P_s^* - \beta_s)]. \end{aligned} \tag{31}$$

There are always two points that the horizontal line intersects with $G_s(t_s)$. When the probability within these two points of intersection equals β_s, the corresponding height of the horizontal line is VaR_{β_s}. Here, P_s^* corresponds to the probability that t_s is less than the point of intersection on the right hand side (P.S.: $F_s^{-1}(P_s^*)$ is the left hand side point of intersection). Proposition 2 generalizes the structure of the value-at-risk for all shipment assignment cases. In fact, it corresponds to the sum of tardiness and earliness costs when shipping lead-time t_s equals $F_s^{-1}(P_s^*)$. However, $F_s^{-1}(P_s^*)$ cannot be obtained from Eq. (31) directly because decision variables $y_{j,l,s}(\forall l \in L, j \in J)$ are involved in Eq. (31) and the expression of the equation is not fixed. In the next subsection, a good approximation expression of $F_s^{-1}(P_s^*)$ is proposed, which provides a critical foundation to obtain the risk-hedging shipment assignment policy and helps understand its principles.

4.3. *Risk-hedging Shipment Assignment Policy*

4.3.1. *Value-at-Risk Reduction by Jobs Combination*

In this section, we mainly focus on the analysis of value-at-risk reduction, which reveals the principles to achieve risk-hedging shipment assignment. Here, due to the irrelevance of jobs combination, we use the individual value-at-risk as the benchmark to do the comparison.

According to Eq.(25) in Proposition 1, Lemma 3 gives the relationship between the joint worst shipping lead-time $F_s^{-1}(P_s^*)$ and the corresponding individual worst shipping lead-times $F_s^{-1}(P_{j,s})(\forall j \in J_s)$ of the jobs assigned to shipment s under the shipment reliability level β_s.

Lemma 3 (Upper and lower bounds). *The joint worst shipping lead-time $F_s^{-1}(P_s^*)$ always has upper and lower bounds, which are determined by the individual worst shipping lead-times $F_s^{-1}(P_{j,s})$ of the jobs $j(\forall j \in J_s)$ assigned to shipment s, i.e., $min_{j \in J_s} F_s^{-1}(P_{j,s}) \leq F_s^{-1}(P_s^*) \leq max_{j \in J_s} F_s^{-1}(P_{j,s})$.*

As Lemma 3 verifies, when the jobs assigned to the same shipment have the same due date and earliness-tardiness cost ratio, there is no difference between (i) the sum of individual value-at-risks $\sum_{j \in J_s} VaR_{\beta_s,j}$ and the joint value-at-risk VaR_{β_s,J_s}, and (ii) the shipment reliability levels that can be achieved in individual and joint cases. In this situation, using the individual value-at-risk is enough to determine whether it is optimal to assign to the same shipment. However, when the parameters of the jobs are different, the simple consideration of individual value-at-risks will ignore the compensation effects facilitated by the jobs combination, and further incur negative impacts on the systems reliability in terms of total costs. In other words, the lowest sum of individual value-at-risks and highest systems reliability do not always match. Accordingly, we have the following corollary.

Corollary 1 (Necessary condition). *The joint value-at-risk is less than the sum of individual value-at-risks only if $\frac{c_{j_1}^{DC}}{c_{j_1}^{TA}} \neq \frac{c_{j_2}^{DC}}{c_{j_2}^{TA}}$ or $d_{j_1} \neq d_{j_2}$ $(\forall j_1, j_2 \in J_s, j_1 \neq j_2)$.*

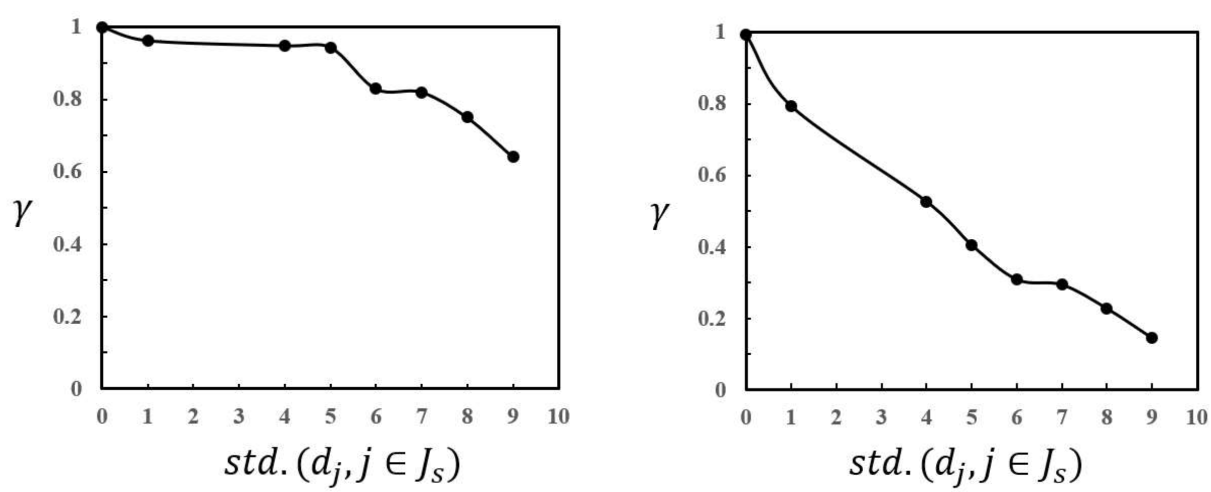
In the following, the situations that joint value-at-risk attain its lowest value with the improved systems reliability will be discussed.We firstly explore the relationship between the joint worst shipping lead-time $F_s^{-1}(P_s^*)$ and the individual worst shipping lead-times $F_s^{-1}(P_{j,s})(j \in J)$.

Proposition 3 (The relationship between the joint and individual worst shipping lead-times). *The two-sided worst shipping lead-times $F_s^{-1}(P_s^*)$ and $F_s^{-1}(P_s^* - \beta_s)$ of shipment s have the following relationship:*

$$
\begin{aligned}
F_s^{-1}(P_s^*) &\triangleq A - \omega_1[A - B - \gamma C], \\
F_s^{-1}(P_s^* - \beta_s) &\triangleq B + \omega_2[A - B - \gamma C],
\end{aligned} \tag{32}
$$

where $A = \frac{\sum_{j\in J_s} c_j^{TA} q_j F_s^{-1}(P_{j,s})}{\sum_{j\in J_s} c_j^{TA} q_j}$,
$B = \frac{\sum_{j\in J_s} c_j^{DC} q_j F_s^{-1}(P_{j,s}-\beta_s)}{\sum_{j\in J_s} c_j^{DC} q_j}$,
$C = min_{j\in J_s} F_s^{-1}(P_{j,s}) - min_{j\in J_s} F_s^{-1}(P_{j,s} - \beta_s)$,
$0 < \omega_1 \leq \frac{min(A - min_{j\in J_s} F_s^{-1}(P_{j,s}),\ max_{j\in J_s} F_s^{-1}(P_{j,s}) - A)}{|A-B-\gamma C|}$,
$0 < \omega_2 \leq \frac{min(B - min_{j\in J_s} F_s^{-1}(P_{j,s}-\beta_s),\ max_{j\in J_s} F_s^{-1}(P_{j,s}-\beta_s) - B)}{|A-B-\gamma C|}$,
and $\gamma = g(std.(d_j, j \in J_s))$.

Here, γ has a negative relationship with the standard deviation of due dates of the jobs assigned to the same shipment s, which can be represented by a non-increasing function of it, i.e., $\gamma = g(std.(d_j, j \in J_s))$. Its boundary condition for the case when $std.(d_j, j \in J_s) = 0$ is $\gamma = g(0) = \frac{A-B}{C}$. Due to the complexity of $F_s^{-1}(P_s^*)$, the explicit expression of function g cannot be obtained in an analytical way. The computational experiments are then conducted to verify the relationship between γ and the deviation of due dates. Fig.6 makes the illustrations under different cases. It is found that the value of γ is decreasing with the standard deviation of due dates. Besides, it is also demonstrated that a larger deviation of the shipping lead-time induces a bigger descending slope.



a.Illustration for the case with smaller shipping lead-time deviation   b.Illustration for the case with larger shipping lead-time deviation

Figure 6: Illustrations of the relationship between γ and $std.(d_j, j \in J_s)$.

In the following, the conditions that a lower joint value-at-risk attains are discussed under two different cases, i.e. Case 1 and Case 2, which further indicate the risk-hedging policy for the shipment assignment.

Case 1: $P_{(d_j - a_s)} < \beta_s\ (\forall j \in J_s)$

When $P_{(d_j-a_s)} < \beta_s (\forall j \in J_s)$, which indicates $F_s^{-1}(P_s^*) \geq (d_j - a_s)\ (\forall j \in J_s)$, according to Proposition

2, the joint value-at-risk of the jobs assigned to shipment s is

$$VaR_{\beta_s,J_s} = \sum_{j\in J_s} c_j^{TA} q_j [F_s^{-1}(P_s^*) + a_s - d_j], \tag{33}$$

where the probability P_s^* is the root of

$$\sum_{j\in J_s} c_j^{TA} q_j [F_s^{-1}(P_s^*) + a_s - d_j] = \sum_{j\in J_s} c_j^{DC} q_j [d_j - a_s - F_s^{-1}(P_s^* - \beta_s)]. \tag{34}$$

According to Proposition 1, the following equation holds:

$$\sum_{j\in J_s} VaR_{\beta_s,j} = \sum_{j\in J_s} c_j^{TA} q_j [F_s^{-1}(P_{j,s}) + a_s - d_j]. \tag{35}$$

In addition, based on Lemma 3, there exists l to make the inequality $F_s^{-1}(P_{j(i)s}) \le F_s^{-1}(P_s^*) \le F_s^{-1}(P_{j(i+1)s})$ hold, in which the jobs are indexed in a non-decreasing order of their corresponding $F_s^{-1}(P_{j,s}) (\forall s \in S)$. Compared with $\sum_{j\in J_s} VaR_{\beta_s,j}$, we set *the possible increased tardiness cost in the extreme case* regarding the shipping lead-time t_s as follows:

$$\overline{\Delta_s} = \sum_{k=1}^{i} c_{j(k)}^{TA} q_{j(k)} [F_s^{-1}(P_s^*) - F_s^{-1}(P_{j(k)s})], \tag{36}$$

and *the possible decreased tardiness cost during "normal" cases* regarding the shipping lead-time t_s is given below:

$$\underline{\Delta_s} = \sum_{k=i+1}^{m} c_{j(k)}^{TA} q_{j(k)} [F_s^{-1}(P_{j(k)s}) - F_s^{-1}(P_s^*)]. \tag{37}$$

Thus, *the difference between the joint value-at-risk VaR_{β_s,J_s} and the sum of individual value-at-risk* $\sum_{j\in J_s} VaR_{\beta_s,j}$ can be expressed as follows:

$$\Delta_s = \underline{\Delta_s} - \overline{\Delta_s}. \tag{38}$$

If $\Delta_s = 0$, such shipment assignment achieves the shipment reliability level β_s without increasing the value-at-risk. If $\Delta_s > 0$, then it implies that the respective shipment assignment realizes the reduction on value-at-risk, and at the same time achieves the shipment reliability level, which is the most desirable solutions we seek for.

Fig. 7. illustrates the case when two jobs are assigned to one shipment. It shows the relative positions of both joint worst shipping lead-time $F_s^{-1}(P_s^*)$ and the individual worst shipping lead-times $F_s^{-1}(P_{j(1)s})$, $F_s^{-1}(P_{j(2)s})$. Given the slopes of the functions, i.e., $c_j^{TA} q_j$ and $c_j^{DC} q_j$ $(j \in \{j(1), j(2)\})$, they co-determine $\overline{\Delta_s}$ and $\underline{\Delta_s}$. Intuitively, if the position of $F_s^{-1}(P_s^*)$ is close to $F_s^{-1}(P_{j(1)s})$, then $\underline{\Delta_s} > \overline{\Delta_s}$, namely, the possible reduction on the tardiness cost of jobs $j(1)$ and $j(2)$ during "normal" cases is larger than the possible increase in the tardiness cost of jobs $j(1)$ and $j(2)$ under the extreme case. In this case, the joint value-at-risk is reduced. However, when the position of $F_s^{-1}(P_s^*)$ is close to $F_s^{-1}(P_{j(2)s})$, we get the opposite situation. So, the value of the joint worst shipping lead-time (i.e.,$F_s^{-1}(P_s^*)$) determines whether the value-at-risk in a joint manner will decrease or not.

The joint value-at-risk is the outcome after striking a balance among the possible increase and decrease

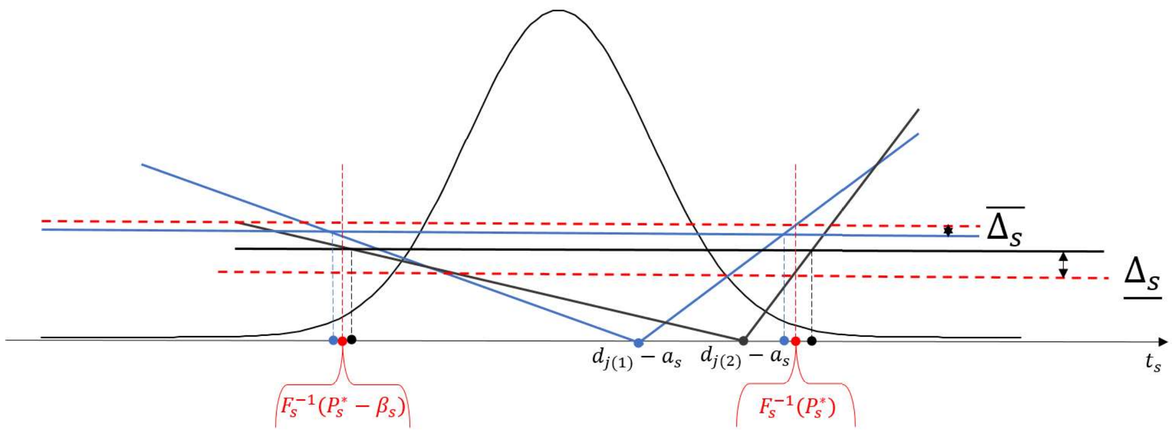


Figure 7: Difference of the joint value-at-risk and the sum of individual value-at-risks, i.e. $\Delta_s = \underline{\Delta_s} - \overline{\Delta_s}$.

in the costs of the jobs assigned together and realizes the best trade-off between the increase and decrease of earliness and tardiness costs so as to improve the systems reliability level and decrease the value-at-risk at system's level.

Based on Eqs. (36) and (37), we know that $\Delta_s > 0$ if and only if

$$F_s^{-1}(P_s^*) < \frac{\sum_{j \in J_s} c_j^{TA} q_j F_s^{-1}(P_{j,s})}{\sum_{j \in J_s} c_j^{TA} q_j}. \tag{39}$$

According to Proposition 3, inequality (39) holds if and only if $A - B > \gamma C$. For a given shipment s, C is a fixed number. The smaller γ is, the more chance for inequality (39) to be true. As γ is a non-increasing function in terms of the deviation of due dates of the jobs assigned together, it indicates that a lower value-at-risk can be achieved when assigning "distinct" jobs together. In addition, given the combined jobs, the deviated value $[A - B - \gamma C]$ turns large when the shipping deviation of the responsible shipment σ_s increases. In other words, when meeting the shipment with high uncertainty, the risk can be hedged through assigning "distinct" jobs together, e.g., the relatively high-penalty late-due-date job and high-storage-cost early-due-date jobs. It further implies that the jobs with similar parameters, such as those with close due dates and same earliness-tardiness cost ratio, are supposed not to be shipped together unless the respective shipment achieves the lowest individual value-at-risk and no other better shipment is available; otherwise, the similar jobs are supposed to be shipped by different small deviated shipments so as to achieve diversification of risk.

Case 2: $P_{(d_j - a_s)} > \beta_s$ $(\exists\, j \in J_s)$

Case 2 here reflects the situation that the on-time delivery probabilities of some of the jobs assigned to the same shipment s are approaching 1, which are at least more than β_s. For the case that there are

$|J_s| - i(1 \leq i < |J_s|)$ jobs with which the on-time delivery probabilities are greater than β_s, the condition $\Delta_s > 0$ holds. For the detailed proof, please refer to Appendix A3.

This result is consistent with the aforementioned findings, i.e., a lower value-at-risk can be achieved by assigning jobs with different due dates together. When extreme cases happen, such as a long time delay, the increased tardiness cost can be offset by the large reduction on the high storage costs of the jobs with which the on-time delivery probability is larger than β_s. On the other hand, for early arrival, the high storage costs of those jobs can be offset by the reduction on the tardiness costs by other jobs.

4.3.2. ***Illustrations under Uniform Distributions***

To explore more properties of the optimal shipment assignment policy, we explore the cases with the uniform distributions to help obtain useful managerial insights. Lemma 4 corresponds to Case 1 i.e., the on-time delivery probabilities of the jobs being assigned to the same shipment s are less than β_s (i.e., $P_{(d_j - a_s)} < \beta_s(\forall j \in J_s)$). If the shipping lead-time of shipment $s(\forall s \in S)$ follows the uniform distribution $U[a, b]$, the value-at-risk is as follows:

Lemma 4 (Uniform-distributed value-at-risk: case 1). Given a set of jobs J_s assigned to the shipment s, the corresponding value-at-risk VaR_{β_s, J_s} is:

$$VaR_{\beta_s, J_s} = \sum_{j \in J_s} c_j^{TA} q_j [\sum_{j \in J_s} w_j d_j - d_j + \frac{\sum_{j \in J_s} c_j^{DC} q_j \beta_s (b - a)}{\sum_{j \in J_s} (c_j^{TA} + c_j^{DC}) q_j}], \tag{40}$$

where $w_j = \frac{q_j (c_j^{TA} + c_j^{DC})}{\sum_{j \in J_s} (c_j^{TA} + c_j^{DC}) q_j}$.

Lemma 5 corresponds to the case when there are $|J_s| - i(1 \leq i < |J_s|)$ jobs with which on-time delivery probabilities are greater than β_s.

Lemma 5 (Uniform-distributed value-at-risk: case 2). Given a set of jobs $|J_s|$ being assigned to the same shipment s, which are sequenced in a non-decreasing order of their due dates, i.e., $j(1), j(2), ..., j(|J_s|)$: When there are $(|J_s| - i)$ jobs with which the on-time delivery probabilities are greater than β_s, i.e., $P_{d_{j(k)} - a_s} > \beta_s(k > i)$, the corresponding value-at-risk VaR_{β_s, J_s} is:

$$\begin{aligned} VaR_{\beta_s, J_s} = & \sum_{k=1}^{i < |J_s|} c_{j(k)}^{TA} q_{j(k)} [\sum_{k=1}^{i} w_{j(k)} d_{j(k)} - d_{j(k)} + \frac{\sum_{k=1}^{|J_s|} c_{j(k)}^{DC} q_{j(k)} \beta_s (b - a)}{\sum_{k=1}^{i} (c_{j(k)}^{TA} + c_{j(k)}^{DC}) q_{j(k)}}] \\ & + \sum_{k=i+1}^{|J_s|} c_{j(k)}^{DC} q_{j(k)} [d_{j(k)} - \sum_{k=1}^{i} w_{j(k)} d_{j(k)} - \frac{\sum_{k=1}^{|J_s|} c_{j(k)}^{DC} q_{j(k)} \beta_s (b - a)}{\sum_{k=1}^{i} (c_{j(k)}^{TA} + c_{j(k)}^{DC}) q_{j(k)}}], \end{aligned} \tag{41}$$

where $w_{j(k)} = \frac{q_{j(k)} (c_{j(k)}^{TA} + c_{j(k)}^{DC})}{\sum_{k=1}^{i} (c_{j(k)}^{TA} + c_{j(k)}^{DC}) q_{j(k)}} (k \leq i)$.

Based on the results of Lemmas 4 and 5, we further analyze the influences of different factors on the value-at-risk incurred by each shipment assignment.

Proposition 4 (Influence of due dates and cost ratios of earliness and tardiness (E-T)).

a. There exist $j_1, j_2 \in J_s(j_1 \neq j_2)$, satisfying $\frac{\partial VaR_{\beta_s,J_s}}{\partial d_{j_1}} > 0$, $\frac{\partial VaR_{\beta_s,J_s}}{\partial d_{j_2}} < 0$;

b. $\frac{\partial VaR_{\beta_s,J_s}}{\partial d_{j_1}} > 0$ *iff* $\frac{c_{j_1}^{DC}}{c_{j_1}^{TA}} > \frac{\sum_{j\in J_s} c_j^{DC} q_j}{\sum_{j\in J_s} c_j^{TA} q_j}$;

c. $\frac{\partial VaR_{\beta_s,J_s}}{\partial d_{j_2}} < 0$ *iff* $\frac{c_{j_2}^{DC}}{c_{j_2}^{TA}} < \frac{\sum_{j\in J_s} c_j^{DC} q_j}{\sum_{j\in J_s} c_j^{TA} q_j}$.

Proposition 4 verifies the impacts of due dates and E-T cost ratios of the jobs being assigned to the same shipment s on the corresponding value-at-risk. First, Proposition 4(a) demonstrates that the optimal strategy to lower the value-at-risk as much as possible is to assign together the jobs with distinct due dates. Later due date jobs and earlier due date jobs are preferred to be put together to lower value-at-risk rather than all late or early due date jobs. This result is consistent with Proposition 3, i.e., a large deviation of the due dates of the jobs being assigned to the same shipment reduces the joint value-at-risk. Propositions 4(b) and 4(c) further present the sufficient and necessary conditions governing what kinds of jobs' due dates should be as early (late) as possible. For instance, if a job $j_1(j_1 \in J_s)$ has an E-T cost ratio $\frac{c_{j_1}^{DC}}{c_{j_1}^{TA}}$ which is large enough, i.e., $\frac{c_{j_1}^{DC}}{c_{j_1}^{TA}} > \frac{\sum_{j\in J_s} c_j^{DC} q_j}{\sum_{j\in J_s} q_j}$, then an earlier due date will further reduce the joint value-at-risk. However, for a job j_1 with a relatively small E-T cost ratio, i.e., $\frac{c_{j_1}^{DC}}{c_{j_1}^{TA}} < \frac{\sum_{j\in J_s} c_j^{DC} q_j}{\sum_{j\in J_s} q_j}$, then a later due date can reduce the joint value-at-risk. For Case 2, similar results can be obtained. To avoid duplication, the corresponding proposition is presented in the supplementary appendix.

Remark: Given the assigned shipment, the optimal combination of the jobs being assigned (together to attain the lowest value-at-risk) is to assign the higher-earliness-cost-earlier-due-date jobs with higher-tardiness-cost-late-due-date jobs.

Proposition 5 (Influence of the standard deviation of the shipping lead-times). Given that the shipping lead-times $t_s(s \in S)$ follow the uniform distribution $U[a,b]$, we have:

a. $\frac{\partial VaR_{\beta_s,J_s}}{\partial a} < 0$ and $\frac{\partial VaR_{\beta_s,J_s}}{\partial b} > 0$ *iff* $\sum_{k=1}^{i} c_{j(k)}^{TA} q_{j(k)} > \sum_{k=i+1}^{|J_s|} c_{j(k)}^{DC} q_{j(k)}$;

b. $\frac{\partial VaR_{\beta_s,J_s}}{\partial a} > 0$ and $\frac{\partial VaR_{\beta_s,J_s}}{\partial b} < 0$ *iff* $\sum_{k=1}^{i} c_{j(k)}^{TA} q_{j(k)} < \sum_{k=i+1}^{|J_s|} c_{j(k)}^{DC} q_{j(k)}$;

for $1 \leq i < |J_s|$, satisfying$P_{(d_{j(i)}-a_s)} < \beta_s, P_{(d_{j(i+1)}-a_s)} > \beta_s$.

Proposition 5 presents the analytical results which relate to the influences brought by the standard deviations of the uniform-distributed shipping lead-times on the joint value-at-risk. Here, the case of $i = |J_s|$ corresponds to Case 1 , which always satisfies the condition in Proposition 5(a). As demonstrated by Proposition 5(a), for common cases when the sum of daily tardiness costs of the jobs (with which the on-time delivery probabilities are less than β_s, i.e., $P_{(d_j-a_s)} < \beta_s$), are greater than the sum of daily earliness costs of the jobs (with which the on-time delivery probabilities are greater than β_s i.e., $P_{(d_j-a_s)} > \beta_s$), the standard deviation of the shipping lead-time has a negative impacts on the joint value-at-risk. However, if some special cases happen, namely, the sum of daily earliness costs of the jobs satisfying $P_{(d_j-a_s)} > \beta_s$ are greater than the sum of daily tardiness costs of the jobs satisfying $P_{(d_j - a_s)} < \beta_s$, then a larger standard deviation of the shipping lead-time may mitigate the joint value-at-risk.

According to the analytical results and findings obtained in Section 4.3 (from both general and illustrative cases), our proposed shipment assignment policy with risk hedging can be stated as follows:

Theorem 1 (The Risk-hedging Shipment Assignment Policy[8]).

i. Diversify shipments when there are enough reliable shipments;

ii. Diversify jobs on the same shipment if shipping with low-reliability shipments is inevitable.

Theorem 1 indicates the interactions between the jobs and the shipments to attain risk mitigated assignments. The ideal situation is that there are enough suitable shipments with on-time delivery. However, when shipping uncertainties exist, the traditional expected method may not be wise. If the shipments being available are highly reliable, then it is better to assign the distinct jobs separately to different reliable shipments. However, when shipping with low-reliability shipments is inevitable, a risk-hedging combination of the jobs become significant. For instance, when facing the peak season of the shipping market and the shipping conditions are getting worse, the shipping uncertainty level gets increased, the jobs with distinct characteristics (i.e.,due dates and E-T cost ratios) should be shipped together for risk hedging. This is especially important for the case when jobs with early due dates have high unit storage costs, and jobs with late due dates have relatively high unit tardiness penalties. However, for those jobs with close or same due dates, they should be shipped separately unless there is no other reliable shipment being available.

5. Linearization Based Approximation Methods and Computational Studies

In this section, the contents are mainly divided into three parts. Firstly, two risk-averse models are proposed for comparison so as to demonstrate the advantage of the proposed risk-hedging shipment assignment policy. They are solved by exact algorithms. The first risk-averse model is based on Proposition 1, which relates to the model proposed in the literature (e.g., see Sun et al. (2018)). In this model, the value-at-risk is considered individually, from the perspective of individual reliability of each job. Thus we call it the individual value-at-risk model (IVaR). The model is presented in details in Section 5.1. The second risk-averse model is based on the proposed risk-hedging policy, which is the approximated deterministic counterpart (ADC) of the formulation in Section 3.3. It reconsiders the value-at-risk from the perspective of jobs-shipment combination with which the joint value-at-risk is involved. The reliability of a job's com-

[8] In our problem, the shipment unit for each order is TEU, i.e., 20-foot ISO container. Thus, the total number of containers to be shipped will not change no matter they are put to ship together or not. On the other hand, the shipment cost is calculated based on the number of containers (Please check the objective function Eq. (9)). The loading and unloading charge relate to number of containers which includes in the freight rate of each order. Therefore, no more specific operations cost will be incurred. We do not consider the case when several orders are put into one container and shipped together.

bination is further taken into account. Hence, the second risk-averse model ADC utilizes a more general risk measure comparing with IVaR. Due to the complexity of ADC, new variables and parameters are introduced in Section 5.2. Computational studies are carried out in Section 5.3 so that more managerial insights can be obtained.

5.1. *Individual Value-at-Risk Model (IVaR)*

In this model, the objective function is stated below:

$$\begin{aligned} \min Z_2 = &\sum_{j\in J} q_j(c_j^{PR} + c_j^{WH}) + \sum_{j\in J}\left\lceil \frac{q_j}{L^{max}} \right\rceil (c_j^{TR} + c_j^{LS}) \\ &+ \omega \sum_{l\in L}\sum_{j\in J}\sum_{s\in S} y_{j,l,s} c_j^{TA} q_j [F_s^{-1}(P_{j,s}) + a_s - d_j] \end{aligned} \tag{42}$$

s.t. (10)-(22),

where $F_s^{-1}(P_{j,s})$ satisfies (25). In the objective function, the only non-linear term is $F_s^{-1}(P_{j,s})$. Based on (25), the model can be linearized by preprocessing $F_s^{-1}(P_{j,s})$ in a computational way and treat it as the input data.

5.2. *Approximated Deterministic Counterpart (ADC)*

Based on Propositions 2 and 3, the objective function (9) can be rewritten into its deterministic equivalence as follows:

$$\begin{aligned} \min Z_3 = &\sum_{j\in J} q_j(c_j^{PR} + c_j^{WH}) + \sum_{j\in J}\left\lceil \frac{q_j}{L^{max}} \right\rceil (c_j^{TR} + c_j^{LS}) \\ &+ \omega(\sum_{l\in L}\sum_{j\in J}\sum_{s\in S} c_j^{TA} q_j y_{j,l,s} [F_s^{-1}(p_s^*) + a_s - d_j]^+ + \sum_{l\in L}\sum_{j\in J}\sum_{s\in S} c_j^{DC} q_j y_{j,l,s} [F_s^{-1}(p_s^*) + a_s - d_j]^-) \end{aligned} \tag{43}$$

s.t.

$$\begin{aligned} &\textstyle\sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{TA} q_j [F_s^{-1}(P_s^*) + a_s - d_j]^+ + \sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{DC} q_j [F_s^{-1}(P_s^*) + a_s - d_j]^- \\ &= \textstyle\sum_{l\in L}\sum_{j\in J} y_{j,l,s} c_j^{DC} q_j [d_j - a_s - F_s^{-1}(P_s^* - \beta_s)], \forall s \in S. \end{aligned} \tag{44}$$

and (10)-(22).

Due to the structure of $F_s^{-1}(P_s^*)$, the objective function (43) is non-convex and there is no equivalent linearization counterpart. We thus approximate $F_s^{-1}(P_s^*)$ with a linear function of decision variables $y_{j,l,s} (\forall l \in L; \forall j \in J; \forall s \in S)$, so that the non-convex problem is converted into a quadratic program. It is further transferred into an integer linear program via the standard linearization techniques (Glover, 1975; Hansen and Meyer, 2009), so as to make it solvable by the optimization solver, e.g., Cplex. As mentioned above, we assume

$$F_s^{-1}(P_s^*) \approx \widehat{F}_s^{-1}(P_s^*) = \frac{\sum_{l\in L}\sum_{j\in J} c_j^{TA} q_j y_{j,l,s} F_s^{-1}(P_{j,s})}{m \cdot Avg.(c_j^{TA} q_j)} + \varepsilon_{s,m},$$

in which m refers to the number of jobs assigned to shipment s, $Avg.(c_j^{TA} q_j)$ is the average penalty cost of total jobs, and $\varepsilon_{s,m}$ is a non-increasing function with respect to the penalty costs of the jobs assigned to