

Optimal Procurement Decision with a Carbon Tax for the Manufacturing Industry

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Abstract

A carbon tax, which has been implemented in several countries, is a cost-effective scheme for reducing carbon emission and developing sustainable supply chains. Two problems, how to make the optimal decision on order quantity and how to select appropriate suppliers for a manufacturer, are studied in this paper in consideration of a carbon tax. For the first problem, a dynamic programming model is developed to study the impact of the carbon tax on calculating the optimal order quantity. In reality, the manufacturer could choose a traditional or a greener supplier. The greener supplier is relatively expensive but yields lower emissions. To obey the emission regulations, the manufacturer should pay for the cost which is incurred by carbon emission. Firstly, in this paper, the expected emission cost is formulated, then, the structural properties of the model are derived. In particular, the optimal order quantity is characterized to minimize the expected total discounted cost. In addition, the effective range of the carbon tax is established to assist government to setup a reasonable carbon tax for a certain industry. For the second problem, a supplier evaluation procedure is proposed to select appropriate suppliers to satisfy the random market demand for the manufacturer. A numerical example from the metal industry is taken to illustrate the properties of the model and the procedure of supplier evaluation. Finally, possible extensions of the model are discussed.

Keywords Carbon tax; Procurement management; Supplier selection; Dynamic programming

1. Introduction

Nowadays, many business firms have realized the need to improve their social responsibility, especially for the carbon-intensive firms. Facing environmental regulations, a firm needs to take a

series of activities, such as, reducing pollutions, carbon emission, to promote sustainable development. In 2013, the Carbon Disclosure Project (CDP) conducted its annual investigation for managing carbon emission. In that project, more than 6,000 suppliers participated in the survey. According to the results of that investigation, an increasing number of suppliers realized the benefits in both monetary savings and emissions reduction. In addition, the results show that 29% of suppliers reported emissions reduction in 2012, while one year ago, it was only 19%. In 2013, 69% of CDP members made investment in reducing greenhouse gas (GHG) emissions while this figure was only 39% in 2011 (Carbon Disclosure Project, 2013).

A carbon tax has been taken as a cost-effective scheme to reduce GHG emissions in a number of countries. As a typical example, British Columbia (B.C.) implemented the carbon tax on July 1, 2008 at a rate of C\$10 per ton of CO₂. In July 2014, the B.C. carbon tax was increased to \$25 per ton of CO₂ (Carbon Tax Center, 2015). The implementation of this scheme could incent companies to improve themselves, such as adopting technologies for cleaner production, and using environment-friendly raw materials or products. It also directly impacts on their cost structure, production planning, procurement management and so on. With respect to the current situation, significant research questions should be considered: (i) how would the carbon tax affect the optimal order quantity for a manufacturer? (ii) how should a government implement an effective range of the carbon tax? (iii) how should a manufacturer select appropriate suppliers to satisfy the production demand? These practical questions are key factors to develop sustainable supply chains for the manufacturing industry.

First of all, in order to comply with the carbon tax scheme, the expected emission cost is formulated to describe the cost of emissions which is incurred during the production processes. The holding cost for leftover products and the penalty cost for backlogging products are also considered. Therefore, to answer these questions, we formulate problems as a dynamic programming model and derive its properties to show the preservation of convexity of objective function after minimization. On the basis of structural properties of the model and the constraint of carbon tax, the optimal order quantity is derived to minimize the expected total discounted cost for the manufacturer. Then, the effective range of the carbon tax is proposed to assist the government to setup a reasonable carbon tax. As the manufacturer could purchase raw materials from multiple

suppliers to satisfy its demand, in this paper, we further established a supplier evaluation procedure to select appropriate suppliers based on the optimal order quantity.

The remainder of this paper is organized as follows. In Section 2, the related literature is provided from three streams including green supply chain management, inventory control with emissions constraints, and green supplier selection. In Section 3, the model setting and its formulation are introduced in detail. In Section 4, the structural properties of the model are analyzed. Then, the optimal ordering quantity and the effective boundary of carbon tax are characterized. In addition, a supplier selection procedure is proposed to select appropriate suppliers to satisfy the random market demand for the manufacturer. A numerical example using the data from the metal industry is presented. In Section 5, the possible extensions of our model are discussed. Section 6 concludes the paper. All the proofs are provided in the Appendix.

2. Literature Review

This section reviews some related papers from three streams including green supply chain management (GSCM), inventory control with emissions constraints, and green supplier selection. In addition, the differences between this paper and other representative papers are pointed out.

The first stream, GSCM, is the motivation of this paper. In reality, the motivation for the introduction of GSCM could be ethical and commercial reasons (Testa and Iraldo, 2010). Starting from the viewpoint of supply chain management (SCM), Seuring and Müller (2008) suggested that sustainable SCM could not only affect management of material, information, and capital flows, but could also achieve the goals of the triple bottom line (environmental, social, and economic). Gavronski and Klassen et al. (2011) proposed that GSCM was the complex mechanisms. These mechanisms can be implemented at the corporate and plant level to assess or improve the environmental performance of a supplier base. Sarkis and Zhu et al. (2011) defined GSCM as integrating environmental concerns into the inter-organizational practices of SCM, for instance, reverse logistics. Given the characteristics of GSCM, Kim and Rhee (2012) integrated four aspects which included green purchasing, green manufacturing/materials management, green distribution/marketing, and reverse logistics into GSCM. Similarly, Srivastava (2007) put forward a more detailed framework which also integrated environmental criteria, including product design, material sourcing and selection, manufacturing processes, delivery of final products to consumers,

and the end-of-life management of the product, into SCM. Recently, Barari and Agarwal et al. (2012) discussed GSCM from the viewpoint of economy and ecology. They also suggested that making profits and achieving an ecological balance were the objectives of GSCM. Around the same time, Zhu and Sarkis et al. (2012) described GSCM from another viewpoint. They pointed that GSCM can be defined as “an emergent environmentally sustainable organizational technological innovation”. Recently, Tseng and Chiu et al. (2013) proposed a similar concept about GSCM with the triple bottom line, from different aspects, they stated that ethics was also important through the whole supply chain. The aforementioned papers described the different characteristics of GSCM, but, they did not focus on a specific industry. Establishing appropriate regulations for a certain industry is desperately needed. For the manufacturing industry, developing sustainable operations can be improved from two aspects including the procurement of environmental raw materials and the optimization of manufacturing processes.

The second stream is inventory control with emission regulations. As pioneers, Hua et al. (2011) studied the economic order quantity (EOQ) model under the mechanism of emissions trading. In their model, a key factor, transfer quantity of carbon emission, was integrated into the traditional EOQ model. In addition, the amount of emissions incurred by transportation and holding processes was described by linear relations with the order size. Based on the basic model, the optimal order quantity, the emission price, and the quotas of carbon emission are derived. Jaber et al. (2013) focused on the GHG emissions from the viewpoint of manufacturing processes. They developed a mathematical programming model with a constant demand, and the objective function of the model consists of the supply chain cost, the emission cost, and the penalty cost for exceeding the allowed limit. The emissions load was expressed as a convex function of the production rate or equipment speed. The cost functions of buyers were derived from the basic EOQ model. Absi et al. (2013) developed a polynomial dynamic programming model to analyze the lot-sizing problems with environmental constraints and a deterministic demand. Four types of constraints including periodic carbon emission constraint, cumulative carbon emission constraint, global carbon emission constraint, and rolling carbon emission constrain were analyzed in their model. Choi (2013) studied supplier selection under the carbon taxation scheme. The carbon emission cost function is mainly related to the distance between suppliers and buyers. The optimal order quantity was derived from the basic newsvendor model while considering the carbon tax.

Emission cost was formulated as a monotonic increasing function with respect to the transportation distance. In addition, some possible extensions of the model were analyzed by Choi (2013) under the buyback contract. Chen X. et al. (2013) studied a carbon-constrained EOQ model, which is described by using a mathematical programming model. The objective function is composed of the traditional cost including setup cost, holding cost, and purchasing cost. However, the emissions were constrained by the carbon cap. Based on the optimal order quantities of the EOQ model and carbon-constrained EOQ model, the carbon prices were further analyzed under different carbon rules. Konur et al. (2014) developed a similar model based on the basic EOQ model. In addition, they compared two transportation scenarios, less-than-truckload (LTL) and trucked (TL), under the influence of different carbon emission regulations. They analyzed the optimal transportation mode under each regulation. However, in our paper, we focus on the emissions incurred from the operations process in the manufacturing industry. The expected emission cost is derived based on a production process with exponential production time. In addition, the total discounted cost was developed with the random demand.

The third stream is green supplier selection. In SCM, evaluation and selection of appropriate suppliers is a significant issue which has attracted many researchers since the 1950s. With respect to green supplier selection, Noci (1997) is a pioneer who proposed a green supplier ranking system to evaluate the environmental performance of suppliers. Lu et al. (2007) integrated the analytic hierarchy process (AHP) and a fuzzy method to select green suppliers. Similarly, Lee et al. (2009) applied fuzzy AHP to select green suppliers for the high-tech industry. In the light of the interrelationships among different criteria, Hsu and Hu (2009) adopted an effective method, analytic network process (ANP) to evaluate green suppliers instead of AHP. Kuo et al. (2010) built an integrated model which considered artificial neural networks (ANN), ANP, and data envelopment analysis (DEA) for green supplier selection. In a case study on a camera manufacturer, they compared integrated methods: ANN–DEA and ANP–DEA. They found that ANN–DEA had a better capability to evaluate supplier performance. Büyüközkan et al. (2012) proposed a novel hybrid green supplier evaluation framework which integrated the fuzzy decision making trial and evaluation laboratory model (DEMATEL), fuzzy ANP and fuzzy TOPSIS. In addition, Zhou et al. (2012) developed a green supplier selection model for the chemical industry, based on ANP, in consideration of the complexity of the evaluation process. After ANP evaluation,

the model incorporated the radial basis function (RBF) neural network into the alternative selection process. This two-stage integrated model improved the dynamic assessment capability. Similarly, a grey ANP-based model was suggested by Dou et al. (2014) to select and improve green suppliers and their performance. Recently, Kumar et al. (2014) developed a green DEA model, which is based on DEA with carbon footprint monitoring, for supplier selection. Ji et al. (2015) exploited evolutionary game theory to study the green procurement relationship between suppliers and manufacturers (buyers). These researchers investigated both qualitative and quantitative aspects of green supplier selection based on suppliers' comprehensive values. In this paper, on the basis of the result of dynamic programming (the optimal order quantity), an evaluation procedure was established to select appropriate suppliers.

3. The Model

In this section, a dynamic programming model is developed. The model aims at seeking the best trade-off between procurement management and inventory control in consideration of the carbon tax. To be more specific, during each time period, in order to minimize its total operating cost, the manufacturer should make several inter-related decisions, including inventory control, production planning, and procurement management. The manufacturer purchases one type of raw materials from several suppliers to produce a product for satisfying its random demand over a finite planning horizon. In order to control carbon emission, some governments have adopted and implemented a series of regulations, such as the carbon tax. Therefore, in the light of the carbon emission from the manufacturing process, the manufacturer has to pay for the carbon tax to obtain the emission right. In reality, until 2014, the carbon tax has been successfully implemented in many countries, such as United Kingdom (\$15.75/tCO₂e), Switzerland (\$68/t CO₂e), Denmark (\$31/tCO₂e), Iceland (\$10/tCO₂e), and, Sweden (\$168/tCO₂e), etc (World Bank, 2014). In the first decision phase, the manufacturer determines the optimal order quantity based on the market demand and operating cost. Then, the manufacturer needs to select appropriate suppliers with respect to green quality and supply capacity. The following sub-sections introduce detailed cost structure of the manufacturer.

Table 1. The notation of variables

Variables	Description
t	index of time period, $t = 1, 2, \dots, N$
p_t	unit penalty cost in period t
h_t	unit holding cost in period t
c_t	unit production cost in period t
δ_t	the parameter of production time
α	the discount rate of emission cost
x_t	inventory level before ordering in period t
y_t	inventory level after ordering in period t
q_t	the order quantity in period t
E	unit emission factor
GQ	the value of green quality of a supplier
ET	the value of carbon tax
ET^U	the upper bound of carbon tax
ET^L	the lower bound of carbon tax
$Z(\cdot)$	the expected loss cost
$EC(\cdot)$	emission cost in a certain period
$EEC(\cdot)$	the expected emission cost over time interval
D_t	unit demand in period t
G_t	total discounted cost (objective function)

3.1 Carbon emission related cost

In each period, the manufacturer purchases one type of raw materials from several suppliers. After manufacturing, the manufacturer sells the final products to satisfy the random market demand. According to the IPCC guidelines (1996), the production-based emission factors are introduced to calculate the amount of GHG emissions which are incurred during the manufacturing processes. In practice, alloy is used to produce many advanced materials, for instance, alloy can be applied to the extremely high temperature service. Production of various types of ferroalloy generates different amounts of CO₂, for example, production of one ton of silicon metal generates 5 tons of CO₂, while, production of one ton of ferrosilicon with 45% silicon generates 2.5 tons of CO₂. In addition, the manufacturer is constrained by the environmental laws or regulations. Therefore, the manufacturer should pay for the carbon tax to get the emission right. In a finite production period, the expected cost of carbon emission is formulated in consideration of the characteristic of manufacturing processes. The cost of carbon emission can be used to help the manufacturer to balance its total cost and to decide the optimal

order quantity. The formulation of emission cost is shown in Proposition 1. Here, we suppose that the production time is exponentially distributed, that is, the manufacturing system is exactly the way it did when the previous state happened. In the next period, the time is still exponentially distributed with the same mean value. Therefore, the manufacturer can make a new decision only based on the change of system state. Ha (1997) also adopted this model setting. In addition, the mean value of exponentially distributed time indicates the average level of the processing time for manufacturing products. The average processing time (or the mean) trends small if δ_t gets larger.

Proposition 1. *For the case of exponential production time with parameter δ_t , the emission cost of producing one product in time period t can be expressed as follows.*

$$EEC(q_t) = E(e^{\alpha t} \cdot EC(q_t)) = \frac{\sum_{t=1}^N \delta_t \cdot EC(q_t)}{\alpha + \sum_{t=1}^N \delta_t}, \quad (1)$$

where $EC(q_t) = E \cdot q_t \cdot ET$ and $q_t = y_t - x_t$.

Proposition 1 describes the analytical expression of the expectation value of the emission cost for the manufacturer. α is a discount rate of emission cost for the scenario of multi-periods. The emissions' cost for an order is denoted as (EC) , which equals the amount of emissions times the unit carbon tax (ET) . The amount of emissions equals the unit emission factor (E) for manufacturing one product times the raw material ordering quantity (q_t) . It is clear to see that the emission cost is mainly decided by the variables of q_t and ET . Thus, the manufacturer should determine the optimal order quantity to minimize the emission cost. In reality, the British Columbia's carbon tax was established since 2008 at C\$10 a ton, this figure has reached C\$30 per ton in 2012 with C\$5 increment per year. Therefore, this phenomenon shows that there exists a monotone-increasing trend of carbon tax. In practice, the manufacturer can implement the production planning without prior knowledge of carbon tax. That is, no matter the structure of carbon tax is linear or nonlinear; it is worth analyzing how carbon tax impacts on ordering quantity in a dynamic scenario. The following content aims to analyze the interaction between the optimal order quantity and the carbon tax.

3.2 Loss cost

During period t , purchasing one type of raw materials incurs ordering cost c_t from the manufacturer. In addition, at the end of each period, leftover raw materials incur holding cost h_t . Besides, the excessive amount of demand will be backlogged. The unit penalty cost for the backlogging products is p_t . In order to avoid ordering nothing as an optimal policy, the penalty cost is assumed to be larger than ordering cost. The demand of raw materials from the manufacturer is a continuous random variable which can be described by using the continuous and strictly positive function. The probability density function (pdf) of each type of raw materials demand is denoted by φ and the cumulative distribution function (cdf) is denoted by Φ . The expression of loss cost is shown as follows.

$$Z(y_t) := \int_0^{y_t} h_t \cdot (y_t - \xi) \cdot \varphi(\xi) d\xi + \int_{y_t}^{\infty} p_t \cdot (\xi - y_t) \cdot \varphi(\xi) d\xi \quad (2)$$

Before ordering, the on-hand inventory level of the raw material is denoted as x_t , and its order quantity in each period is denoted as y_t . Based on the above analysis, the expected holding and shortage costs function can be modeled in Equation (2). The decision variable in Equation (2) is y_t . When y_t is smaller than D_t , the holding cost h_t is charged. When D_t is smaller than y_t , the penalty cost p_t is charged.

3.3 Model Formulation

The above two sub-sections introduced the carbon emission related cost and loss cost. Based on them, a dynamic cost framework can be developed as follows. Let $G_t(x_t)$ represents the total expected discounted cost with parameter x_t in period t , and $G_{t+1}(x_t)$ denotes the terminal cost, which consists of the opportunity cost and the emission cost. The opportunity cost means the value of unsold products in the terminal period. If the stock level in the terminal period is zero, then the related cost is zero. If the stock level is larger than zero, the emission cost and the holding costs should be charged. Suppose the manufacturer is risk neutral, then, the operating cost over N periods can be expressed by using the discounted cost with a discount factor β ($0 < \beta < 1$). The expression of $G_t(x_t)$ is shown in Equation (3).

$$G_t(x_t) = \min_{y_t \geq x_t} \{c_t \cdot (y_t - x_t) + Z(y_t) + EEC(q_t) + \beta \int_0^{\infty} G_{t+1}(y_{t+1} - \xi) \varphi(\xi) d\xi\}, \quad (3)$$

where

$$G_{t+1}(x) := -(c_{t+1} + E \cdot EF)x. \quad (4)$$

The first term in Equation (3) is the ordering cost of a product in time period t . The second term, as shown in Equation (3), is the expected holding and shortage costs. The third term is the expected emission cost which is illustrated in Proposition 1. The last term in Equation (3) is the discounted value of the cost in time period $t + 1$. In order to minimize the total cost of the manufacturer and to get optimal order quantity, the expression of $G_t(x_t)$ can be formulated as shown in Equation (5) and Equation (6).

$$G_t(y_t) = \min_{y_t \geq x_t} \left\{ H(y_t) - \left(c_t + \frac{E \cdot ET \cdot \sum_{t=1}^N \delta_t}{\alpha + \delta_t} \right) \cdot x_t \right\} \quad (5)$$

where

$$H(y_t) := c_t \cdot y_t + \frac{E \cdot ET \cdot \sum_{t=1}^N \delta_t}{\alpha + \delta_t} \cdot y_t + Z(y_t) + \beta \int_0^\infty G_{t+1}(y_{t+1} - \xi) \cdot \varphi(\xi) d\xi \quad (6)$$

In each time period, a manufacturer needs to make the optimal decision to solve the issue of how to coordinate inventory control, procurement management, and production planning, that is, the manufacturer should determine the optimal purchasing quantity of raw materials and select appropriate number of suppliers to minimize its operating cost under the carbon tax plan. In the following section, the structural properties of the model and the optimal order quantity of the manufacturer are studied.

4. Model discussions

4.1 The structural properties

In this section, the structural properties of the model in Equation (5) and Equation (6) are studied. First of all, the convexity of cost function in a single period is illustrated. Then, this property is proved in the condition of finite horizon. Based on these properties, the optimal order quantity is derived. In addition, the effective range of the carbon tax is established. Theorem 1 is applied to prove the convexity of cost function.

Theorem 1. *Let $f(x)$ is a convex function with second derivative defined on \mathbf{R} . $F(x)$ is a function, $F(x) := \mathbf{E}_D f(x - D)$, on \mathbf{R} , D is a random variable with pdf φ . Thus, $F(x)$ is a convex function on \mathbf{R} .*

Proposition 2. *If G_{t+1} is a convex function,*

(a) $H(y_t)$ is a convex function;

(b) in period t , there is an optimal ordering quantity, which can minimize the cost including ordering cost, emission cost, and holding cost.

According to the structural property of Equation (5), the minimal value of $G_t(x_t)$ is predominantly determined by the value of $H(y_t)$ and the on-hand inventory level x_t . Proposition 2 firstly examines the convexity of $H(y_t)$. Then, the existence of the optimal order quantity (y_t) is proved. In part (a) of Proposition 2, Theorem 1 is applied to verify the convexity of $H(y_t)$, which is the fundamental step to analyze the optimal order quantity for a certain period. In each single time period, $H(y_t)$ reflects a series of cost factors including ordering cost, holding cost, shortage cost, emission cost, and discounted cost. These costs are all incurred by the order quantity. In addition, the convexity of $H(y_t)$ indicates that there is a corresponding lower bound of the total cost, which means that there is a potential opportunity for the manufacturer to save money. With respect to the operations process, part (b) examines the existence of the optimal decision for the manufacturer. The logic of solving the optimal order quantity is as follows. For a certain period t , let y_t^* denote the optimal order quantity, then, the set of $\{y_t^*, H(y_t^*)\}$ is the lowest point of the objective function. If the purchasing decision is smaller (larger) than y_t^* , then, the manufacturer should increase (decrease) the ordering quantity to y_t^* . Thus, y_t^* is the optimal decision for the manufacturer to minimize its total cost in time period t .

The optimal order quantity of the manufacturer specifies that the order quantity from suppliers is determined by the results of the optimization result in Equation (5). Based on Proposition 2, in time period t , although the first item in the objective function is convex, this optimal order quantity cannot ensure that this value is the optimal choice for a finite horizon. Therefore, in the following analysis, whether the optimal order quantity could minimize the value of the objective function for multiple periods is verified.

Proposition 3. For $t = 1, 2, \dots, N$, if G_{t+1} is a convex function, then, G_t is a convex function and there exists the optimal order quantity in a finite horizon.

With respect to the structure of the G_t , the characteristic of convexity of Equation (6) is proved first. Because the summation of two convex functions is still a convex function, thus, G_t is convex. In addition, the convexity of G_t discloses a multi-period optimization issue. Based on the results in part (a) of Proposition 2, the convexity of G_t and the terminal cost function (G_{t+1}), the optimal order quantity y_t is also an optimal solution for the entire finite time horizon because the convexity preservation exists under the situation of minimization. Therefore, during the operations process, the implementation of recursive procedure can be used to make the optimal ordering decisions over a finite horizon.

4.2 The operational decisions

The above propositions analyze the properties of the dynamic model in Equation (5) and prove the existence of the optimal order quantity under the carbon tax in a finite horizon. With respect to the uncertainty of demands and the carbon tax, the mathematical formulation for the optimal order quantity and the effective range of the carbon tax are derived.

Proposition 4. For $t = 1, 2, \dots, N$, the optimal order level \tilde{y}_t minimizes $H(y_t)$ of the manufacturer over the finite period,

(a) the optimal order quantity is

$$q_t^* = \Phi^{-1}\left(\frac{p_t + \beta \cdot e \cdot ET - (1 - \beta) \cdot c_t - \frac{\sum_{t=1}^N \delta_t \cdot E \cdot ET}{\alpha + \sum_{t=1}^N \delta_t}}{h_t + p_t}\right) - x_t; \quad (7)$$

(b) the effective range of the carbon tax is

$$\frac{h_t + (1 - \beta) \cdot c_t}{\sum_{t=1}^N \delta_t \cdot E / (\alpha + \sum_{t=1}^N \delta_t) - \beta \cdot E} \leq ET \leq \frac{p_t + (1 - \beta) \cdot c_t}{\sum_{t=1}^N \delta_t \cdot E / (\alpha + \sum_{t=1}^N \delta_t) - \beta \cdot E} \quad (8)$$

Proposition 4 aims at solving the mathematical structure of the optimal order quantity which can minimize the $H(y_t)$ for the manufacturer. The optimal order quantities of $H(y_t)$ can be obtained with the critical quantile as shown in Equation (9).

$$\Phi(y_t) = \frac{p_t + \beta \cdot (c_t + e \cdot ET) - c_t - \frac{\sum_{i=1}^N \delta_i \cdot E \cdot EF}{\alpha + \sum_{i=1}^N \delta_i}}{h_t + p_t} \quad (9)$$

The critical quantile in Equation (9) should locate in a closed interval which is from zero to one. Therefore, the numerator in Equation (9) should be larger than zero and smaller than the denominator. As a result of the simplification, the sufficient requirement ($ET^L < ET < ET^U$) can be obtained with respect to the constraint of the carbon tax, where ET^L and ET^U denote the lower and the upper bound of carbon tax, respectively. In addition, the effective range of the carbon tax can provide meaningful suggestions for the governments to design efficient and reasonable taxation schemes with various emissions factors, especially for different carbon-intensive industries. Establishing different types of taxation schemes could make implementation of the carbon tax more reasonable.

Based on the optimal order quantity, the manufacturer needs to select appropriate suppliers to satisfy its demand. Facing a loose supplier market, the manufacturer should not only consider the criterion of supply capacity, but also focus on the green quality of suppliers. In order to select appropriate suppliers, an evaluation procedure is developed with respect to two aspects including supply capacity and the green quality. The green quality of each supplier can be represented by a real number. In the literature review section, we discussed that various decision making methods have been applied to determine the green quality of suppliers. The following evaluation procedure predominantly integrates the green quality and the carbon tax to select appropriate suppliers.

Step 1. The decision maker sorts the set of suppliers by taking the green quality as a criterion. Different suppliers may have the same quality; the updated set will be a partially ordered set which is denoted by \mathbf{S} . If \mathbf{S} does not contain an unordered pair of elements, then \mathbf{S} is a chain.

Step 2. The set \mathbf{S} is filtered by using the relationship between the green quality and the carbon tax. The form of this relationship can be defined as $GQ := \log_2(ET / ET^U - ET)$. Eliminating suppliers whose green quality is no less than the value of GQ .

Step 3. Set up initial values for the variables as follows, $i = 1$, $t = 1$, $\Delta S = S_i$, $\Delta q_{i,t} = q_{i,t}$. Putting ΔS and $\Delta q_{i,t}$ into the set S' and Q , respectively. Delete S_i from S . Let $i = i + 1$.

Step 4. If the summation of all elements in the set Q and the last period inventory level is smaller than the optimal order quantity of the manufacturer, and S is a nonempty set, then, go to **Step 3**; otherwise, the inventory level in period t is updated, let $t = t + 1$.

Step 5. Check whether t is greater than or equal to N . If t is greater than or equal to N , then, the evaluation procedure is stopped. Otherwise, go to **Step 3**, until t reaches N .

Step 6. The elements in the set S' are the selected suppliers. The elements in the set Q are the optimal order quantity from each selected supplier during the whole period N .

The meaning of each step of the evaluation procedure is as follows. The first step sorts the set of alternative suppliers according to its green quality. A supplier with a larger value of green quality has higher priority to be selected. This type of suppliers can get larger business share than others, which could induce suppliers supply environmentally-friendly raw materials to reduce their emissions. In addition, the decision maker needs to normalize the value of green quality of each supplier. The normalization scale of green quality can be observed by following $GQ := \log_2(ET / ET^U - ET)$ when carbon tax takes values in its effective interval. The second step filters alternative suppliers based on the relationship between green quality and carbon tax. This step further narrows the selection space based a threshold which is mainly determined by the upper boundary of carbon tax. Within the constraint of carbon tax scheme, the manufacturer has to cooperate with greener suppliers. The third step, as a iterative process, determines setup parameters and the stack area for optimal results. In this step, ΔS and $\Delta q_{i,t}$ are counters for the selected supplier and its supply capability, respectively. They are just for updating the selected supplier and its supply capability during the iterative process. The fourth step matches the demand quantity of the manufacturer and order quantities from qualified suppliers. If the demand is satisfied and S is a nonempty set in time period t , then, the procedure goes to the next period; otherwise, the procedure goes back to the third step. Step five ensures the terminal condition of the algorithm for finite periods. The output results are presented in the final step of the algorithm. The output results include the selected suppliers as shown in set S and the order quantity as shown in set Q .

4.3 An application

In this numerical analysis, we take a metal industry as an example. In practice, ferrosilicon is used to produce carbon steel and stainless steel. Ferrosilicon is a ferroalloy, which is produced by iron and silicon with different silicon content from 15% to 90%. However, production of different types of ferrosilicon emits different amount of CO₂. The ferrosilicon with higher silicon content incurs higher amount of CO₂, IPCC reported that production of one ton of ferrosilicon with 45% Si and 90% Si emits 2.5 tons and 4.8 tons CO₂, respectively. For a certain period, assume the market information of the ferrosilicon with 90% Si is as follows, $c = 1.1$ \$/kg, $h = 0.01$ \$/kg, $p = 2.3$ \$/kg, $\alpha = 0.6$, $\beta = 0.5$, $\delta = 0.025$, $E = 0.048$ kg CO₂/kg per product. Based on the initial setting of parameters and the result in part (b) of Proposition 4, the range of the carbon tax per kilogram ferrosilicon with 90% Si is from 26.65 to 31.10 per CO₂ton. Facing the tight pool of suppliers, the manufacturer has no choice and should cooperate with limited suppliers to satisfy its demand. However, in the loose situation, the manufacturer needs to select appropriate suppliers with different standards. Next, we will further analyze the issue of supplier selection by adopting the above evaluation procedure. The initial values of each supplier are shown in Table 1.

Table 2. The initial parameters of suppliers

Suppliers	S1	S2	S3	S4	S5	S6	S7	S8	S9
Criteria									
green quality	1.7	1.9	3.8	4.9	3.2	3.8	2.3	2.6	4.1
supply capability	47	44	25	20	28	26	40	30	22

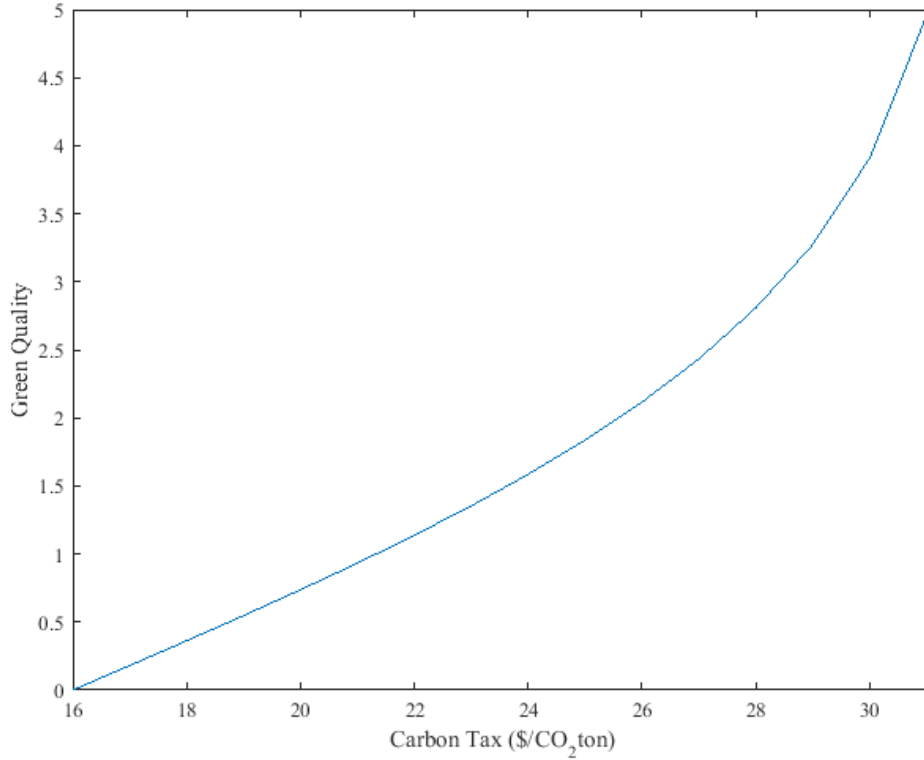


Figure 1. The relationship between the green quality and the carbon tax

Figure 1 indicates that there exists a positive correlation between the green quality and carbon tax. The manufacturer is willing to cooperate with suppliers who have higher green quality when the carbon tax is increasing. The greener suppliers can provide cleaner raw materials or production equipment for the manufacturer to reduce her/his production emissions. In addition, from Figure 1, we can find that if the carbon tax is equal to the half (\$16/ton) of upper bound of emissions tax (\$32/ton), the manufacturer will not bother much the green quality of its suppliers. But, if the emissions tax is continuously increasing, then, the increasing of emissions tax will compel the manufacturer to cooperate with the supplier with higher green quality. Taking the last period as an example, assume the carbon tax is 26\$/ton CO₂ and $x_N = 0$, thus, its corresponding green quality is 2.1155. The manufacturer is going to filter suppliers with lower green qualities which are no less than 2.1155. After updating, the cardinality of the updated set **S** is 7 and the elements are S4, S9, S3, S6, S5, S8, and S7. In addition, the slight change of the carbon tax also can affect the cardinality of **S** (alternative numbers of suppliers). For instance, in July 2014, the B.C. carbon tax was \$25/ton of CO₂, taking it as a criterion, the corresponding green quality is 1.8365, the cardinality of **S** is 8, and, the elements are S4, S9, S3, S6, S5, S8, S7, and S2. Based

on the filtered set of suppliers, the manufacturer needs to select appropriate suppliers to satisfy demand during the whole production horizon.

The demand and the carbon tax are two significant variables that can influence the manufacturer's procurement decision. Therefore, four scenarios including (1) high demand and high carbon tax, (2) high demand and low carbon tax, (3) low demand and high carbon tax, and (4) low demand and low carbon tax are presented respectively to illustrate how these two factors impact the manufacturer's decisions. We assume that the demand follows the normal distribution with mean value, 80 or 100, and variance, 20 or 15. Based on the initial parameters setting and Equation (7), the order quantity in the last period can be calculated by using MATLAB. The values of the optimal order quantity are 55.8503 (for low demand) and 105.1394 (for high demand). We assume the lower and higher carbon tax values are 22 and 28, respectively. Thus, the corresponding green qualities are 1.1375 and 2.8074, respectively. The manufacturer is going to filter suppliers with lower green qualities which are no less than 1.1375 and 2.1155 in different scenarios. That is, the selection pools of suppliers are different with respect to the green quality. For the green quality equal 1.1375, the set of potential suppliers is {S4, S9, S3, S6, S5, S8, S7}. If the green quality equal 1.1375, the set of potential suppliers is {S4, S9, S3, S6, S5}. Following the procedure, the selection results including the selected suppliers and order profile are presented in Table 3. We can see that the results for scenarios (1) and (2) are the same and the results for scenarios (3) and (4) are the same, this is because the scenario with lower green quality can provide a larger selection pool for the manufacturer.

Table 3. The selection results

Scenarios	Selected Suppliers	Order Profile
(1)	S4, S9, S3, S6, S5	20, 22, 25, 26, 8
(2)	S4, S9, S3, S6, S5	20, 22, 25, 26, 8
(3)	S4, S9, S3	20, 22, 7
(4)	S4, S9, S3	20, 22, 7

5. Model extensions

In practice, there exist some other uncertain factors which could influence decisions making of the operating processes. Therefore, this section discusses the potential extensions of the model from four aspects including demand, supply, carbon emission regulations, and environmental production technology.

5.1 Demands

The above sections studied the decision making process for the optimal order quantity with a deterministic demand distribution. This subsection deals with the same decision issue but with the incomplete demand information. Assume the demands in each period are independent and identically distributed. Let $\varphi(\xi|\theta)$ denotes the demand density, where θ is an unknown parameter which can be used to classify the different types of raw materials. In addition, a manufacturer has a prior density function (g) on the unknown parameter θ , assume $g(\theta) \sim \Gamma(\theta|a, S)$. Based on the Bayes' rule, the predictive demand density function can be described by using the conjugate priors as follows. The demand of conjugate priors distribution includes an exponential distribution (θ is unknown) and the prior on θ follows a gamma distribution (a and S are known).

$$\begin{aligned}\phi(\xi | a, S) &= \int_0^\infty \varphi(\xi | \theta) g(\theta | a, S) d\theta - \int_0^\infty \theta e^{-\theta\xi} \frac{S^a \theta^{a-1} e^{-\theta S}}{\Gamma(a)} d\theta \\ &= \frac{S^a}{\Gamma(a)} \int_0^\infty \theta^a e^{-(\xi+S)\theta} d\theta - \frac{-S^a}{(\xi+S)\Gamma(a)} \int_0^\infty \theta^a d e^{-(\xi+S)\theta} \\ &= \frac{aS^a}{(\xi+S)^{a+1}}\end{aligned}\tag{10}$$

5.2 Supply market

In consideration of the cost of carbon emission, a manufacturer not only should improve his/her operations strategies, but also should encourage his/her suppliers to supply environmentally-friendly raw materials and to provide cleaner production equipment. In order to achieve the goal of reducing carbon emission, the manufacturer needs to cooperate with more professional suppliers. The raw material suppliers should be encouraged to adopt the advanced production technology, which could reduce the carbon-intensity of the whole raw material market. On the other hand, suppliers of production equipment should improve their awareness on the technology with lower carbon emission. Suppliers should establish the cooperation relationships with manufacturers. Although the investments on implementation of new production equipment and technologies will incur additional cost to suppliers, these suppliers have advantages and could benefit once these technologies become the necessity.

5.3 Emissions trading

Besides the carbon tax, an emissions trading policy is another fundamental scheme to reduce greenhouse gas emissions. As a typical example, the European Union (EU) has successfully

launched the EU emissions trading system (EU-ETS) since 2005. The EU-ETS limits emissions from more than 11,000 heavy energy-using installations in power generation and manufacturing industry (European Union, 2013). Under the EU-ETS, each agent can decide their trading volume based on their emission allowances, demand, and carbon pricing, etc. This scheme also will incur the additional cost. The current research results show that there are two main types of emissions trading cost. The first one is predominantly determined by the carbon price and the amount of carbon emission (Hua et al., 2010). Another one was analyzed by Gong and Zhou (2013), and the trading price was taken as a Markov chain in their paper.

5.4 Carbon capture and storage

Instead of some financial instruments, engineers developed another effective method, carbon capture and storage (CCS), to reduce carbon emission, especially for those carbon-intensive industries, such as the cement sector, the iron and steel sector, and the refinery industry. The investment on CCS could weaken the influence of the factors of carbon emission. As the value of the factor of emissions decreases, the value of the carbon tax is going to be a fixed value based on the upper bound of carbon tax (ET^U). For the long run, the application of CCS could cut the amount of emissions for a manufacturer or an industry. On the other hand, this technology also changes the total structure of a manufacturer. Once the manufacturer decides to apply CCS technology, its cost structure will be very complicated, which not only includes the carbon tax, but also should cover other series of sub-cost, such as the costs are incurred from CO₂ capture technology, CO₂ transportation by using pipeline, and CO₂ storage. So before investment, the manufacturer should balance the cost between financial instruments and technology investment in the long-run.

6. Conclusions

This paper studied an issue of dynamic procurement planning under the carbon tax in a supply chain. A manufacturer needs to select appropriate suppliers to satisfy the random demand. Under the regulation of carbon tax, the manufacturer has to pay for the additional cost which is incurred by the carbon emission from the production processes. With respect to the characteristic of tractable for production processes, we studied the expected emission cost under the scenario of

the exponential production time. Then, a dynamic programming model was developed to describe the expected total discounted cost in a finite horizon.

Firstly, the optimal order quantity was derived to minimize the expected total discounted cost for a certain period. The convexity of the cost function, which is the key part of the objective function, is proved. It indicates that there is a corresponding lower bound of the total cost, which could provide opportunities for the manufacturer to save money. In the following analysis, with respect to the multi-period decisions making, the optimal order quantity for minimizing the total expected cost was proved. That is, the manufacturer can make order planning based on the optimal order quantity and the value of carbon tax in each single period. Then, implement the iterations backward by following the sequence $t = N, N-1, \dots, 1$. The manufacturer will observe its optimal order quantity in a finite period. In iterative processes, the optimal order quantity can be easily revised by taking different values of carbon tax. This is an easy way for the manufacturer to adapt to the dynamic environments. In addition, from the perspective of government, adopting reasonable carbon tax can effectively motivate manufacturers to walk through the road to sustainable development. For example, manufacturers could adopt cleaner production technologies or cooperate with greener suppliers to reduce carbon emission.

Based on the aforementioned analysis, a supplier evaluation procedure was established to help the manufacturer to select appropriate suppliers when facing a loose supply market. Given the optimal order quantity and the value of carbon tax, the manufacturer can filter potential suppliers and observe the optimal order profile. The metal industry was selected to illustrate the numerical example. In our model, we consider only a single product with a deterministic demand distribution. Relaxing any of assumptions could lead to different results. In addition, from the modeling aspect, other potential factors were discussed including demands, supply market, emissions trading, and carbon capture and storage. These are also the research directions in the future that can overcome our limitations.

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APPENDICES

PROOF OF PROPOSITION 1.

Because t follows exponential distribution with parameter δ_t , thus, the expected emission cost can be derived as follows.

$$\begin{aligned}
 EEC(q_t) &= \mathbb{E}(e^{\alpha t} \cdot EC(q_t)) \\
 &= \int_0^\infty EC(q_t) e^{\alpha t} \cdot \delta_t \cdot e^{-\delta_t t} dt \\
 &= \sum_{t=0}^\infty EC(q_t) \delta_t \int_0^\infty e^{-(\alpha + \delta_t)t} dt \\
 &= \frac{-1}{\alpha + \sum_{t=0}^\infty \delta_t} e^{-(\alpha + \delta_t)t} \Big|_0^\infty \sum_{t=0}^\infty EC(y_t) \delta_t \\
 &= \frac{\sum_{t=1}^N \delta_t \cdot EC(q_t)}{\alpha + \sum_{t=1}^N \delta_t}.
 \end{aligned} \tag{A.1}$$

Q.E.D.

PROOF OF THEOREM 1.

$F(x)$ can be expanded as follows

$$F(x) = \mathbb{E}_D f(x - D) = \int_0^x f(x - D) \varphi(D) dD \tag{A.2}$$

Then, taking the first-order derivative and second-order derivative of $F(x)$ with respect to x , respectively, we can get

$$\frac{\partial F(x)}{\partial x} = \int_0^x \frac{\partial f(x - D)}{\partial x} \varphi(D) dD = \int_0^x f'(x - D) \varphi(D) dD, \tag{A.3}$$

$$\frac{\partial^2 F(x)}{\partial x^2} = \int_0^x \frac{\partial^2 f(x - D)}{\partial x^2} \varphi(D) dD = \int_0^x f''(x - D) \varphi(D) dD, \tag{A.4}$$

because $f''(x - D)$ is non-negative, then, $F''(x)$ is also non-negative. Thus, according to the definition of the convex function, $F(x)$ is a convex function on \mathbf{R} .

Q.E.D.

PROOF OF PROPOSITION 2.

(a) $Z(y_t)$ can be expanded as follows

$$\begin{aligned}
 Z(y_t) &= \int_0^{y_t} h_t \cdot (y_t - \xi) \cdot \varphi(\xi) d\xi + \int_{y_t}^\infty p_t \cdot (\xi - y_t) \cdot \varphi(\xi) d\xi \\
 &= h_t \cdot y_t \int_0^{y_t} \varphi(\xi) d\xi - h_t \int_0^{y_t} \xi \cdot \varphi(\xi) d\xi + p_t \int_{y_t}^\infty \xi \cdot \varphi(\xi) d\xi - p_t \cdot y_t \int_{y_t}^\infty \varphi(\xi) d\xi
 \end{aligned} \tag{A.5}$$

Taking the first-order derivative of $Z(y_t)$ with respect to y_t , we can get

$$\begin{aligned}
\frac{\partial Z(y_t)}{\partial y_t} &= h_t \int_0^{y_t} \varphi(\xi) d\xi - p_t \int_{y_t}^{\infty} \varphi(\xi) d\xi \\
&= h_t \int_0^{y_t} \varphi(\xi) d\xi - p_t (1 - \int_0^{y_t} \varphi(\xi) d\xi) \\
&= (h_t + p_t) \int_0^{y_t} \varphi(\xi) d\xi - p_t
\end{aligned} \tag{A.6}$$

The above proof processes show that $Z(y_t)$ is a convex function. In addition, by applying Theorem 1 and β is larger than zero, the fourth term of the Equation (6) is convex. The first two terms in Equation (6) are also convex, because these two parts have positive relationships with y_t and correlation coefficients are larger than zero. Thus, $H(y_t)$ is a convex function.

(b) With respect to part (b), based on the property of the convex function, there is an optimal order quantity y_t^* , which can minimize $H(y_t)$. Thus, y_t^* is the optimal solution for time period t .
Q.E.D.

PROOF OF PROPOSITION 3.

It is sufficient to verify that G_t is a convex function on a convex set. Then, on the basis of the convexity preservation under minimization, which was proved by Heyman and Sobel, the optimal result of G_t is an optimal choice in each period for a finite horizon. The convexity of G_t is prerequisite conditions by applying convexity preservation under minimization. Let $w_t(x_t) = -c_t x_t$,

because c_t is a positive number, thus, $w_t(x_t)$ can be taken as a convex function and its range is a convex set. Based on Proposition 2, $H(y_t)$ is a convex function. The sum of two convex functions is still a convex function. Next, we will present that G_t is defined on a convex set. Let y_t denote the inventory level after ordering in period t , its domain field denote $\mathbf{Y} = [y, \infty]$, where y is a positive number. Assuming there are m points y_i in \mathbf{Y} , $i = 1, 2, \dots, m$. If \mathbf{Y} is a convex set, then

$$\sum_{i=1}^m a_i + y_i \in \mathbf{Y}, \text{ where } a_i \geq 0 \text{ and } \sum_{i=1}^m a_i = 1.$$

Step 1. If $m = 2$, the results are obvious.

Step 2. Assume the results are hold if $m = k$.

Setp 3. If $m = k + 1$,

$$\sum_{i=1}^{k+1} a_i x_i = \sum_{i=1}^k a_i x_i + a_{k+1} x_{k+1} = (1 - a_{k+1}) \sum_{i=1}^k \frac{a_i x_i}{(1 - a_{k+1})} + a_{k+1} x_{k+1} \tag{A.8}$$

$$\text{because } 1 - a_{k+1} = \sum_{i=1}^k a_i, \text{ thus, } \sum_{i=1}^k \frac{a_i}{(1 - a_{k+1})} = 1, \text{ and } \sum_{i=1}^k \frac{a_i x_i}{(1 - a_{k+1})} \in \mathbf{Y}.$$

Based on the definition of convex set,

$$(1 - a_{k+1}) \sum_{i=1}^k \frac{a_i x_i}{(1 - a_{k+1})} + a_{k+1} x_{k+1} \in \mathbf{Y}. \tag{A.9}$$

Therefore, \mathbf{Y} is a convex set. Similarly, the domain field of x_t , which is defined on $[0, x_t]$, can be proved to be a convex set. In $H(y_t)$, the correlation coefficients are larger than zero, thus, the range of $H(y_t)$ is larger than zero, which is also a convex set. The range of $w_t(x_t)$ is also a convex.

The intersection of two ranges is a convex set. Therefore, G_t is defined on a convex set. Based on the proposition of the convexity preservation under minimization, which was proved by Heyman and Sobel (2003), the optimal result of G_t is an optimal choice in each period for a finite horizon. That is, the optimal ordering policy in Proposition 2 is optimal for a finite horizon.

Q.E.D.

PROOF OF PROPOSITION 4.

(a) The convexity of $H(y_t)$ has been proved in Proposition 2, thus, we can rearrange $H(y_t)$ as follows.

$$\begin{aligned}
H(y_t) &= c_t y_t + Z(y_t) + EEC(y_t) + \beta \int_0^\infty G_{t+1}(y_t - \xi) \phi(\xi) d\xi \\
&= c_t y_t + Z(y_t) + EEC(y_t) - \beta \int_0^\infty (c_t + E \cdot ET)(y_t - \xi) \phi(\xi) d\xi \\
&= c_t y_t + Z(y_t) + EEC(y_t) - \beta (c_t + E \cdot ET) y_t \int_0^\infty \phi(\xi) d\xi + \beta (c_t + E \cdot ET) \int_0^\infty \xi \phi(\xi) d\xi \\
&= c_t y_t + Z(y_t) + EEC(y_t) - \beta (c_t + E \cdot ET) y_t + \beta (c_t + E \cdot ET) \mu.
\end{aligned} \tag{A.10}$$

In order to derive the optimal q_t^* , we take the first-order derivative of function $H(y_t)$ with respect to variable y_t .

$$\begin{aligned}
\frac{\partial H(y_t)}{\partial y_t} &= c_t + \frac{\partial Z(y_t)}{\partial y_t} + \frac{\partial EEC(y_t)}{\partial y_t} + \beta \int_0^\infty \frac{\partial G_{t+1}(y_t - \xi)}{\partial y_t} \phi(\xi) d\xi \\
&= c_t + (h_t + p_t) \int_{y_t}^0 \phi(\xi) d\xi - p_t + \frac{\sum_{t=1}^N \delta_t \cdot E \cdot ET}{\alpha + \sum_{t=1}^N \delta_t} - \beta (c_t + E \cdot ET)
\end{aligned} \tag{A.11}$$

Let $\frac{\partial H(y_t)}{\partial y_t} = 0$, then, we can get

$$\Phi(y_t) = \frac{p_t + \beta (c_t + E \cdot ET) - c_t - \frac{\sum_{t=1}^N \delta_t \cdot E \cdot ET}{\alpha + \sum_{t=1}^N \delta_t}}{h_t + p_t} \tag{A.12}$$

Therefore, the optimal order quantity can be derived.

$$q_t^* = \Phi^{-1} \left(\frac{p_t + \beta \cdot e \cdot ET - (1 - \beta) \cdot c_t - \frac{\sum_{t=1}^N \delta_t \cdot E \cdot ET}{\alpha + \sum_{t=1}^N \delta_t}}{h_t + p_t} \right) - x_t \tag{A.13}$$

(b) The critical quantile in $\Phi(y_t)$ should locate in a closed interval which is from zero to one. Therefore, the numerator of $\Phi(y_t)$ should be larger than zero and smaller than the denominator. Let ET^L and ET^U denote the lower bound and the upper bound of ET , respectively.

$$ET^L = \frac{h_t + (1 - \beta) c_t}{\sum_{t=1}^N \delta_t \cdot E / (\alpha + \sum_{t=1}^N \delta_t) - \beta \cdot E} \tag{A.14}$$

$$ET^U = \frac{p_t + (\beta - 1) c_t}{\sum_{t=1}^N \delta_t \cdot E / (\alpha + \sum_{t=1}^N \delta_t) - \beta \cdot E} \tag{A.15}$$

Then the sufficient requirement ($ET^L \leq ET \leq ET^U$) can be derived with respect to the constraint of carbon tax.

Q.E.D.

Reference for the Appendix

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