# Modeling the effects of airline slot hoarding behavior under the grandfather rights with use-it-or-lose-it rule

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#### **Abstract**

The prevalent airport slot policy, based on the grandfather rights and use-it-or-lose-it rule, may induce the so-called slot hoarding behavior, i.e., airline intentionally operates excessive or even unprofitable flights. This paper develops a vertical-structured model to explore the effects of such slot policy on airline's service decisions (flight frequency, aircraft size and airfare) and airline profit, and the resultant implications for a profit-maximizing airport or a welfare-maximizing airport. The effects of airline competition on slot hoarding behavior are also examined with an oligopoly competition model in which the carriers provide horizontally differentiated flight services. Analytical solutions are derived and compared with the "no slot policy" scenario. We find that the claimed negative effects of the slot policy on airport congestion may be overstated since an airline chooses to hoard slots if and only if the demand/capacity ratio is significantly low. When the airline has to hoard slots by operating excessive flights, it would use smaller aircraft, charge a higher airfare and serve more passengers. For a private airport, the slot policy may increase the airport's profit by allowing the airport to transfer some of the negative effects of weak travel demand to airlines. For a public airport, the slot policy does not decrease social welfare unless passengers' valuation toward frequency benefit is low. Finally, for airlines with equal access to airport slots, as the substitutability among airlines and/or the number of competing airlines increases, the incentive of slot hoarding decreases. Hence, regulators may expect a much milder negative effect of slot hoarding in a competitive aviation market.

**Keywords**: Grandfather rights; use-it-or-lose-it; slot hoarding; airline competition.

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### 1. Introduction

With the substantial growth in air traffic over the past decades, major airports around the world are operating at full capacity. In order to keep congestion and delays at a reasonable level and to ensure efficient utilization of airport infrastructure, airports are classified into three categories according to their levels of congestion. A Level-1 airport (non-coordinated airport) means that the capacity is generally enough to satisfy the demands at all times, whereas a Level-2 airport (schedules facilitated airport) implies that there is a potential for congestion during some periods of the day, week or season, which can be alleviated by schedule adjustments mutually agreed between the airlines and facilitator. An airport is called a Level-3 airport (coordinated airport) if travel demand exceeds capacity due to insufficient infrastructure or conditions imposed by the government.

As of summer 2018, there are 204 Level-3 airports in the world, of which about 51% are in Europe (IATA, 2018). All airlines operating at a Level-3 airport must have allocated slots, which are defined as "the permission to use the full range of airport infrastructure for scheduled time of arrival or departure on a specific date". In other words, it refers to the right to land at or take off from a congested airport. Slots at Level-3 airports, especially those in the peak periods, are valuable assets for airlines. In 2008, Continental Airlines paid \$209 million for four pairs of slots at Heathrow airport in order to start services from Houston and Newark airports. In the same year, Southwest Airlines paid \$7.5 million for 14 slots at LaGuardia Airport (Sheng et al., 2015). With global air traffic expected to grow at 4.4% annually for the next 20 years (AirBus, 2017), it is of great importance that airport slots are allocated and utilized as efficiently as possible.

For the majority of the busiest airports outside the US<sup>1</sup>, the slot allocation scheme operates within the framework of the IATA worldwide slot guidelines (WSG) and any local

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<sup>&</sup>lt;sup>1</sup> Almost all US airports are non-slot-controlled. The runway access is allocated on a 'first come, first served' basis, which enables full utilization of capacity at all times. Slot controls are only imposed on some high traffic-density airports. Historically, 4-5 airports were subject to slot control under the high density rule. Currently, there are seven US airports that are under slot control. Among these airports, only the slot allocation of the John F. Kennedy airport (JFK) generally follows the IATA WSG (FAA, 2018). For more detailed discussions of country-specific slot controls, see for example Czerny et al. (2008).

regulations that are in place (e.g., the European Council Regulation No. 95/93 and its subsequent amendments). As shown in Fig.1, an independent slot coordinator first determines the total number of available slots by considering the airport infrastructure of runways, taxiways, terminals and other possible constrains, such as noise regulations and night curfews. It then prioritizes historic holders (e.g., incumbent airlines) by allowing them to inherit slots which were used in the previous winter/summer scheduling season (i.e., the grandfather right). However, if these slots are to be successfully inherited, they must be used for at least 80% of the time in the previous scheduling season. Otherwise, the carrier would lose the underutilized slots, which may be reallocated to its competitors in the next scheduling season (i.e., the use-it-or-lose-it rule).

Once the grandfather rights have been confirmed and the corresponding slots have been allocated to the airlines, the remaining slots are grouped into a "slot pool", of which up to 50% of the pooled slots are reserved for new entrants. The rest are allocated to incumbents for free. Such an administrative slot allocation scheme, primarily based on the grandfather rights with use-it-or-lose-it rule (hereafter referred to as the grandfather policy), tends to minimize the number of flight changes from season to season. This provides incumbent airlines with a maximum degree of certainty over future slot holdings (DotEcon, 2001). Hence, it reduces the complexity of slot allocation/coordination process and ensures the consistence and stability of airline's schedule planning.

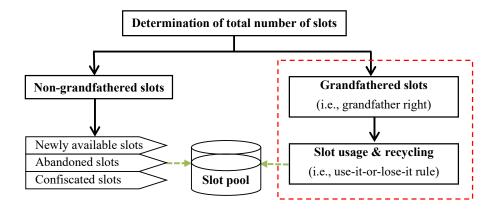


Fig. 1. The basic procedure for slot allocation (adapted from Madas and Zografos, 2008).

In the meanwhile, however, the current practice has been criticized for inefficiency because it entitles carriers to use the slots for only 80% of the time during the scheduling season (i.e., insufficient usage). More importantly, it may also give rise to the so-called slot hoarding behavior, i.e. in order to comply with the use-it-or-lose-it rule and prevent the underutilized slots from being confiscated, incumbent airlines have strong incentives to intentionally operate excessive or even unprofitable flights (i.e., inefficient usage). Indeed, the turnover of slots in major airports is typically very small (DotEcon, 2001). Statistics from the European air transport market in summer 2008 indicated that the proportion of slots withdrawn at the EU's major airports was quite low, reaching 2.3% at Frankfurt, 2.6% at Munich, 2.4% at Paris-CDG, 2.0% at Paris-Orly and only 0.4% at Heathrow (European Commission, 2011). The study of Fukui (2012) on the US market also confirmed that the incumbent airlines, especially those dominant carriers with large slot portfolios, have obvious slot hoarding behavior at slot controlled airports like LaGuardia and O'Hare. Therefore, for most practical cases, the expected benefit of continuing to use the slots is larger than that of losing them. In this sense, the "use-it-or-lose-it" rule actually works like a "use-it" rule.

Despite the claimed economic efficiency of introducing market-based mechanisms to allocate airport slots, so far none of them has been successfully implemented in a large scale. This is probably due to two major reasons. First, in any of the proposed market-based schemes carriers have to pay for acquiring airport slots, which are instead grandfathered for free in the current allocation scheme. For instance, in 2008, the US Federal Aviation Administration (FAA) planned to auction some slots at the airports in New York. The proposal was eventually postponed indefinitely due to the strong resistance from stakeholders (i.e., the airports and airlines). Second, airline competition could be restricted because major airlines may be able to afford higher slot price than small operators or new entrants (i.e., the "deep pocket" effect). For example, in China's first airport slot auction trial conducted at Guangzhou Baiyun airport in 2015, the nation's four largest airlines (i.e., Air China, China Eastern, China Southern, Hainan Airlines) and their affiliates won all the nine pair of slots by paying a total

of 550 million RMB.<sup>2</sup> These major carriers became successful bidders despite the presence of many other small-sized or privately-owned carriers. In this regard, the market-based schemes may cause serious competition concerns.

Given that the grandfather policy can ensure the consistence and stability of airline's schedule planning with relatively low implementation cost, it is expected to prevail for the foreseeable future with market-based schemes serving as effective supplements. It is somewhat surprising, despite the prevalence of the grandfather policy to allocate airport slots, few theoretical studies in the existing economic literature have been made upon its impacts on airlines' service decisions as well as the profit and the social welfare of the whole system. To our best knowledge, Sieg (2010) was the only study which presented a formal theoretical model to investigate how a monopolistic airline and a monopolistic airport strategically respond to the grandfather policy. He found that an airport can take advantage of airlines' slot hoarding behavior through transferring some of the negative effects (e.g., a drop in air travel demand) to airlines. In such a case, the current grandfather policy made airports achieve higher profit, but airlines' profit and the system's social welfare were decreased. However, his study had two major limitations. First, airfare was the only decision variable of the airline in response to the grandfather policy. The carrier's operational decisions on flight frequency and/or aircraft size have been ignored. In fact, flight frequency has important implications in the context of the grandfather policy: (i) flight frequency determines the schedule delay (i.e., the difference between passengers' preferred departure time and the nearest available flight time) perceived by air passengers (e.g., Flores-Fillol, 2009; Brueckner and Zhang, 2010; Lin, 2012; Wang et al., 2014; Alfonso et al., 2016). Higher frequency also gives connecting passengers more flexibility for schedule coordination or less apprehension over missed connections due to low punctuality or reliability of air travel. Therefore, flight frequency plays an important role in both determining passenger volume and evaluating passenger benefits/consumer surplus in the social welfare analysis. (ii) By explicitly modeling the decision of flight frequency, one could simultaneously obtain carriers' choices of aircraft size. This allows us to provide some theoretical explanations for the recent empirical finding of why airlines tend to operate more

<sup>&</sup>lt;sup>2</sup> "RMB" is the Chinese currency, Renminbi, and US\$1 approximates RMB6.51 as of Jan 1, 2018.

flights with smaller aircraft at slot-controlled airports (Fukui, 2012; GAO, 2012). Second, Sieg (2010) considered a monopolistic airline in the downstream aviation market and assumed that the airline always hoarded slots for fear of losing market power. In the civil aviation industry, however, airline competition has been widely discussed for a few decades since the liberalization of market access (e.g., Fu et al., 2010; Li et al., 2010; Adler et al., 2014). Whether an airline decides to hoard slots or not depends on the expected benefits from continuing to use the slots. Such expected benefits are essentially affected by the competition among airlines (Sheng et al., 2015).

In light of the above, this study attempts to answer the following questions: How does the grandfather policy affect airline's decisions on flight frequency, airfare, aircraft size and the resultant airline profit? What are the grandfather policy's profit implications for a private airport and its welfare implications for a public airport? How does airline competition affect airlines' slot usage behavior? To this end, we first present a simple basic vertical-structured model to describe the relationship between an airport and an airline. In the upstream, a profit or welfare maximizing airport decides the charge for its aeronautical services. Airport's demand is derived by the equilibrium in the downstream market, where a profit-maximizing airline, subject to the grandfather policy and taking upstream airport charge as given, decides the number of passengers and flight frequency. The airfare and aircraft size are derived endogenously. We then extend the basic model to consider airline competition in an oligopolistic market with n ( $n \ge 2$ ) carriers providing differentiated flight services. The model results are benchmarked against the case without the grandfather policy, thus allowing us to analyze the effects of the prevalent grandfather policy on airline's service level, profit and the social welfare of the whole system.

The remainder of the paper is organized as follows. Section 2 reviews related literature. Section 3 sets up the basic model in which an airline and an airport interact under the grandfather policy. Market equilibrium under the grandfather policy is analytically solved and benchmarked against the case without the grandfather policy. Section 4 extends the basic

model to consider airline competition in an oligopolistic market with n ( $n \ge 2$ ) carriers. Section 5 concludes the paper and identifies the areas for future studies.

### 2. Literature review

Many of the existing relevant studies are qualitative and explanatory. They have focused on assessment of the current administrative slot allocation scheme, including the discussions on the sources of inefficiency and the identifications of potential market-based slot allocation alternatives (e.g., DotEcon, 2001; NERA, 2004; MacDonald, 2006; Madas and Zografos, 2008, 2010; De Wit and Burghouwt, 2008; Emission Commission, 2011). These previous studies have received significant attention from both the operations research and economics literature. On the operations research side, large-scale optimization models have been proposed to improve the scheduling efficiency of the current administrative slot allocation scheme. These slot scheduling models were typically formalized by minimizing the total schedule displacement, i.e., the difference between the requested and the allocated slot times to airlines, subject to various constraints such as slot allocation constraints (e.g., the set of rules contained in the IATA WSG), airline network connectivity constraints, uncertain airport capacity constraints (see for example, Castelli et al., 2012; Zografos et al., 2012; Corolli et al., 2014). A comprehensive and critical review of relevant studies on airport slot scheduling could be found in Zografos et al. (2017) and Jacquillat and Odoni (2018). More recently, Pellegrini et al. (2017) extended the previous single airport slot scheduling models by simultaneously allocating slots on a network scale while explicitly incorporating the aircraft rotation constraints. Alternative multi-objective models have also been proposed to capture the trade-off between total and acceptable schedule displacements (Zografos et al., 2018) or between the numbers of slots rejected and displaced (Ribeiro et al., 2018).

On the economics literature side, the proposed alternative market-based schemes, such as slot auction, congestion pricing and slot trading, have gained much attention due to their claimed distributive efficiency, i.e., allocating slots to carriers who attach the highest value to them. As a result, the vast majority of economic studies were dedicated to investigating the social

welfare implications of different market-based schemes with microeconomic models, most of which were generally built within the framework in which a single congested airport was served by airlines engaging in Cournot competition. On the issue of congestion pricing, Daniel (1995) raised the possibility of self-internalization by airlines at an airport. Brueckner (2002) found that airlines internalize only the congestion they impose on themselves. As a result, he proposed a differentiated congestion pricing scheme that captures the uninternalized portion of congestion in order to reach the social optimum. Brueckner (2009) and Verhoef (2010) further compared the welfare implications of the price-based (differential/uniform pricing) approaches and the quantity-based (slot trading/auction) approaches for a public congested airport. In contrast to the public airport assumption, Basso and Zhang (2010) compared congestion pricing and slot policy when profit matters to a private airport. Different from the above deterministic models, Czerny (2010) incorporated the uncertainty of passenger benefits and congestion costs when comparing different schemes. There were also a few empirical studies conducted on some specific markets. Due to data availability, however, the related studies were limited to examining the competition effects of slot trading in the US and UK (Fukui, 2010, 2014). More recently, Valdes and Gillen (2018) examined the consumer welfare effects of slots concentration and reallocation from a legacy carrier to low cost carriers (LCCs) using the panel data of domestic flights from the Mexico City airport.

Despite the prevalence of the administrative scheme in allocating airport slots, most of the above mentioned economic studies focused on evaluating different market-based alternatives. In this respect, we aim to fill the gap in the economics literature by investigating the impacts of the current administrative slot allocation scheme (i.e., the grandfather policy) on airlines and airports. Unlike the above-mentioned OR studies addressing the optimal slot allocation issue, this paper focuses on examining airlines' strategic slot utilization behavior resulted from the grandfather policy. Specifically, we analyze how an airline's service decisions (airfare, frequency, and aircraft size) and profit are affected when the airline hoards slots by intentionally operating excessive flights. Such slot hoarding behavior distorts an airline's slot requests, which have been given as input of the slot allocation models developed in the OR literature. What's more, we also compare the associated implications of the slot grandfather

policy for a profit-maximizing airport and for a welfare-maximizing airport. With such analysis, this paper complements previous studies by allowing regulatory authorities to evaluate the possible effects of the grandfather policy on relevant stakeholders.

#### 3. The basic model

#### 3.1. Airline's decisions under the grandfather policy

It is common that an airline possesses monopoly power on a particular route. For instance, it was reported in 2013 that there were 2780 airport-pairs connected within the US domestic market, among which 77% were served by a single carrier.<sup>3</sup> Therefore, we first consider an airline as a monopolistic supplier in a particular origin-destination (OD) market. The airline faces the following linear demand function

$$q = \alpha - \theta(p - \gamma f),\tag{1}$$

where the intercept  $\alpha$  denotes the potential market size and the slope  $\theta$  denotes the price sensitivity, measuring how demand q reacts to a change in the generalized travel cost/full price perceived by air passengers. The full price equals the airfare p minus the frequency benefit ( $\gamma f$ ), where f is the flight frequency and  $\gamma$  measures the marginal benefit from an increase in flight frequency. An increase in flight frequency offers passengers more departure/arrival time choices, and thus reduces the schedule delay and benefits passengers. In accordance with the formulations adopted in Heimer and Shy (2006), Flores-Fillol (2009), Lin (2012), and D'Alfonso et al. (2016), the frequency benefit has been introduced additively in the demand function. For presentation purpose, the linear demand function in Eq. (1) can be rewritten as the following inverse demand function

$$p - \gamma f = D - bq, \tag{2}$$

where  $b=1/\theta$  and  $D=\alpha/\theta=b\alpha$ . Obviously, b is the reciprocal of price sensitivity, measuring how full price reacts to a change in demand. D denotes the maximum willingness

<sup>&</sup>lt;sup>3</sup> https://www.anna.aero/2013/12/04/77pc-us-domestic-air-routes-monopoly-nine-routes-least-five-airlines-fighting-passenger s/. Note that it does not imply that these markets were all served by a monopolistic airline, because passengers may have the option of taking connection flights. For instance, if BOS-LAX is served by an airline, another airline serving BOS-DEN and DEN-LAX can compete on this OD market.

to pay for travel and is positively proportional to potential market size  $\alpha$ . The two concepts are closely related and can be used interchangeably, as stated in Xiao et al. (2013) and Jiang and Zhang (2014). Without loss of generality, parameter D is referred as market size in the following analysis.

Turning to the supply side, an airline's operating cost can be divided into three major cost components (Belobaba et al., 2015). First, the aircraft operation cost, which accounts for the largest proportion of an airline's expenses, includes fuel cost, flight crew and attendants cost, maintenance cost, and gate rental cost etc. The cost per aircraft departure is assumed to be kf instead of a constant k, implying that the cost per aircraft departure increases with flight frequency. Such a specification reflects the decreasing returns for a carrier operating at a congested airport because a busy airport will also have intense use of gates and baggage systems (in addition to runways), which are well subject to decreasing returns at high utilization levels (Flores-Fillol, 2009; Brueckner, 2009). Note the economies of using large aircraft hold here (i.e., economies of traffic density present if congestion effects are controlled for). Assuming that for a given load factor l, aircraft size s can be determined by traffic volume divided by flight frequency and load factor, i.e., s = q/(lf) (for similar specifications, see for example Flores-Fillol, 2009; Brueckner and Zhang, 2010; Lin, 2012). The cost per seat can thus be calculated as  $kf/s = kq/(ls^2)$ , which obviously decreases with aircraft size s. Since the total aircraft departure/flight frequency is f, the total aircraft departure cost is thus calculated as  $f \cdot kf = kf^2$ . The same formulation has been adopted in relevant studies as well (e.g., Heimer and Shy, 2006; Flores-Fillol, 2009). The second cost component is the ground operating cost associated with airport aeronautical service charges, which is generally incurred in handling aircraft and processing passengers, such as the landing charge and the passenger service charge etc. In this paper, these airport-related charges are expressed by a charge  $\omega$  per passenger because of the assumption of exogenous load factor (Barbot, 2004; Verhoef, 2010). The third cost component is an in-flight passenger service cost, which includes items such as entertainment, food and beverage on board. The marginal cost per passenger is normalized to zero for simplicity, an assumption routinely used in the

aviation economics literature. In this sense, the airfare can be interpreted as the price-cost margin (Barbot, 2004). Given the above specifications, the airline chooses the traffic quantity q and flight frequency f to maximize its own profit  $\pi$ , which is computed by adding revenue and subtracting cost, expressed as

$$\max_{q,f} \pi = (p - \omega)q - kf^2 = (D - bq + \gamma f - \omega)q - kf^2.$$
(3)

Since the airline is subject to the grandfather policy, the scheduled flight frequency is required to satisfy the following constraint

$$\delta M \le f \le M \,, \tag{4}$$

where M is the slot capacity or the number of slots that the airline grandfathered from the airport being considered, and  $\delta$  (0 <  $\delta$  < 1) specifies the minimum slot usage proportion required by the use-it-or-lose-it rule (e.g., 80% defined in the IATA worldwide slot guidelines). The lower bound implies that the carrier must ensure slot uses are above the threshold in order to comply with the use-it-or-lose-it rule (Sieg, 2010). In this respect, the "use-it-or-lose-it" rule is actually formulated as a "use-it" rule. The rationale behind it is that incumbent airlines are generally not willing to lose any underutilized slots for two major reasons: 1) those slots may be re-allocated to its potential competitors in the next scheduling season, thus leading its market power to be weakened; and 2) although retaining the underutilized slots may involve operating unprofitable flights at present, these slots could be useful and profitable if considering a potential traffic growth in the future. As mentioned earlier, empirical findings also suggested that the turnover of slots in major airports was indeed very small. It is noteworthy that if  $\delta = 0$ , it corresponds to the scenario where the grandfather policy does not exist. In such a case, there is no minimum slot utilization requirement. It is equivalent that the airline owns the slots (e.g., slot ownership by carrier) and can use them in the future even if they are not used in the current scheduling season.

Given the above specifications, the airline's optimal decisions can be obtained by deriving the Kuhn-Tucker conditions of the following Lagrangian function

$$L(q, f, \lambda_1, \lambda_2) = Dq + \gamma fq - bq^2 - \omega q - kf^2 + \lambda_1 (M - f) + \lambda_2 (f - \delta M), \qquad (5)$$

where  $\lambda_1$  and  $\lambda_2$  are the nonnegative Lagrangian multipliers of the upper bound constraint and lower bound constraint of inequality (4).

The detailed derivation is given in Appendix A. For the convenience of readers, Table 1 summarizes the solutions in the presence/absence of the grandfather policy. The associated equilibrium variables are denoted by the star/tilde superscripts, respectively. To facilitate representation of the boundary conditions of each case in Table 1, two parameters  $N_1 = D - (4bk - \gamma^2)M/\gamma$  and  $N_2 = D - (4bk - \gamma^2)\delta M/\gamma$  are defined and introduced. Given  $0 < \delta < 1$ , it is obvious that  $N_1 < N_2$ . More importantly, as shown in Table 1,  $N_1$  and  $N_2$ are closely related to the shadow prices (i.e., the Lagrangian multipliers) of the upper bound constraint and lower bound constraint of Eq. (4). Case 1 and Case 4 correspond to the situations where the airport charge is low enough thus that the airline would use as many slots as possible. But the maximum number of available slots is M, which is the number of slots grandfathered by the airline. In other words, the upper bound of the flight frequency constraint (see Eq. (4)) is binding at the optimum. Case 3 implies that the airport charge is high enough thus that the airline tends to use as few slots as possible. But in order to comply with the 'use-it-or-lose-it' rule and avoid losing slots, the airline has to use  $\delta M$  slots. In other words, the lower bound of the frequency constraint (see Eq. (4)) is binding at the optimum. In the other cases the airport charge is moderate so that the optimal solutions lie within the feasible region.

**Table 1.** A summary of the optimal solutions with/without the grandfather policy.

With the grandfather policy ( $0 < \delta < 1$ )		
Case 1: $\omega \in [0, N_1)$	$f^* = M, q^* = \frac{D - \omega + \gamma M}{2b}, \lambda_1^* = \frac{\gamma(N_1 - \omega)}{2b}, \lambda_2^* = 0$	
Case 2: $\omega \in [N_1, N_2]$	$f^* = \frac{\gamma(D-\omega)}{4bk-\gamma^2}, q^* = \frac{2kf^*}{\gamma}, \lambda_1^* = \lambda_2^* = 0$	
Case 3: $\omega \in (N_2, D)$	$f^* = \delta M, q^* = \frac{D - \omega + \gamma \delta M}{2b}, \lambda_1^* = 0, \lambda_2^* = \frac{\gamma(\omega - N_2)}{2b}$	
Without the grandfather policy ( $\delta = 0$ )		
Case 4: $\omega \in [0, N_1)$		
Case 5: $\omega \in [N_1, D)$	$ \int_{0}^{\infty} \frac{\gamma(D-\omega)}{4bk-\gamma^{2}}, \chi_{0}^{\infty} = \frac{2k_{0}^{2}}{\gamma}, \chi_{1}^{0} = \chi_{2}^{0} = 0 $	

Note: To focus on positive solutions, it is assumed that  $D-\omega>0$  and  $4bk-\gamma^2>0$ .

The equilibrium solutions in Table 1 have quite intuitive implications. Consider the non-binding Case 2 (or Case 5) as an example, differentiating the optimal flight frequency with respect to the marginal benefit from higher frequency or the marginal cost of aircraft departure, it is easy to obtain  $\partial f^*/\partial \gamma = \frac{1}{4bk-\gamma^2} + \frac{2\gamma^2}{(4bk-\gamma^2)^2} > 0$  and  $\partial f^*/\partial k < 0$ . The results imply that the carrier would operate more flights if passengers attach a higher value to the perceived frequency benefit or if the marginal aircraft operating cost is low. Given the inverse relationship between aircraft size and flight frequency (i.e.,  $s^* = q^*/(lf^*) = 2k/(l\gamma)$ ), the opposite conclusions apply to the optimal aircraft size. That is, the optimal aircraft size increases in the marginal operating cost parameter k and decreases in the passenger frequency benefit parameter  $\gamma$ .<sup>4</sup>

<sup>4</sup> Fu et al. (2015) noted that during 2002-2008, the average aircraft size of flights linking the largest Chinese airports actually decreased despite substantially increases in flight frequency. The authors claimed that this was likely due to Chinese carriers' efforts to construct hub-and-spoke networks and the desire to increase airline

Comparing Case 2 (and Case 3) with Case 5, it is easy to find that the grandfather policy takes effect if and only if  $\omega \in (N_2, D)$  or equivalently  $\frac{D-\omega}{M} < \frac{(4bk-\gamma^2)\delta}{\gamma}$  (see Case 3). It is interesting to further examine the airline's decisions in response to the grandfather policy:

$$\Delta f = f^* - \int_0^{\infty} ds \, ds - \frac{\gamma(D - \omega)}{4bk - \gamma^2} = \frac{\delta M(4bk - \gamma^2) - \gamma(D - \omega)}{4bk - \gamma^2} > 0, \tag{6}$$

$$\Delta s = s^* - \Re \sigma = \frac{q^*}{lf^*} - \frac{\Re \sigma}{lf'^0} = -\frac{\delta M (4bk - \gamma^2) - \gamma (D - \omega)}{2lb\gamma \delta M} < 0, \tag{7}$$

$$\Delta q = q^* - 2 / \omega = \frac{D - \omega + \gamma \delta M}{2b} - \frac{2k(D - \omega)}{4bk - \gamma^2} = \frac{\gamma \left(\delta M (4bk - \gamma^2) - \gamma (D - \omega)\right)}{2b(4bk - \gamma^2)} > 0, \tag{8}$$

$$\Delta p = p^* - p = \gamma (f^* - f) + b(p - q^*) = \frac{\gamma (\delta M (4bk - \gamma^2) - \gamma (D - \omega))}{2(4bk - \gamma^2)} > 0.$$
 (9)

The implications of Eqs. (6)-(9) are explained as follows. First, in the presence of the grandfather policy, the monopolistic airline chooses to hoard slots by operating more flights than what would be necessary in the absence of such a policy, i.e.,  $\Delta f > 0$  as shown in Eq. (6). This is consistent with the findings of previous studies. But such slot hoarding behavior happens only when the demand/capacity ratio is significantly small (i.e.,  $(D-\omega)/M$  is less than  $(4bk - \gamma^2)\delta/\gamma$  in our study). This is because if the market demand D is low and/or the slot capacity M is large, the airline's optimal flight frequency (increasing in D), in the absence of the grandfather policy, can not satisfy the minimum slot utilization requirement (increasing in M) given in Eq. (4). In order to avoid losing the underutilized slots, the airline has to intentionally operate excessive flights than what is actually needed so that it can successfully inherit those slots in the next scheduling season. It is noteworthy that although both low market demand D and large slot capacity M (i.e., the quantity of grandfathered slots) could induce slot hoarding behavior, they have different impacts on airport congestion. On one hand, if the number of slots grandfathered from the airport is relatively large (e.g., the incumbent/dominant carrier holds the majority of airport slots), the slot hoarding behavior implies operating excessive flights at the Level-3 airport, which may sometimes give rise to

undesirable airport congestion. Such negative effects could be amplified in periods of bad weather, during which the actual airport capacity may be much lower than the declared airport capacity. This is one of the reasons why the grandfather policy has been criticized in the literature (e.g., DotEcon, 2001; NERA, 2004). On the other hand, however, if the slot hoarding behavior results from relatively low market demand (e.g., in the off-season) instead of large slot capacity, operating more flights may not cause serious congestion at the airport due to the overall low market demand. The negative effects of the grandfather policy on airport congestion may be overstated in this particular case. It is therefore important in practice to identify which factor gives rise to the airline slot hoarding behavior.

Second, the airline would choose to operate smaller aircraft when it has to hoard slots, i.e.,  $\Delta s < 0$  as shown in Eq. (7). Since the airline has to intentionally schedule more flights than what is optimal in terms of profit maximization, it will operate smaller aircraft to minimize the profit loss. It is noteworthy that the result is derived from a purely economic perspective. In practice, however, in certain markets regulators may impose additional constraints that limit carriers' choice of smaller aircraft. For instance, the FAA once required airlines serving the LaGuardia airport to maintain a certain seat capacity (FAA, 2006). Airlines failing to meet the minimum standard would lose the slots for their smaller flights. This new theoretical result, ignored by Sieg (2010), supports the recent empirical findings by Fukui (2012) and GAO (2012). With regression analysis conducted on the US market, Fukui (2012) found that the average passenger throughput per flight at the slot constrained airports was significantly lower than that of the slot unconstrained airports with similar conditions. The statistical analysis conducted by GAO (2012) also showed that flights at the slot-controlled airports were 75% more likely to be scheduled using an aircraft with fewer than 100 seats than flights at other like-sized airports that were not slot controlled. Unfortunately, similar analysis cannot be conducted on the European aviation market, where all major airports are slot controlled. Therefore, these empirical results cannot be directly extended to the EU market and should be interpreted with caution.

Third, according to Eqs. (8) and (9), more passengers are served and a higher airfare is charged when the airline hoards slots. It should be noted that the results are only partially consistent with the findings in Sieg (2010), who concluded that the monopolistic carrier attracted more passengers by lowering the airfare when it decides to hoard slots. The reason that a higher airfare is observed in our analysis is that we have explicitly modeled the effect of flight frequency and the associated schedule delay perceived by air passengers, which has been ignored in the study of Sieg (2010). Recalling the demand function defined in Eq. (1), a higher flight frequency decreases passengers' schedule delay and thus allowing the airline to charge a higher airfare for better quality of air service. The passenger volume increase due to increased frequency benefit outweighs the passenger volume decrease due to price elasticity of demand. Therefore, the carrier could simultaneously schedule more flights and charge a higher airfare while serving more passengers. This result has important implications for examining the effect of the grandfather policy on airport profit (as shown in Section 3.2).

Finally, the contribution to airline's profit by tightening the lower bound of the constraint by one unit can be derived form Eq. (5), i.e.,  $\partial \pi/\partial (\delta M) = -\lambda_2$ . Since the Lagrangian multiplier  $\lambda_2^*$  is positive in Case 3, the decrease in marginal airline profit due to stricter grandfather policy is given by  $((4bk - \gamma^2)\delta M - \gamma(D - \omega))/2b$ . Summarizing the main conclusions of the above analysis yields leads to the following proposition:

**Proposition 1.** (i) In the presence of the grandfather policy, the monopolistic airline chooses to hoard slots if and only if the demand/capacity ratio is lower than  $(4bk - \gamma^2)\delta/\gamma$ . (ii) When the airline has to hoard slots, it would simultaneously use smaller aircraft, charge a higher airfare and serve more passengers. The resultant airline profit is decreased.

#### 3.2. Effects of the grandfather policy on profit-maximizing airport

We first investigate the impacts of the grandfather policy on a profit-maximizing upstream airport. Such an investigation is important for two reasons. First, the privatization of the

British Airports Authority in Britain in 1997 has induced more and more countries to partially or completely privatize their airports. Second, due to the airport commercialization movement in recent years, even publicly owned airports have suffered from increasing pressure to be financially more self-sufficient and less reliant on government subsidy. Profit-maximizing airports have thus been modelled as an important benchmark although in practice airports may be under alternative forms of regulations on their pricing and operations (Zhang and Zhang, 2003; Fu and Zhang, 2010; Basso and Zhang, 2010; Yang and Fu, 2015; Xiao et al. 2017). The airport revenues generally consist of the aeronautical service revenues and the non-aeronautical service revenues. The aeronautical service charge ( $\omega$ ) has been taken as a cost component for the downstream airline as shown in Eq. (3). The non-aeronautical service revenues generally depend upon commercial service in airport terminals, such as duty-free shopping, food catering, car parking and rental. Since the non-aeronautical revenues rely heavily on passenger throughput of an airport, it is assumed that the airport earns an exogenous average net profit h from commercial service for each passenger (Fu and Zhang, 2010; Xiao et al., 2013).<sup>5</sup> Again, the airport's operational cost is assumed to be zero. Given the equilibrium derived in the downstream airline market, the upstream airport decides the aeronautical service charge  $\omega$  to maximize its profit  $\Pi$ , given as

$$\max_{\omega \in [0,D)} \Pi = (\omega + h)q. \tag{10}$$

The airport's demand is equal to the downstream equilibrium passenger volume q, which is a function of the upstream aeronautical service charge  $\omega$ . According to Table 1, the grandfather policy doesn't take effect for any given  $\omega \in [0, N_2]$ . In this case, the airport profit doesn't change no matter whether the grandfather policy is implemented or not. For any given  $\omega \in (N_2, D)$  (see Case 3 in Table 1), however, the downstream airline decides to hoard slots under the grandfather policy and one of the resultant choices is to serve more passengers, i.e.,  $q^* > \mathcal{U}$  as indicated in Eq. (8). The associated airport profit, defined in Eq. (10), is thus always greater than that in the absence of the grandfather policy for the same level of  $\omega$ .

<sup>&</sup>lt;sup>5</sup> For alternative modelling of commercial services, see Czerny (2006) and Yang and Zhang (2011).

Therefore, as long as the optimal airport charge, in the absence of the grandfather policy, falls into the interval  $(N_2, D)$ , we could assert that the grandfather policy would increase the maximum airport profit.

From Eq. (10) and Table 1, it can be proved that if the potential air travel demand is significantly low, i.e.,

$$D < \frac{2\delta M(4bk - \gamma^2)}{\gamma} - h,\tag{11}$$

then the grandfather policy increases the airport profit. The detailed proof is given in Appendix B, and the intuition behind it can be drawn as follows. When the potential air travel demand is high enough (e.g., during the high season), the carrier itself is willing to schedule more flights to cater for the travel needs even without the grandfather policy. On the contrary, if the potential air travel demand is low, the airline has no incentive to operate as many flights as possible and it may be unable to comply with the use-it-or-lose-it rule. In such a case, the grandfather policy forces the airline to operate excessive flights and carry more passengers, leading to an increase in both the aeronautical revenues and the non-aviation revenues for the airport. The result further confirms the viewpoint of Sieg (2010) that the grandfather policy allows the airport to transfer some of the negative effects (i.e., drop in travel demand) to airlines. From this perspective, it partly explains why the airport prefers the grandfather policy over slot ownership by air carriers.<sup>6</sup> What's more, it is straightforward to observe that the right-hand side of Eq. (11) increases in the required slot usage proportion  $\delta$  and decreases in the marginal frequency benefit  $\gamma$ . Therefore, the probability that the grandfather policy increases the airport profit becomes greater if stricter use-it-or-lose-it rule is implemented and/or if passengers' valuation toward additional flight frequency is lower (e.g., with a larger proportion of leisure passengers). The explanation is that in both situations, it is

<sup>&</sup>lt;sup>6</sup> The property rights of slots have not been clearly defined from a legal perspective yet. All stakeholders want to claim slot ownership. The airports claim to own the slots since they own the infrastructure (e.g., runways) necessary to operate a flight.

The airlines treat slots as their possessions since they have put great effort and investment into air services, and they could inherit slots after the grandfather policy. The governments also argue that they own the slots since the nation's airspace is being used. For more discussions on this issue in the US context, see for example Berardino (2010).

more likely that carriers fail to comply with the use-it-or-lost-it rule. The next proposition states the effects of the grandfather policy on airport profit.

**Proposition 2.** The grandfather policy can increase airport profit when the potential air travel demand is lower than  $2\delta M(4bk-\gamma^2)/\gamma - h$ . A stricter use-it-or-lose-it rule and/or lower marginal frequency benefit can increase the probability that the grandfather policy raises the airport profit.

#### 3.3. Effects of the grandfather policy on a welfare-maximizing airport

This subsection is devoted to analyzing the social welfare implications of the grandfather policy for a welfare-maximizing airport. The social welfare is defined as the sum of the airport profit, the airline profit and the consumer surplus. The payments of the aeronautical charges and airfares are cancelled out in the social welfare computation since they just represent a transfer from the airline to the airport and from the passengers to the airline, respectively. Thus, the social welfare is given as

$$\max_{\omega \in [0,D)} SW = (\omega + h)q + (p - \omega)q - kf^{2} + \int_{0}^{q} (D + \gamma f - bq)dq - pq$$

$$= -\frac{b}{2}q^{2} + (D + h)q + \gamma fq - kf^{2},$$
(12)

where both passenger volume q and flight frequency f are a function of airport charge  $\omega$ . Substituting the corresponding variables in Table 1 into Eq. (12) yields the social welfare expressions associated with each case. The social welfare can be graphically plotted as the piecewise functions as shown in Fig. 2, where  $\omega^* = D - \frac{(D+h)(4bk-\gamma^2)}{2bk-\gamma^2}$ . The properties of

Fig. 2 (i.e., the monotonicity and the maximum value of the social welfare function) are given in Appendix C.

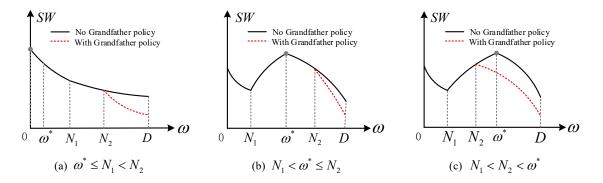


Fig. 2. Social welfare in the presence/absence of the grandfather policy.

The red dotted line and the black solid line in Fig. 2 represent the social welfare with and without the grandfather policy, respectively. Without the grandfather policy, the maximum social welfare is obtained when aeronautical service charge  $\omega = 0$  if  $\omega^* \leq N_1$  (see the black solid line in Fig. 2(a)) and  $\omega = \omega^*$  if  $\omega^* > N_1$  (see the black solid line in Fig. 2(b) or Fig. 2(c)). According to Table 1, the grandfather policy doesn't take effect for any given  $\omega \in [0, N_2]$ . Therefore, the red dotted curve and the black solid curve overlap with each other over the interval  $[0, N_2]$ . For any given  $\omega \in (N_2, D)$ , the social welfare always decreases in the presence of the grandfather policy. However, this doesn't necessarily imply that the grandfather policy must decrease the maximum social welfare. In Fig. 2(a) and 2(b), the maximum social welfare is obtained when aeronautical service charge  $\omega = 0$  and  $\omega = \omega^*$ , irrespective of whether the grandfather policy is implemented or not. In other words, the grandfather policy doesn't change the maximum social welfare that can be obtained. In Fig. 2(c), however, the maximum social welfare is no longer achieved at  $\omega^*$  but reached at either  $\omega = 0$  or  $\omega = N_2$  due to the implementation of the grandfather policy. In either case, the maximum social welfare is decreased. Thus, it is in Fig. (2c) that the grandfather policy could decrease the social welfare. This requires the condition  $\omega^* > N_2 > N_1$  to be satisfied, which leads to

$$0 < \gamma < \gamma_A = \frac{-(D+h) + \sqrt{(D+h)^2 + 8bk(\delta M)^2}}{2\delta M}.$$
 (13)

In contrast to Sieg (2010), the above result highlights the importance of incorporating frequency benefit in analyzing the social welfare implications of the grandfather policy. Although the motivation behind airline slot hoarding behavior may be simply to prevent from losing slots, the positive consequence is to operate more flights and serve more passengers. More importantly, the increase in flight frequency offers passengers more departure time choices, which leads to a decrease in passengers' schedule delay cost. If passengers' valuation toward such delay cost is high, the additional consumer benefits brought by the grandfather policy are able to offset its negative effects. Therefore, the grandfather policy doesn't necessarily decrease the social welfare unless the marginal benefit from higher frequency is significantly low.<sup>7</sup>

Since the grandfather policy may increase the airport profit according to **Proposition 2**, this raises another interesting question: whether the airport profit could be increased without decreasing the social welfare? Unfortunately, in the presence of the grandfather policy, none of the optimal airport charges in Fig. 2 falls into the interval  $(N_2, D)$ . However, it is only in interval  $(N_2, D)$  that the airport profit can be increased by the grandfather policy. The social welfare effects of the grandfather policy are presented as follows.

**Proposition 3.** The grandfather policy can decrease the social welfare if passengers' valuation toward frequency benefit is lower than  $\frac{-(D+h)+\sqrt{(D+h)^2+8bk(\delta M)^2}}{2\delta M}.$ 

With the grandfather policy, there does not exist an airport charge which increases the airport profit without decreasing the social welfare.

<sup>&</sup>lt;sup>7</sup> With somewhat similar intuition, Richard (2003) analytically demonstrated that airline mergers may be welfare-increasing. This is because although a merger increases airline market power, consumers may benefit from increased flight frequency.

### 4. Extensions to oligopoly competition

This section extends the basic model to explicitly consider airline competition over two consecutive time periods in the context of the grandfather policy. Each competing airline maximizes its total profit over two periods by deciding on how many slots to use in each period while anticipating its rival's decisions. The two consecutive periods are interrelated by the use-it-or-lose-it rule, i.e., the maximum available number of slots in period two depends on the number of the slots actually used in period one. In other words, the underutilized slots, failing to meet the minimum slot utilization requirement (e.g., 80% of the time during the scheduling season), are lost and may be reallocated to other airlines serving other air markets. The airline cannot get the lost slots back in the next period. We first consider airline competition between two symmetric carriers, and then oligopolistic competition market with n ( $n \ge 2$ ) symmetric carriers. The carriers are symmetric in the sense that they are the same except that they provide horizontally differentiated flight service to air passengers.

Consider two competing airlines with equal access to airport slots, offering differentiated flight services over a single OD pair. The demand system for air transport in a specific period is linear in the following form:

$$p_{it} - \gamma f_{it} = D_t - q_{it} - \beta q_{it}, \forall i, j \in (A, B), i \neq j, t = 1, 2,$$
(14)

where the subscript i (i = A, B) denotes airlines and subscript t (t = 1, 2) denotes time period. Parameter  $\beta$  measures the service substitutability or the degree of horizontal product differentiation between the two carriers.  $\beta$  ranges from zero (e.g., the flight services are independent products) to one (e.g., the flight services are considered as perfect substitutes). The other notations are the same as in the basic model. The above inverse demand functions, derived from maximizing a quadratic and strictly concave utility function as proposed by Dixit (1979) and Singh and Vives (1984), have been widely used in oligopoly studies in the air transport literature (e.g., Fu et al., 2006; Fu and Zhang, 2010; Lin, 2012; Xiao et al., 2013; D'Alfonso et al., 2016). Empirical evidences indicate that airline competition is consistent

with Cournot behavior (Brander and Zhang, 1993; D'Alfonso et al., 2016). Therefore, we consider the case in which the duopoly airlines engage in Cournot competition by simultaneously choosing the passenger volume and flight frequency in each period to maximize their own total profit over the two periods concerned, i.e.,

$$\max_{(q_{i1}, q_{i2}, f_{i1}, f_{i,2})} \pi_i = \sum_{t=1}^2 ((p_{it} - \omega)q_{it} - kf_{it}^2), \forall i \in (A, B).$$
(15)

In the absence of the grandfather policy, the airlines can use slots in the next period even if some of the slots are not used in the current period. Therefore, the profits over two consecutive periods are independent. The carriers can independently optimize its operations and profit for the two periods. Imposing symmetry on the first order condition of Eq. (15), the optimal solutions for each airline in each period can be easily obtained. In the presence of the grandfather policy, however, the two consecutive periods become interrelated. According to the use-it-or-lose-it rule, the slot usage in period one determines the maximum available number of slots that can be used in period two, i.e.,  $f_{i2} \le f_{i1}/\delta$ . By applying the same solution procedure as outlined in Appendix A, the constrained optimization problem could be solved. Table 2 summarizes the solutions of the duopoly case in the presence/absence of the grandfather policy. As in the basic model, given the optimal passenger volume and flight frequency, the airfare is then calculated with the inverse demand function and the aircraft size is determined by traffic volume divided by flight frequency and load factor.

**Table 2.** Summary of the optimal solutions of the duopoly competition case.

With the grandfather policy			
Case 1: $D_1 \ge D_2  \text{or}  0 < \delta \le \frac{D_1 - \omega}{D_2 - \omega}$	$f_{it}^* = \frac{\gamma(D_t - \omega)}{2k(2+\beta) - \gamma^2}, q_{it}^* = \frac{2k(D_t - \omega)}{2k(2+\beta) - \gamma^2}$		
Case 2: $D_1 < D_2 \ \text{ and } \ \frac{D_1 - \omega}{D_2 - \omega} < \delta < 1$	$f_{i1}^* = \frac{\delta \gamma \left(\delta(D_1 - \omega) + (D_2 - \omega)\right)}{\left(2k(2+\beta) - \gamma^2\right)(1+\delta^2)}, f_{i2}^* = \delta f_{i1}^*$ $q_{i1}^* = \frac{\gamma^2 \left(\delta(D_2 - \omega) - (D_1 - \omega)\right) + 2k(1+\delta^2)(2+\beta)(D_1 - \omega)}{\left(2k(2+\beta) - \gamma^2\right)(1+\delta^2)(2+\beta)}$ $q_{i2}^* = \frac{\delta \gamma^2 \left((D_1 - \omega) - \delta(D_2 - \omega)\right) + 2k(1+\delta^2)(2+\beta)(D_2 - \omega)}{\left(2k(2+\beta) - \gamma^2\right)(1+\delta^2)(2+\beta)}$		
Without the grandfather policy			

$$\tilde{f}_{it} = \frac{\gamma(D_t - \omega)}{2k(2+\beta) - \gamma^2}, \quad \mathcal{Y}_{il} = \frac{2k(D_t - \omega)}{2k(2+\beta) - \gamma^2}$$

Note: 1) To focus on positive solutions, we assume  $D_i - \omega > 0$  and  $2k(2+\beta) - \gamma^2 > 0$ ; 2) i = A,B and t = 0

It is straightforward to observe that the grandfather policy takes effect if and only if  $D_1 < D_2$ 

and  $\frac{D_1 - \omega}{D_2 - \omega} < \delta < 1$  (see Case 2 in Table 2). In other words, if  $D_1$  is smaller than  $D_2$  and

the required slot usage ratio  $\delta$  is large enough, the optimal slot usage number in period one (in terms of maximizing the profit of period one) can not ensure enough slots to be used in period two. In such a case, the airlines intentionally use more slots in period one than what is optimal in the absence of the grandfather policy. It is straightforward to verify that the effects of slot hoarding on airline's service decisions found in the basic model are still valid in the extended model. We are nevertheless more interested in the effects of airline competition on slot usage, which is the motivation for the extended model. Let  $\Delta f_{ii} = f_{ii}^* - f_{ii}^*$  (i = A, B and t= 1,2) represent the slot usage difference in each period due to the implementation of the grandfather policy. It is found that

$$\begin{cases}
\Delta f_{i1} = \frac{\gamma \left(\delta(D_2 - \omega) - (D_1 - \omega)\right)}{\left(2k(2 + \beta) - \gamma^2\right)(1 + \delta^2)} > 0, \\
\Delta f_{i2} = \frac{\gamma \left(\delta^3(D_1 - \omega) - (D_2 - \omega)\right)}{\left(2k(2 + \beta) - \gamma^2\right)(1 + \delta^2)} < 0.
\end{cases}$$
(16)

**Remark.** Eq. (16) implies that each airline, in the presence of the grandfather policy, intentionally uses more slots in period one, i.e., slot hoarding behaviour occurs. Although the slot hoarding behavior decreases the profit generated in period one, it secures certain number of slots for period two, which increases the profit in period two. This process ends until the marginal profit decrease in period one equals the marginal profit increase in period two. More importantly, the number of hoarded slots in period one decreases with the degree of substitutability between the two carriers, i.e.,  $\partial \Delta f_{i1}/\partial \beta < 0$ . In other words, the slot hoarding behavior would be weakened if there is a stronger competition between the two airlines. This is because in a highly competitive air transport market, airlines are not willing to use excessive slots in period one because this would not only cause more profit loss in this period, but also makes airlines engage in more intense competition in period two thus lowers the profits of both airlines.

It is interesting to further investigate an oligopoly market with  $n \ (n \ge 2)$  symmetric carriers. To be consistent with the previous analysis, airline i faces the following inverse linear demand functions

$$p_{it} - \gamma f_{it} = D_t - q_{it} - \beta \sum_{j \neq i}^{n} q_{jt}, \forall i = 1, 2..., n, t = 1, 2.$$
(17)

Similar to the above symmetric duopoly case, for each airline i, the number of hoarded slots in period one is calculated as  $\Delta f_{i1} = \frac{\gamma \left(\delta(D_2 - \omega) - (D_1 - \omega)\right)}{\left(2k\left(2 + \beta(n-1)\right) - \gamma^2\right)(1 + \delta^2)}$ . What's more, the limits

of  $\Delta f_{i1}$  and  $\sum_{i=1}^{n} \Delta f_{i1}$ , as *n* approaches infinity, can be calculated as

$$\lim_{n \to \infty} \Delta f_{i1} = 0 \quad \text{and} \quad \lim_{n \to \infty} \sum_{i=1}^{n} \Delta f_{i1} = \frac{\gamma \left( \delta(D_2 - \omega) - (D_1 - \omega) \right)}{2k\beta(1 + \delta^2)} > 0.$$
 (18)

Eq. (18) implies that in a competitive market with many competing airlines, no individual carrier has an incentive to hoard slots (i.e.,  $\lim_{n\to\infty} \Delta f_{i1} = 0$  holds). For the whole system, however, the total number of hoarded slots converges to a positive constant, i.e.,  $\lim_{n\to\infty} \sum_{i=1}^n \Delta f_{i1} > 0$ . These results are summarized as follows.

**Proposition 4.** (i) In a competitive airline market with equal access to airport slots, as the substitutability among airline service and/or the number of competing airlines increases, airlines' incentive of slot hoarding decreases. (ii) When the market is perfectly competitive with many competing airlines, the effects of the slot hoarding behavior caused by the grandfather policy are trivial from airlines' perspective since the competition makes slot hoarding costly. But as a whole, the total number of hoarded slots should be considered from the regulator's perspective.

#### 5. Conclusion and further studies

In this paper, a vertical-structured model was presented to explore the airline's slot hoarding behavior under the grandfather policy, and the resultant profit and social welfare implications. The main theoretical contributions of this paper are two-fold. First, we complement previous theoretical studies by explicitly modeling flight frequency and the associated schedule delay, which allows us to investigate airlines' strategic choices of flight frequency and aircraft size in addition to airfare or air traffic volume in the presence of grandfather policy. As a result, the subsequent analysis on airline/airport profit and social welfare implications can be conducted in a more comprehensive and realistic way. Second, the effect of airline competition on slot hoarding behavior is further investigated in an oligopoly market with  $n \in \mathbb{Z}$  carriers offering horizontally differentiated services. We examined both the effects of substitutability among carriers and the number of competing carriers on each airline's slot

hoarding behavior. The resultant effect on the total number of hoarded slots is also explored from the system's perspective.

In terms of the main findings and insights obtained in this paper, we conclude that: (i) instead of assuming that the carriers always choose to use excessive slots, our analytical results suggest that the grandfather policy takes effect if and only if the demand/capacity ratio is sufficiently low. Therefore, the negative effects of the grandfather policy, such as inducing excessive flights and intensifying airport congestion, may be overstated for certain markets. (ii) As an extension of the very few relevant studies, schedule delay and the associated frequency benefits perceived by passengers have been explicitly modeled. This allows us to simultaneously derive airlines' strategic choices of flight frequency, aircraft size, and airfare or traffic volume. More importantly, the profit and social welfare implications can thus be evaluated in a more comprehensive and realistic way. Our analytical results suggest that when the airline has to hoard slots, it chooses to operate more flights with smaller aircraft, charge a higher airfare and serve more passengers. The resultant airline profit is decreased. In comparison with the quite relevant study of Sieg (2010), our conclusions supplement and correct some of his results. On the one hand, we find that one of the carrier's responses is to operate smaller aircraft in addition to scheduling more flights. This finding, not captured by Sieg (2010), provides a theoretical explanation for the recent empirical findings that airlines tend to operate more flights with smaller aircraft at slot-controlled airports (Fukui, 2012; GAO, 2012). On the other hand, a higher airfare, instead of a lower airfare found in Sieg (2010), is charged when the airline decides to hoard slots. This is because we have explicitly modeled passengers' schedule delay and the increased flight frequency, whereas a higher frequency allows the airline to charge a higher airfare for better quality of air service. (iii) In terms of airport profit, it has been shown that the grandfather policy can increase airport profit when the potential air travel demand is low. This indicates that the grandfather policy allows the airport to transfer some of the negative effects of a drop in travel demand to airlines. As above, it explains why the airport prefers the grandfather policy to the slot ownership scheme. For the whole system, the grandfather policy does not necessarily decrease the social welfare unless the marginal benefit from higher frequency is sufficiently low. Such a finding

highlights the importance to incorporate frequency benefits in assessing the social welfare implications of the grandfather policy. (iv) By extending the basic monopoly model to the case of oligopoly airline market, the effects of airline competition on slot hoarding behavior is also examined. It is found that airlines with equal access to airport slots would hoard fewer slots if their flight services are less differentiated. In a perfectly competitive market with many competing carriers, although no individual airline has an incentive to hoard slots, there exists a certain number of excessively used slots for the whole system. Hence, in a very competitive air transport market, the slot hoarding behavior caused by the grandfather policy could be ignored from individual airline's perspective. As a whole, however, they can not be ignored from the system's perspective.

Although this study offers fresh insights into the impacts of the grandfather policy, it can be further extended in several directions. First, slot trading was officially recognized by the European Commission on April 30, 2008 as an acceptable way to swap slots among airlines. In the U.S., the "buy and sell" rule was approved as well for domestic flights operating at High Density Traffic Airports (HDTA). This leads to many important questions. For example, how is the airlines' slot hoarding behavior affected by slot trading in the secondary markets? How could the secondary slot trading help to remedy the efficiency loss caused by the grandfather policy in the primary market? Incorporating slots trading in our model is expected to offer answers to these questions. Second, one of the major motivations behind slot hoarding behavior is to secure available slots for the future (e.g., option behavior). Therefore, future demand uncertainty plays an important role in determining the airline's slot hoarding behavior. In a two-period dynamic game, for example, airlines in period one may acquire excessive slots and operate sub-optimally in order to make more expected profit in period two. Such behavior is certainly affected by the demand uncertainty in period two. In contrast to the deterministic model proposed in this paper, a stochastic multi-period model incorporating demand uncertainty is worth exploring. Third, there is a complementarity between slots at different slot controlled airports. Once a flight's departure time is decided at the origin airport, a slot at the destination airport needs to be secured around the scheduled time of arrival. Such a complementarity in time and space among slots may influence airlines' slot requests.

Different from the single-airport setting, this would require the modeling of slot-bundles at OD airports, or model slots to be time-specific so that the network-wide effects of slot hoarding can be examined. Finally, when considering the effects of competition on slot hoarding behavior, we assume that the competing airlines have equal access to airport slots. In practice, however, airlines slot hoarding behavior may act as a strategic way to deter potential rivals' entry. Therefore, it is meaningful to also investigate the market entry deterrence problem jointly with airline slot hoarding behavior. We hope our analysis could lead to more advanced studies on these important issues.

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## Appendix A. Solution procedure of the constrained problem

The Lagrangian function of profit maximization model (3) with constraint condition (4) can be formulated as

$$L(q, f, \lambda_1, \lambda_2) = Dq + \gamma fq - bq^2 - \omega q - kf^2 + \lambda_1 (M - f) + \lambda_2 (f - \delta M). \tag{A.1}$$

The corresponding Kuhn-Tucker conditions are as follows.

$$\begin{cases} \frac{\partial L}{\partial q} = D + \gamma f - 2bq - \omega \le 0, q \ge 0, & q \frac{\partial L}{\partial q} = 0, \\ \frac{\partial L}{\partial f} = \gamma q - 2kf - \lambda_1 + \lambda_2 \le 0, f \ge 0, & f \frac{\partial L}{\partial f} = 0, \\ \frac{\partial L}{\partial \lambda_1} = M - f \ge 0, \lambda_1 \ge 0, & \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \\ \frac{\partial L}{\partial \lambda_2} = f - \delta M \ge 0, \lambda_2 \ge 0, & \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0. \end{cases}$$
(A.2)

After ruling out the trivial solution (i.e., q = 0), we solve (A.2) and obtain

Case 1: if  $\lambda_1 > 0$  and  $\lambda_2 = 0$ , we then have

$$\begin{cases} D + \gamma f - 2bq - \omega = 0, \\ \gamma q - 2kf - \lambda_1 = 0, \\ f = M. \end{cases} \Rightarrow \begin{cases} q = \frac{D - \omega + \gamma M}{2b}, \\ f = M, \\ \lambda_1 = \frac{\gamma (D - \omega) - (4bk - \gamma^2)M}{2b}. \end{cases}$$
(A.3)

Case 2: if  $\lambda_1 = 0$  and  $\lambda_2 = 0$ , we then have

$$\delta M < f < M \Rightarrow \begin{cases} D + \gamma f - 2bq - \omega = 0, \\ \gamma q - 2kf = 0. \end{cases} \Rightarrow \begin{cases} q = \frac{2kf}{\gamma}, \\ f = \frac{\gamma(D - \omega)}{4bk - \gamma^2}. \end{cases}$$
(A.4)

Case 3: if  $\lambda_1 = 0$  and  $\lambda_2 > 0$ , we then have

$$\begin{cases} D + \gamma f - 2bq - \omega = 0, \\ \gamma q - 2kf + \lambda_2 = 0, \\ f = \delta M. \end{cases} \Rightarrow \begin{cases} q = \frac{D - \omega + \gamma \delta M}{2b}, \\ f = \delta M, \\ \lambda_2 = \frac{(4bk - \gamma^2)\delta M - \gamma(D - \omega)}{2b}. \end{cases}$$
(A.5)

To focus on positive solutions, it is assumed that  $D-\omega>0$  and  $4bk-\gamma^2>0$ . In order to represent the boundary conditions of each case in a more concise way, we define two parameters:  $N_1=D-(4bk-\gamma^2)M/\gamma$  and  $N_2=D-(4bk-\gamma^2)\delta M/\gamma$ . Given that  $0<\delta<1$ , it is obvious that  $N_1< N_2$ . Case 1 requires  $\lambda_1>0$ , or equivalently  $0\leq \omega < N_1$ . Case 2 requires  $\delta M< f< M$ , or  $N_1\leq \omega \leq N_2$ . Case 3 requires  $\lambda_2>0$ , or  $D>\omega>N_2$ .

## **Appendix B. Proof of Proposition 2**

The objective is to find out under what condition the optimal airport charge, in the absence of the grandfather policy, will fall into the interval  $(N_2,D)$ . Hence, we consider the cases without the grandfather policy, i.e., Case 4 and Case 5 in Table 1. The airport profit can be sketched as the piecewise functions as shown in Fig 3, where  $\hat{\omega} = (D-h)/2$  and  $\overline{\omega} = (D-h+\gamma M)/2$  ( $\hat{\omega} < \overline{\omega}$ ). Generally, the optimal airport charge reflects a trade-off between the aeronautical service revenues and the non-aeronautical revenues.

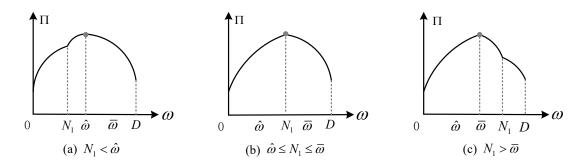


Fig. 3. Airport profit in the absence of the grandfather policy.

Since  $N_1 < N_2$ , it is thus impossible that the optimal airport charge in Fig. 3(b) or Fig. 3(c) falls into the interval  $(N_2, D)$ . In Fig. 3(a), however, the optimal airport charge  $\hat{\omega}$  can fall into the interval  $(N_2, D)$  if the condition  $N_1 < N_2 < \hat{\omega}$  holds, which yields

$$N_2 < \hat{\omega} \Leftrightarrow D - (4bk - \gamma^2)\delta M/\gamma < (D-h)/2 \Rightarrow D < \frac{2\delta M(4bk - \gamma^2)}{\gamma} - h.$$

## **Appendix C. Properties of Fig. 2**

We first derive the social welfare function in the absence of the grandfather policy (see the black solid line in Fig. 2). According to the equilibrium solutions obtained in Case 4 and Case 5 in Table 1, we have the following results

$$\begin{cases} \omega \in [0, N_1) \Rightarrow f = M, q = \frac{D - \omega + \gamma M}{2b} \Rightarrow \frac{\partial SW}{\partial \omega} = -\frac{D + \omega + 2h + \gamma M}{4b} < 0, \\ \omega \in [N_1, D) \Rightarrow f = \frac{\gamma (D - \omega)}{4bk - \gamma^2}, q = \frac{2k(D - \omega)}{4bk - \gamma^2} \Rightarrow \frac{\partial SW}{\partial \omega} = \frac{2k\left((2bk - \gamma^2)(D - \omega) - (D + h)(4bk - \gamma^2)\right)}{4bk - \gamma^2}. \end{cases}$$
(C.1)

Therefore, the social welfare function is decreasing in the interval  $[0, N_1)$ .

In order to examine its monotonicity in the interval  $[N_1, D]$ , we have the following

$$\begin{cases}
2bk - \gamma^{2} \leq \frac{\gamma(D+h)}{M} \Rightarrow \frac{\partial SW}{\partial \omega} < 0, \\
2bk - \gamma^{2} > \frac{\gamma(D+h)}{M} \Rightarrow \begin{cases}
\frac{\partial SW}{\partial \omega} > 0 & \text{if } \omega \in [N_{1}, \omega^{*}) \\
\frac{\partial SW}{\partial \omega} < 0 & \text{if } \omega \in [\omega^{*}, D)
\end{cases}, & \text{where } \omega^{*} = D - \frac{(D+h)(4bk - \gamma^{2})}{2bk - \gamma^{2}}.$$
(C.2)

Given the above results, the social welfare function decreases in the interval [0,D) if  $\omega^* \leq N_1$ ; Otherwise, it first decreases in the interval  $[0,N_1)$ , then increases in  $[N_1,\omega^*)$  and finally decreases in  $[\omega^*,D)$ .

Next, we prove that the maximum social welfare in the absence of the grandfather policy is obtained at  $\omega = \omega^*$  as shown in Fig. 2(b) or Fig. 2(c) when  $N_1 < \omega^*$  (or equivalently

$$2bk - \gamma^2 > \gamma(D+h)/M$$
). When  $\omega = \omega^* = D - \frac{(D+h)(4bk - \gamma^2)}{2bk - \gamma^2}$ , we have  $f = \frac{\gamma(D-\omega^*)}{4bk - \gamma^2}$ 

and  $q = \frac{2k(D - \omega^*)}{4bk - \gamma^2}$  according to Case 5 in Table 1. Substituting the value of  $\omega^*$  into the expressions, the resultant values of f and q are calculated as

$$f = \frac{(D+h)\gamma}{2bk-\gamma^2} \quad \text{and} \quad q = \frac{2k(D+h)}{2bk-\gamma^2}.$$
 (C.3)

On the other hand, the Hessian matrix H of the unconstrained social welfare maximization problem Eq. (12) (treating q and f as independent decision variables) is

$$H = \begin{pmatrix} \frac{\partial SW}{\partial q^2} & \frac{\partial SW}{\partial q \partial f} \\ \frac{\partial SW}{\partial f \partial q} & \frac{\partial SW}{\partial f^2} \end{pmatrix} = \begin{pmatrix} -b & \gamma \\ \gamma & -2k \end{pmatrix}. \tag{C.4}$$

The order principal minor determinants of matrix H are

$$|-b| = -b < 0$$
, and  $|H| = 2bk - \gamma^2 > \gamma(D+h)/M > 0$ . (C.5)

Consequently, the Hessian matrix H is negative definite, and thus the maximization problem has a unique optimal solution, which is exactly the same as that in Eq. (C.3). Therefore, the maximum social welfare in the absence of the grandfather policy must be achieved at  $\omega = \omega^*$ .