The following publication Shaban, I. A., Wang, Z. X., Chan, F. T. S., Chung, S. H., & Qu, T. (2019). An extra-baggage service price setting with reference to cargo prices using multi-item newsvendor model. Computers and Industrial Engineering, 128, 877–885 is available at https://doi.org/10.1016/j.cie.2018.10.041.

An Extra-Baggage Service Price Setting with Reference to Cargo Prices Using Multi-Item Newsvendor Model

Abstract

The world is now considered as a small village, and with increasing commercial and cultural adverse between nations, air transport continues to boom and passenger numbers are continuously increasing. This increase leads to extensive use of wide-bodied aircraft which have large space in the aircraft belly-hold resulting in cargo sector having overcapacity. In this regard, the belly-hold overcapacity is discussed with two objectives to fill the unused spaces. Firstly, designing the extra-baggage service. Secondly, conferring price setting for the new service with reference to the cargo price. We adopt the multi-item newsvendor based pricing model to derive a price formula for the extra-baggage service as a function of cargo price. The stochastic extra-baggage and deterministic cargo demands were firstly modeled. The results showed that the optimal extra-baggage price can be determined as a function of cargo price in terms of the base price plus a premium price, but it gave an exaggerated extra-baggage price. Therefore, we modeled the cargo demand in stochastic form, and the second model showed better results because it cancels the effect of cargo penalties in deterministic cargo demand. Moreover, the stochastic cargo model gives the airline the opportunity to switch between two different pricing strategies either market penetration or pure premium strategy. Finally, the model can be generalized for any two products that share common features and serve in the same market.

Keywords: Pricing, Air cargo, Newsvendor, Multi-item newsvendor, Extra-baggage service.

1 Introduction

Air freight transportation is major transportation mode that contributes to the growth of global trade by facilitating goods movement around the world. Air freight covers over a third of the trade in the world by value (IATA, 2017a). The freight business has recovered its growth by 3.8% in 2016, relative to 2015, and compared to the number of passengers in 2016, the passenger demand is also expected to increase to nearly the double in 2035 (IATA, 2016). This number of passengers motivates airlines to use wide-bodied aircraft. Accordingly, the airlines gain more space in their belly-holds which leads an overcapacity in the cargo sector, although the cargo demand is increasing, (IATA, 2017b). Moreover, relative to the current wide-bodied aircraft capacities, the cargo demand is not enough to fill them (Economist, 2016).

This situation differs from the last decade, where the problem was to allocate the cargo in a constrained capacity. Therefore, capacity allocation models were developed to solve such constrained capacity. Air cargo capacity allocation had a different perspective in the past, where, Hellermann, (2006) showed that capacity option pricing in the long-term and in the spot market involves three players, the shipper, the forwarder as an intermediary and the airline as the asset provider. He also used the Stackelberg game to solve the relation between the single airline which offers the main capacity in the long-term to a single intermediary (freight forwarder) and suggested that the spot market price premium policy be used rather than the reservation spot prices. Hellermann's model was extended by Tao et al., (2017) to develop a capacity selling and reservation model in long-term contracting between one airline and several freight forwarders. The authors also used the Stackelberg model to define and simulate the game between the airline and the different forwarders.

The booking horizon starts twelve months before the flight departure, and the booking request comes from several forwarders individually, so the accumulated cargo demand from the different forwarders along the booking horizon should be considered. The decision required is whether to accept or reject the forwarder's request for the existing capacity and the forwarder allowed space (Amaruchkul & Lorchirachoonkul, 2011). Gupta, (2008) proposed two different capacity contracting schemes between a single airline and a single forwarder, suggesting that the carrier may deal with the freight rate exogenously, and the forwarder reserve the capacity with fixed fees in advance. Second, he proposed a similar scheme, but with zero reservation fees. The two schemes give the carrier the power to control the contract clauses and parameters. However, these schemes decrease the forwarders opportunity costs, and the forwarder needs to make many games to select a suitable carrier to reserve capacity, because the carrier still has the whole power to decide the freight rate.

The abovementioned studies give good solutions to capacity allocation under constrained capacity and also whether to accept or reject the forwarders request according to the accumulated demand, and hence allocation. The allocation process under constrained capacity leads to two possible conditions. First, the airline decides to sell the full space without any buffering, but some forwarders do not show up or cancel their booking, hence, the airline loses the opportunity cost due to the shortage. Second, the airline tries to avoid the shortage problem in an unwise way and they sell the full capacity with a large buffer to avoid the shortage, but the overestimated buffer leads to overbooking problems and the airline incurs a penalty cost due to the offloaded cargo amounts. Thus, studying the optimal overbooking levels is crucial for airlines to estimate accurate booking limits. Kasilingam, (1997) for example, developed a one-dimensional cargo overbooking model under random aircraft capacity, formulated for both discrete and continuous probability distributions. Wannakrairot and Phumchusri, (2016) formulated air cargo overbooking in two dimensions (weight and volume) under uncertain overbooking and show -up rates, and booking orders density. Overbooking studies are crucial when the capacity is less than the accumulated demand. However, as aforementioned, the actual situation on many routes is that airlines suffer an overcapacity in the aircraft belly-hold, and hence cargo overcapacity is a new problem.

Nevertheless, the overcapacity problem is general on many routes and it changes with the season, with some routes receiving demand excess of their capacity. This leads to unbalance between the different routes; some are underutilized, and others have a hot-selling (over-demand) situation (Feng et al., 2015). To solve this problem, they adopted strategic foreclosure to develop a closed form model in order to set a bundling mechanism and balance the demand between the hot-selling routes and the underutilized routes. The authors divided the forwarders into two categories; partners and excluded forwarders. They assumed that the partners are the forwarders who have the largest demand, have the priority for allocation in the hot-selling routes, but they also have more space in the underutilized routes. The excluded forwarders were allocated directly to the underutilized routes if the partners had already occupied the hot-selling routes. In this connection, Feng's study gives a partial solution for the overcapacity problem because of two reasons; first, the model solves the seasonal problem, but it does not solve the other periods over the year which encounters the large unused space in the aircraft belly-hold. Second, even if the study maintains the balance between the hot-selling routes and the underutilized routes, the capacity allocation does not increase the demand and the bundling mechanism cannot guarantee full utilization of the under underutilized routes.

In this paper, we believe that airlines need to open new markets in order to receive new demand and fill the belly-hold space as much as possible. As mentioned above, however, the airline allocates the passengers baggage, and the cargo to the wide-bodied aircraft belly-hold, and a large part of the space in the belly-hold remains unused during the flight. Regarding the passengers' baggage, US-based airlines gained more than 4 billion USD from the baggage fees (Bureau of Transportation Statistics, 2016). Moreover, twenty percent of Britons are charged for overweight baggage, on average £395 million per year (DailyMail, 2017). Despite the big profits from baggage, particularly overweight baggage, many passengers try to set their baggage weight to meet the regular checked in baggage requirement. To the best of our knowledge, passengers' excess baggage have not attracted much research attention, with only Wong et al., (2009) discussing the passenger regular checked-in baggage limits.

The contribution of this paper is represented in proposing a new service to motivate passengers to bring all their luggage to the airline and be charged a feasible price, instead of paying high penalty fees. The service is named "Extra-baggage service". This service can be treated as a special cargo service, because the extra-baggage and cargo services have many common features, see **Section 2.** The common features between the extra-baggage and the cargo services inspired us to set the new service (extra-baggage) price with the aid of the existing service (cargo), so we adopted the multi-item newsvendor model in order to formulate the extra-baggage price with reference to the cargo price. The newsvendor model fits our objective because its scope is the same as of our work, and it can be used to set diverse pricing strategies for the suppliers. It also includes different demand statements, and it is commonly used to determine the optimum quantity stock (Khouja, 1999). The newsvendor model can be formulated as a function of price, for example, Whitin, (1955) was the first to introduce the newsvendor-based price model but in a single-period problem, while Thowsen, (1975) proved that be it could be used in dynamic pricing. The newsvendor was then extended to solve the multi-item problem, for instance Lau and Lua, (1996) provided a solution for multiple product news-vendor model under single and multiple capacity constraints. Erlebacher, (2000) developed a heuristic solution for the multiple product newsvendor in the case where the optimality condition was violated. In addition, Abdel-Aal and Selim, (2017) proposed a non-linear integer programming formulation to the multiple product newsevendor model in the case of full and partial market access under the risk-averse condition. In terms of the application of the newsvendor model in the cargo industry, Hellermann, (2006) discussed the validity of the classical newsvendor model to decide on the cargo amounts that the airline can provide to the freight forwarders and/or the shippers. Furthermore, Wong et al., (2009) applied the multi-item newsvendor model in order to determine the passenger baggage limits in the aircraft belly-hold.

In this paper, we apply the multi-product newsvendor model in a new use, to determine the optimum price of the extra-baggage service with the reference of the cargo price. At first, as the extra-baggage is a new service, its demands modeled in stochastic form and the cargo demand modeled in deterministic form (stochastic-deterministic (S-D) model). Then because of the limitations of the deterministic cargo demand model, we defined the cargo demand in stochastic form (stochastic -stochastic (S-S) model). The model showed that the results of the S-S model cope the (S-D) model limitations. Therefore, this model is the nucleus of a set of other models, which is concerned with formulating the price of new product/service with reference to an existing product/service, provided that the new and the old products have common features, and serve in the same market.

The rest of this research is organized as follows. The next section shows a detailed description for the extra-baggage service, and the common and the different features between the cargo and the extra-baggage services. Section 3 encompasses theoretical and numerical analysis for the multi-item newsvendor model. Section 4, contains the conclusions and recommendations for future work.

2 Excess baggage, extra-baggage, and cargo services

In this section, we discuss in detail the extra-baggage service and its operation, and the common features between the cargo and the extra-baggage services. The extra-baggage service is proposed to replace the current excessive baggage scheme in the airlines.

The traveler pays for the excess baggage or overweight baggage, when the permitted baggage limits are broken. The allowed baggage differs among airlines, for example, some airlines allow only one hold bag assigned to the aircraft belly-hold and one handbag. Others allow two hold bags and one in the cabin. The hold bag weight ranges between 20 to 35 kg, while the hand-held bags range from 7 to 23 kg. for instance, Emirates offers up to 35 kg in the hold, and not more than 7kg in economy class (Emirates, 2018). Any excess weight over the sum of the handbags and the hold bags must be paid as excess baggage. The excess baggage charge is complex and confusing for many passengers. This is because the excess baggage fees differ from airline to airline, from one route to another, and from one ticket class to another. For example, some carriers provide near to 30 different charging sets. Not only is the charge rate complexity the main dilemma in the current excess baggage regime, but also the value of the charge itself. For instance, Lufthansa, in economy class on certain route charges 300 USD for any excess weight over the allowed baggage weight, and the excess weight should not exceed 23kg., while if the passenger wants to book excess weight in advance, he/she should pay 200 USD for 23kg, and 650 USD to book more weight, but cannot book more than 32kg (Lufthansa, 2017). All these reasons cause the passengers to check their permitted baggage allowance carefully, resulting in unused space in the wide-bodied aircraft belly-hold.

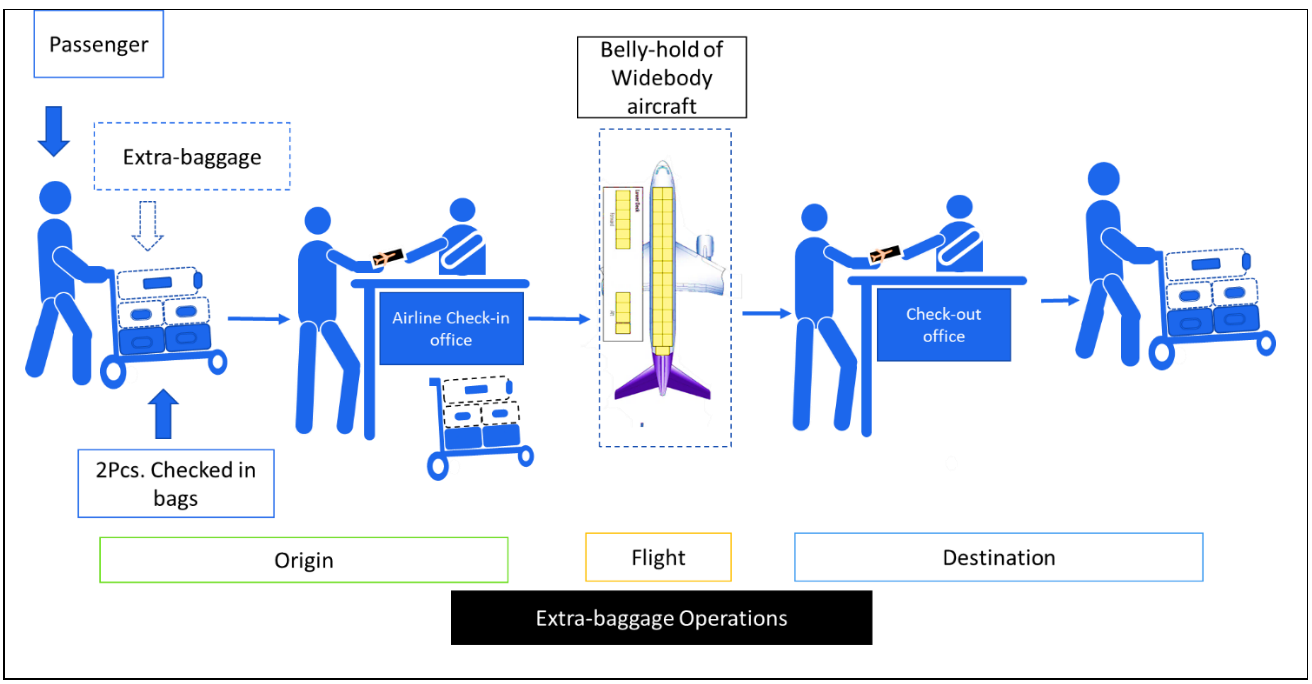
In this aspect, we propose a new excess baggage service rather than the current scheme under the name "Extra-baggage service." The extra-baggage service can be considered as a special cargo service, where the extra-baggage and the cargo services have some common features, especially when they serve the same market. **Figure 1** shows the main common features and the basic differences between the extra-baggage and cargo services.

The extra-baggage operations start from the passenger who brings his/her belongings to the airport when traveling to a new destination. The airline can offer two payment options for the extra-baggage. First, the passenger books extra-weight during the air ticket booking or through the airline's website, and the extra-baggage will be added to the tickets. Second, he/she pays directly for any extra-weight over the allowed baggage in the airline check-in office, and the rate for the second case should be equal to the first one plus penalty costs for each kilogram, and hence the airline can avoid an inconvenient situation as in the current excess bag scheme. Passengers check up with their allowed bags and extra-baggage in the airport at the airline check-in office, similar to the regular check-in process. The extra-baggage and the allowed baggage for the passenger are loaded together into the wide-bodied aircraft belly-hold. Then the passenger picks all of his/her baggage (allowed, and extra-baggage) when arriving at the destination.

On the other hand, the cargo operations are more complicated than the extra-baggage, where the cargo operations start from the shippers who either buy a space directly from the airline, usually for large shippers, or the shippers send their goods to a freight forwarder that reserves space in the aircraft. The freight forwarder works as a mediator between the shippers and the airline. The freight forwarder consolidates the freight from different shippers and sends it to the combination airline (The combination airline carries both passengers and cargo). The freight forwarder at the destination receives the freight from the airline and sends it to the consignee. Also, the consignee may be the person who receives the freight directly from the airline.

The shipper can be a passenger, where the passenger who has many belongings sends them to a freight forwarder, the freight forwarder in turns takes the freight to the destination. Therefore, the extra-baggage switches to cargo and becomes the whole cargo operations. The cargo rate is based on freight weight and volume, and it is measured in FTK (freight ton kilometers). Thus, the extra-baggage can be considered as a special cargo, the passenger brings the belongings to his flight, and the airline offers a rate without any negotiation, the same as for air tickets.

Based on these common features between air cargo and extra-baggage, we adopted the multi-product newsvendor to set the extra-baggage price as a function of cargo price. The model has been formulated in two different demand combination environments; first, stochastic extra-baggage and deterministic cargo demand; second, stochastic extra-baggage and stochastic demand.



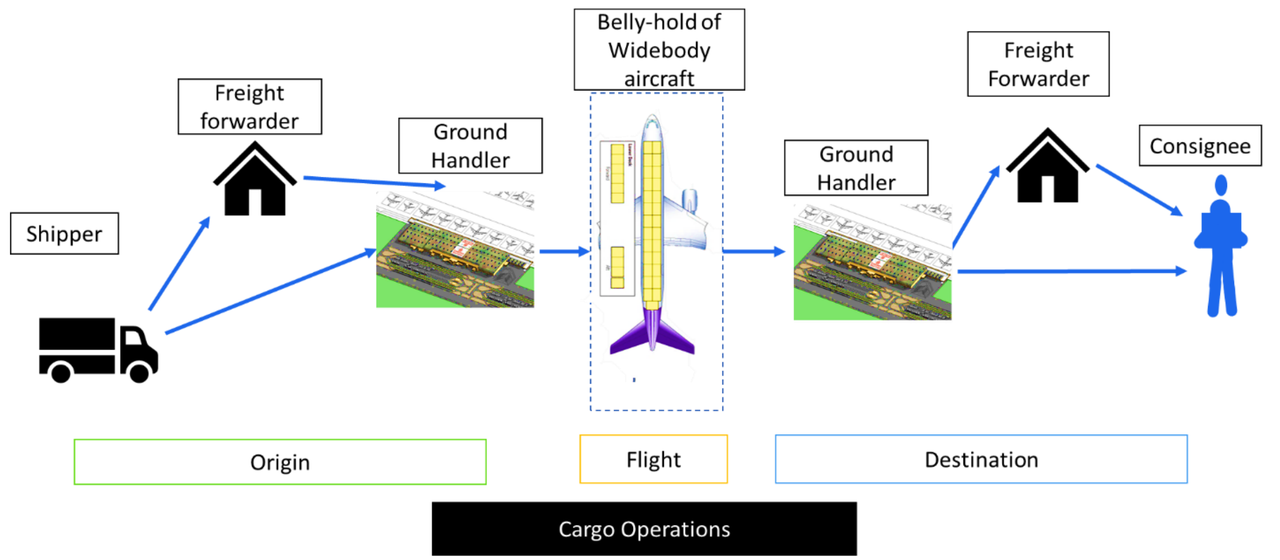


Figure 1 The basic operations of extra-baggage and the cargo services

3 The Extra-baggage optimal price

In the above section, the extra-baggage service was introduced, and the relationship between the extra-baggage and the cargo was also discussed. In this section, the theoretical formulation of the extra-baggage price is developed. The formulation is undertaken in two steps; in the first step, because the extra-baggage is a new service, its demand unpredictable, so the extra-baggage demand is formulated in a stochastic form, while the cargo service demand is formulated in a deterministic environment. Formulating the cargo under deterministic demand is due to the fact that air cargo is a stable industry, thus it can be predicted. In the second step, the model is then enhanced by formulating both extra-baggage and the cargo under stochastic demand.

3.1 Stochastic-deterministic (S-D) model

Consider a combination airline offers cargo service j at a price p_j, and it plans to add a new extra-baggage service i to its services list. The airline plans to set the price p_i for the new extra-baggage service with reference to the cargo price p_j. Also, it aims to determine the space to offer in the new service x_i, in order to maximize the overall expected profit. Both services have price-dependent demand functions. We assume that the demand environment of product j is deterministic, the product i demand is random, and noise does not depend on its price. In the literature, the randomness of the price demand function is modelled in two forms, typically additive and multiplicative. Mills, (1959) defined the additive form as $D(p,\varepsilon) = y(p) + \varepsilon$, and Karlin and Carr, (1962) defined the multiplicative form as $D(p,\varepsilon) = y(p)\varepsilon$, where $y(p)$ is the function which depicts the decreasing relationship between the demand and the price, and ε is the random variable which may be defined in the range $[A, B]$. In economics literature, $y(p)$ is represented in linear form $y(p_i) = a - bp,$ when the demand function is additive, and in iso-elastic form $y(p_i) = ap_i{}^{-b}$ when the demand function is multiplicative (Kocabıyıkoğlu & Popescu, 2011). To interpret the relationship between $D(p,\varepsilon)$ and $y(p)$, the second term is the deterministic demand curve, and this term changes in stochastic demand in the first term, by adding or multiplying the scaling factor ε which represents the random market size.

In this model, we suppose that the cargo market is more stable so as to make it predictable, thus its demand may be represented in deterministic form $y(p_j)$, while the extra-baggage market size is still difficult to predict, so it is better to be represented in the stochastic form $D(p_i,\varepsilon_i)$. As abovementioned, both additive and multiplicative approaches are used in the literature. However, the extra-baggage is proposed as a solution for the overcapacity problem, which is unstable on different routes, while the demand in some routes exceeds its capacity, especially in seasonal periods. Therefore, the extra-baggage demand is preferably formulated in iso-elastic form, and hence, we adopt the multiplicative model to formulate the extra-baggage service, in the multiplicative demand case, $D(p_i,\varepsilon_i) = y(p_i)\varepsilon_i$, where $y(p_i)$ can be replaced by $y(p_i) = ap_i{}^{-b}$. In the single period problem, the airline offers quantity x_j of the cargo service j at unit operational cost c_j, and x_i of the extra-baggage i at unit operational cost c_i. The operational cost of both cargo and extra-baggage can be written as in equation (1):

$$C = c_i x_i + c_j x_j \tag{1}$$

If the offered quantity of the cargo during the period exceeds the forecasted demand, then the quantity difference stands for the leftover $x_j - y(p_j)$ at unit overbooking cost h_j; similarly, the leftover in the extra-baggage i is $x_i - D(p_i,\varepsilon_i)$ at unit overbooking cost h_i; where $h_i \geq c_i$, and $h_j \geq c_j$. On the other hand, if the forecasted demand exceeds the offered quantities x_i, x_j then the airline will incur unit shortage "opportunity" costs, s_i, s_j respectively. The total flight revenue is $p_i D(p_i,\varepsilon_i) + p_j y(p_j)$. The profit function can be expressed in terms of quantity and price, as in equation (2);

$$\Pi(x_i,x_j,p_i,p_j) = \begin{cases} p_i D(p_i,\varepsilon_i) + p_j y(p_j) - c_i x_j - c_j x_j - h_j\big(x_i - D(p_i,\varepsilon_i)\big) \\ \qquad -h_J\big(x_j - y(p_j)\big); \qquad\qquad D(p_i,\varepsilon_i) + y(p_j) \leq x_i + x_j \\ p_i x_i + p_j x_j - c_i x_i - c_j x_j - s_i(D(p_i,\varepsilon_i) - x_i) \\ \qquad -s_J\big(y(p_j) - x_j\big); \qquad\qquad D(p_i,\varepsilon_i) + y(p_j) > x_i + x_j \end{cases} \tag{2}$$

Also, assuming that the total demand of the extra-baggage i and cargo j equals the aircraft belly-hold capacity $\emptyset$, as in equation (3),

$$D(p_i,\varepsilon_i) + y(p_j) = \emptyset \tag{3}$$

The proper form of the demand of extra-baggage i in this profit equation is $D(p_i,\varepsilon_i) = y(p_i)\varepsilon_i$, and $y(p_j) = \emptyset - y(p_i)\varepsilon_i$, identifying the q value for each service by $q_i = x_i/y(p_i)$, and $q_j = x_j/(\emptyset - y(p_i)\varepsilon_i)$.

$$\Pi(q_i, q_j, p_i, p_j) = \begin{cases} p_i y(p_i)\varepsilon_i + p_j(\emptyset - y(p_i)\varepsilon_i) - c_i q_i y(p_i) - c_j q_j(\emptyset - y(p_i)\varepsilon_i) \\ -h_i y(p_i)(q_i - \varepsilon_i) - h_j(q_i - 1)(\emptyset - y(p_i)\varepsilon_i); & \varepsilon_i \le q_i \\ p_i y(p_i) q_i + p_j q_j(\emptyset - y(p_i)\varepsilon_i) - c_i q_i y(p_i) - c_j(\emptyset - y(p_i)\varepsilon_i) - \\ s_i y(p_i)(\varepsilon_i - q_i) - s_j(\emptyset - y(p_i)\varepsilon_i)(1 - q_i); & \varepsilon_i > q_i \end{cases} \tag{4}$$

These variable transformations solve the problem of the relationship between the sum of the demand and the total quantity because interpretation of the new transformation is only related to the random variable of extra-baggage ε_i , and thus, the value q_i. The study aims to set the price and the offered quantity of the extra-baggage with reference to the existing cargo service when they have some common features. In this case, the shortage in product i occurs when ε_i exceeds the q_i value, and the airline experiences leftover if ε_i is less than the q_i value. Regarding the leftover and shortage in cargo j, they can be determined based on the likelihood of the shortage and the leftover of the extra-baggage i. Therefore, the corresponding optimal capacity offering and pricing policy is to offer $x_i{}^* = y(p_i{}^*)q_i{}^*$ units in the aircraft belly-hold for the extra-baggage and sell it at unit price $p_i{}^*$ which is function of air cargo j price p_j, where $q_i{}^*$ and $p_i{}^*$, maximize the expected profit.

$$\begin{aligned} &E[\Pi(q_i, p_i, q_j, p_j)] \\ &\quad = (p_i - c_i - p_j + c_j q_j)y(p_j)\mu_i + (p_j - c_j q_j)\emptyset \\ &\quad - (c_i + h_i)y(p_i)\int_A^{q_i}(q_i - \varepsilon_i)f(\varepsilon_i)d\varepsilon_i - h_c\emptyset(q_j - 1)\int_A^{q_i} f(\varepsilon_i)\,d\varepsilon_i \\ &\quad + h_j(q_j - 1)y(p_i)\int_A^{q_i}\varepsilon_i\, f(\varepsilon_i)d\varepsilon_i - (p_i + s_i - c_i)y(p_i)\int_{q_i}^{B}(\varepsilon_i - q_i)\, f(\varepsilon_i)d\varepsilon_i \\ &\quad - (p_j + s_j)(1 - q_j)\int_{q_I}^{B} f(\varepsilon_i)\,d\varepsilon_i + (p_j - s_j)(1 - q_j)y(p_i)\int_{q_i}^{B}\varepsilon_i\, f(\varepsilon_i)d\varepsilon_i \end{aligned} \tag{5}$$

Defining $\Lambda(q_i) = \int_A^{q_I}(q_i - \varepsilon_i)f(\varepsilon_i)d\varepsilon_i$; and $\Theta(q_i) = \int_{q_i}^{B}(\varepsilon_i - q_i)\, f(\varepsilon_i)d\varepsilon_i$; $\varpi(q_j) = \int_A^{q_i}\varepsilon_i\, f(\varepsilon_i)d\varepsilon_i$, and $\xi(q_j) = \int_{q_i}^{B}\varepsilon_i\, f(\varepsilon_i)d\varepsilon_i$, equation (5) can be written as equation (6)

$$E[\Pi(q_i, p_i)] = \psi(p_i, p_j) - L(q_i, q_j, p_i, p_j) \tag{6}$$

where

$$\psi(p_i, p_j) = (p_i - c_i - p_j + c_j q_j)y(p_i)\mu_i + (p_j - c_j q_j)\emptyset \tag{7}$$

and

$$\begin{aligned} L(q_i, q_j, p_i, p_j) &= (c_i + h_i)y(p_i)\,\Lambda(q_i) + h_j\emptyset(q_j - 1)\int_A^{q_i} f(\varepsilon_i)\,d\varepsilon_i - h_j(q_j - 1)y(p_i)\,\varpi(q_j) \\ &+ (p_i + s_i - c_i)y(p_i)\Theta(q_i) + (p_j + s_j)(1 - q_j)\int_{q_I}^{B} f(\varepsilon_i)\,d\varepsilon_i \\ &- (p_j - s_j)(1 - q_j)y(p_i)\xi(q_j) \end{aligned} \tag{8}$$

Mills, (1959) defined the interpretation of the riskless profit function, in equation (7), as a deterministic profit value when replacing the uncertainty value of the product value ε_i by the mean value μ_i. In this model, the profit function holds extra-baggage and cargo and thus, the profit is a function of the two items prices. Lemma 1 can be derived from equation (7).

Lemma 1 *For extra-baggage service with stochastic demand and cargo service with deterministic demand the riskless profit of a flight which carries both extra-baggage and cargo can be estimated by*

$$\psi(p_i, p_j) = (p_i - c_i)y(p_i)\mu_i + (p_j - c_j)y(p_j)$$

Proof

Equation (7) is derived from the transformed objective function (5), and the equation can be divided into two terms; the first is related to the extra-baggage service i and the second to the cargo service j.

$\psi(p_i, p_j) = \{(p_i - c_i)y(p_i)\mu_i\} + \{(c_j q_j - p_j)y(p_i)\mu_i + (p_j - c_j)\emptyset\}$, and as previously mentioned,

when changing the stochastic demand to deterministic form the ε is replaced by μ and therefore, we can move to the rule, $y(p_i)\mu_i = (\emptyset - y(p_j))$. □

Lemma 1 proves that the model keeps the basic meaning of the profit function which is defined by the difference between the total revenue and the total costs. This also ensures model robustness and simplicity.

Equation (8) is the loss function according to the definition of Silver and Peterson, (1985), which evaluates the leftover cost $(c_i + h_i)$ for each of $\Lambda(q_i)y(p_i)$ of extra-baggage i, the expected leftover when too large value of q_i is selected; in addition to $h_j\emptyset$ for each likelihood of the leftover in extra-baggage minus the mean value of h_j in the range $[A, q_i]$; if the value of q_j is chosen more than one, and the underage costs for product i is $(p_i + s_i - c_i)$ for each $\Theta(q_i)y(p_i)$ expected shortages when too small value q_i is selected. The shortage costs of the cargo service are $(p_j + s_j)$ for the likelihood extra-baggage quantity minus $(p_j - s_j)$for expected extra-baggage quantity; if the value of q_j is chosen less than one. The expected profit is depicted in (6), and the riskless profit occurs in certain selected demands with no uncertainty, and the uncertainty factor in the model is added to the expected penalties.

The objective of the model is to maximize the expected profit in (7):

$$\underset{q_i, p_i}{\text{Max}imize} \; E[(\Pi(q_i, p_i)]. \tag{9}$$

The first and the second partial derivatives of $E[(\Pi(q_i, p_i)]$ are taken with respect to q_i and p_i

$$\frac{\partial E[(\Pi(q_i, p_i)]}{\partial q_i} = -(c_i + h_i)y(p_i)F(q_i) + (p_i + s_i - c_i)y(p_i)[1 - F(q_i)] \tag{10}$$

$$\frac{\partial^2 E[(\Pi(q_i, p_i)]}{\partial {q_i}^2} = -[(c_i + h_i) + (p_i + s_i - c_i)]y(p_i)f(q_i) \tag{11}$$

Equations (10) and (11) prove that the expected profit function is concave in product i quantity when equation (10) is equal to zero. Similarly, the overall expected profit is concave in both extra-baggage i and cargo j, equation (12)

$$\frac{\partial E[(\Pi(q_i,p_i)]}{\partial p_i{}^*}=0 \tag{12}$$

Lemma 2 follows equation (12):

Lemma 2 *For fixed extra-baggage and cargo quantities, the optimal price of extra-baggage i is determined uniquely as a function of the cargo service j and the mixed quantities of the two services:*

$$p_i{}^*=\frac{\mu_i p_i{}^o}{\mu_i-\Theta(q_i)}+\frac{b[(c_i+h_i)\Lambda(q_i)-h_j(q_j-1)\varpi(q_i)+(s_i-c_i)\Theta(q_i)-(p_j-s_j)(1-q_j)\xi(q_i)]}{(b-1)(\mu_i-\Theta(q_i))}$$

where $p_i{}^o=\frac{b(c_i+p_j-c_jq_j)}{b-1}$

Proof. for the multiplicative demand of extra-baggage, $p_i{}^o$ is the optimal riskless price, which maximizes the riskless profit $\psi(p_i,p_j)=(p_i-c_i-p_j+c_jq_j)y(p_i)\mu_i+(p_j-c_jq_j)\emptyset$, where $y(p_i)=ap_i{}^{-b}$, by definition. The maximum value of the riskless profit function can be obtained when equating the first derivative w.r.t p_i to zero, Thus, letting:

$$\frac{\partial\psi(p_i,p_j)}{\partial p_i}=(p_i-c_i-p_j+c_jq_j)y'(p_i)\mu_i+y(p_i)\mu_i;$$

$$a\mu_i p_i{}^{-b-1}\left[b(c_i+p_j-c_jq_j)-(b-1)p_i{}^o\right]=0$$

Therefore, the maximum value of $\psi(p_i,p_j)$ is at $p_i{}^o=\frac{b(c_i+p_j-c_jq_j)}{b-1}$,

Next, regarding the overall expected profit function in equation (6), determine the optimal price of extra-baggage as a function of the air cargo, and maximize the expected profit. We need to equate the first differentiation of the (6) w.r.t p_i to zero;

$$\begin{aligned}\frac{\partial E[(\Pi(q_i,p_i)]}{\partial p_i{}^*}&=(p_i-c_i-p_j+c_jq_j)y'(p_i)\mu_i+y(p_i)\mu_i-(c_i+h_i)y'(p_i)\Lambda(q_i)\\&+h_j(q_j-1)y'(p_i)\varpi(q_i)-(p_i+s_i-c_i)y'(p_i)\Theta(q_i)-y(p_i)\Theta(q_i)\\&+(p_j+s_j)(1-q_j)y'^{(p_i)}\xi(q_i)\end{aligned}$$

hence;

$$\begin{aligned}-abp_i{}^{-b-1}&(p_i{}^*-c_i-p_j+c_jq_j)\mu_i+ap_i{}^{-b}\mu_i+abp_i{}^{-b-1}(c_i+h_i)\Lambda(q_i)-abp_i{}^{-b-1}h_j(q_j-1)\varpi(q_i)\\&+abp_i{}^{-b-1}(p_i{}^*+s_i-c_i)\Theta(q_i)-ap_i{}^{-b-1}\Theta(q_i)-p_i{}^*\Theta(q_i)\\&-ab\,p_i{}^{-b-1}(p_j+s_j)(1-q_j)\xi(q_i)=0\end{aligned}$$

Thus,

$$p_i{}^*=\frac{\mu_i p_i{}^o}{\mu_i-\Theta(q_i)}+\frac{b[(c_i+h_i)\Lambda(q_i)-h_j(q_j-1)\varpi(q_i)+(s_i-c_i)\Theta(q_i)-(p_j-s_j)(1-q_j)\xi(q_i)]}{(b-1)(\mu_i-\Theta(q_i))}. \quad \square$$

The riskless price $p_i{}^o$ is concave in the cargo price, where the extra-baggage riskless price increases with the increase of cargo price until it reaches a turn down point then the extra-baggage riskless price decreases. The airline can forecast short-term market demand, andis most likely a combination of the Gamma and normal distributions as mentioned by Swan, (2002). Thus, the random variable is normally distributed in the application of numerical analysis with $\mu_i=0.6, and\ \ \sigma_i=0.2$. See **Figure 2.**

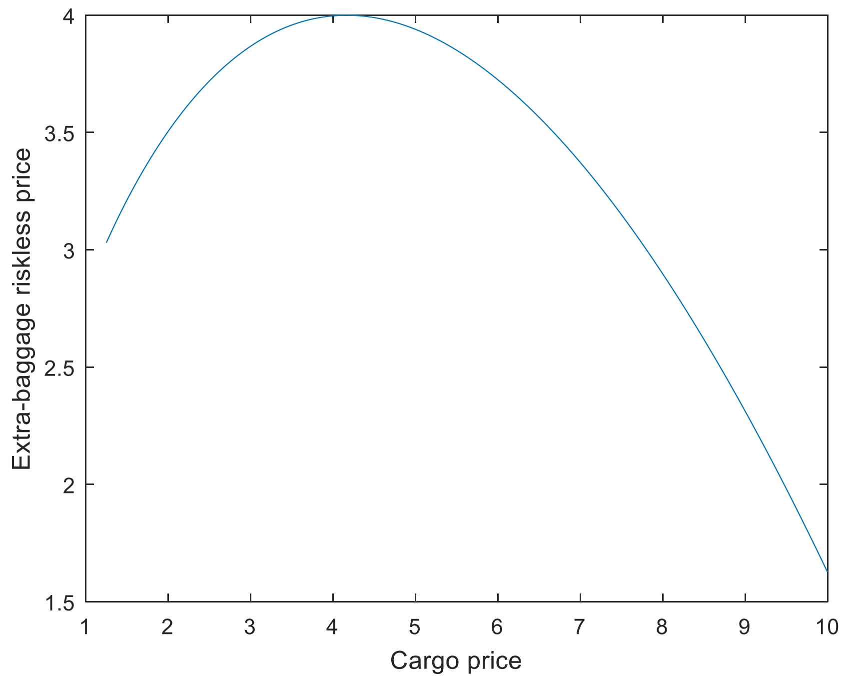


Figure 2 A plot of extra- baggage riskless price p_i^0 as a function of cargo price p_j ranging from 0.5 to 10 in increment 0.025, the cargo quantity is assumed as 16000, a_1=20000, b_1=1.5, and b_2=1.25

The extra-baggage price elasticity b sets the maxima of the extra-baggage riskless price, as shown in **Figure 3**, where the riskless price of the extra-baggage decreases exponentially with increase of the extra-baggage price elasticity.

Lemma 2 captures the optimal price of the extra-baggage i as a function of the cargo price and mixed quatities of both services. The price equation containes three terms; each term expresses an important concern in order to set the price for extra-baggage with reference to the cargo price. The next theorem summarizes these three terms.

Theorem 1 *For given extra-baggage and cargo quantities,setting the optimal price for the extra-baggage with reference of cargo price j requires the airline to define three terms;*

i. *The safety factor* $\frac{\mu_i}{\mu_i-\Theta(q_i)}$

ii. *The riskless price* p_i^o

iii. *The premium value* $\frac{b\left[(c_i+h_i)\Lambda(q_i)-h_j(q_j-1)\varpi(q_i)+(s_i-c_i)\Theta(q_i)-(p_j-s_j)(1-q_j)\xi(q_i)\right]}{(b-1)(\mu_i-\Theta(q_i))}$

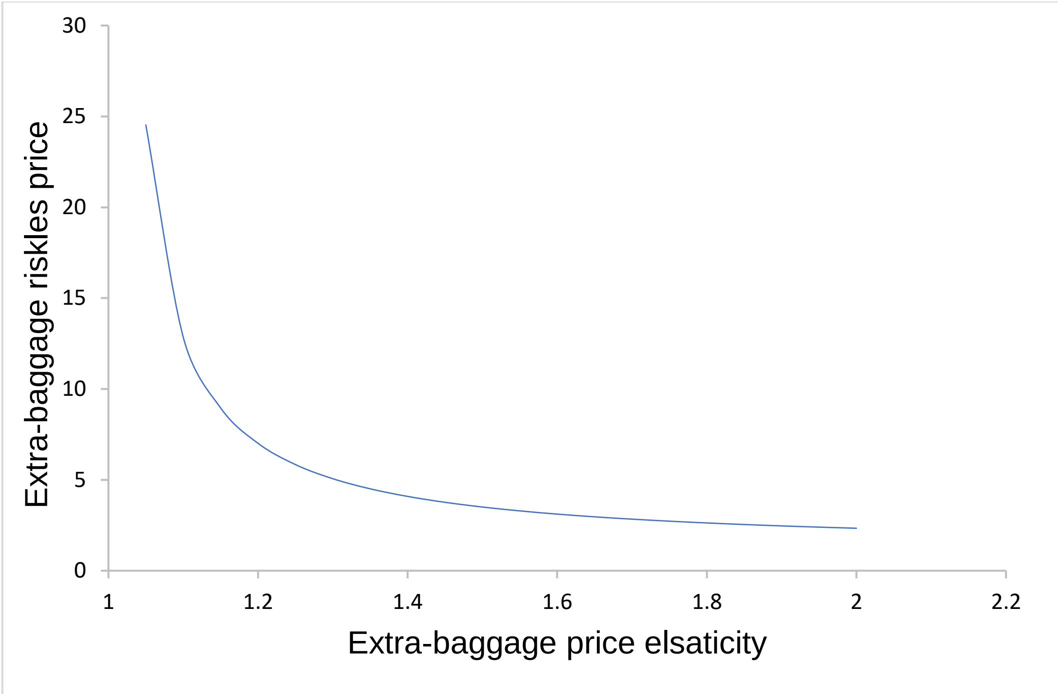


Figure 3 A plot of extra-baggage riskless price as a function of price elasticities, (a) Cargo price elasticity and, (b) Extra-baggage price elasticity.

The theorem explains the main theme of the pricing scheme which can be followed to set the price of the extra-baggage with the aid of the cargo price. This model is inspired from Petruzzi and Dada, (1999), who's model considers the use of single period newsvendor model to set the price of a single product. The authors defined the optimal price of the product as the sum of the base price and the premium amount in the multiplicative demand function, whereas our model uses product price information to set a different product price. Theorem 1 and lemma 2 define the base as price equaling the riskless price of the extra-baggage multiplied by a safety factor and the premium value which is a function of the overall expected shortage and the overall expected leftover amount, and the expected sales of the extra-baggage. Hence, the extra-baggage optimal price can be expressed by equation (13),

$$p_i^* = p_{B_i} + \frac{b\left[(c_i+h_i)E[leftover\ (q_i,p_i)]+(s_i-c_i)E[shortage\ (q_i,p_i)]-[h_j[overage\ of\ q_j]+\left(p_j-s_j\right)E[shortage\ of\ q_j]\right]}{(b-1)E[sales(q_i,p_i)]} \quad (13)$$

Therefore, the interpretation of the base and premium prices may be described next; the base price is obtainable from estimating the total costs of the extra-baggage service multiplied by the safety factor "SF" which ensures that the riskless price is not underestimated by dividing the mean demand over the expected sales, where $SF \geq 1$, The base price is also concave in the cargo price, see **Figure 4.**

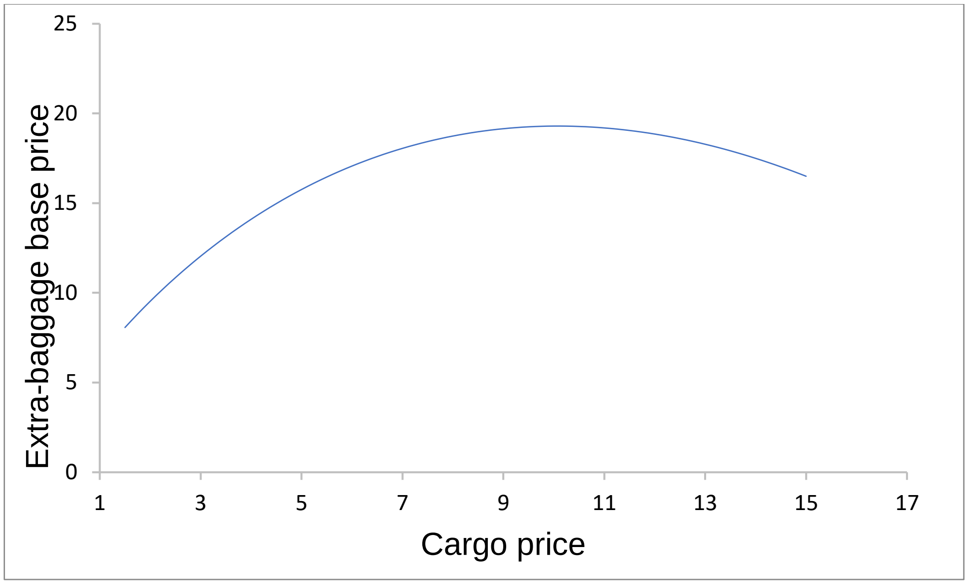


Figure 4 A plot of extra-baggage base price as a function of cargo prices.

The base price concavity is more flat in the actual range of cargo prices, so we assumed a larger range of extra-baggage at cargo prices to show the curve behavior. The airline can manage the safety factor by studying the demand average and the expected shortage. Moreover, the premium value in selling price for the extra-baggage is based on the formula which considers the overall expected leftover of the extra-baggage, in addition to the overall expected shortage costs of the same service. The result agrees` with Petruzzi's results, but this model holds a defined service, cargo service, which is the airline's main service, so the cargo service affects the price of the new extra-baggage service. This is because of the demand uncertainty in the extra-baggage, thus the sum of the expected penalties of the j cargo service is subtracted from the expected penalties of the i extra-baggage and divied on the overall expected extra-baggage sales. The cargo deterministic demand represents a big limitation in this model, where the premium value increases exponentially with respect to the increase of the cargo price, see **Figure 5**.

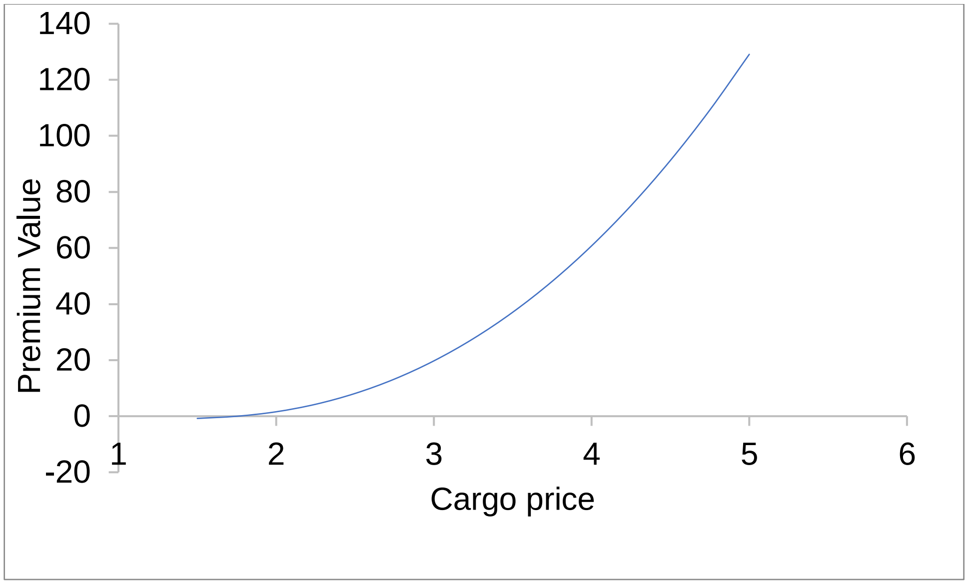


Figure 5 A plot of Premium value as a function of cargo price in the stochastic-deterministic model where, x_1=16000, x_2=18000, b_1=1.25, $\mu = 0.6$, and $\sigma = 0.2$

This vast upsurge in premium value leads to an overestimated optimum prices and converts the concave behavior of the base price to a monotonic form at the optimum price, see **Figure 6**.

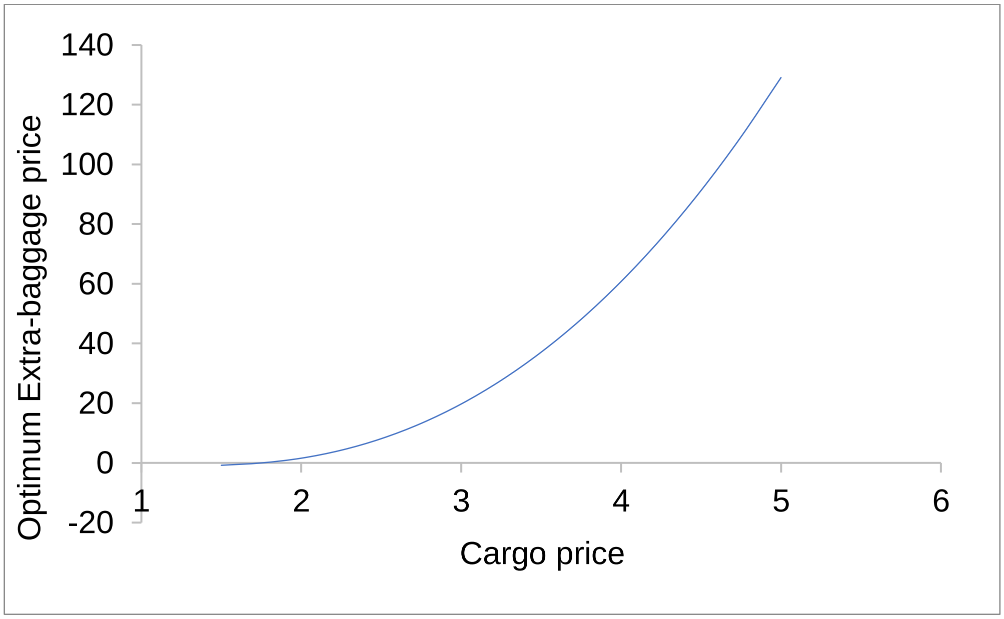


Figure 6 A plot of extra-baggage optimum price as a function of cargo price

3.2 Stochastic-stochastic model

In the previous section, an extra-baggage pricing newsvendor model was developed based on a stochastic extra-baggage demand and deterministic cargo demand. Because of the limitation of the stochastic-deterministic (S-D) model, which appeared the exponential increase in the premium value, in this model, the cargo demand is also formulated in stochastic form, assuming the demand function of the cargo as,

$$D(p_j, \varepsilon_j) = y(p_j) + \varepsilon_j \tag{14}$$

and thus, the profit function in equation (4) can be changed to;

$$\Pi(q_i, q_j, p_i, p_j) = \begin{cases} p_i y(p_i)\varepsilon_i + p_j(\emptyset - y(p_i)\varepsilon_i) - c_i q_i y(p_i) - c_j \emptyset + c_j y(p_i)\varepsilon_i - c_j(q_j - \varepsilon_j) \\ -h_i y(p_i)(q_i - \varepsilon_i) - h_j(q_i - \varepsilon_j); & \varepsilon_i \leq q_i, \varepsilon_j \leq q_j \\ p_i y(p_i) q_i + p_j \emptyset - p_j(\varepsilon_j - q_j) - p_j y(p_i)\varepsilon_i - c_i q_i y(p_i) - c_j(\varepsilon_j - q_j) - c_j \emptyset - \\ c_j y(p_i)\varepsilon_i - s_i y(p_i)(\varepsilon_i - q_i) - s_j(\varepsilon_j - q_j); & \varepsilon_i > q_i, \varepsilon_j > q_j \end{cases} \tag{15}$$

The advantage of modeling the cargo in the formulation removes the effect of the extra-baggage noise on the cargo penalties, which also decreases the price of the extra-baggage. Thus, the corresponding optimal capacity offering and pricing policy changes, where $x_i^* = y(p_i^*)q_i^*$ but under the random cargo demand, also the optimal extra-baggage price p_i^* is also a function of cargo price. These values q_i^*, p_i^*can be determined from the new expected profit;

$$\begin{aligned} E[(\Pi(q_i, p_i, q_j, p_j)] &= [p_i - c_i - p_j + c_j] y(p_i)\mu_i + (p_j - c_j)\emptyset \\ &\quad - (c_i + h_i) y(p_i) \int_{A_1}^{q_i} [q_i - \varepsilon_i]\, f(\varepsilon_i)\varepsilon_i - (c_j + h_j) \int_{A_2}^{q_j} [q_j - \varepsilon_j]\, f(\varepsilon_j) d\varepsilon_j \\ &\quad - [p_i + s_i - c_i]\, y(p_i) \int_{q_i}^{B_1} [\varepsilon_i - q_i]\, f(\varepsilon_i) d\varepsilon_i \\ &\quad - [p_j + s_j - c_j] \int_{q_j}^{B_2} [\varepsilon_j - q_j]\, f(\varepsilon_j) d\varepsilon_j \end{aligned} \tag{16}$$

Equation (16) can be written in terms of riskless profit and loss in a different form than equation (6) as;

$$E[(\Pi(q_i, p_i, q_j, p_j)] = \psi(p_i, p_j) + L(q_i, p_i) + L(q_j, p_j) \tag{17}$$

where the riskless profit is

$$\psi(p_i, p_j) = [p_i - c_i - p_j + c_j] y(p_i)\mu_i + (p_j - c_j)\emptyset \tag{18}$$

Lemma 3 *For two stochastic demand items, if the extra-baggage is the first item and its demand is formulated in multiplicative form, and the cargo is the second item which is modeled in additive demand form, then the riskless profit can be estimated by the sum of mean profit of the extra-baggage service and the mean profit of the cargo service,*

$$\psi(p_i, p_j) = [p_i - c_i] y(p_i)\mu_i + (p_j - c_j)[y(p_j) + \mu_j]$$

Proof: The proof of this lemma can be derived as far as lemma 1.

Regarding the loss function, and unlike equation (6), which describe the losses of both extra-baggage and cargo in a form depending on the extra-baggage status, the loss function in (17) is the sum of the expected losses of extra-baggage and cargo losses where the extra-baggage losses can be expressed as;

$$L(q_i, p_i) = [(c_i + h_i)\Lambda(q_i) + [p_i + s_i - c_i]\Theta(q_i)]y(p_i)$$

and the loss function with respect to the cargo is;

$$L(q_j, p_j) = (c_j + h_j)\Lambda(q_j) + [p_j + s_j - c_j]\,\Theta(q_j)$$

As shown in the loss functions of both the extra-baggage and cargo, the formulas are not interrelated with each other, which means that penalties have a different interpretation than in the old model. The shortage and the leftover of the extra-baggage does not change, but the cargo penalty cost is the shortage $[p_j + s_j - c_j]$ when too small a value of q_j is chosen over the $\Theta(q_j)$ range, and the penalty is the overbooking cost $(c_j + h_j)$ when the airline selects a too large q_j over the $\Lambda(q_j)$ range.

The objective also can be represented by equation (7), and the optimum quantity of extra-baggage can also be obtained through equation (10) by equating the equation to zero, and similarly for the optimal extra-baggage price in equation (12). Lemma 4 can be inferred from equation (12) and (16);

Lemma 4 *For a fixed extra-baggage quantity and cargo quantity, the optimal price of the extra-baggage is uniquely determined as a function of the cargo price and the mixed extra-baggage and cargo quantities:*

$$p_i^* = \frac{\mu_i p_i^o}{\mu_i - \Theta(q_i)} + \frac{b}{(b-1)}\frac{[(c_i + h_i)\Lambda(q_i) + (s_i - c_i)\Theta(q_i)]}{\mu_i - \Theta(q_i)}$$

Proof: for the multiplicative demand of the extra-baggage, and additive demand of the cargo, p_i^o is the optimal riskless price, which maximizes the riskless profit $\psi(p_i, p_j) = (p_i - c_i - p_j + c_j)y(p_i)\mu_i + (p_j - c_j)\emptyset$, where $y(p_i) = ap_i^{-b}$, by definition, and the maximum value of the riskless profit function can be obtained when equating the first derivative w.r.t p_i to zero, Thus:

$$\frac{\partial\psi(p_i, p_j)}{\partial p_i} = (p_i - c_i - p_j + c_j)y'(p_i)\mu_i + y(p_i)\mu_i;$$

$$a\mu_i(b-1)p_i^{-b-1}\left[\frac{b}{(b-1)}(c_i + p_j - c_j) - p_i^o\right] = 0$$

Therefore, the maximum value of $\psi(p_i, p_j)$ is at $p_i^o = \frac{b}{b-1}(c_i + p_j - c_j)$,

Next, regarding the overall expected profit function in equation (6), determine the optimal price of extra-baggage as a function of the air cargo, and maximize the expected profit. By equating the first differentiate of the (6) w.r.t p_i to zero:

$$\frac{\partial E[(\Pi(q_i,p_i)]}{\partial p_i{}^*} = \left(p_i - c_i - p_j + c_j q_j\right) y'(p_i)\mu_i + y(p_i)\mu_i - (c_i + h_i) y'(p_i)\Lambda(q_i)$$
$$+ h_j\left(q_j - 1\right) y'(p_i)\varpi(q_i) - (p_i + s_i - c_i) y'(p_i)\Theta(q_i) - y(p_i)\Theta(q_i)$$
$$+ \left(p_j + s_j\right)\left(1 - q_j\right) y'(p_i)\xi(q_i)$$

hence;

$$a(b-1)p_i{}^{-b-1}[\mu_i - \Theta(q_i)]\left[\frac{\mu_i}{\mu_i - \Theta(q_i)} p_i{}^o + \frac{b}{(b-1)} \frac{[(c_i + h_i)\Lambda(q_i) + (s_i - c_i)\Theta(q_i)]}{\mu_i - \Theta(q_i)} - p_i{}^*\right] = 0$$

Thus,

$$p_i{}^* = \frac{\mu_i p_i{}^o}{\mu_i - \Theta(q_i)} + \frac{b}{(b-1)} \frac{[(c_i+h_i)\Lambda(q_i)+(s_i-c_i)\Theta(q_i)]}{(\mu_i - \Theta(q_i))}. \quad \square$$

Lemma 4 depicts the optimal price of the extra-baggage as a function of the cargo price and mixed quantities of both services. The extra-baggage riskless price in this model is a linear function of cargo price. The riskless price equals the extra-baggage operational costs plus the cargo profit, and this price can set the demand elasticity factor $b/(b-1)$. **Figure 7**, and **Figure 8**, show the effect of price elsticity on the riskless price, where the extra-baggage riskless price decreases exponetially with the increase in the demand elsaticity. The price equation contains three terms; each term expresses an important concern in order to set the price for the extra-baggage with reference to the cargo price. The next theorem summarizes these three terms.

Theorem 2 *For given extra-baggage and cargo quantities q_i and q_j, respectively, setting the optimal price for the extra-baggage with reference of product j requires the airline to define three terms;*

i. *The safety factor SF =* $\frac{\mu_i}{\mu_i - \Theta(q_i)}$
ii. *The riskless price* $p_i{}^o$
iii. *The premium value =* $\frac{b}{(b-1)} \frac{[(c_i+h_i)\Lambda(q_i)+(s_i-c_i)\Theta(q_i)]}{(\mu_i - \Theta(q_i))}$

The theorem explains the main theme of the pricing scheme which can be followed to set the price of the extra-baggage with the aid of the cargo price. Theorem 2 and lemma 4 define the extra-baggage base price as equal to the riskless price multiplied by the safety factor, and the premium value is a function of the overall expected shortage and the overall expected leftover amount, and the expected sales of the extra-baggage. So, the extra-baggage optimal price can be expressed by equation (19),

$$p_i{}^* = p_{B_i} + \frac{b}{(b-1)} \frac{\left[(c_i+h_i)E[leftover\ (q_i,p_i)] + (s_i - c_i)E[shortage\ (q_i,p_i)]\right]}{E[sales(q_i,p_i)]} \quad (19)$$

Therefore, in this theorem, the base price is obtainable from the estimation of the total costs of the extra-baggage service multiplied by the safety factor “SF” which is related the expected sales of the extra-baggage, and the premium selling price for the extra-baggage based on the formula which takes the overall expected leftover of extra-baggage into account, in addition to the overall expected shortage costs of the same service. This is because the demand is uncertain between the two services, and thus the sum of the expected penalties of the cargo service is deducted from the expected penalties in theorem 2, and the results are divided into the overall expected sales of extra-baggage i .

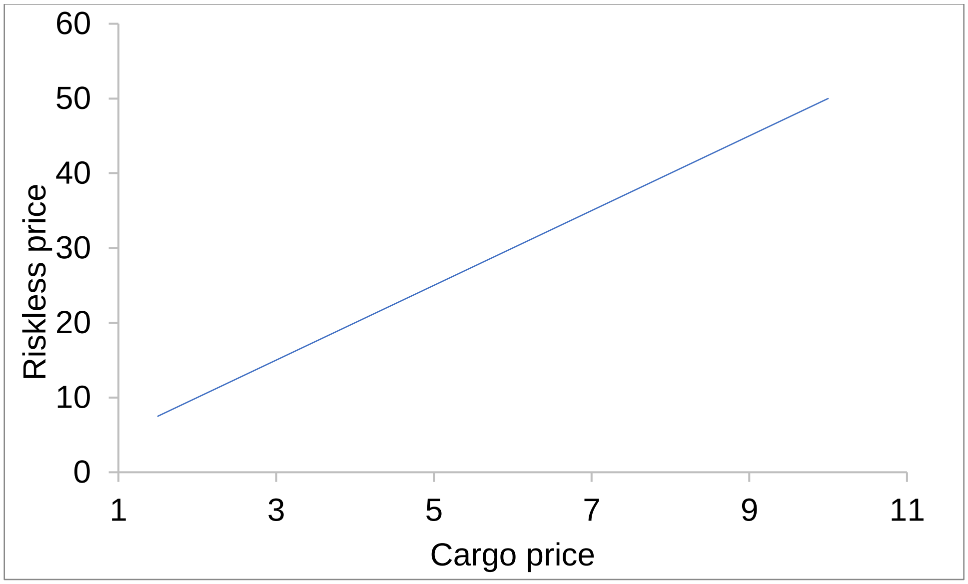


Figure 7 A plot of extra-baggage riskless price as a function of cargo price when the extra-baggage price elasticity b=1.25

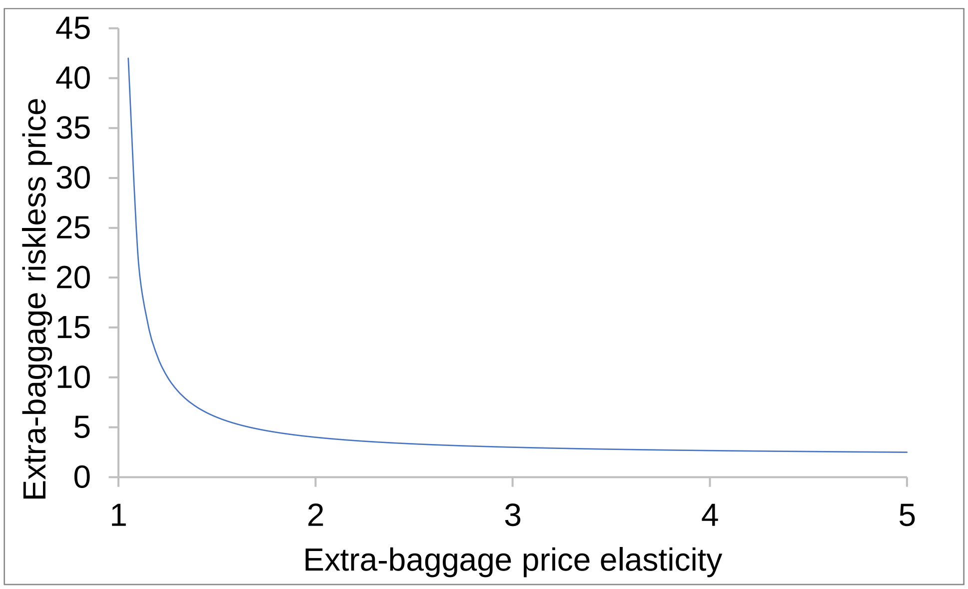


Figure 8 The effect of the extra-baggage price elasticity on the riskless price

Equation (19) shows the difference between formulating the cargo in deterministic or stochastic forms. It can be induced that the deterministic formulation of the cargo shows the way the extra-baggage uncertainty affects the cargo, see equation (5). However, when the cargo uncertainty is included in the model, it neutralizes the extra-baggage effect, and therefore cargo penalties cannot be involved in the extra-baggage price. Therefore, the limitation of the previous model is avoided, more over the penalty can be negative or positive, see **Figure 9**.

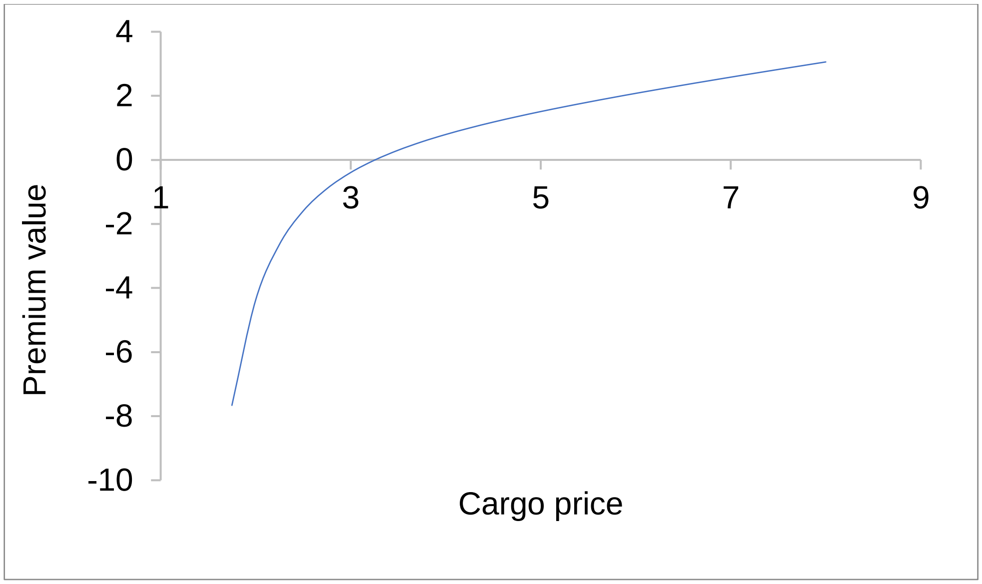


Figure 9 A plot of Premium value as a function of cargo price in the stochastic-stochastic model, x_1=16000, x_2=18000, b_1=1.25, μ=0.6, and σ=0.2

As shown in **Figure 9**, the premium value is not always positive, and this leads to the result that the optimum price of the extra-baggage is not always bigger than the base price. However, the premium value is negative when the cargo price is low, and hence the extra-baggage optimum price will be less than the base price and in some cases less than the cargo price. and vice versa for positive premium value. The logic contradicts this behavior, because the smaller the cargo price means the larger the cargo demand and less space remains in the aircraft belly-hold. Thus, the airline must increase the extra-baggage price. Therefore, the optimum extra-baggage can be estimated by

$$p_i^* = p_{B_i} + [-premium\ value]$$

On the other hand, if the airline needs to penetrate the market, so they may offer less extra-baggage prices to accelerate the extra-baggage adoption they can then use the derived equation

$$p_i^* = p_{B_i} + [premium\ value]$$

4 Conclusions

In this paper, a new excess baggage and the overweight scheme are identified and discussed as an extra-baggage service in a combination airline. The service is described and compared with both the current excess baggage schemes in the different airlines, and with the cargo scheme side. The extra-baggage scheme is proposed as a solution for overcapacity resulting from the extensive use of wide-bodied aircraft and the reduction in the sea shipping rates. The extra-baggage is treated as a special cargo service, and we show that the extra-baggage can be considered as a cargo if the passenger acts as a shipper and sends luggage to a freight forwarder. The frieght forwader inturn forwards them to the airline which assign these luggage to the aircraft belly-hold, in addition to permitted baggage.

In this aspect, we adopt the multi-item newsvendor model to set the extra-baggage price with reference to the cargo price. The model is formulated in a stochastic-deterministic (S-D) environment, and because of the model limitation, it is then formulated in stochastic-stochastic (S-S) form, where the premium value in the second model (S-S) shows better results over the first oneb (S-D). The extra-baggage price can be set with reference to the cargo prices, in terms of base price, and premium value. The extra-baggage price is the sum of the base price and the estimated premium value. The premium value is the expected penalties over the expected sold capacity and can be either positive or negative. This means that the optimum price may be larger than the base price, or may be less than the base price,but it cannot be less than the riskless price.

Finally, the two-items newsvendor model can be used to set a new product and/or service price with reference to another product and/or service, provided that the two services and/or products share some common features and serve in the same market.

A further investigation in the future will involve conducting a market investigation model for the proposed extra-baggage service, to investigate suitable price policies for the extra-baggage service, to examine the effect of seasonality on the offered prices and to discuss price discrimination between the different flight classes.

Acknowledgments

The work described in this paper was supported by grants from The Natural Science Foundation of China (Grant No. 71471158); The Hong Kong Polytechnic University under student account code RUMZ. The authors also would like to thank The Hong Kong Polytechnic University Research Committee for financial and technical support.

References

Abdel-Aal, M. A. M., & Selim, S. Z. (2017). Risk-averse multi-product selective newsvendor problem with different market entry scenarios under CVaR criterion. *Computers & Industrial Engineering, 103*, 250-261. doi:https://doi.org/10.1016/j.cie.2016.11.026.

Amaruchkul, K., & Lorchirachoonkul, V. (2011). Air-cargo capacity allocation for multiple freight forwarders. *Transportation Research Part E: Logistics and Transportation Review, 47*(1), 30-40. doi:https://doi.org/10.1016/j.tre.2010.07.008.

Bureau of Transportation Statistics. (2016). Baggage fees by airline 2016. Retrieved from https://www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/subject_areas/airline_information/baggage_fees/html/2016.html/ Accessed 13 February 2018.

DailyMail. (2017). Revealed: Brits pay out nearly £400million in excess baggage charges each year - because of baffling rules. Retrieved from http://www.dailymail.co.uk/travel/travel_news/article-4530100/Brits-pay-395million-excess-baggage-charges.html/ Accessed 21 January 2018.

Erlebacher, S. J. (2000). Optimal and heuristic solutions for the multi-item newsvendor problemwith a single capacity constraint, *Production and Operations Management, 9*(3), 303-318. doi:10.1111/j.1937-5956.2000.tb00139.x

Feng, B., Li, Y., & Shen, H. (2015). Tying mechanism for airlines' air cargo capacity allocation. *European Journal of Operational Research, 244*(1), 322-330. doi:http://dx.doi.org/10.1016/j.ejor.2015.01.014.

Gupta, D. (2008). Flexible carrier–forwarder contracts for air cargo business. *Journal of Revenue & Pricing Management, 7*(4), 341-356. doi:10.1057/rpm.2008.29.

Hellermann, R. (2006). *Capacity options for revenue management : theory and applications in the air cargo industry*. Thesis (doctoral)--WHU, Otto Beisheim School of Management., Berlin.

IATA. (2016). IATA Forecasts passenger demand to double over 20 years. Retrieved from http://www.iata.org/pressroom/pr/Pages/2016-10-18-02.aspx/ Accessed 28 June 2017.

IATA. (2017a). Enabling global trade. air cargo. Retrieved from http://www.iata.org/whatwedo/cargo/pages/index.aspx/ Accessed 13 November 2017.

IATA.(2017b). IATA cargo strategy. Retrieved from http://www.iata.org/whatwedo/cargo/Documents/cargo-strategy.pdf/ Accessed 13 December, 2017.

Karlin, S., & Carr, C. R. (1962). Prices and optimal inventory policy. *Studies in applied probability and management science, 4,* 159-172.

Kasilingam, R. G. (1997). An economic model for air cargo overbooking under stochastic capacity. *Computers & Industrial Engineering, 32*(1), 221-226. doi:http://dx.doi.org/10.1016/S0360-8352(96)00211-2.

Khouja, M. (1999). The single-period (news-vendor) problem: literature review and suggestions for future research. *Omega, 27*(5), 537-553. doi:http://dx.doi.org/10.1016/S0305-0483(99)00017-1.

Kocabıyıkoğlu, A., & Popescu, I. (2011). An elasticity approach to the newsvendor with price-sensitive demand. *Operations Research, 59*(2), 301-312. doi:10.1287/opre.1100.0890

Lau, H.-S., & Hing-Ling Lau, A. (1996). The newsstand problem: a capacitated multiple-product single-period inventory problem. *European Journal of Operational Research, 94*(1), 29-42. doi:http://dx.doi.org/10.1016/0377-2217(95)00192-1.

Lufthansa. (2017). Flat rates for excess baggage.Retrieved from https://www.lufthansa.com/us/en/Flat-rates-for-excess-baggage/ 25 February, 2018.

Mills, E. S. (1959). Uncertainty and price theory. *The Quarterly Journal of Economics, 73*(1), 116-130. doi:10.2307/1883828

Petruzzi, N. C., & Dada, M. (1999). Pricing and the newsvendor problem: a review with extensions. *Operations Research, 47*(2), 183-194. doi:10.1287/opre.47.2.183.

Silver Edward, A., & Peterson, R. (1985). *Decision systems for inventory management and production planning* (2nd ed.). New York: Wiley.

Swan, W. M. (2002). Airline demand distributions: passenger revenue management and spill. *Transportation Research Part E: Logistics and Transportation Review, 38*(3), 253-263. doi:https://doi.org/10.1016/S1366-5545(02)00009-1.

Tao, Y., Chew, E. P., Lee, L. H., & Wang, L. (2017). A capacity pricing and reservation problem under option contract in the air cargo freight industry. *Computers & Industrial Engineering, 110*, 560-572. doi:https://doi.org/10.1016/j.cie.2017.04.029.

The Economist. (2016). Too little freight, too much space; Air cargo.(demand for air-cargo businesses). https://www.economist.com/news/business/21695013-overcapacity-hits-another-part-transport-industry-too-little-freight-too-much-space/ Accessed 21 January 2018.

Thowsen, G. T. (1975). A dynamic, nonstationary inventory problem for a price/quantity setting firm. *Naval Research Logistics Quarterly, 22*(3), 461-476. doi:10.1002/nav.3800220306.

Wannakrairot, A., & Phumchusri, N. (2016). Two-dimensional air cargo overbooking models under stochastic booking request level, show-up rate and booking request density. *Computers & Industrial Engineering, 100*(Supplement C), 1-12. doi:https://doi.org/10.1016/j.cie.2016.08.001

Whitin, T. M. (1955). Inventory control and price theory. *Management Science, 2*(1), 61-68.

Wong, W., Zhang, A., Hui, Y., & Leung, L. (2009). Optimal baggage-limit policy: Airline passenger and cargo allocation. *Transportation Science, 43*(3), 355-369. doi:10.1287/trsc.1090.0266.