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Improved Laplacian Matrix based power flow solver for DC distribution networks

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Abstract

Distribution networks feature distinct topologies than transmission networks, such as radial or weakly meshed structures with tens of thousands of nodes. They have more points of power injection owing to the integration of distributed generators and high R/X ratios. Furthermore, there has recently been a surge of interest in DC distribution networks. In the planning and operation of modern distribution systems, load flow needs to be executed in series considering short intervals of time in the order of minutes or even less. Hence, these networks require a load flow solver that can converge fast with low computational burden. In this paper, we propose a unique iterative power flow solver based on graph theory for DC distribution networks. The proposed formulation is flexible and can handle both radial and mesh configurations with just one connectivity matrix. To validate the proposed method, we used the IEEE 33 bus test feeder and compared the results with an existing methodology. Results suggest that the proposed method is robust and possesses fast convergence.

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Keywords: DC distribution networks; DC loads; Fast convergence; Graph theory; Meshed networks; Laplacian Matrix; Load flow analysis

1. Introduction

There is an increasing proliferation of direct current (DC) powered appliances in our everyday lives, such as mobile phones, tablets, laptops, personal computers, autos (electric vehicles), LED lights, and electronic loads [1]. Furthermore, most of the distributed generators (DGs) connected to distribution systems have inherently DC output [2]. All this has invoked an interest in distributing electrical power in DC paradigm [3–5].

In a direct current distribution network (DC-DN), a transformer is replaced by a converter which typically operates in three modes: constant voltage, constant current, and constant power [6]. When a converter operates in constant power mode, it supplies power to a constant power load (CPL) [7]. In this case, the load flow (LF) constraint

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becomes nonlinear and requires an iterative LF solution [8]. The sheer number of DGs in modern distribution networks requires fast, robust and generic LF solvers.

Recently, different techniques have been proposed to address LF solution in DC grids. For instance, a LF solver based on loop analysis with linear formulation is proposed in [9]. But the approximations made in this study for linearization can sacrifice accuracy, especially under practical conditions. Another study proposes an LF formulation for DC-DN with radial configurations using numerical techniques based on successive approximation and Taylor series expansion [10]. Another Taylor series based linear formulation is proposed in [11] for DC-DNs, which considers both radial and meshed configurations, but the accuracy is limited by loading conditions. A comprehensive overview and comparative analysis of LF studies for DC-DNs can be found in [12,13]. Most of these studies are based on linear methods and demonstrations are mostly performed for radial networks. Moreover, owing to the non-linearities in LF equations due to CPLs, the LF solvers are prone to diverge or converge to unrealistic solutions. Therefore, the convergence cannot be taken for granted [14]. An iterative LF solution based on Laplacian Matrix (LM) formulation using graph theory is proposed in [15] for DC-DNs, where two connectivity matrices are utilized to handle meshed configurations. In addition, this study provides the conditions for which the convergence of the proposed method can be guaranteed. Even though the LM algorithm is faster than direct load flow approaches, there is still a need and room for improvement in speed and convergence given the daunting complexity of modern distribution networks. The main objective of this research is to develop an LF algorithm for DC-DNs that can achieve fast convergence while maintaining the solution's uniqueness. In this paper, we reformulate the LM method by using only one connectivity matrix to improve its convergence characteristics. The key contributions of this paper are listed as follows:

- A reformulation of LM-based LF using graph theory for DC-DNs hosting CPLs for both radial and meshed topologies, called the Improved Laplacian Matrix (ILM) method is proposed where only one connectivity matrix is required to solve both radial and mesh configurations. The reason for using one connectivity matrix is to reduce the number of LF iterations.
- The proposed formulation is adaptable, allowing for modifications to the network parameters easily.
- The uniqueness of the solution is demonstrated by Banach-fixed point theorem using non-linear mapping.

A comparison is made in terms of processing time and the number of iterations required with the existing methodology [15]. The rest of this paper is laid out as follows. Section 2 contains a reformulation of the LM algorithm and its proof of convergence. The validation of the proposed method is presented in Section 3 with the help of simulation results and discussion. Finally, in Section 4, conclusions are drawn.

2. Method formulation

The reformulation of the LM algorithm [15] to improve its convergence characteristics is presented in this section. Consider the arbitrary DC-DN illustrated in Fig. 1 with $\boldsymbol{B}$ number of buses and $\boldsymbol{L}$ number of lines to demonstrate the formulation. The dotted lines in Fig. 1 are tie lines used to create a mesh configuration.

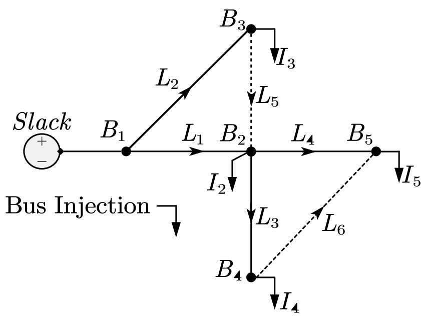

Fig. 1. Arbitrary DC distribution network.

2.1. Algorithm to form connectivity matrix

As indicated before, in a previous study, two connection matrices were utilized to solve a meshed DC network. In this study, just one generalized connectivity matrix is employed, which allows accounting for both radial and meshed networks. A single connection matrix will be used instead of two, which will result in a reduction in computing burden. The connection matrix needs to be built only once and before the execution of the LF algorithm for any given network, and it will remain constant throughout the iterative process.

The algorithm to construct the connectivity matrix $C \in \mathbb{R}^{B\times L}$ is given in Fig. 2.

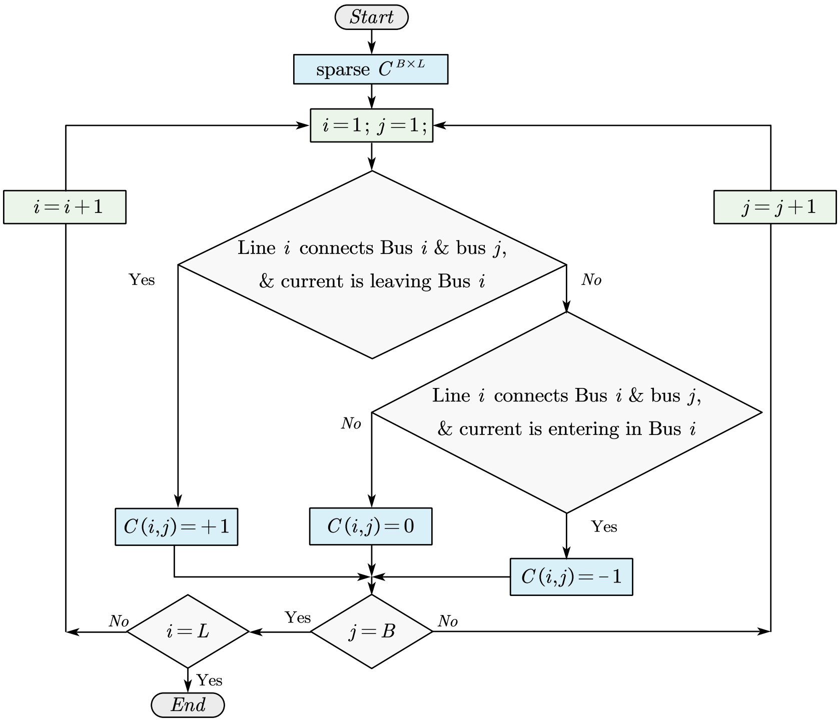

Fig. 2. Algorithm to form connectivity matrix.

If the grid connections are assumed to be as shown in Fig. 1, the connectivity matrix for a radial network (ignoring dotted lines) will be as follows.

$$C = \begin{bmatrix} +1 & +1 & 0 & 0 \\ -1 & 0 & +1 & +1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{1}$$

Now, if we consider the dotted lines in Fig. 1, which transform the system into a meshed configuration, the modified connectivity will take the following form.

$$C = \begin{bmatrix} +1 & +1 & 0 & 0 & 0 & 0 \\ -1 & 0 & +1 & +1 & -1 & 0 \\ 0 & -1 & 0 & 0 & +1 & 0 \\ 0 & 0 & -1 & 0 & 0 & +1 \\ 0 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} \tag{2}$$

The fundamental difference between the LM and ILM methods is that the LM technique requires a separate connectivity matrix to accommodate loops, whereas the ILM method allows radial and meshed configurations to be combined in one connectivity matrix. This generic connection matrix can handle radial and meshed configurations because the algorithm to create the connectivity matrix is the same for both configurations. DC-DNs normally have three types of buses: viz; 1- Constant Voltage Bus (υ), 2- Constant Power Bus (ρ), 3- Constant Current Bus (i). According to the bus type, we can split the connectivity matrix into submatrices as follows:

$$C = \begin{bmatrix} C_\upsilon & C_i & C_\rho \end{bmatrix} \tag{3}$$

Here our focus is to deal with constant voltage and constant power buses in the LF solution, so if there is no constant current bus in the system, we can rewrite (3) as follows:

$$C = \begin{bmatrix} C_\upsilon & C_\rho \end{bmatrix} \tag{4}$$

where,$C_\upsilon \in \mathbb{R}^{\upsilon \times L}$, $C_\rho \in \mathbb{R}^{\rho \times L}$ and $B = \upsilon + \rho$

From now onwards, "υ" refers to constant voltage buses, and "ρ" refers to buses with constant power demand. We need to seek a solution around the constraint that V_υ is perfectly known (1 pu), but V_ρ is an undetermined quantity. The relationship between branch currents and the bus injected currents can also be built using a connectivity matrix as follows:

$$I = C \times K \tag{5}$$

where, I is a vector of bus current injections ($I \in \mathbb{R}^{B\times 1}$) and K is a vector of branch currents ($K \in \mathbb{R}^{L\times 1}$).

The vector of bus injection currents I can be split into two sub-matrices as well, using the connectivity matrix corresponding to different bus types.

$$I = \begin{bmatrix} I_\upsilon & I_\rho \end{bmatrix} \tag{6}$$

where I_υ is the vector of the source currents of ideal voltage sources and I_ρ is the vector of the demand current of constant power buses.

Let us define the DC-DN primitive resistance matrix $\in \mathbb{R}^{L\times L}$ as follows:

$$\Re = diag\,[R_1, R_2, \ldots, R_L] \tag{7}$$

We can write an expression for line voltage drops as follows:

$$\Delta V = \Re \times K \tag{8}$$

If G = $\Re^{-1}$, then network resistance-weighted Laplacian Matrix (LM) can be constructed as follows:

$$\Phi_{\rho\upsilon} = C_\rho \times G \times C_\upsilon^T \tag{9}$$

$$\Phi_{\rho\rho} = C_\rho \times G \times C_\rho^T \tag{10}$$

$\Phi_{\rho\upsilon}$ is the all conductive coupling between constant voltage buses and constant power buses.

$\Phi_{\rho\rho}$ is the conductive coupling associated with constant power buses, also known as the demand-to-demand matrix. Using expressions (5), (6), (7), (9) and (10) following result can be achieved:

$$I_\rho = V_\rho \times \Phi_{\rho\upsilon} + V_\rho \times \Phi_{\rho\rho} \tag{11}$$

Our goal is to solve for unknown voltages, i.e., V_ρ. So, we can write an expression for V_ρ in terms of systems known quantities as follows:

$$V_\rho = \left(\frac{1}{\Phi_{\rho\rho}} \times \frac{P_\rho}{diag(V_\rho)}\right) - \left(\frac{\Phi_{\rho\upsilon} \times V_\upsilon}{\Phi_{\rho\rho}}\right) \tag{12}$$

DGs are becoming highly prevalent in modern distribution networks. The power of a DG (P_{dg}), can be incorporated in the formulation easily as given below.

$$V_\rho = \left(\frac{1}{\Phi_{\rho\rho}} \times \frac{P_{dg} - P_\rho}{diag(V_\rho)}\right) - \left(\frac{\Phi_{\rho\upsilon} \times V_\upsilon}{\Phi_{\rho\rho}}\right) \tag{13}$$

With the presence of CPL in the network, expression (13) becomes nonlinear and must be solved iteratively. Let us augment (13) with an iterative counter (t) that begins with a flat start and iterates until the requisite convergence tolerance is met.

$$V_{\rho}^{(t+1)} = \left(\frac{1}{\Phi_{\rho\rho}} \times \frac{P_{dg} - P_{\rho}}{diag(V_{\rho}^{t})} \right) - \left(\frac{\Phi_{\rho v} \times V_{v}}{\Phi_{\rho\rho}} \right) \tag{14}$$

Note that the connectivity matrix and Laplacian matrices for any system only need to be built once before starting the LF solution, and they will remain constant throughout the iterative process. It is also worth noting that in [8], two connectivity matrices were used to solve a meshed network, whereas just one connectivity matrix is required in this upgraded reformulation. Another benefit of this revised formulation over the prior formulation is that it can easily handle multiple voltage-controlled nodes. The proposed algorithm is given in Fig. 3.

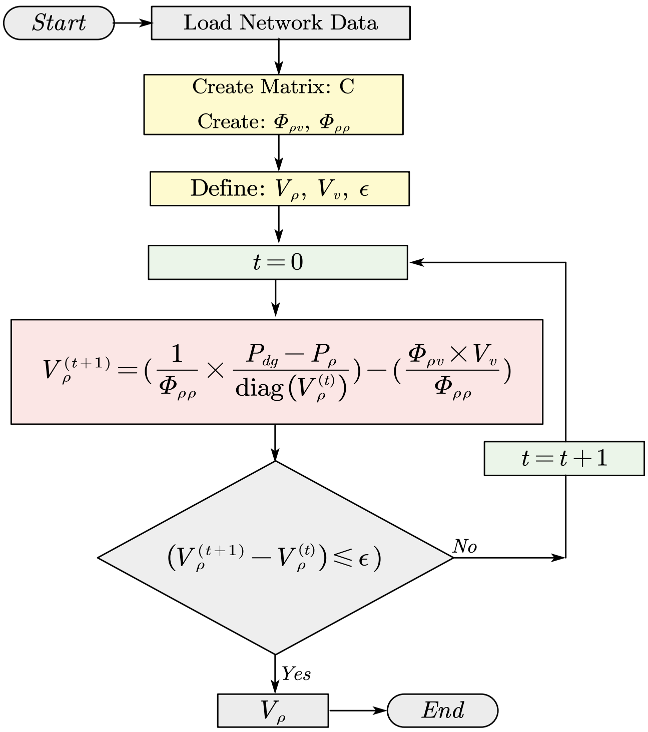

Fig. 3. Proposed ILM load flow algorithm.

2.2. Convergence proof

The proof to guarantee the convergence given in [15] holds true for this formulation as well, provided that the criteria established in [15] are met, furthermore $\Phi_{\rho\rho}$, is a diagonal dominant matrix. For more detail, readers should refer to [15]. The contraction constant can be written as follows:

$$\psi = max \left\{ \frac{\frac{|P_{\rho(i)}|}{v_{min}}}{\frac{v_{min}}{\Re_{thv(i,i)}}} \right\} \forall i \in \rho \tag{15}$$

where, $\mathcal{R}_{thv(i,i)} = \Phi_{\rho(i,i)}^{-1}$ is the Thevenin equivalent resistance at each node, ψ: contraction constant of non-linear mapping (Banach-fixed point theorem)$v_{\min}$: minimum voltage at any bus for a specific loading.

During normal operation, the load current is always less than the short-circuit current, which guarantees that $0 \leq \psi \leq 1$ This implies that this recursive formulation converges to the LF solution, and that the solution will be unique.

This observation is quite valuable, especially when several iterations are required for an LF solution. The contraction constant contains information on the loading in terms of system characteristics, for which the proposed method can ensure convergence. It is worth noting that the contraction constant can be calculated prior to execute the entire LF algorithm. Convergence cannot be ensured using the proposed method for any value of the contraction constant greater than unity. If this situation occurs, one approach is to decrease load or increase generation and find an appropriate combination that ensures convergence. An updated LF algorithm to confirm the convergence of a given system is given in Fig. 4.

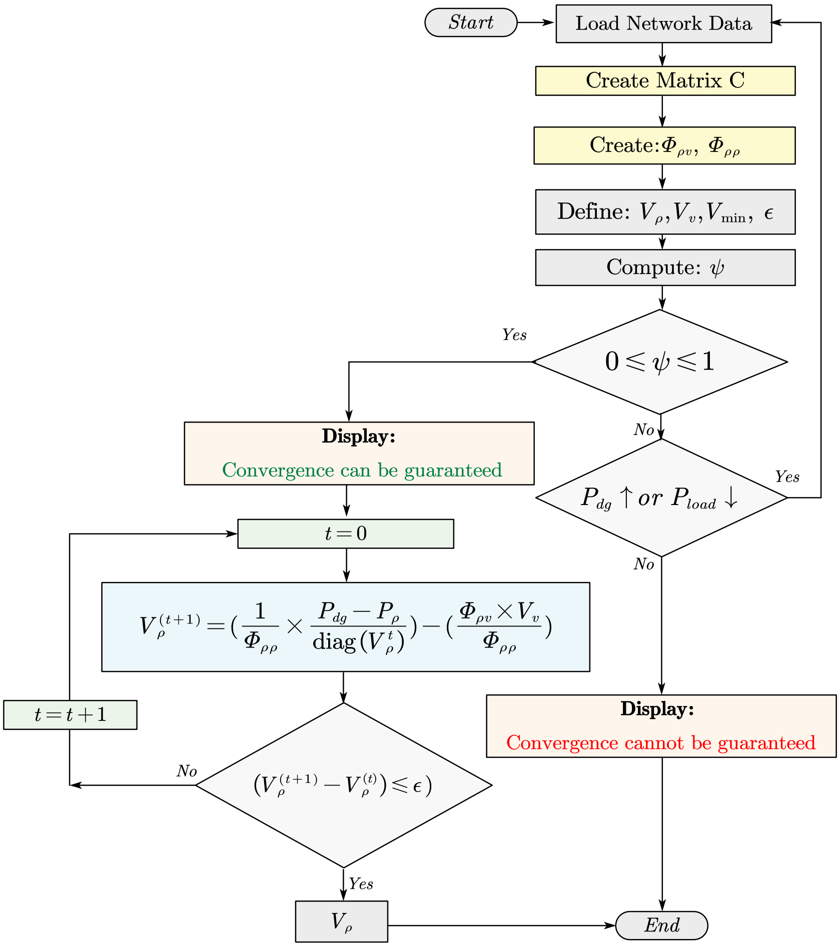

Fig. 4. Updated algorithm to confirm convergence.

3. Results and discussion

A modified IEEE 33 bus test feeder is utilized to validate the proposed method. The modified test feeder and its data can be found in [15]. Under various loading scenarios, we compared the proposed approach to the LM method in terms of CPU processing time and the number of iterations required. Note that all simulations are executed in MATLAB 2020b on a desktop PC with the following specifications: CPU: Intel core i7 @ 3.21 and 3.19 GHz, 16 GB RAM, 64-bit, Windows 10. The convergence tolerance was set up to 10 decimal places.

Fig. 5 shows the error in terms of number of iterations at nominal loading for both radial and meshed configurations. For radial configuration, both algorithms almost have the same performance, but for a meshed network, ILM outperforms the LM method in terms of number of iterations required to achieve the required accuracy threshold. Because both algorithms employ a single connectivity matrix for radial network, the difference in performance is since LM uses two connectivity matrices whereas ILM only requires one connectivity matrix to solve a meshed network. This difference is more prominent under heavy loading conditions, as shown in Fig. 6.

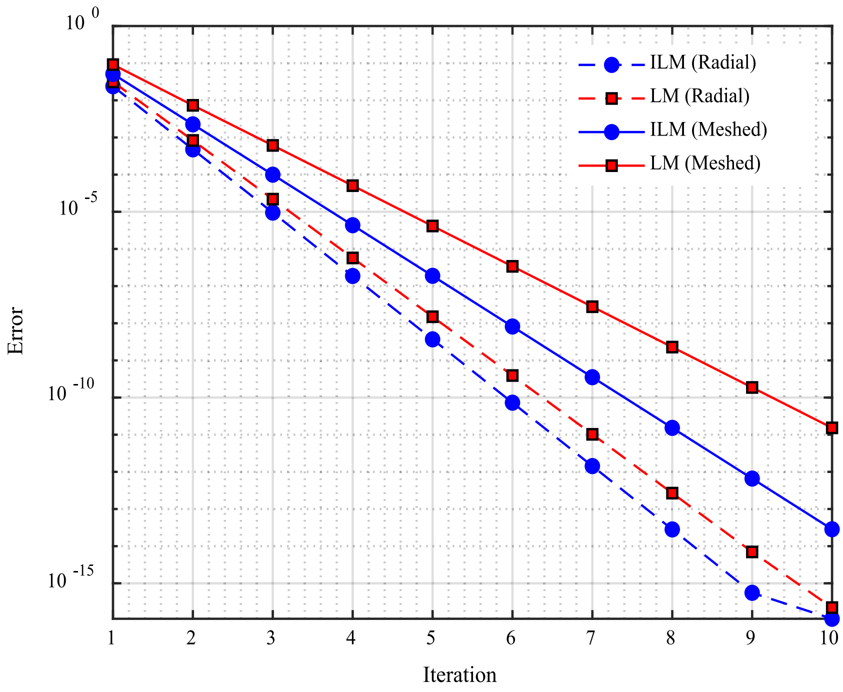

Fig. 5. Error as a function of number of iterations (Nominal Loading).

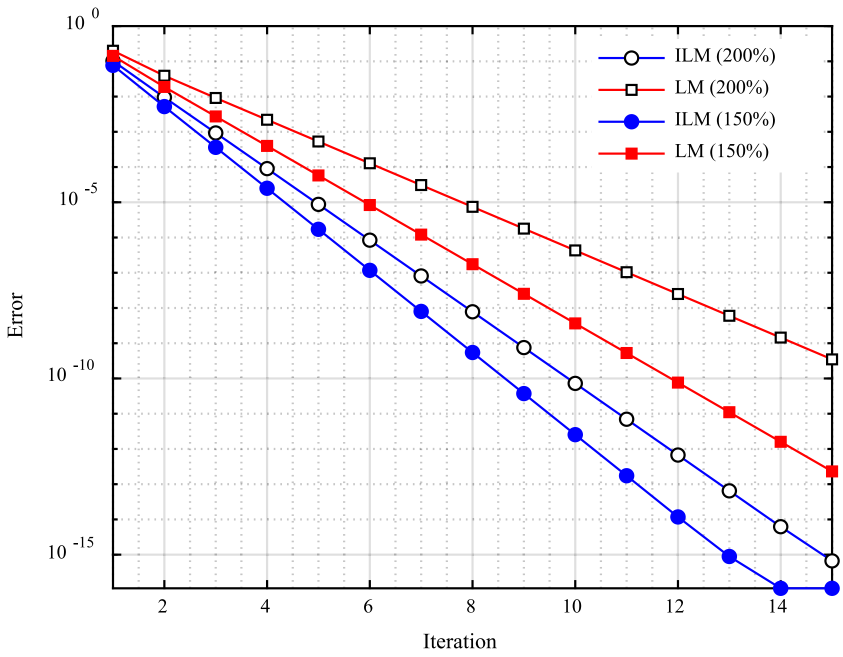

Fig. 6. Error as a function of iterations for meshed configuration at different loadings.

The detailed comparison of ILM with LM method is given in Table 1. The proposed ILM method converges faster than the LM method for a meshed configuration because the LM method uses two connectivity matrices to solve a meshed network, whereas ILM method solves it with only one matrix. We plotted the contraction constant

Table 1. Detailed performance comparison of ILM and LM.

Loading (%)	Configuration	LF method	Iterations	Time (ms)
100	Radial	LM	7	1.815
		ILM	6	1.382
	Meshed	LM	10	1.904
		ILM	8	1.574
150	Radial	LM	8	1.957
		ILM	7	1.584
	Meshed	LM	12	2.104
		ILM	9	1.673
200	Radial	LM	10	2.016
		ILM	9	1.654
	Meshed	LM	16	2.517
		ILM	10	1.798

under various loadings to ensure that the proposed method is convergent, and the solution is unique. As the load is raised, the value of the contraction constant ascended towards unity.

The results shown in Fig. 7 imply that the proposed method can guarantee convergence even at heavy loading as long as the contraction constant value is within the limits.

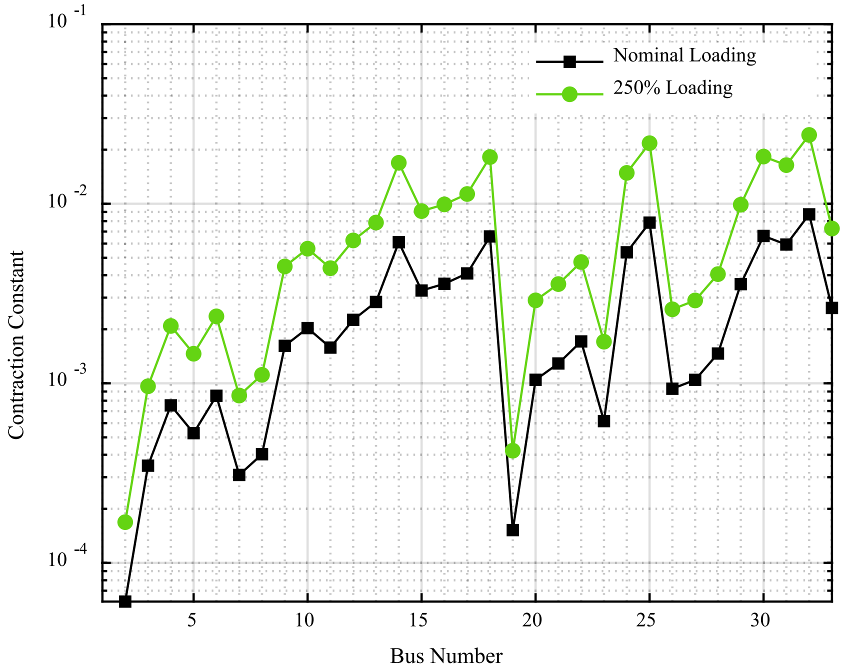

Fig. 7. Value of contraction constant at each bus (ILM, Meshed).

Fig. 7depicts the value of contraction constant under a different loading from which the loading range can be defined for which the proposed method can guarantee the convergence. A bad initial guess can be one of the reasons for algorithm divergence. So, to evaluate the robustness of the proposed algorithm, we also tested it without a flat start, i.e., with a random initial guess. Three random points ranging from 0.5 to 1.5 per unit (pu) are considered, and for these random initial guesses, the LF algorithm is executed for nominal loading and for 200% loading as well as shown in Fig. 8.

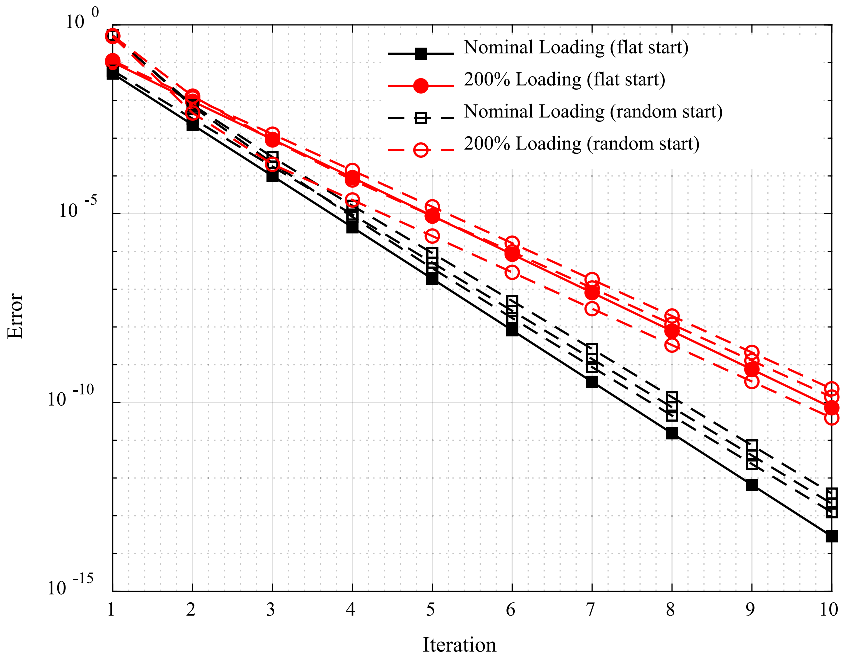

Fig. 8. Convergence from random initial guess.

The proposed method converges well with random initial guesses as well, under both nominal and high loading conditions, as shown in Fig. 8. This implies that the proposed algorithm is robust and can converge even without a flat start.

4. Conclusions

In this paper, an LF solver is proposed based on graph theory by reformulating the LM used in LF iterations to improve the convergence characteristics. Results suggests that the proposed ILM outperforms the LM method in terms of number of iterations and processing time required to achieve the same level of accuracy. The reason for this improvement is that ILM can solve meshed networks with only one connectivity matrix, whereas LM requires two connectivity matrices to solve a meshed network. Convergence of the proposed algorithm has also been taken into consideration with the Banach fixed-point theorem. This is a very useful observation that can be utilized to check the convergence criterion where a large number of iterations are required for an LF solution. The convergence results are in agreement with the analytically evaluated guarantee of convergence using the Banach fixed-point theorem for the proposed LF algorithm. Furthermore, to test the robustness of the ILM, the LF algorithm has also been executed from a flat start and from a random initial guess for different loading conditions. In both cases, the ILM successfully converges. This implies that the ILM is faster than the LM method to solve a meshed network, and it is a suggested option where high accuracy and speed are needed. Considering the different R/X ratios of the AC distribution network with ILM is an interesting research topic for future studies.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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