Life-cycle probabilistic loss and resilience quantification of civil infrastructure under extreme events

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Abstract

Resilience and loss assessment plays an important role in hazard risk management and mitigation. As a paramount indicator, resilience has been widely utilized to account for robustness and recovery capability of civil infrastructure. However, previous studies evaluate the resilience conditioned on a single hazard event, which is less likely to incorporate long-term uncertainty. Although the long-term loss was commonly computed in previous research, quantification of loss has been restricted to the expectation and the stationary occurrence model. Statistical moments, such as standard deviation, skewness, and kurtosis of long-term loss interpreting uncertainty characteristics, were rarely discussed. Uncertainty associated time-dependent characteristics with respect to stochastic occurrence and intensity have not been taken into account. To address these concerns, an integrated long-term resilience and loss assessment framework is developed to aid the life-cycle management and decision making process of civil infrastructure.

Keywords: Long-term resilience, loss assessment, renewal process, life-cycle.

1 Introduction

Civil infrastructure is threatened by hazards such as earthquakes, hurricanes, and structural deterioration due to incessant exposure over the lifetime, thereby causing catastrophic damages and severe financial and societal losses. Mitigating potential loss and improving resilience of civil infrastructure under such hazards have become primary concerns of decision-makers. Consequently, it plays an essential role to quantify resilience and loss over the service life.

In previous studies, resilience has been widely investigated in hazard management and mitigation. The quantification of resilience is primarily conditioned on the structural functionality and recovery capability under an individual hazard event, whereas the associated long-term uncertainty has been ignored. During the service life of civil infrastructure, there is substantial uncertainty affecting the performance of civil infrastructure. For instance, under earthquake hazards, uncertainty may spring from earthquake activities, structural performance, and consequence evaluation, etc. In particular, it is necessary to consider uncertainty in terms of hazard occurrence and intensity over a long time period. Though such long-term effect was underlined in the loss assessment in previous research, the quantification of long-term loss was limited to the expectation. Uncertainty indicated by the other statistical moments such as standard deviation, skewness, and kurtosis were rarely evaluated. Furthermore, the quantification of long-term loss is mainly based on stationary occurrence. For instance, the homogeneous Poisson process has been widely used to model the occurrence of earthquakes incorporating uncertainty of the ground motion activities [1, 2]. Nevertheless, the time-varying characteristics are underrated Such negligence and underestimate may imply inadequate structural performance levels and result in adverse decisions. Therefore, the development of a long-term resilience and loss assessment framework is of significance for the life-cycle management civil infrastructure.

To address these concerns, an integrated framework is developed to quantify the long-term resilience and loss of civil infrastructure subjected to natural hazards. A stochastic renewal process is employed to model the stochastic hazard occurrence. The expectation of long-term resilience and the first four moments of long-term loss can be derived based on the renewal model. Long-term uncertainty springing from time-independent and time-varying occurrence is successfully assessed by using different models of the inter-event time. The proposed framework is expected to aid decision-makers in the life-cycle management of civil infrastructure under various extreme events.

2 Stochastic model of hazards

The renewal process and renewal theory have gained increasing attention in hazard management and risk assessment, especially in the field of earthquake engineering. For instance, the random occurrence of earthquakes is commonly modeled by a Poisson renewal process, which is more widely known as the homogeneous Poisson process (HPP). An HPP is a renewal process with the inter-arrival time following exponential distribution. Additionally, in recent investigations of long-term seismic analysis, the renewal process with a Brownian model is increasingly used [3]. This model indicates that the inter-arrival time between two earthquakes follows a Brownian passage-time (BPT) distribution. In this section, definitions and essential properties of stochastic renewal process are introduced.

2.1 Renewal process

A renewal process $\{N(t),\ t > 0\}$ is a stochastic process and it can be used to model random occurrence of hazard events. The inter-arrival times of a renewal process are independently identically distributed (IID). For instance, the inter-arrival time is a sequence of positive Random Variables (RVs), denoted as $\{W_1, W_2, \ldots, W_k\}$. Alternatively, a renewal process can be defined by an arrival process, in which the arriving times are non-negative RVs $\{T_1, T_2, \ldots, T_k\}$. Within a time period $(0, t]$, the arriving time of the kth event ($k \geq 1$) equals to the sum of k IID time intervals $T_k = W_1 + W_2 + \ldots + W_k$. The cumulative distribution function (CDF) of arriving time T_k can be defined as $F_{T_k}(t)$, representing the probability of T_k smaller or equal to time t

$$F_{T_k}(t) = \mathrm{P}[T_k \leq t] \tag{1}$$

Based on the relationship of $T_k = W_1 + W_2 + \ldots + W_k$, the CDF of arriving time can be described by the CDF of inter-arrival time $F_W(t)$, referring to a k-fold convolution $F_W^{(k)}(t)$ of the CDF of the inter-arrival time

$$\begin{aligned} F_{T_k}(t) &= F_W{}^{(k)}(t) \\ &= \mathrm{P}\left[W_1 + W_2 + \cdots + W_k \leq t\right] \end{aligned} \tag{2}$$

In a renewal process, the number of events $N(t)$ within time interval $(0, t]$ can be denoted as the largest integer of k

$$N(t) = \max\left\{k:\ T_k \leq t\right\} \tag{3}$$

The definition of a renewal process states that the number of renewals up to time t is greater than or equal to k if and only if the kth event arrives at or earlier than time t [4]

$$N(t) \geq k \Leftrightarrow T_k \leq t \tag{4}$$

An illustrative diagram of a renewal process is shown in Figure 1.

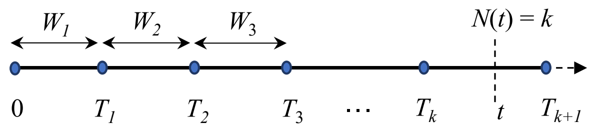

Figure 1. Stochastic occurrence of hazards under a renewal process

2.2 Renewal function

One of the most significant properties in the renewal theory is the renewal function. A renewal function refers to the expected value of renewals in a process. With the biconditional statement of Eq. (4), renewal function $m(t)$ in the time domain $(0, t]$ can be written as

$$\begin{aligned} m(t) = E[N(t)] &= \sum_{k=1}^{\infty} \mathrm{P}[N(t) \geq k] \\ &= \sum_{k=1}^{\infty} \mathrm{P}[T_k \leq t] = \sum_{k=1}^{\infty} F_W{}^{(k)}(t) \end{aligned} \tag{5}$$

The renewal function satisfies an integral equation conditioning on the first arrival time y. The CDF of inter-arrival time $F_W(t)$ is assumed to be continuous. Hence, the integration denoted by a density function yielding at the first renewal $dF_W(y)$ can be written as

$$\begin{aligned} m(t) &= E[N(t)] \\ &= \int_0^{\infty} E[N(t) \mid W_1 = y] dF_W(y) \end{aligned} \tag{6}$$

The first renewal time is also the first arrival time. The integral function depending on the first renewal time y has two possible cases: one is that less than k events occur within $(0, t]$ when $y > t$

$$E[N(t) \mid W_1 = y > t] = 0 \tag{7}$$

On the other hand, there would be precisely k events up to time t if the first renewal time follows $y \leq t$. Renewal theory defines that a renewal process restarts after each event occurrence. This gives that the number of renewals up to time t is identical to one plus the number of renewals between time interval $t - y$ [4]. Therefore,

$$\begin{aligned} E[N(t) \mid W_1 = y \leq t] &= 1 + E[N(t-y)] \\ &= 1 + m(t-y) \end{aligned} \tag{8}$$

Substituting Eq. (8) into Eq. (6), the renewal function becomes

$$\begin{aligned} m(t) &= \int_0^{\infty} E[N(t) \mid W_1 = y] dF_W(y) \\ &= \int_0^{t} [1 + m(t-y)] dF_W(y) \\ &= F_W(t) + \int_0^{t} m(t-y) dF_W(y) \end{aligned} \tag{9}$$

The integral form can be interpreted into the convolution form

$$\begin{aligned} m(t) &= \sum_{k=1}^{\infty} F_W{}^{(k)}(t) \\ &= F_W{}^{(1)}(t) + \sum_{k=2}^{\infty} F_W{}^{(k)}(t) \\ &= F_W(t) + \sum_{k=1}^{\infty} \int_0^{t} F_W{}^{(k)}(t-y) dF_W(y) \end{aligned} \tag{10}$$

Eqs. (10) and (11) interpret the essential property of a renewal process. This property is applied to formulate and derive long-term resilience and loss in the following section.

3 Long-term resilience and loss

Based on the stochastic renewal model of hazard occurrence, the long-term resilience and loss can be formulated.

3.1 Long-term resilience

The long-term resilience is the sum of resilience of the civil infrastructure under all hazard events. Denote the service life of civil infrastructure as $(0, t_{int}]$. The total number of arrivals of hazard events is $N(t_{int})$. Within the investigated period t_{int}, the long-term resilience $RE(t_{\mathrm{int}})$ can be defined as

$$RE(t_{\mathrm{int}}) = \sum_{k=1}^{N(t_{\mathrm{int}})} R_k \tag{11}$$

in which R_k is the resilience of civil infrastructure with respect to a single hazard. The long-term resilience is a compound random variable; hence, it should be denoted by the expectation. According to the definition of renewal function, the expected long-term resilience can be computed as

$$E[RE(t_{\text{int}})] = E[R]m(t_{\text{int}}) \tag{12}$$

in which $E[R]$ is the expectation of resilience of infrastructure. The value of $E[R]$ can be computed using the resilience model provided by Bruneau *et al*. [5]

$$R_r = \frac{1}{\Delta t_r} \int_{t_0}^{t_0+\Delta t_r} Q(t)dt \tag{13}$$

in which $Q(t)$ is the functionality of infrastructure under the recovery function at time t; t_0 is the initial investigated time; and Δt_r is the investigated time interval. The recovery model is based on the approach as shown in ATC [6]. Other resilience models can also be employed.

3.2 Long-term loss

The long-term loss represents the hazard-induced financial loss within the life-cycle of civil infrastructure. It can be computed by summing up the repair loss of infrastructure subjected to all hazard events. The long-term loss $LTL(t_{\text{int}})$ can be written as

$$LTL(t_{\text{int}}) = \sum_{k=1}^{N(t_{\text{int}})} L_k e^{-rT_k} \tag{14}$$

in which r is the monetary discount rate, as the long-term economic loss requires to discount to the present value. L_k is the repair loss, which is the product of vulnerability and the repair cost. The long-term loss $LTL(t_{\text{int}})$ is also a compound random variable. As long-term loss refers to an economic indicator, only assessing its expectation may misestimate the potential risk. Therefore, variance and higher-order moments (i.e., skewness and kurtosis) of long-term loss are also significant. Statistical moments of long-term loss can be derived using the renewal function based on the following recursive function

$$E\left[LTL^i(t_{\text{int}})\right] = \sum_{k=0}^{i-1} \binom{i}{k} E[L^{i-k}] \int_0^{t_{\text{int}}} e^{-irs} E\left[LTL^k(t_{\text{int}} - s)\right] dm(s) \tag{15}$$

where $LTL^i(t_{int})$ refers to the ith order moment of long-term loss. $E[L^{i-k}]$ indicates the i - k moment of repair loss. The detailed derivation of the recursive function Eq. (15) can be found in Li *et al*. [7].

4 Case study

An illustrative example of the proposed long-term resilience and loss assessment framework is applied to an RC highway bridge subjected to earthquake hazards [7]. The service life of the bridge is 75 years. The resilience of the bridge during the investigated time interval Δt_r e.g., 365 days, is 0.89. The repair loss of the bridge can be computed as 156,310 USD [7]. The bridge is assumed to be fully restored after the recovery actions. For illustrative purpose, the repair loss is assumed to follow an exponential distribution. The occurrence rate of earthquakes is given as 0.1109. The monetary discount rate is taken as 2%. The long-term resilience and loss under a stationary HPP and time-dependent renewal-BPT model are assessed.

The Poisson renewal process (e.g., an HPP) is employed to model the stochastic occurrence of earthquakes [8]. The probability of having $N(t_{int}) = k$ number of earthquakes during the investigated time period (0, t_{int}] follows a Poisson distribution

$$\mathrm{P}[N(t_{\text{int}})] = \frac{(\lambda t_{\text{int}})^k e^{-\lambda t_{\text{int}}}}{k!}, k = 1, 2, \cdots \tag{16}$$

The inter-arrival time follows exponential distribution $W_k \sim \text{EXP}(\lambda)$ with mean $E[W] = 1/\lambda$. The CDF of the inter-arrival time is given as

$$F_W(x) = 1 - e^{-\lambda x} \tag{17}$$

in which λ is the occurrence rate. Based on the definition, the renewal function of an HPP can be calculated as

$$m(t_{\text{int}}) = E[N(t_{\text{int}})] = \lambda t_{\text{int}} \quad (18)$$

Accordingly, the long-term resilience of the investigated bridge is computed as 7.40 using Eq. (12). According to Eq. (15), the first four moments of long-term loss can be determined. The expectation, standard deviation, skewness, and kurtosis are calculated to be 673,343 USD, 358,796 USD, 0.91, and 4.20, respectively. The Monte Carlo simulation is conducted to validate the renewal-based analytical approach. It is been noted that the simulation approach is inefficient and time-consuming compared to the analytical analysis.

The results associated with long-term resilience and loss can be different if a time-dependent inter-arrival model is adopted. For instance, if the occurrence of earthquakes follows a renewal process with a BPT model, the PDF of the inter arrival time is given as

$$f_W(t) = \left(\frac{\mu}{2\pi\alpha^2 t^3}\right)^{1/2} \exp\left\{-\frac{(t-\mu)^2}{2\mu\alpha^2 t}\right\} \quad (19)$$

in which μ is the mean of return period and α is the coefficient of variation (COV). For illustrative purposes, μ is computed as the reciprocal of occurrence rate. The COV α is assumed to be 1. Given the PDF, the renewal function under a renewal BPT model can be computed by Eq. (9). Under the circumstance, the long-term resilience of bridge remains as 7.40, whereas the expected long-term loss is reduced to 669,220 USD. The other statistical moments also decrease. The standard deviation, skewness, and kurtosis of long-term loss change to 344,080 USD, 0.79, and 3.89, respectively. For the investigated case, uncertainty associated with the time-dependent stochastic occurrence does not have an impact on the long-term resilience, whereas the long-term loss is reduced. Such changes may result in different decisions during the decision making process.

In addition to the resilience and loss assessment, a sensitivity analysis is performed to identify the impact of potential uncertainty associated with repair ratios (RRs) on the repair loss and consequently the long-term seismic loss. Padgett *et al*. [9] suggested that seismic loss of bridges due to repair could be sensitive to the change of RRs. The repair loss of the bridge is the product of the rebuilding cost and probability of failure. The fragility curves demonstrating failure probabilities being in different damage states under intensity measures are shown in Figure 2. The rebuilding cost was 2,306 USD/m^2. Other parameters remain the same as [7]. For instance, the mean values of stochastic RRs of slight, moderate, and major damage state are assumed identical to the previous deterministic values as 0.03, 0.25, and 0.75, respectively.

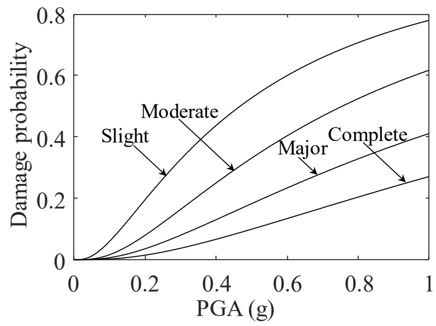

Figure 2. Fragility curves associated with fur damage states, adapted from Li et al. [7]

To incorporate uncertainty in RRs, three damage states, including slight, moderate, and major, are assumed to follow lognormal distributions with a COV of 0.5, while the ratio for no damage and complete damage remains as 0 and 1, respectively. The repair loss can be computed as 156,240 USD, which does not show much difference to the deterministic case with 156,310 USD. The impact of stochastic RRs on the long-term loss is less prominent. The repair loss remains an exponential distribution. The first four moments of long-term loss are 673041 USD, 358,635 USD, 0.91, and 4.20, respectively. Therefore, for the investigated case, stochastic RRs have little impact on the long-term seismic loss.

5 Conclusions

This paper provides a systematic framework to assess the long-term resilience and loss of civil infrastructure based on the stochastic renewal process model. The expected long-term resilience and statistical moments (i.e., expectation, standard deviation, skewness, and kurtosis) of long-term loss can be effectively formulated using the proposed renewal-based approach. Time-dependent characteristics can be captured by the stochastic renewal process by incorporating different inter-arrival time models. The proposed framework is applied to a highway bridge subjected to earthquake hazards. Results show that uncertainty associated with the stochastic occurrence may affect the long-term resilience and loss assessment. Uncertainty resulting from stochastic repair ratios is less likely to influence the loss estimation.

6 References

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