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Assessing investment value of privately-owned public rental housing projects with multiple options

Dezhi Li [a,*], Kai Guo [a], Jia You [a], Eddie Chi-Man Hui [b]

[a] Department of Construction and Real Estate, Southeast University, PR China [b] Department of Building and Real Estate, The Hong Kong Polytechnic University, Hong Kong

* Corresponding author.
E-mail address: njldz@seu.edu.cn (D. Li).

[1] See: http://house.ifeng.com/pinglun/detail_2012_05/09/14409008_0.shtml

a b s t r a c t

Public rental housing (PRH) has been officially designed as the mainstream of China's affordable housing. However, very few private developers treat PRH as an ideal investment target on account that its investment value seems too low on the basis of traditional valuation method represented by the Net Present Value (NPV) method. Thus, disregarding for financial and operational issues of PRH projects, the normal role of a private developer in the supply of PRH is an agent which earns meagre but stable agent fee. Consequently, the government has to bear a heavy financial and operational burden in PRH projects aiming to keep the sustainability of PRH projects. To improve this situation, a privately-owned PRH provision mode, Building-Own-Operation-Concession (BOOC) mode, is proposed, where the private developer is in charge of the life cycle management of PRH projects through a concession contract from the government, including the fund-raising, construction, operation and even demolition. Besides complying with all PRH relevant regulations, the private developer is entitled with multiple options to abandon, transfer and expand of targeted PRH project. Based on the NPV method and Real Option Pricing Model, a method for assessing the investment value of a PRH project in BOOC mode is proposed. Then, the proposed method is exemplified in a hypothetical privately-owned PRH project in Nanjing city of China. The proposed provision mode BOOC and valuation method are expected to show a new perspective for accelerating the sustainable development of PRH.

Keywords:
Public rental housing
Multiple options
Investment value
NPV
Real option pricing model

1. Introduction

Since its economic reform and opening up, China's housing price has been rising rapidly and remains obstinately high, which makes medium and low income families hardly afford it. In order to solve this problem, the government puts forward a housing system mainly comprising low-rent housing (LRH) and economically affordable housing (EAH), which however have many inevitable shortcomings. On one hand, the selling price of EAH is still unaffordable for most medium-low income families. On the other hand, LRH is only available for families with the lowest income. There are few families can fit the standard of LRH and EAH. For example, from 2006 through 2010, only 11.40 million urban lowest income households are supplied with LRH, and 3.60 million urban medium-low income households are supplied with EAH in the whole China.[1] Considering the relatively high urbanization rate of China, around 7% of urban households are scarcely covered by LRH and EAH together. What's worse, it seems more embarrassing for those in a “sandwich layer”, who can neither meet the standard of LRH nor afford EAH. Along with the accelerating pace of urbanization, quite a large number of university graduates join the “sandwich layer”, as a matter of course, PRH came into being with the aim of providing affordable housing in 2009. Till 2013, Chinese central government announced PRH projects as the mainstream of the affordable housing system. In 2014, LRH projects were converted into PRH projects. In the near future, it is expectable that PRH will be blossoming everywhere.

However, it is shown that there is not enough funding for the implementation of PRH projects nationally, so the sustainability of PRH hinges on the financial support from the government (Li et al., 2014). Moreover, local residents take it for granted that government rent is lower than market rent. Say, it is investigated that government rent is 20% lower than market rent in Beijing, 40% in Chongqing,10% in Shanghai, and 30% in Shenzhen (Zeng, 2013). The investment and payment of PRH isn't in direct proportion, which is hardly attractive to private developers. Therefore, the government has to bear the cost solely, which inevitably increases the burden of the public financial expenditure. In the near future, provided that the government fails to afford PRH project and abolish it, the needed probably won't access to PRH projects. This frustrating tendency is yet under the assumption that private developers just construct and operate the PRH projects in a formalistic way, and the assessment of PRH projects is using the traditional method NPV, regardless of its limitations. If private developers implement PRH projects flexibly in terms of abandoning, transferring and expanding PRH projects, will they take part in the provision of PRH projects? Will those PRH projects turn sustainable? This paper, aiming to give a satisfactory answer to the above-mentioned questions, tries to construct a new mode for the implementation of PRH projects under the guidance of real option theory, which has been flexibly applied in various industries, such as vaccination strategies for clinical therapeutics (Favato, Baio, Capone, Marcellusi, & Mennini, 2013), financial application of real estate (Pezeshkian, Lashgari, & Stiller, 2014), and photovoltaic systems of power industry (Biondi & Moretto, 2014). Imaginably, a new PRH provision mode proposed, private developers would make more profit than in traditional modes.

This paper is carried out according to the following structure. An overall review of the domestic and foreign relevant researches on China's PRH and real option theory are made in Section “Literature review”. On the basis of it, Section “The BOOC mode and its embodied options” discusses current PRH construction modes and analyzes their deficiencies accordingly. Then, the application of real option theory in PRH projects and the options in BOOC mode are summarized in details. On top of it, Section “The Pricing Model of BOOC Mode” demonstrates the valuation process of BOOC mode as well as real option pricing model. A case study to present the application of proposed BOOC mode and the relevant evaluation methods is conducted in Section “Case study”. Lastly, Section “Conclusions” draws conclusions and puts forward suggestions for the further researches.

2. Literature review

Affordable housing is one of the main approaches to solving housing problem world-wide. As one important kind of affordable housing, PRH project has become popular in many countries. A lot of scholars have conducted researches on affordable housing and its improvement approaches from different perspectives. For example, Ha (2008) investigated the characteristics and problems of social housing estates in South Korea, pointing out the necessity for the private sector, civil society and the government to work together in close partnership to work out a more practical housing and community scheme. Horn, Ellen, and Schwartz (2014) explored the influence of The Housing Choice Voucher Program (HCV) on low income households, spotting the inefficiency of HCV in providing low income households with well-equipped residential communities. Verdugo (2014) studied the relationship between immigrants' location choices and affordable housing supply and found that immigrants benefit from better housing conditions in cities with more affordable housing supply.

In the light of above literature, it is obvious that affordable housing plays a critical role in guaranteeing the stability and living quality of a region, therefore, the sustainability of affordable housing becomes a focus of attention in modern society. Chen, Stephens, and Man (2013) pointed out that it is impossible to sustain an affordable housing program without widespread public support. Hwang, Zhao, and Ng (2013) identified the critical factors affecting schedule performance of affordable housing projects in Singapore and found out that “coordination among various parties” is one of the top three factors to assure the implementation of project schedule objectives. Disney and Luo (2015) investigated the impact of Right to Buy (RTB) policy on social welfare in UK, which allows tenants to purchase affordable houses at a substantially dropped price and mitigates the financial burden of affordable housing projects. Chaskin and Joseph (2014) stated that Chicago has been implementing the greatest effort to redevelop urban residential communities and addressing urban poverty through publicprivate partnership (PPP) modes in the United States. Regarding Chinese scholars, Ma, Wang, and Ma (2014), Gao (2014), Luo (2014) and Xiao and Liu (2014) introduced Real Estate Investment Trusts (REITS) as a new way for PRH financing. Wang and Zhu (2014) believed that governmental spending on PRH projects is still the best way. Yao (2015) suggested a Public-Private-Partnership (PPP) mode to make a winewin situation between government and private developers in PRH project. Yu (2014) found shortage of funds to be the major problem in the sustainable development of PRH projects in China.

With regard to real option theory, it was firstly introduced by Professor Stewart Myers of the MIT Sloan School of Management in 1977 (Myers, 1977). Since then, a lot of scholars apply real option theory into decision-making in circumstances with uncertainty and flexibility. Among them, Karsak and Ozogul (2005)€ , casting away the traditional cash flow methodology, proposed a real option evaluation approach to evaluate a flexible manufacturing system investment. Liu (2013) analyzed the influencing factors of foreign banks entering into China using real option theory. Santos, Soares, Mendes, and Ferreira (2014) compared real option approach with traditional methods, Net Present Value in particular, assessing an investment in energy production under uncertain circumstances. Park, Kim, and Kim (2014) investigated how uncertain energy policies affect the financial viability of an offset project using a real option-based model, and expected to assist private entities in establishing proper investment strategies for Clean Development Mechanism (CDM) projects under uncertain energy policies. Many scholars also have conducted researches on real option in real estate development. For example, Titman (1985) estimated the value of the undeveloped land where the future price of building units was uncertain using the option pricing model. Rocha, Salles, Garcia, Sardinha, and Teixeira (2007) introduced real option method into real estate and identified the optimal strategy and timing for construction phrase. Lee and Jou (2007), by means of real options, investigated how to choose a density ceiling and how a regulator makes policies forcing developers to develop the property less densely. Liu (2009, 2010) assessed the investment value of real estate using real option valuation. Hui, Ng, and Lo (2011) evaluated Kwun Tong Town Center, the largest urban redevelopment project in Hong Kong, using the real option model Monte Carlo method and found real option-pricing model an efficient way to appraise the optimal timing and its feasibility. Yao and Pretorius (2014) developed a long-dated American call option pricing model for valuing development land under leasehold. Using the BlackeScholes model modified with value leakage, Li et al. (2014) assessed the investment value of a privately-owned PRH project with deferral option.

Based on the above literature, it is widely acknowledged that getting private sector involved effectively is the major solution to the sustainable development of PRH projects. Yet, the strategies to attract private developers vary from different regions. In addition, real option theory is widely used in uncertain circumstances, such as energy, foreign investment and commercial housing investment decisions. Furthermore, very few scholars put forward privatelyowned PRH and corresponding valuation model with modified BlackeScholes model. But, given more flexibility, the private sector may enjoy more real options in privately-owned PRH projects, and thus privately-owned projects may become more profitable. Considering all, this paper tries to propose a new privately-owned mode, entitled Building-Own-Operation-Concession (BOOC) mode, as well as a new valuation model based on the binomial option pricing model (BOPM).

3. The BOOC mode and its embodied options

3.1. Existing construction modes of PRH

There are mainly three kinds of construction modes of PRH in China, including Government-led mode, Cooperation mode and Private sector-led mode. The cardinal difference of these three modes is mainly caused by the investment proportion of government and private sector. The advantages and disadvantages of each mode are shown in Table 1.

From this table, it can be seen that there are many deficiencies in the modes existing currently. As to the government-oriented mode, the construction cost is a very heavy burden on government. In terms of private-sector-oriented mode, private developers seldom or never look to future increase considering its unsatisfactory profit, just because of this, it is financially impracticable to implement PRH with the above two modes. Under the strict rule of the cooperation mode, commissioned developers fail to develop PRH projects flexibly. As a matter of course, given no full ownership, these private developers have no chance to bring their talents and abilities to implement PRH projects (Li et al., 2014). Therefore, it is a great necessity to innovate and improve the cooperation mode to mitigate the financial burden on the government and enable private developers to be fully involved in PRH with their managerial experience.

3.2. The proposed BOOC mode

Concerning the cooperation mode between the government and the private sector, PPP is the most popular mode in many areas of construction, including roads, bridges, rail transits, harbors, water supply, prison and public houses (Taiwo, 2015). In reality, PPP can be further classified into various categories, such as PrivateFinance-Initiative (PFI), Build-Transfer (BT), Build-OperateTransfer (BOT), Build-Transfer-Operate (BTO), Build-Own-Operate (BOO) Rehabilitate-Operate-Transfer (ROT), among which BOO entrusts full ownership to the private sector upon completion of the targeted project and empowers the private sector with independent management of the targeted project. Nevertheless, the private sector still cannot abandon, transfer or expand the targeted project flexibly.

In order to improve the existing construction modes of PRH projects in China, traditional BOO mode is modified to be BuildingOwn-Operation-Concession (BOOC) mode, as shown in Fig. 1. The added Concession (C) refers to a concession contract (Fig. 2) between the government and the private sector, which gives an official recognition to the independent rights of the private sector, including ownership and rights of operation, abandonment, transfer and expansion. Then, holding these rights and possessing the managerial experience in the past commercial real estate construction, the private sector can make a flexible and independent decision. In addition, the private sector has priority to make

Table 1
Advantages and disadvantages of existing constructions modes.

Mode	Advantages	Disadvantages
Government-led mode	Government supervise directly construct efficiently	Too much financial burden on government no private sector involved
Cooperation mode	Less financial burden on government private sector involved	Too restrictive regulations economically unsustainable
Private Sector-led mode	No burden on government fully get private sector involved	Meager profit long payback period less private developers want to construct

specific options in the implementation of BOOC mode.

3.2.1. Option to abandon

At present, due to the irreversibility of investment, private developers hesitate to undertake PRH projects. Once a developer is involved in a PRH project, it cannot but to carry through the project even if the project doesn't necessarily make profits. In view of this problem, the BOOC mode gives the private developer right to abandon. In other words, if a developer has conducted PRH project for several years but seen the low profit, it has an option to abandon the project and gives it back to the government. It's also necessary for the government remedy and compensate for the financial loss of the developer to a certain extent so as to minimize the amount of losses.

3.2.2. Option to transfer

As mentioned before, the private developer has the ownership of a PRH project according to the contract of concession signed with the government. Then, the developer has an option to transfer its ownership to other enterprises. With the stable returns of PRH project, the private has right to transfer the PRH project to other enterprises at a ready market price on the condition that the transferee should carry through the PRH project under the instruction of PRH regulations further on.

3.2.3. Option to expand

Besides the options to abandon and transfer, developer's option to expand refers to his right to increase investment and scale up the PRH projects after several years' operation. It goes beyond doubt that the scale of expanding must conforms to the authorized level. However, the expansion of PRH projects is not only charged by the individual will and right, but also determined by the commercial facilities in the surroundings in consideration to the integrated construction.

4. The pricing model of BOOC mode

4.1. Traditional pricing model

Traditionally, the discounted cash flow (DCF) method is applied to value a project, company or asset and weigh the time value of money at a discounted rate. The sum of all discounted cash flows of each year is the net present value (NPV), which can be calculated as:

$$NPV = \sum_{i=0}^{n} (CI_t - CO_t)(1+q)^{-t} \quad (1)$$

Where t is the time of the cash flow, q is the discount rate, CI_t and CO_t are cash inflow and outflow at time t respectively.

A positive NPV value refers to the underlying asset as economically profitable and vice versa. With its simplicity and understandability, NPV is universally acknowledged as an effective method in evaluation area. But, the NPV method implicitly assumes that management is passive, and the uncertainty in the future generally decreases the investment value. By contrast, real option valuation assumes that management is actively changing according to market changes. Given these different treatments, the real options value of a project would be typically higher than the NPV. As for the private developers in BOOC mode, they may face many uncertainties during PRH project, but they can exercise many options, as mentioned in Section 3.2, to handle those uncertainties. With rich experiences in

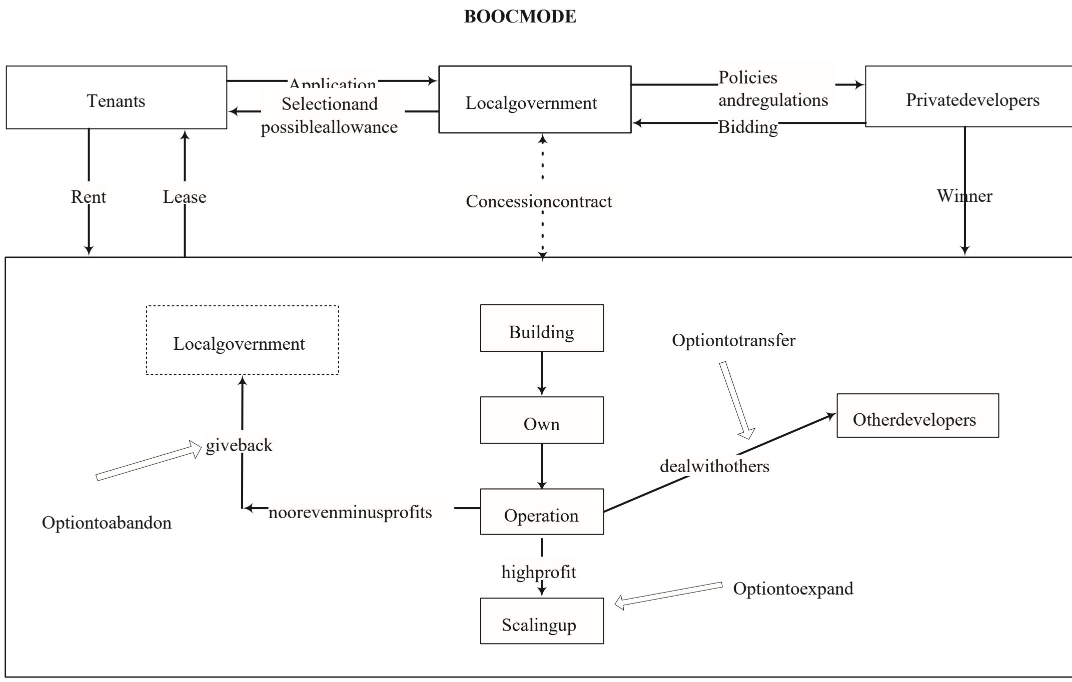


Fig. 1. BOOC contract process.

commercial housing projects and multiple options in PRH projects, the private developers may adapt to and even benefit from some uncertainties. As a result, to better assess the value of PRH projects in BOOC mode, the real option theory and its pricing model are introduced.

4.2. Real option pricing model

Originated from financial options, a real option means that private developers has the right, but no obligation, to undertake certain business initiatives, such as deferring, abandoning, expanding, staging or contracting a capital investment project in the future. Real option pricing model is a multi-disciplinary concept and two classic option pricing models is constituted by the BlackeScholes option pricing model (Black & Scholes,1973) and the CoxRoss-Rubenstein binomial option pricing model (CRR BOPM) (Cox, Ross, & Rubinstein, 1979). Among them, the BlackeScholes option pricing model theoretically estimate the price of European-style

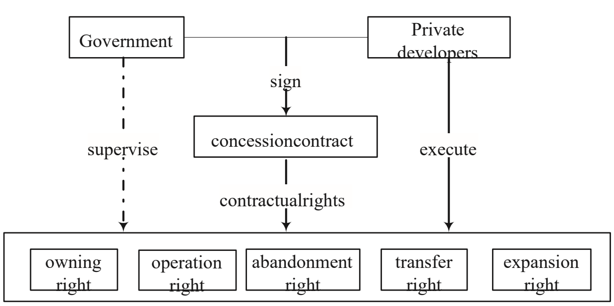


Fig. 2. Concession contract.

options based on a partial differential equation, which may be exercised only at the expiration date of the option, i.e. at a single pre-defined point in time. That is to say, the option cannot be executed before the expired time even if a particular early execution would have a greater profit than the execution of expiration date. As to BOOC mode, although the concession contract is signed according to a certain fixed number of years, all the options in it can be executed at any time, as long as this execution could bring a maximum profit within the contract term. Therefore, applying BlackeScholes model to pricing BOOC mode is not very appropriate.

As a variation of BlackeScholes model, CRR BOPM constructs binomial tree model and considers the underlying asset in a period of time, not at just one expired time. Hence, the CRR BOPM model is particularly efficient in assessing American style options, which can be exercised at any time up to expiration. In the proposed BOOC model and its concession contract, the concessionary private developer is permitted to execute real options at any time in view of the market/operation situation. So, the options in the BOOC mode can be classified as an American-style option, and CRR BOPM seems more suitable than BlackeScholes model here.

5. CRR BOPM of BOOC mode

5.1. Binomial tree model of CRR BOPM

The CRR BOPM is based on the description of an underlying instrument over a period of time rather than a single point, which determines it being used to value American options exercisable at any time in a given time interval. When the time interval is short enough, it is can be assumed that the price fluctuation of underlying asset just goes in two possible ways: above (u > 1, u is the rising proportion) or below (d < 1, d is the decreasing proportion). Thus, for one-step binomial model, S_0 is the current price, the price at the expired time will either be $S_{up} = S_0*u$ or $S_{down} = S_0*d$. The option price at expired time can be calculated as: $f_u = S_0*u-K$ ($f_u = 0$, if $S_o*u < K$), $f_d = S_0*d-K$ ($f_d = 0$, if $S_0*d < K$), where K is the executed price, and the final option price is f. The process is shown in Fig. 3.

Then the option price at present f can be calculated from f_u and f_d, the formula is:

$$f = e^{-rT}[pf_u + (1-p)f_d] \qquad (2)$$

where $p = \frac{e^{rT}-d}{u-d}$, $u = e^{\sigma\sqrt{T}}$, $d = \frac{1}{u}$, r_1 is the risk-free rate, T is time duration, σ_1 is volatility, p is the possibility of rising, and 1-p is the possibility of decreasing.

In general, the life cycle of a real project is made up of many time intervals, for which the option price of a project needs to be assessed with the help of an N-steps binomial tree (as shown in Fig. 4) deduced from one-step binomial model.

Then, the option price $f_{i,j}$ at any executed node – i.e. at expired time of the option – can be easily calculated, which is $f_{i,j} = \max(0, S_{i,j} - K_{i,j})$ for a call option, or $f_{i,j} = \max(0, K_{i,j} - S_{i,j})$ for a put option, where $K_{i,j}$ is execute price, $S_{i,j}$ is the price of underlying asset at the node (i, j) (i = l, 2, ..., n; j = 0, l,... i counted at node i from up to down, and n is the number of steps of the binomial tree). $\Delta t (= T/n)$ is the time duration of every step. After the option price $f_{i,j}$ is obtained, every earlier node's option price can be calculated using the formula:

$$f_{i-1,j-1} = e^{-r\Delta t}[pf_{i,j} + (1-p)f_{i,j-1}] \qquad (3)$$

Thus the option value can be found for each node, starting from the executed node, working back to the first node of the tree, thus the last calculated result is the final value of the option.

5.2. Binomial tree model of BOOC mode

In general, the concession contract of a PRH project lasts for at least decades, thus it is necessary to construct an N-step pricing model for the BOOC mode. Applying the basic procedures in Section "Binomial tree model of CRR BOPM" and considering the characteristics of PRH projects, the binomial tree of a PRH project in its life cycle can be drawn as Fig. 5, which demonstrates some nodes to exercise the option to abandon, the option to transfer or the option to expand. Although all of these three options are exercisable at any time during the contract period, the choice is determined by the price of each option. Specifically, the option with the biggest price would be exercised. For example, if the option price of the option to transfer is bigger than those option prices of the option to abandon and to expand, the option to transfer should be exercised. Consequently, it is essential to estimate the option price of these three options.

5.3. The execution of option to abandon

When the project profit is not ideal, like at the node of $S_{1,0}$, $S_{2,0}$ and $S_{3,1}$,the private developer would lose money for keeping the project. Then the developer can choose to abandon the project to reduce the loss. The abandonment option is a kind of American put option, whose execute price should be K_a, where K_a is the

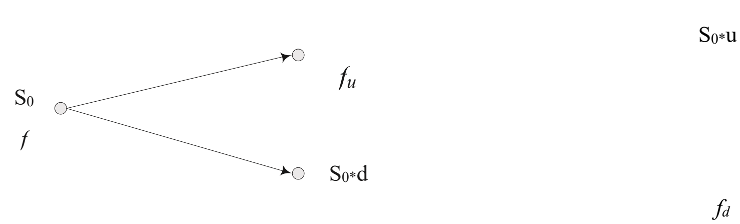


Fig. 3. One estep binomial tree.

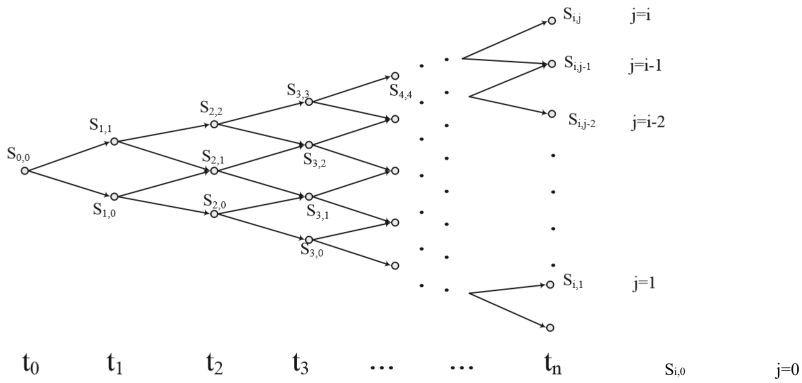


Fig. 4. N-steps binomial tree.

construction fee C. That is to say, if the developer isn't benefited from the project, the construction cost it invested can be compensated. As a result, from the perspective of the private developer, this option greatly reduces the risk of PRH projects and increases the PRH project's investment value.

5.4. The execution of option to transfer

When a PRH project's profit is good enough to attract other private developers, like at the node of $S_{1,1}$, $S_{2,1}$, and $S_{3,2}$, the existing concessionary developer can choose to execute the option and transfer it. This option can be classified as a kind of American call option, and its execute price can be calculated as K_t ð1 þ dÞ C, where C is the construction cost, and d is the transfer coefficient, up to negotiation between the concessionary developer and the buyer. If d is negative, the concessionary developer would execute the abandonment option. On the contrary, if d is greater than the average industry profit rate, the private developer would keep its PRH projects for more profit. So, d is normally greater than 0 and less than the average industry profit rate. With this option, the developer can build and operate its PRH projects in a more flexible and profitable way, which certainly increases the PRH project's investment value.

5.5. The execution of option to expand

When a project's operation is really good, like at the node of $S_{3,3}$, $S_{3,2}$ and $S_{i,j}$, the private developer can execute the expansion option, scaling the project up for more profit. This expansion option is a

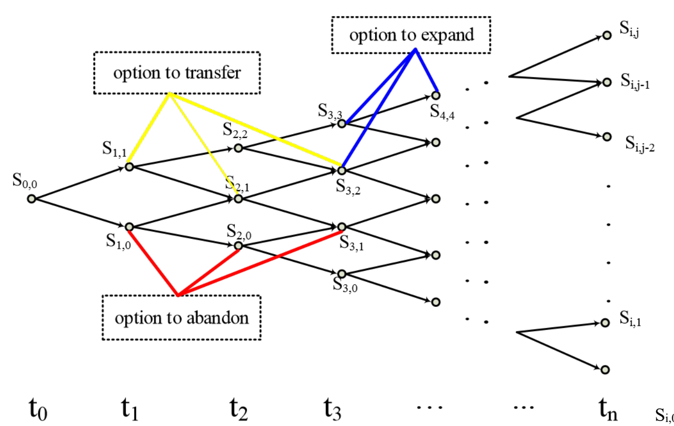


Fig. 5. Pricing tree for BOOC mode.

kind of American call option, and the execute price K_e can be estimated as K_e ð1 þ dÞ*C, where v is the expansion coefficient, normally the average profit rate of real estate industry, and C is the construction cost. This option to expand endows the private developers a chance to make more profit from PRH projects, thus it increases the project's investment value.

5.6. The investment valuation of BOOC mode

Trigeorgis (2005) put forward a new expanded NPV criterion to capture the additional value of managerial operating flexibility and other strategic interactions: Expanded (or strategic) NPV ¼ passive NPV þ Option Premium (ROV) (Flexibility value and Strategic value).

On the basis of this criterion, the investment value (IV) should be NPV plus ROV. Since the private developer is expected to make the best decision according to market situation, this is the advantage of BOOC mode. The concession contract endows the private developer with the options to adapt to the market. That means the private developer will choose the most profitable option to execute, thus BOOC mode brings more value for the PRH projects. Then IV can be assessed as:

$$IV = NPV + Max(ROV_a, ROV_t, ROV_e, 0) \quad (4)$$

Based on this criteria and from above analysis, it is not difficult to find out the max one of ROV_a, ROV_t, and ROV_e, then the final option price is easily calculated for the investment.

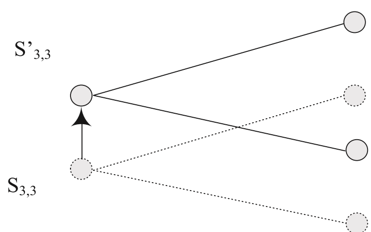
For abandonment option, it is one kind of put option, thus the option price at the executed node can be obtained, which is $Max[(S - K_a), 0]$. K_a and S are the execute price and valuation of the project at the executed node. After that, the option price for abandonment option ROV_a can be calculated from formula (3).

For transfer option, it is one kind of call option, thus the option price at the executed node can be obtained, which is $Max[(S - K_t), 0]$. S and K_t are the valuation of the project and the execute price at the executed node respectively. Then the option price for transfer option ROV_t can be calculated from formula (3).

Special attention should be paid to here for the calculation of expansion option. It is assumed that the private developer executes the expansion option at node $S_{3,3}$. With this expansion, the private developer puts in another investment. This means that the developer will possess a new property of PRH project in addition to the existing one, which of course would also spend more on construction. Thus the underlying asset valuation and the construction cost at node $S_{3,3}$ have a significant increase, as shown in Fig. 6.

So, it can be seen as two PRH projects together, that means when calculating the expansion option price at node $S_{3,3}$, the underlying asset valuation $S'_{3,3}$ should be the existing one ($S_{original}$) plus the one of the new investment (S_{new}), then the option price at the executed node is $Max[(S_{original} + S_{new}) - (K_e + C_{new}), 0]$, K_e is the execute price and C_{new} is the new construction cost. Then, ROV_e can be calculated from formula (3).

When the three options prices are found, the investment value



(IV) of the PRH projects can be obtained from formula (4). That is to say, the IV is equal to the NPV value plus the biggest one of the three option prices, namely ROV_a, ROV_t and ROV_e.

6. Case study

6.1. Project background

To test the effectiveness of the proposed BOOC mode and corresponding pricing model, an existing PRH project is exemplified. This project is located in Nanjing city, Jiangsu province of China. The ownership is held by the local government, and the developers are responsible for construction. It is now hypothesized the project to be built in BOOC mode. The project is of a high accessibility with two highways nearby, urban main-road and secondary main-road crossed, and a lot of bus routes. Meanwhile, the public service facilities have been completed, and education resources are abundant. The building indices of the PRH project are derived from real case and shown in Table 2, where the residential area means the building area of residential buildings, and commercial area represents the building area for commercial purpose. In addition, civil defense area is the underground protective construction used to protect personnel and supplies in wartime, while the equipment area is the area for installing water supply equipment, central heating equipment, fire pool, etc. There are some relationships among these building indices. For example, the construction area is the product of volume rate and land area, which is also the sum of residential area, commercial area, civil defense area and equipment area.

6.2. NPV assessment

Although the project is built in an urban-rural fringe area, the related facilities are well constructed. It is assumed that the occupancy rate of the project is 100%, especially when the rent is at a relatively low level. The market survey shows that the rent for residential area is determined at 25 CNY/m^2/month, since the similar private rental housing nearby is around 35 CNY/m^2/month. It is assumed that the rent will go up 10% every 5 years based on the growth trend of local housing price, inflation rate and interview with potential investors.

According to the NPV formula (1), some parameters are necessary for the assessment. First of all, the discount rate (q), is set at 10% on the basis of interviews with decision makers of the real estate industry. Second, the time of the cash flow (T), causes the residential limits in China is 70 years. Thus, it is conservatively assumed that the contract time period (T) is 40 years. The last parameter is the net cash flow (C_i), i.e. cash inflow minus cash outflow. For this PRH project, the cash inflow comes from rental, and the cash outflow is construction costs and operation costs. From these data, cash inflow and outflow can be reckoned respectively, i.e. 756.24 million CNY and 784.58million CNY. Then NPV is calculated, i.e. minus 28.35 million CNY. Fig. 7 shows the

Fig. 6. The tree node for expansion.

Table 2
Building indices of the studied PRH project.

Index	Value	Index	Value
Land property	residential land	Land area (m^2)	53547
Structure	frame-shear wall	Construction area (m^2)	250000
Building height (m)	93	Residential area (m^2)	203478
Number of households	4000	Commercial area (m^2)	10710
Volume rate	4	Civil defense area (m^2)	15800
Greening rate	30%	Equipment area (m^2)	20012

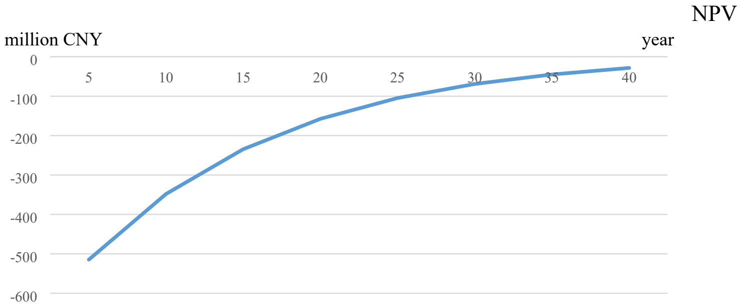


Fig. 7. NPV trend.

Table 3
Parameters in empirical test.

Variables	Value
q	10%
T	40 years
r_1	5.41%
s_1	29.18%
d	5.41%
v	11.30%
r_2	5.41%
s_2	29.18%

trend of NPV. From this assessment, the project investment valuation is all negative at any point in the next 40 years. Hence, the developers should not construct it.

6.3. BOPM assessment

6.3.1. Step 1: creation of the binomial tree

First there are parameters needed to be determined in real option pricing model. It is assumed that T is the contract time period of 40 years. If one step is regulated as every 5 years, there will be 8 steps. With regard to r-risk free rate, it is set at 5.41%, which is the interest rate for more than 5 years. The volatility s is 29.18%, which is calculated in view of house prices in recent years. There are also parameters needed for BOPM (Table 3), the specific explanations are shown in corresponding parts. Then the binomial tree of this project can be created as in Fig. 8.

6.3.2. Step 2: calculation of the option price

Although, it is expected that the flexible options in BOOC mode can make PRH projects profitable to private developers, the nature of PRH, which can only be rented, determines that it is difficult to be profitable in the short run. Therefore, it is highly recommended to keep the PRH projects for a longer term and make profit from flexible policies endowed by the concession contract. It is hypothesized that the private developer executes the options at step 5. This is because a shorter term does not completely reflect the value of PRH projects brought by BOOC. Yet, the concession contract somehow protects the private developer from financial loss to a certain extent. And within a longer timeframe, it is can be seen obviously from the calculation below that, the longer the private developers keep the project, the more profit they can make. Thus the executed binomial tree is shown in Fig. 9. Assume that abandonment option, transfer option, and expansion option are executed respectively at node 3, node 2, and node 1. Clearly, the time of node 3 is that the operation of PRH project is very not good. If that happened, it is the time when the private developer would execute the option to abandon. And at node 2, the operation is good, it is expected to make some profit from the PRH project, there may have some other enterprises interested in taking over it, or the concessionary developer wants cash flow, then node 2 is a good timing to execute option to transfer it. And for node 1, it is obvious the operation is very smooth. In this case, the private developer can earn a great income by keeping it. Thus, it is very likely for him to choose the option to expand for more profit.

(1) Calculation of abandonment option

As mentioned in Section 5.6, the option price of abandonment option at node 3 can be calculated, that is the valuation of the PRH project at node 3 subtract the execute price 3. Then using formula (3) to calculate step by step ahead in the binomial tree, ROV_a can be calculated. The process is shown in Fig. 10. The result at the beginning of the tree is ROV_a, which is calculated as 52.16 million CNY.

(2) Calculation of transfer option

Some parameters need to be assumed for calculation. As mentioned in Section 5.6, for the transfer price $K_t (K_t (1 + d)\ C)$, the transfer coefficient d should be greater than 0 and lower than the average profit rate in the real estate industry, so it is hypothesized that d is equal to the risk-free rate of 5.41%. Then the option price and ROV_t can be counted in the tree, as shown in Fig.11. The result at the beginning of the tree is ROV_t, which is calculated as 36.10 million CNY.

(3) Calculation of expansion option

The calculation for expansion option is special. For the valuation at node 1, it is $Max[(S_{original} + S_{new}) - (K_e + C_{new}); 0]$. For testing the effectiveness of expansion option, it is hypothesized that the

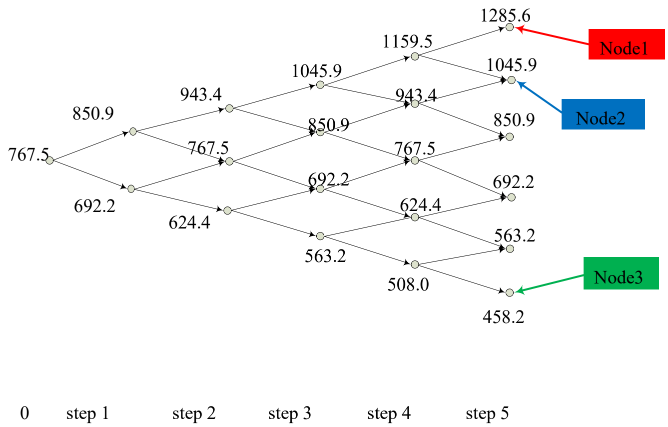


Fig. 9. Binomial tree for calculation.

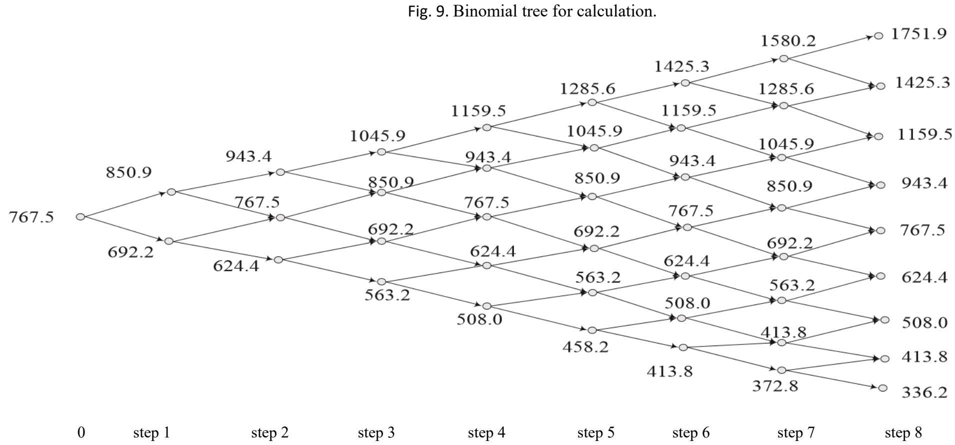


Fig. 8. Binomial tree for empirical test.

private developer invests in building a new PRH project with the same scale and construction cost. Then C_{new} is the same as $C_{original}$. For $K_e = (1 + v)C$, the average profit rate v in the real estate industry is 11.3%, according to Rand Consulting Company. As to S_{new}, obviously a new PRH project would be most preferred to be built in BOOC mode. However, it is not necessary for constructing another binomial tree for valuation, although it is more accurate. Considering the new concession contract for the secondary investment as a whole, it can only be executed at the contract expired time. Then it can be seen as European call options, thus the integral valuation can be estimated by BlackScholes equation.

$$V = SN(d_1) - N(d_2)Ke^{-rt} \quad (5)$$

Where V is the option value for S_{new}, S is current underlying asset price for a new project. K is the execute price, assumed as the construction fee plus the operation fee. And T is contract time of 40 years. And r_2 is risk-free interest rate of 5.41%, σ_2 is the volatility of returns of the underlying asset at 29.18%. Therefore, the new investment valuation $S_{new}(S_{new} = S + V)$ can be calculated. Also, the option price ROV_e for expansion can be worked out in the binomial tree in Fig. 12. Thus the result at the beginning of the tree is ROV_e, which is calculated as 105.58 million CNY.

6.3.3. Step 3: assessing the investment value

According to the criterion in Section "The investment valuation

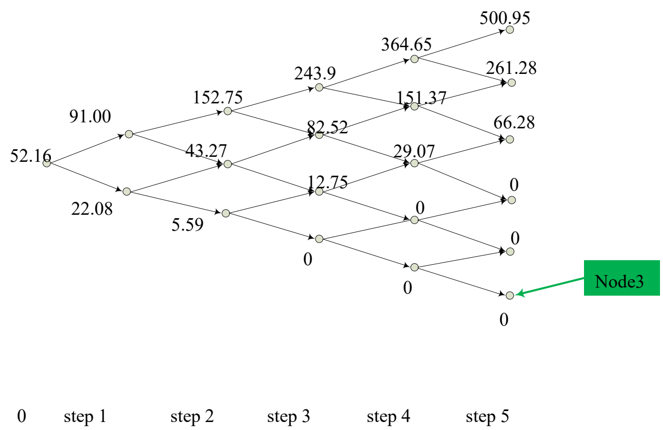


Fig. 10. Option price of abandonment option (million CNY).

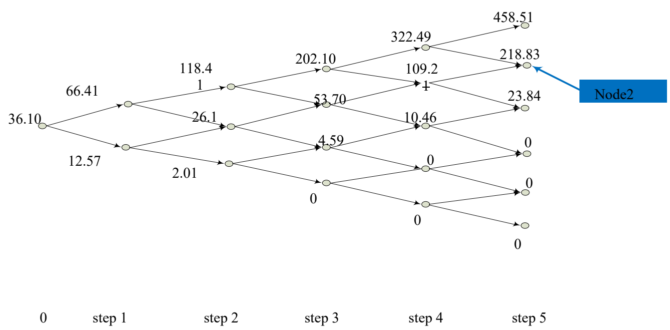


Fig. 11. Option price of transfer option (million CNY).

of BOOC mode", investment value (IV) can be assessed by the formula (4) as:

$$IV = NPV + Max(ROV_a; ROV_t; ROV_e; 0) = NPV + ROV_e = 28.35 + 105.58 = 77.23 \text{ million CNY}$$

7. Discussions

It has been evidently presented in empirical test that BOOC mode is significantly applicable. Just for the PRH project, NPV result is estimated to be negative at 28.35 million CNY, for which the project should not be kicked off and no private developer would like to take it from the economic perspective. However, the results from BOOC mode show that keeping the project is profitable and is even more profitable by executing the option to expand, which is at 77.23 million CNY. In this case, it is probable that some private developers will build and operate it. Thus the financial burden of PRH project on the government will be greatly alleviated as a matter of course.

How to choose the best time node to execute one of the options will greatly affect the results. Although the chosen node for verification in this article already reflects the benefit of BOOC mode, it seems more complex and changeful in actual situations how private developers adapt themselves and make flexible decisions according to the changing circumstances of market. With BOOC concession contract, PRH projects never fail to bring high returns for private developers in the short run and it is equally sensible to take the PRH projects in the long run. Seen from the binomial tree for BOOC mode, PHR projects using efficient strategies to execute option will be profitable without the slightest doubt.

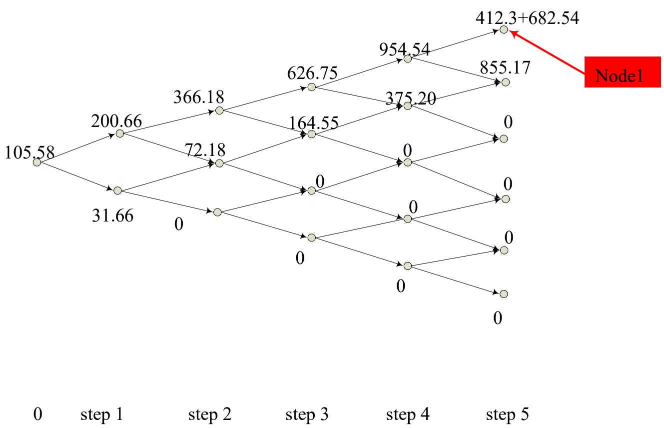


Fig. 12. Option price of expansion option (million CNY).

8. Conclusions

Nowadays, government has attached great importance to solving the housing problem of low-income residents', PRH project seems ready to play its role under this circumstance. What's more, the demand and supply of PRH promote each other mutually with the rapid pace of urbanization. Especially in China, PRH project has been announced as a major part of the affordable housing system. However, the currently existing PRH projects depend greatly on the government. What's worse, construction modes assessed with traditional methods at presents seem unattractive to private developers due to the low rate of return. Numerous studies have been done to improve such situation through education resources, communications, and other public service facilities, but there is still a deficiency in constructing PRH sustainably and effectively. Finding a new approach to developing PHR projects and getting social capitals fully involved in the implementation of PHR projects seem extremely urgent to solve residents' housing problems and develop a stable and sustainable community.

This article has applied the concepts of real option theory to analyze the characters of PRH projects by making the project lifecycle multiple-stages with options in BOOC mode. In the concession contract of BOOC, the concessionary developers are entitled with three options. Expectably in this case, private developers can flexibly choose and execute one of the options at any time within the contract period according to the market situation. Through the empirical test, it is evidently proved that the concession contract of BOOC mode is efficient. Possessing the options from the concession and rich experience in construction, private developers are very likely to be profited from the project. Thus BOOC mode does not only alleviate the great financial burden on governments, but also turn the construction of prone-to-deficit PRH projects into a sustainable way.

From this empirical test, it goes beyond doubt that BOOC mode is practicable. BOOC mode, on the basis of real option theory, enables private developers to build and operate the projects flexibly without restrictive regulations and motivate them to make investment in PRH without any hesitation. As to pricing model, this article mainly uses real option pricing modeleCRR BOPMe to assess the value of a PRH project. Options in BOOC mode can be executed by private developers, perfectly conforming to the feature of CRR BOPM and executed at any time before the expired time. That is to say, CRR BOPM reflects the potential value of PRH projects to a great extent. Undoubtedly, application of CRR BOPM in actual situations is more complex and changeful. In the future, it is expected to include more factors and flexible policies to a BOOC concession contract, such as option to delay or option to switch. Besides, the pricing model needs to be intensively studied and applied in more projects.

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