1   **Mathematical programming models for construction site layout problems**

2

3   **Abstract**

4   We address the construction site layout problem that determines the locations of

5   temporary facilities. Mathematical programming models for the site layout problem are

6   proposed, which can be solved by state-of-the-art solvers to optimality. A number of safety,

7   health and environmental concerns, such as falling objects, dusts, and noise, are incorporated

8   in the extensions of the mathematical models. We demonstrate, using numerical experiments,

9   the superiority of our proposed mathematical programming model over existing heuristics in

10   terms of solution optimality and the wide applicability in terms of handling practical

11   considerations. Based on the results of conducted experiments, the proposed method achieved

12   a 3-19% improvement on optimality over those of the existing heuristics methods. The

13   contribution of this research work includes the advanced development of a mathematical

14   programming model incorporating extended concerns and solving site layout problems within

15   reasonable time.

16   **Key Words:** construction site layout; construction management; mathematical programming;

17   bi-objective optimization

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1 Introduction

Construction site layout is a planning activity that determines the locations of temporary facilities (TFs) to improve construction efficiency. TFs include warehouses, fabrication shops, maintenance shops, batch plants, administration offices, tool trailers, staging areas, maintenance areas, labor residence facilities, and tower cranes. The types of TFs required vary for projects of different sizes, locations, and other characteristics. The layout of TFs affects the construction time (e.g., for construction sites of limited space), costs (e.g., for large sites with a large amount of material transported between different TFs), safety (e.g., there must be some clearance around a tower crane), and environment (e.g., it is better not to position a noisy workshop close to an office for health and safety reasons). A main objective of the TF layout is to minimize travel times, to remove unnecessary movement of resources, and to reduce the frequency of handling of materials. A well-designed site layout saves considerable nonproductive time that is caused by inefficient coordination of the resources employed, especially for large projects where traveling between facilities can be considerably time consuming. Deriving the best construction site layout is a difficult problem because there are many possible alternatives and it is impractical to evaluation all of the alternatives. In the conventional approach, site layout planning is done based on the experience of planners. The level of accuracy that can be achieved varies among different projects and lacks of stabilities. Site managers and related contractors still need to make further adjustments based on the actual information acquired from sites. The planning activities and qualities of plans seriously rely on experience and accuracy of site information. To improve the current circumstance, the development of computational approaches to guarantee the objective optimality and reliability is necessary.

1.1 Literature review

Because of the practical importance and the computational complexity of TF layout problems, a large number of attempts for them have been made by researchers. The TF layout

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45 problem can be classified in different dimensions: (i) discrete vs. continuous construction site

46 layout, (ii) static vs. dynamic site layout, and (iii) layout construction vs. layout improvement.

47 (i) Discrete layout (or called 'facility to location assignment' by Osman et al., 2003) means

48 there are a finite number of candidate locations in a construction site and each TF needs to be

49 assigned to one of the candidate locations. Continuous layout (or called 'facility to site

50 assignment' by Osman et al., 2003) refers to the situation where TFs can be placed at any

51 empty space of a construction site. The continuous empty space is sometimes discretized in

52 solution algorithms for continuous layout problems. (ii) Static layout problems assume that

53 all the TFs are set up and removed at the same time; as a result, a TF cannot overlap spatially

54 with permanent facilities or other TFs. Dynamic layout problems account for the time

55 dimension of the construction of the permanent facilities, and hence, after completing their

56 tasks, some TFs will be removed to make room for new TFs to be set up. (iii) Layout

57 construction is to design the layout of TFs from scratch, whereas layout improvement deals

58 with conducting some but not fundamental changes to an initial layout, which is likely to be

59 designed by site managers based on experience.

60   A large variety of heuristic and meta-heuristic algorithms have been developed to

61 generate solutions for construction site layout problems. We present the stream of studies on

62 discrete site layout in this paragraph and the stream of works on continuous site layout in the

63 next paragraph. Yeh (1995) studied a static discrete site layout problem with the objective of

64 minimizing the sum of construction costs of TFs at candidate locations and interactive costs

65 between TFs. An annealed neural network method was developed and tested over two case

66 studies with 12 TFs and 12 candidate locations. Li and Love (1998) formulated a static

67 discrete site layout model that minimizes total traveling distance of site personnel between

68 facilities. They proposed a genetic algorithm to solve an example with 11 TFs and 11

69 candidate locations. Li and Love (2000) extended the above study to an unequal-area facility

70 layout problem, where some candidate locations are too small to accommodate large TFs.

71 Unlike the above studies which require deterministic cost parameters in the models, Lam et al.

72 (2007) applied fuzzy reasoning to calculate the closeness relation of TFs in the objective

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73 function to account for the uncertainty of the parameters, and proposed an ant colony

74 optimization (ACO) algorithm to solve the problem. This research was extended in Lam et al.

75 (2009), which proposed a max-min ant system algorithm. Unlike the above static site layout

76 problems, Xu and Li (2012) proposed a multi-objective particle swarm optimization

77 algorithm for a dynamic site layout problem with two objectives: one is minimizing the total

78 transportation costs and the other is maximizing the distances between high-risk facilities.

79   In the stream of literature on continuous construction site layout, Mawdesley et al. (2002)

80 applied a genetic algorithm to a problem setting in which the inter-facility distance (minimum

81 and maximum distance between two TFs) is account for. Zouein et al. (2002) further took

82 into account the orientations of TFs. In addition to factors considered in the above two studies,

83 Osman et al. (2003) developed a computer-aided design (CAD) platform to visually observe

84 the layout; another characteristic of Osman et al. (2003) is that it minimizes the "relative

85 proximity weight" instead of transportation costs as the values of transportation costs are

86 difficult to obtain. Elbeltagi et al. (2004) and Sanad et al. (2008) have also investigated

87 continuous site layout problems using genetic algorithm. In the two studies, Elbeltagi et al.

88 (2004) addressed the dynamic site layout problem and Sanad et al. (2008) took into account

89 many safety and environment considerations, such as safety zones around construction areas,

90 noisy workshops and facilities that emit gases, and effect of harmful TFs on other facilities.

91 Zhou et al. (2009) integrated genetic algorithm and simulation to address continuous

92 construction site layout for tunnels. Ning et al. (2010) proposed a max-min ant system

93 algorithm to address the continuous construction site layout problem with two objectives:

94 safety/environment concerns and total transportation cost of flows between the facilities. The

95 models and algorithms were integrated into a decision-making system in Ning et al. (2011).

96 To find Pareto-optimal solutions for site layout, Ning and Lam (2013) presented a modified

97 Pareto-based ant colony optimization algorithm and Yahya and Saka (2014) developed a

98 multi-objective artificial bee colony algorithm. As opposed to the above studies which have

99 taken into account solely quantitative factors, Ning et al. (2016) applied the intuitionistic

100 fuzzy set theory to account for qualitative factors in construction site layout.

101   There are few studies that have applied mathematical optimization to construction site

102   layout problems. Easa and Hossain (2008) developed a mathematical optimization model for

103   a static continuous site layout problem. The model was solved using LINGO software for an

104   instance of four TFs. By contrast, we take advantage of state-of-the-art mixed-integer

105   programming techniques and solve larger problems much more efficiently. Said and El-Rayes

106   (2013) designed an approximate dynamic programming method for a dynamic continuous

107   site layout problem. Approximate dynamic programming, by nature, is a heuristic and cannot

108   guarantee optimal solutions, in contrast to our exact mixed-integer programming approach.

109   Wong et al. (2010) used integer programming to solve site layout problems to minimize

110   transport costs. Our model distinguishes from theirs in three aspects: first, we consider many

111   more realistic factors; second, they applied a big-M method to linearize constraints, whereas

112   our linearization approach does not require the big-M; third, we consider two objectives and

113   propose methods to obtain the exact Pareto-frontier for the two objectives. Hammad et al.

114   (2016) applied mathematical programming technique for site layout to minimize noise

115   pollution and transport costs. Their model is inherently nonconvex and solvers cannot

116   guarantee global optimality. Furthermore, they proposed an epsilon-constraint method to

117   identify an approximate Pareto-frontier for the two objectives, while our study takes

118   advantage of the discrete nature of the decision variables and proposes cutting planes to

119   obtain the exact Pareto-frontier.

120

121   **1.2   Objectives and contributions**

122   The objective of this paper is to develop mathematical models for TF layout problems

123   that can be solved by state-of-the-art solvers to optimality. The contribution of the paper is

124   two-fold. First, for a long time, the construction management community has considered it

125   unrealistic to use mathematical programming to address layout problems due to their large set

126   of feasible solutions. However, in the last 30 years, the spectacular advances in computational

127   power and in integer optimization methods have made it possible to address problems that

128 were once thought to be intractable in practical settings (Bertsimas and Shioda, 2007). Hence,

129 in contrast to most of the existing literature, which tries to develop heuristic or meta-heuristic

130 approaches, we take advantage of the latest development of mathematical programming

131 techniques in developing mathematical models to solve the problems to optimality within

132 reasonable time. Second, we demonstrate how mathematical models can be applied to handle

133 many practical considerations, especially those that can hardly be dealt with by heuristic

134 methods.

135   The remainder of this paper is organized as follows. Section 2 presents a mathematical

136 programming model for a basic static discrete construction site layout problem. Section 3

137 demonstrates how mathematical programming can be applied to handle a large variety of

138 practical considerations. Section 4 reports the results of numerical experiments that are used

139 to validate the effectiveness of the proposed models. Conclusions are presented in Section 5.

140 The symbols used in the paper are listed in Table 1.

141

142 **Table 1** Symbols

Sets	
Ω_k	Set of candidate locations suitable for TF $k = 1, 2, ..., K$ due to closeness requirement to fixed facilities, $\Omega_k \subseteq \{1, 2, \cdots, I\}$

Indices	
i, j $= 1, 2, ..., I$	A candidate location for temporary facilities
k, l $= 1, 2, ..., K$	A temporary facility

Parameters	
δ_{ik}	Binary parameter that equals 1 if candidate location i is large enough to accommodate TF k and 0 otherwise.
δ'_{ik}	Binary parameter that equals 0 if TF k cannot be set up at candidate location i due to safety considerations and 1 otherwise.
δ''_{ik}	Binary parameter that equals 0 if TF k cannot be set up at candidate location i due to health/environmental considerations and 1 otherwise.
δ''_{ijkl}	Binary parameter that equals 0 if simultaneously setting up TF k at

	candidate location i and TF l at candidate location j, $k<l$, is not allowed due to health/environmental considerations, and 1 otherwise.
c_{ik}	Safety, health and environmental damages posed to the neighbors by TF k if it is set up at candidate location i.
c_{ijkl}	Safety, health and environmental damages posed if TFs k and l, $k<l$, are simultaneously set up at candidate locations i and j, respectively.
D_{ij}	Distance between candidate locations i and j
I	Total number of candidate locations for temporary facilities
K	Total number of temporary facilities
K'	Total number of fixed facilities
$\sigma(k)$	The location where fixed facilitate k must be set up
q_{kl}	Amount of flow from TF k to TF l, $q_{kk=0}$

Functions of the Decision Variables

$C(\mathbf{x})$	Transportation cost for layout plan $\mathbf{x}$
$C(\mathbf{z})$	Transportation cost for layout plan $\mathbf{z}$
$D(\mathbf{x},\mathbf{z})$	Safety, health and environmental damages caused by layout plan $\mathbf{x}$ (or represented as $\mathbf{z}$)

Decision Variables

x_{ijkl}	Binary variable that equals 1 if TF k is set up at candidate location i and TF l is set up at candidate location j, $k<l$, and 0 otherwise.
$\mathbf{x}$	A layout plan: $\mathbf{x}=(x_{ijkl}, i,j=1,2,...,I, k=1,...,K-1, l=k+1,...,K)$.
z_{ik}	Binary variable that equals 1 if TF k is set up at candidate location i, and 0 otherwise.
$\mathbf{z}$	A layout plan: $\mathbf{z}=(z_{ik}, i=1,2,...,I, k=1,2,...,K)$.

143 **2 Mathematical Programming Model for Construction Site Layout**

144   In this section, we present a mathematical programming model for a basic static discrete

145 construction site layout problem. Then we investigate the computational complexity of the

146 problem and finally we propose techniques to transform the mathematical programming

147 model into an integer linear programming formulation.

2.1 A nonlinear optimization model

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149   A static discrete construction site-level layout problem is concerned with assigning a

150 number of predetermined TFs to a number of predetermined locations. The number of

151 predetermined places is equal to or greater than the number of predetermined TFs. We

152 assume that each of the predetermined places is capable of accommodating any of the TFs,

153 i.e., we consider an equal-area facility layout problem. To minimize the transportation costs

154 between TFs, the problem can be formulated as a nonlinear optimization model:

155   [P1]
$$\min_{\mathbf{z}} C(\mathbf{z}) = \sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{k=1}^{K} \sum_{l=1}^{K} D_{ij} q_{kl} z_{ik} z_{jl} \tag{1}$$

156 subject to:

157
$$\sum_{i=1}^{I} z_{ik} = 1, \quad k = 1, 2, ..., K \tag{2}$$

158
$$\sum_{k=1}^{K} z_{ik} \leq 1, \quad i = 1, 2, ..., I \tag{3}$$

159
$$z_{ik} \in \{0,1\}, \quad i = 1, 2, ..., I, k = 1, 2, ..., K \tag{4}$$

160 The objective function (1) minimizes the total transportation costs between TFs. Constraints

161 (2) impose that each TF is assigned to exactly one location. Constraints (3) enforce that each

162 location can accommodate at most one TF. Constraints (4) define the z_{ik} 's to be binary

163 decision variables. As model [P1] has multiplication terms of decision variables, $z_{ik} z_{jl}$, it is

164 inherently nonlinear and cannot be directly solved by off-the-shelf mixed-integer (or integer)

165 linear programming solvers.

166   Some studies in the literature assume that the number of predetermined locations is equal

167 to the number of predetermined TFs, which simply means $I = K$ in model [P1]. Note that

168 when the number of predetermined locations is greater than the number of predetermined TFs,

169 as opposed to studies in the literature, the mathematical model [P1] does not need to have

170 extra 'dummy' facilities to make both numbers equal.

171 ## 2.2 Hardness of the problem

172   It has been stated in the literature (Lam et al., 2007, 2009; Yahya and Saka, 2014) that
173 the TF layout problem is strongly NP-hard. However, the NP-hardness is not demonstrated.
174 Below, we provide a rigorous proof. The key idea is to demonstrate SLP is at least as difficult
175 as an NP-hard problem, the travelling salesman problem (TSP).

176   The decision version of the TF layout problem of [P1] is in NP; that is, given an
177 assignment of TFs to the candidate locations, it can be determined in polynomial time
178 whether the total transportation cost is larger than a given constant Θ. We then show the NP-
179 completeness of the problem by reducing a well-known strongly NP-hard problem, the
180 travelling salesman problem, into a TF layout problem. The decision version of the TSP is
181 defined as follows: Given a set of nodes $\{1, 2, ..., I\}$, distances D_{ij} from node i to node j,
182 and constant Θ, determine whether there is a circle that starts from node 1, visits each of the
183 other nodes once, and returns to node 1, such that the length of the circle is not greater than
184 Θ.

185 **Theorem 1**: The decision version of the TF layout problem is strongly NP-hard.

186 **Proof**:

187   Suppose that the number of TFs is equal to the number of candidate locations, $K = I$;
188 $q_{1,2} = q_{2,3} = ... = q_{I,1} = 1$ and the other q_{kl}'s are 0. The objective function (1) in model [P1]
189 becomes $\sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{k=1}^{I} D_{ij} x_{ijk,k+1}$, where $x_{ijI,I+1}$ is defined to be $x_{ijI,1}$. It follows easily that a
190 solution to the above TF layout problem will have a transportation cost not greater than Θ if
191 and only if the TSP has a circle whose length is not greater than Θ. Thus, the TSP can be
192 solved by solving a TF layout problem. □

193 ## 2.3 Linearization

194   The nonlinear model [P1] cannot be directly solved by off-the-shelf mixed-integer (or
195 integer) linear programming solvers. In order to take advantage of the state-of-the-art
196 progress in the computation of mixed-integer linear programs, we introduce binary decision
197 variables x_{ijkl} that equals 1 if TF k is set up at candidate location i and TF l is set up at

198 candidate location j, $k < l$, and 0 otherwise. Then model [P1] can be transformed to the

199 following integer linear programming formulation:

200 [P2]
$$\min_{\mathbf{x}} C(\mathbf{x}) = \sum_{i=1}^{I} \sum_{j=1}^{I} \sum_{k=1}^{K-1} \sum_{l=k+1}^{K} (D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl} \qquad (5)$$

201 subject to:

202 $$x_{ijkl} \geq z_{ik} + z_{jl} - 1, \quad i = 1, 2, ..., I, j = 1, 2, ..., I, k = 1, 2, ..., K-1, l = k+1, k+2, ..., K \qquad (6)$$

203 $$x_{ijkl} \leq z_{ik}, \quad i = 1, 2, ..., I, j = 1, 2, ..., I, k = 1, 2, ..., K-1, l = k+1, k+2, ..., K \qquad (7)$$

204 $$x_{ijkl} \leq z_{jl}, \quad i = 1, 2, ..., I, j = 1, 2, ..., I, k = 1, 2, ..., K-1, l = k+1, k+2, ..., K \qquad (8)$$

205 $$x_{ijkl} \in \{0,1\}, \quad i = 1, 2, ..., I, j = 1, 2, ..., I, k = 1, 2, ..., K-1, l = k+1, k+2, ..., K \qquad (9)$$

206 and Constraints (2)–(4). Constraints (6) require that $x_{ijkl} \geq 1$ if both z_{ik} and z_{jl} are 1. Since

207 Constraints (9) enforce the condition that x_{ijkl} is either 0 or 1, x_{ijkl} must be 1 if both z_{ik} and

208 z_{jl} are 1. Constraints (7) and (8) impose that x_{ijkl} must be 0 if either z_{ik} or z_{jl} is 1.

209 Altogether, Constraints (6)–(9) formulate the requirement that x_{ijkl} is 1 if and only if both z_{ik}

210 and z_{jl} are 1.

211 Three issues regarding the integer linear programming model [P2] are noteworthy. First,

212 in the definition of x_{ijkl}, we require $k < l$ to eliminate symmetry. As a result, in objective

213 function (5) the coefficient of x_{ijkl} is $D_{ij}q_{kl} + D_{ji}q_{lk}$, which consists of the transportation cost

214 from TF k to TF l and the transportation cost from l to k. Second, the relation between

215 decision variables x_{ijkl}, z_{ik} and z_{jl} is $x_{ijkl} = z_{ik}z_{jl}$, i.e., x_{ijkl} is equal to 1 if and only if both z_{ik}

216 and z_{jl} are 1. However, since we minimize x_{ijkl} in objective function (5), x_{ijkl} will

217 automatically take the smallest possible value in the optimal solution. As a result, constraints

218 (7) and (8) can be removed without losing optimality. Third, as model [P2] is an integer

219 linear program, it can be solved by state-of-the-art solvers such as CPLEX and Gurobi.

220 **3   Practical Extensions**

221 Models [P1] and [P2] are basic and their purpose is to show the NP-hardness of the

222 problem, the mathematical formulation, and how the mathematical formulation can be

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223 linearized and solved by solvers. In this section, we demonstrate how mathematical

224 programming can be applied to handle a large variety of practical considerations.

3.1 Unequal area TF layout problem

226 Model [P2] assumes that all of the candidate locations can accommodate all of the TFs.

227 If some of the predetermined locations are too small to accommodate some large TFs, then

228 the problem becomes an unequal-area TF layout problem. The unequal-area facility layout

229 problem can be easily handled by setting some of the z_{ik} decision variables at 0. Define

230 binary parameter δ_{ik} that equals 1 if candidate location i is large enough to accommodate TF

231 k and 0 otherwise. Then the unequal-area TF layout problem can be formulated as:

232 [P3] the same objective function as [P2]

233 subject to:

$$z_{ik} \leq \delta_{ik}, \quad i = 1, 2, ..., I, k = 1, 2, ..., K \tag{10}$$

235 and constraints (2)–(4), (6) and (9). Constraints (10) ensure that if candidate location i cannot

236 accommodate TF k, i.e., $\delta_{ik} = 0$, then z_{ik} has to be 0, meaning that TF k will not be set up

237 at location i in the optimal solution.

3.2 Safety considerations

239 Safety is also a factor that should be taken into account in construction facility layout.

240 For instance, proper safety zones around construction areas can protect workers and entities

241 from falling objects. Some clearance from buildings or structures must be kept clear. This

242 consideration can be handled as follows: define binary parameter δ'_{ik} that equals 0 if TF k

243 cannot be set up at candidate location i due to safety considerations and 1 otherwise. Then

244 adding the following constraints to model [P3] will account for the safety considerations:

$$z_{ik} \leq \delta'_{ik}, \quad i = 1, 2, ..., I, k = 1, 2, ..., K \tag{11}$$

246 The safety concerns can be considered in practice according to the determinations and

247 judgements by occupational health and safety supervisors, preliminary risk analysis of

248 construction site, or historical hazard and accident records in similar projects. During the

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249 construction stage, related safety concerns can also be identified through inspections or

250 sensor readings for further layout improvement processes.

3.3 Health/environmental considerations

252 In a construction site, some TFs are of high harmful effects, for example, noisy

253 workshops and those emit harmful substances such as gases, dusts, and radiation. These TFs

254 will present health hazards to neighbors, especially if they are positioned adjacent to hospital,

255 schools, or residential buildings. To exclude layout plans that may lead to such health hazards

256 to the neighboring fixed facilities, we define binary parameter δ''_{ik} that equals 0 if TF k

257 cannot be set up at candidate location i due to environmental considerations and 1 otherwise.

258 Then adding the following constraints to model [P3] will account for the

259 health/environmental impact on the neighboring fixed facilities:

$$z_{ik} \leq \delta''_{ik}, \quad i = 1, 2, ..., I, k = 1, 2, ..., K \tag{12}$$

261 Another health/environmental consideration is the dangerous interaction between two

262 TFs. For instance, TFs such as noisy workshops and those emit harmful substances should be

263 kept a far distance away from sensitive TFs such as temporary administration offices and

264 labor residence facilities. To take into account this type of environmental consideration, we

265 define binary parameter δ''_{ijkl} that equals 0 if simultaneously setting up TF k at candidate

266 location i and TF l at candidate location j, $k < l$, is not allowed due to

267 health/environmental considerations, and 1 otherwise. Then adding the following constraints

268 to model [P3] will account for the health/environmental considerations of two TFs:

$$x_{ijkl} \leq \delta''_{ijkl}, \quad i = 1, 2, ..., I, j = 1, 2, ..., I, k = 1, 2, ..., K-1, l = k+1, k+2, ..., K \tag{13}$$

3.4 Inter-facility distance for convenience of construction

271 Some facilities may need to be positioned near each other. For example, a crane must be

272 placed close to the building it serves. To reflect this constraint, we define the set of candidate

273 locations suitable for TF $k = 1, 2, ..., K$, due to closeness requirement to fixed facilities, as

274 $\Omega_k \subseteq \{1, 2, \cdots, I\}$. The following constraints can be added to model [P3] to allow only some

275 specific locations for each TF:

$$z_{ik} = 0, \quad i \notin \Omega_k, k = 1, 2, ..., K \tag{14}$$

3.5 Fixed facilities

278   In the above models as well as in many models in the literature, the transportation costs

279 between TFs have been taken into account, whereas the transportation costs between TFs and

280 fixed facilities in the construction site are not incorporated, or at least not incorporated

281 quantitatively (constraints (13) have accounted for the transportation costs between TFs and

282 fixed facilities in a qualitative manner). We can slightly modify the models to include the

283 transportation costs between TFs and fixed facilities in objective functions. Specifically, we

284 will treat a fixed facility as a temporary one except that its location is pre-determined. Define

285 K' as the number of fixed facilities. Redefine K as the total number of temporary facilities

286 and fixed facilities, where $1, 2, ..., K'$ represent fixed facilities and $K'+1, K'+2, ..., K$

287 represent TFs. Redefine I as the number of candidate locations for temporary facilities plus

288 the number of fixed facilities. Define $\sigma(k)$ as the location where fixed facilitate k must be set

289 up, $\sigma(k) = 1, 2, ..., I$. Then the TF layout problem considering fixed facilities can be formulated

290 as:

291 [P4]   the same objective function as [P3]

292 subject to:

$$z_{\sigma(k),k} = 1, \quad k = 1, 2, ..., K' \tag{15}$$

294 and constraints (2)–(4), (6), and (9)–(14). Constraints (15) ensure that a fixed facility

295 $k = 1, 2, ..., K'$ must be set up at location $\sigma(k)$.

3.6 Layout improvement

297   The construction site manager may already have a layout plan and he does not intend to

298 have the plan fundamentally changed. Let $\hat{\mathbf{z}} = (\hat{z}_{ik}, i = 1, 2, ..., I, k = 1, 2, ..., K)$ represent the

299 manager's plan and suppose that the manager allows a change of at most U TFs' locations

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300 (e.g., $U = 3$). Then the following constraint can be added to model [P3] to obtain a layout

301 improvement plan:

$$\sum_{i=1}^{I}\sum_{k=1}^{K} \hat{z}_{ik} z_{ik} \geq K - U \tag{16}$$

303 Constraint (16) guarantees that there are at least $K - U$ TFs are set up at the same locations

304 as they are in plan $\hat{\mathbf{z}}$.

305 **3.7   Multi-objective optimization**

306   The transportation costs are not the solely concern in construction site layout. As

307 mentioned above, the safety, health and environmental concerns are also important factors to

308 be taken into account. Sections 3.2 and 3.3 have formulated the safety, health and

309 environmental concerns in a 'yes/no' manner, i.e., a TF can either be set up at a location

310 without causing any safety, health and environmental problem, or is not allowed to be set up

311 at the location at all. This 'yes/no' may be too restrictive in practical applications.

312   A more reasonable way to model the safety, health and environmental concerns is to

313 adopt a multi-objective approach: one objective is the transportation cost, and the other

314 objective is the safety, health and environmental damage. In this setting, even if a TF k can

315 be set up at candidate location i ($\delta'_{ik} = 1$ and $\delta''_{ik} = 1$), it may still cause some safety, health

316 and environmental damages.

317   To formulate such a bi-objective model, we need to define some new parameters: c_{ik} is

318 the safety, health and environmental damages posed to the neighbors by TF k if it is set up at

319 candidate location i; c_{ijkl} is the safety, health and environmental damages posed if TFs k and

320 l, $k < l$, are simultaneously set up at candidate locations i and j, respectively. Then the

321 following bi-objective model can be formulated:

322 [P5]
$$\min_{\mathbf{x},\mathbf{z}} \begin{pmatrix} C(\mathbf{x}) \\ D(\mathbf{x},\mathbf{z}) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} (D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl} \\ \sum_{i=1}^{I}\sum_{k=1}^{K} c_{ik} z_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} c_{ijkl} x_{ijkl} \end{pmatrix} \tag{17}$$

323 subject to constraints (2)–(4), (6), and (9)–(15).

324   It is unlikely for model [P5] to have a solution $(\mathbf{x}, \mathbf{z})$ that minimizes both objectives

325 simultaneously. The purpose of formulating the bi-objective model [P5] is for us to develop

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326 algorithms to identify Pareto optimal solutions and the Pareto frontier of the two objective

327 functions. The Pareto frontier can be shown to construction managers to decide which

328 solution to adopt. Because of the discrete nature of the decision variables, we propose the

329 following algorithms to identify all the Pareto optimal solutions:

330 **Algorithm: Identifying all the Pareto optimal solutions**

331 Step 0. Define Ψ as the set that saves the Pareto optimal solutions to be generated; $\Psi \leftarrow \varnothing$.

332 Solve [P4] and obtain an optimal solution denoted by $\mathbf{z}^0$ (note that we only mention

333 the $\mathbf{z}$-component of the solution because the $\mathbf{x}$-component is implied by the $\mathbf{z}$-

334 component). Then $\mathbf{z}^0$ has the smallest transportation costs.

335 Step 1. Solve the following model and its optimal solution is denoted by $\mathbf{z}^*$.

336 [P6]
$$\min_{\mathbf{x},\mathbf{z}} D(\mathbf{x},\mathbf{z}) = \sum_{i=1}^{I}\sum_{k=1}^{K} c_{ik} z_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} c_{ijkl} x_{ijkl} \tag{18}$$

337 subject to:

338
$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} (D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl} \leq \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} (D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl}^0 \tag{19}$$

339 where x_{ijkl}^0 is element in the solution $\mathbf{z}^0$, and subject to the other relevant constraints.

340 Note that because of constraint (19), $\mathbf{z}^*$ must have the smallest transportation costs;

341 because we minimize the safety, health and environmental damages in objective

342 function (18), $\mathbf{z}^*$ has the smallest damage among all of the solutions that minimize

343 the transportation costs. Therefore $\mathbf{z}^*$ is a Pareto-optimal solution. Set $\Psi \leftarrow \Psi \cup \{\mathbf{z}^*\}$.

344 Step 2. Solve the following model:

345 [P7]
$$\min_{\mathbf{x},\mathbf{z}} C(\mathbf{x}) = \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} (D_{ij}q_{kl} + D_{ji}q_{lk})x_{ijkl} \tag{20}$$

346 subject to:

347
$$\sum_{i=1}^{I}\sum_{k=1}^{K} c_{ik} z_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} c_{ijkl} x_{ijkl} \leq \sum_{i=1}^{I}\sum_{k=1}^{K} c_{ik} \hat{z}_{ik} + \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} c_{ijkl} \hat{x}_{ijkl}, \hat{\mathbf{z}} \in \Psi \tag{21}$$

348
$$\sum_{i=1}^{I}\sum_{k=1}^{K} (1-\hat{z}_{ik})z_{ik} \geq 2, \hat{\mathbf{z}} \in \Psi \tag{22}$$

15

349   and the other relevant constraints. If model [P7] has no feasible solution, go to Step 3;

350   otherwise, its optimal solution is denoted by $\mathbf{z}^*$. Note that because of constraint (21),

351   the safety, health and environmental damages of $\mathbf{z}^*$ is not greater than any solution in

352   Ψ; constraints (22) make sure that $\mathbf{z}^*$ is different from any solution in Ψ; in other

353   words, solutions in Ψ will not be generated again. The objective function (20)

354   minimizes the transportation costs. It should be highlighted that $\mathbf{z}^*$ is not necessarily a

355   Pareto-optimal solution because it is possible that a solution in Ψ has the same

356   damage as $\mathbf{z}^*$ but a lower transportation cost. However, we still let $\Psi \leftarrow \Psi \bigcup \{\mathbf{z}^*\}$ for

357   the moment and repeat Step 2.

358 Step 3. Compare each pair of solutions in Ψ and remove those solutions that are not Pareto

359   optimal from Ψ. Then Ψ is the complete set of Pareto optimal solutions. □

360 **4   Numerical experiments**

361   In this section, we report the results of numerical experiments that are used to validate

362 the effectiveness of the proposed models. The experiments are run on a PC equipped with

363 3.60GHz of Intel Core i7 CPU and 16GB of RAM. The integer programming models are all

364 solved by CPLEX 12.6.3.

365 **4.1   Comparison of solution quality with the literature**

366   To test the superiority of our proposed mathematical programming models over existing

367 heuristics in terms of solution optimality, we solve four case studies in three papers in the

368 literature and compare our solutions with theirs. The results are shown in Table 2.

369   In the first case study of Yeh (1995), the total "interactive cost" between TFs was solved

370 to be 93 using the annealed neural network method. Our method finds the optimal solution

371 with the total interact cost at 90, a reduction of 3%. For the second case of Yeh (1995), our

372 model improves the objective value from −133 to −143. In the case of Li and Love (1998),

373 the genetic algorithm obtains an objective of 15,090, while our model improves it to 12,284,

374 a reduction of 19%. Similarly, the objective value for the case of Li and Love (2000) is also

375 improved by 19%. These comparisons clearly show the superiority of our proposed

376 mathematical programming models over existing heuristics in terms of solution optimality.

16

Moreover, the last column of Table 2 demonstrates that the optimal solutions are obtained within reasonable time, as the TF layout problem is a tactical-level decision problem and does not require real-time calculation.

**Table 2** Comparisons with the literature

Literature		Our models		
Cases	Objective value obtained	Optimal objective value	Optimal solution	CPU time (s)
Case 1 of Yeh (1995)	93	90	[9,12,6,10,8,11,7,4,5,2,1,3]^	0.04
Case 2 of Yeh (1995)	−133*	−143*	[10,12,8,2,7,1,9,11,5,4,3,6]	8.38
Li and Love (1998)	15,090	12,284	[11,10,6,3,2,4,7,8,5,1,9]	61.06
Li and Love (2000)	15,160	12,320	[11,2,5,8,9,6,4,3,7,10,1]	626.52

*: The objective values are negative because the case allows the "interactive costs" between TFs to be negative.

^: This solution means that TF 1 is set up at location 9, TF 2 is set up at location 12, etc.

4.2 Comprehensive case study

We then report the result of a case study to show the wide applicability of the proposed models. We consider a site shown in Figure 1(a). It has 11 candidate locations: A, B, C, D, E, F, G, H, I, J, and K; the distances (D_{ij}) between the locations are shown in Table 3, which are symmetric, meaning that $D_{ij} = D_{ji}$. Eight TFs need to be set up at these locations; the amount of flow (q_{kl}) between the TFs is shown in Table 4. In practice, the amount of flow between the TFs can be estimated in advance based on material procurement plans, construction schedules, equipment/tools operation protocols, resource allocation dependencies (Ma et al., 2005), or even historical records of similar projects judged by experienced field experts, depending on different functionalities the TFs served as. Though possible objective logistical effort and cost could be inferred given different resources features (human, equipment, and material) and environmental effects (e.g. weather), this can be served as one of the limitation and is necessary to be investigated in the future.
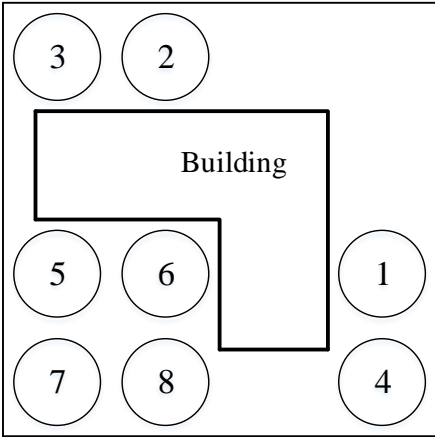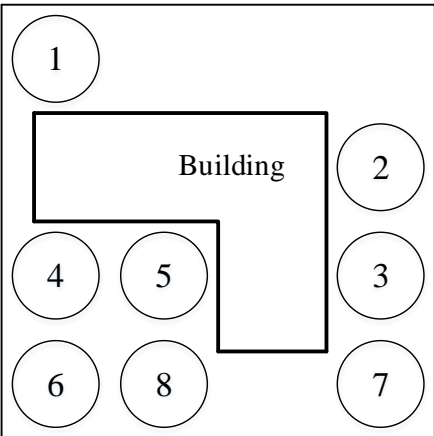
17
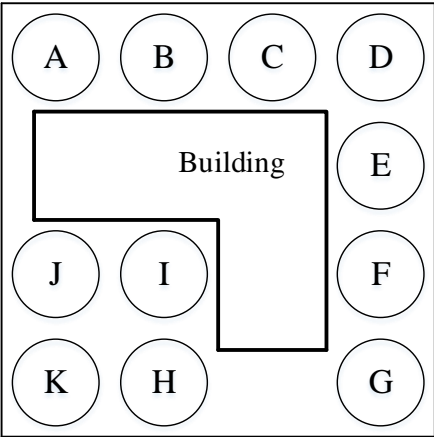
(a): Layout of the candidate locations
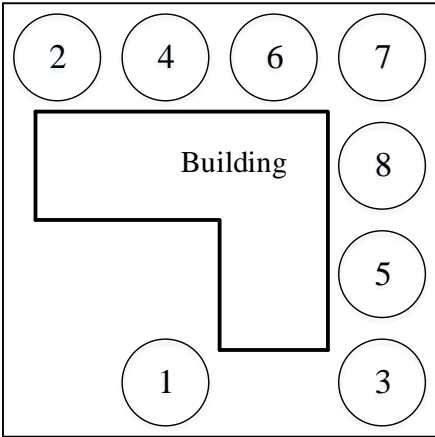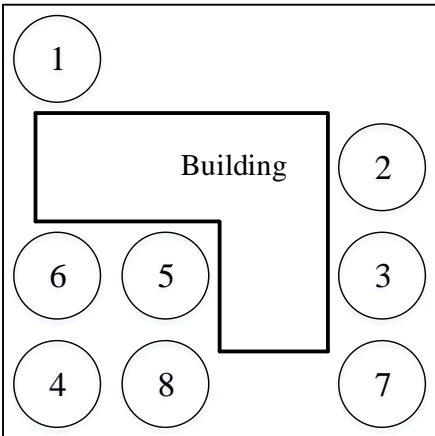
(b): Optimal solution for [P2]
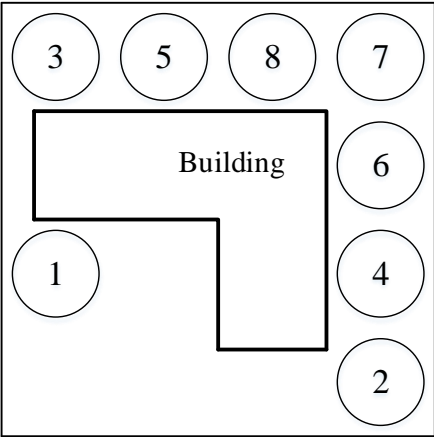
(c): Optimal solution for [P3]

(d): Optimal solution with safety considerations

(e): Optimal solution considering health/
environment impact on the neighours

(f): Optimal solution considering health/environment
impact caused by interaction between two TFs

Fig. 1 Site layout in the case study

403 **Table 3** Distances between the candidate locations

	A	B	C	D	E	F	G	H	I	J	K
A	0	2	4	6	8	10	12	8	6	4	6
B	2	0	2	4	6	8	10	10	8	6	8
C	4	2	0	2	4	6	8	12	10	8	10
D	6	4	2	0	2	4	6	10	12	10	12
E	8	6	4	2	0	2	4	8	10	12	10
F	10	8	6	4	2	0	2	6	8	10	8
G	12	10	8	6	4	2	0	4	6	8	6
H	8	10	12	10	8	6	4	0	2	4	2
I	6	8	10	12	10	8	6	2	0	2	4
J	4	6	8	10	12	10	8	4	2	0	2
K	6	8	10	12	10	8	6	2	4	2	0

404

405 **Table 4** Amount of flow between the temporary facilities

	1	2	3	4	5	6	7	8
1	0	3	4	5	6	7	8	9
2	3	0	5	6	7	8	9	10
3	4	5	0	7	8	9	10	11
4	5	6	7	0	9	10	11	12
5	6	7	8	9	0	11	12	13
6	7	8	9	10	11	0	13	14
7	8	9	10	11	12	13	0	15
8	9	10	11	12	13	14	15	0

406

407   We first solve model [P2], which addresses the basic construction site layout problem.

408 The optimal objective value (transportation cost) is 2,784, and the optimal layout is shown in

409 Figure 1(b). It can be seen that TFs 6, 7, and 8 are close to each other because the amount of

410 flow between them is very large. It can also be found that TF 1 is far away from the other TFs

411 because only 7 candidate locations (A to G) are close and TF 1 is excluded from these

412 locations because its amount of flow to other TFs is the smallest.

413   Suppose that location H is too small to accommodate TF 1 and location E is too small for

414 TF 8, i.e., $\delta_{H1} = \delta_{E8} = 0$. We then solve model [P3] and the minimum transportation cost is

415 still 2,784. The optimal layout is shown in Figure 1(c). We find that because the 11 candidate

416 locations are symmetric, the layout in Figure 1(c) is actually the "mirror" of the layout in

417 Figure 1(b), and hence their transportation costs are identical.

418   Suppose further that because of safety considerations, TF 8 cannot be assigned to any of

419   the locations A, B, C, D, E, F, or G, i.e., $\delta'_{A8} = \delta'_{B8} = ... = \delta'_{G8} = 0$. The minimum

420   transportation cost is then increased to 2,856. The optimal layout is shown in Figure 1(d). TF

421   8 has to be set up at a location among H, I, J, and K. Because there is large amount of flow

422   between TF 8 and TFs 5, 6, 7, these three TFs are all assigned to the locations near TF 8. The

423   other four TFs are then assigned to locations near TFs 5, 6, 7, and 8.

424   Suppose further that because of health/environment impact on the neighbors, TF 7

425   cannot be assigned to any of the locations H, I, J, or K, i.e., $\delta''_{H7} = \delta''_{I7} = \delta''_{J7} = \delta''_{K7} = 0$. The

426   minimum transportation cost is then increased to 2,904. The optimal layout is shown in

427   Figure 1(e). Now TF 7 and TF 8 cannot be close to each other. As a result, TFs 4, 5, and 6 are

428   assigned to be close to TF 8 because there is large amount of flow between TF 8 and these

429   three TFs.

430   Suppose further that TF 6 and TF 8 must be set up at locations whose distance is more

431   than 2 due to health/environmental considerations. This implies that $\delta''_{ij,6,8} = 0$ if $D_{ij} = 2$. The

432   minimum transportation cost is increased to 2,920. The optimal layout is shown in Figure 1(f).

433   We find that the layout in Figure 1(f) is very similar to that in Figure 1(e) with a small

434   adjustment of the distance between TF 6 and TF 8.

435   Next, we examine the application of the bi-objective optimization model [P5]. Suppose

436   that $c_{ik} = 0$ and suppose that c_{ijkl} is as follows:

437
$$c_{ijkl} = \begin{cases} kl & \text{if } D_{ij} = 2 \\ 0, & \text{if } D_{ij} > 2 \end{cases} \tag{23}$$

438   That is, c_{ijkl} is not 0 if $D_{ij} = 2$. For example, $c_{KH,4,8} = 4 \times 8 = 32$. All of the constraints in the

439   above are still used. Applying the proposed algorithm, we computed the Pareto frontier,

440   which is plotted in Figure 2. As expected, the minimum total transportation cost is 2,920; at

441   the same time, the total safety, health and environmental damage is 150. If we just minimize

442   the total safety, health and environmental damage, then it will be 22; however, the

443   transportation cost will be increased to 3,504. It should be noted that because of the discrete

444   nature of the problem, the Pareto Frontier curve is not convex. In sum, the Pareto frontier in

445   Figure 2 can be used to assist construction site manager to design a layout plan that balances

446 the tradeoff between the transportation cost and the total safety, health and environmental

447 damage.

448
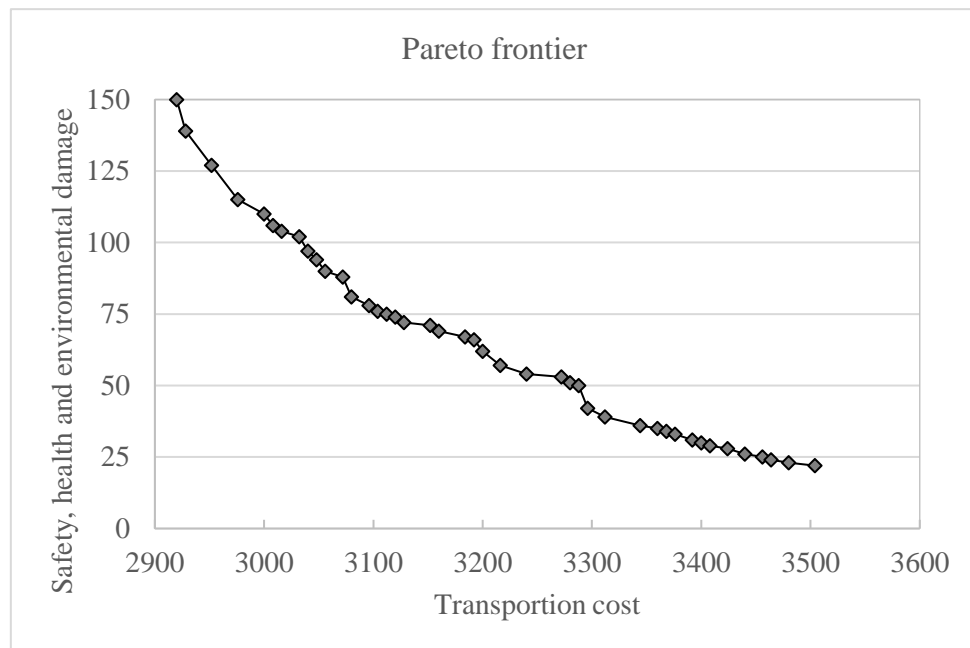
Fig. 2 Pareto frontier of the case study

449

450

451

4.3 Discussion of the research limitations

453 According to the results obtained, a superiority on site layout optimality has been

454 identified by using the proposed mathematical programming model and its optimization

455 processes. It tackled the concerns of mathematical programming which is used to be

456 considered unrealistic in site layout planning. It further suggested layout improvement in

457 using the proposed approach is possible given the case study shows the layout problems can

458 be solved within reasonable time.

459 While the proposed mathematical programming model considering extensive practical

460 factors in solving site layout planning problems, the estimation of safety, health and

461 environmental concerns require objective and predetermined information of the site in order

462 to perform an accurate planning. Related estimation techniques, such as unbiased judgements,

463   historical data in similar projects, and detailed site information, are necessary and they are

464   seriously influence the performance of the optimization in practice.

465   The proposed model targets on solving static and discrete site layout planning problem

466   which can be less practical if the predetermined locations for TPs are not applicable.

467   However, a possible site layout planning procedure is proposed accordingly given the

468   efficient performance the proposed approach can achieve. A preliminary identification for

469   TPs' locations can always be decided judged by experienced site managers. Through the

470   optimization process using our approach, a preliminary result can be offered in advance.

471   Then there are rapid but gradual adjustments, by updating field information to amend the

472   numbers, locations and sizes of potential TFs if needed, for the existing site layout. Moreover,

473   site managers may have several layout plans in mind and are unsure of which plan to use. The

474   proposed model can then consider all locations in the plans as candidate locations and obtain

475   a globally optimal layout based on these candidate plans.

476

477

478   **5   Conclusions**

479   This paper has examined the construction site layout problem that determines the

480   locations of temporary facilities. A mathematical model was proposed, which can be solved

481   by state-of-the-art solvers to optimality. This contrasts to the heuristic methods in the

482   literature, which may obtain good but not necessarily optimal solutions. We further

483   demonstrated how mathematical programming can be applied to handle a large variety of

484   practical considerations. Specifically, a number of safety, health and environmental concerns,

485   can be incorporated in the proposed mathematical programming models, either as hard

486   constraints or in a bi-objective formulation. Numerical experiments show that the optimal

487   solutions obtained by the proposed mathematical programming models improve over the ones

488   by existing heuristics for as large as 19%. A case study demonstrates the wide applicability of

489   the proposed models in terms of handling practical considerations. Further efforts will be

490   directed at using mathematical programming to address dynamic construction site layout

491   problems.

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