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# Constant-ductility-based energy factor demands of oscillators with modified Clough hysteretic model

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***Abstract:*** This study focuses on the energy factor demand of oscillators with the modified Clough hysteretic model subjected to pulse-like near-fault ground motions. First, the definition of the energy factor demand of modified-Clough oscillators was clarified. Then, oscillators covering a reasonable range of nonlinear parameters and dynamic properties were analysed with a constant-ductility-based method, and a ground motion database characterising pulse-like near-fault ground earthquake motions was used as seismic excitations. The influence of the hysteretic quantities on the energy factor demand of the modified-Clough oscillators was discussed based on the statistical results. Comparison between the energy factor demand from the analysis database and the counterparts computed based on the widely used Newmark-Hall model was made. Recognising the limitations of the current design model, an alternative design equation for estimating the mean energy factor demand of oscillators with the modified Clough hysteretic model was proposed using nonlinear regression analyses. The improved accuracy of the developed model was demonstrated. The research findings of this work provide a basis for energy-balance-based seismic design of structures whose hysteretic force-displacement response can be idealised with the modified-Clough model.

***Keywords:*** energy factor, modified Clough model, near-fault ground motion, seismic design

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## 1. Introduction

In performance-based seismic engineering, appropriate determination of inelastic seismic demands of structures subjected to expected earthquake events has been reckoned as an essential task. In particular, examinations of the strength demand indices [1-6], e.g. the strength reduction factor, of various systems have been conducted in recent decades to offer a basis for the force-capacity-based seismic design. Recognising the significance of inelastic deformation capacity of a system, research communities were involved in studies of various deformation demand indices [7-9], and developed the corresponding displacement-based earthquake design methods. Recently, it was found that the inelastic seismic demand can also be reasonably quantified using the energy-balance concept, and an energy factor [10, 11] deduced from a modified Housner equation [12] is able to concurrently prescribe the strength and the deformation demand of a system responding into the inelastic stage under earthquake attacks. In particular, to extend the Housner principle to short-period and long-period systems, Lee and Goel [10, 11] derived the energy factor defined by the covered area of the skeleton force-displacement pushover curve of an inelastic system to that of an elastic system with the identical elastic dynamic properties (i.e. period, mass and damping) using the single-degree-of-freedom (SDOF) system following the elasto-perfectly-plastic (EPP) hysteretic law. Later, the inelastic design spectra developed by Newmark and Hall [13] quantifying the relationship between the strength demand (i.e. strength reduction factor) and deformation demand (i.e. ductility factor) of an EPP system was substituted to the energy factor, and the seismic energy factor was utilised in seismic design of ductile structures. In particular, a direct-design method based on the energy factor of inelastic SDOF systems and a preselected yielding mechanism, i.e. the performance-based plastic design (PBPD) method [14], was developed. It is worth noting that the PBPD method can produce structures with a more controllable manner when experiencing severe ground motions, and the design approach retains practical attractiveness since the energy-factor-based governing equation can be further reduced to a force-equilibrium equation, which can be readily used by practitioners. The potential of the energy factor is verified by successful applications in seismic design of ductile systems such as moment resisting frame systems [15], steel plate shear wall systems [16], steel frames equipped with buckling restrained braces [17] and special truss moment resisting frames [18, 19]. In parallel with the works of using the SDOF analogy for representation of low-to-medium rise structures, Jiang et al. [20] proposed an energy-factor-based multi-mode pushover analysis method based on the notion of modal SDOF system, and the potential of extending the energy factor in multi-mode-sensitive structures was demonstrated. More recently, Ke et al. [21-23] explored the energy factor of oscillators with the bilinear model with significant post-yielding stiffness ratio and the counterparts following the trilinear kinematic hysteretic law. Research findings from these works showed that the cascading effect between the ground motion characteristics and the hysteretic law could appreciably influence the energy factor demand.

The primary objective of the study was to provide a comprehensive understanding of the energy factor demand of oscillators with the modified Clough hysteretic model [24, 25]. Using a constant-ductility-based method [21-23, 26], the energy factor demand spectra of SDOF systems of the modified Clough hysteretic model subjected to an ensemble of near-fault ground motions were developed, and a practical range of hysteretic parameters was covered by more than 210 thousand inelastic spectral analyses. The applicability of the current design equations of the energy factor deduced from EPP systems for quantifying the energy factor demand of oscillators with the modified Clough hysteretic law was visited. Lastly, an alternative design model based on nonlinear regression analysis was developed using the available data, and the accuracy of the design model for quantifying the energy factor demand of the modified-Clough oscillators subjected to near-fault pulse-like ground motions was justified.

## 2. Energy factor demand of oscillators with the modified Clough hysteretic model

The nonlinear force versus displacement feature of a modified-Clough SDOF system accompanied with the definition of the energy factor is schematically given in Fig. 1. In particular, the energy factor is the ratio of the area covered by the skeleton pushover response and the horizontal coordinate axis till the expected inelastic deformation (i.e. $S_{ine}$ in Fig. 1) to that of the corresponding elastic system (i.e. $S_e$ in Fig. 1) under a seismic event. Note that the hysteretic model is a modification of the original Clough model [24, 25]. It is worth noting that the model applies to systems showing stiffness degradation subjected hysteretic loading conditions, and can reasonably describe the cyclic behaviour of concrete structures governed by flexural action. The energy factor of a modified-Clough oscillator is given by

$$\gamma_{MC} = 2\varpi\mu_s - \varpi + \alpha\varpi(\mu_s - 1)^2 \tag{1}$$

$$\varpi(T;\mu_s;\alpha;\xi) = (\frac{F_y}{F_e})^2 \tag{2}$$

where $T$ = structural period; $\mu_s$ = $\delta/\delta_y$ defined as ductility factor (where $\delta$ and $\delta_y$ are the maximum displacement and the yield displacement of the oscillator subjected to an earthquake ground motion, respectively); $\alpha$ = post-yielding stiffness ratio; $F_y$ = yield force; $F_e$ = maximum force of the corresponding elastic oscillator with the identical elastic characteristics as the modified-Clough oscillator; $\xi$ = damping ratio. The computation of the energy factor demand of a modified-Clough oscillator requires a constant-ductility-based method [21-23, 26]. Note that in the modified-Clough model, the initial slope of the unloading branch (e.g. path ⑦-⑧ in Fig. 1) was set equal to the initial elastic stiffness, and cyclic stiffness degradation was reflected by further compromising the stiffness in the later stage. The rationale of the modified-Clough model and detailed hysteretic law of the loading-unloading-reloading path is documented in [24, 25].

## 3. Energy factor spectra of modified-Clough oscillators subjected to pulse-like near-fault ground motions

### *3.1. Seismic input and parameter matrix*

In this study, an earthquake ground motion database including one hundred near-fault ground motion records, which was developed by Hatzigeorgiou [9], was used as seismic excitations. It is worth mentioning that the earthquake ensemble can reasonably characterise pulse-like near-fault ground motions, and detailed information about the rationale of the earthquake samples and the corresponding accelerograms is documented in [9].

The main parameters in the numerical investigation include structural period ($T$), ductility factor ($\mu_s$), and post-yielding stiffness ratio ($\alpha$). In particular, $T$ was varied from 0.1 s to 5 s to capture a spectrum of modified-Clough oscillators with various dynamic properties. To offer an in-depth understanding of the influence of hysteretic parameters, $\mu_s$ ranging from 2 to 8 and six levels of $\alpha$ varying from 0 to 0.05 were included in the parameter matrix. The damping ratio of 5% was used in all the analyses. In the constant-ductility-based procedure, iteration started by giving an initial value of $F_y$ (approaching $F_e$) to the modified-Clough oscillator and performing a nonlinear dynamic analysis of the system with an earthquake excitation. In each analysis, the actual ductility of the oscillator was compared with the specified target ductility factor, and repetition was initiated with decreasing $F_y$. To ensure the accuracy of the results, the convergence criterion in the iterations was given by

$$\left|\frac{\mu-\mu_s}{\mu_s}\right| \leq \frac{1}{1000} \tag{3}$$

where $\mu$ = actual ductility computed in the iterations during a constant-ductility-based method.

*3.2. Energy factor demand spectra of modified-Clough oscillators*

Representative energy factor demand spectra of the modified-Clough SDOF systems are given in Fig. 2. For clarity, the energy factor spectra under an individual ground motion and the mean spectra of one hundred ground motions are shown in the figure. As a general remark, the mean energy factor demand evidently increases with increasing ductility factor for short-period systems (i.e. $T \leq 0.3$ s), but the opposite trend was observed for the modified-Clough oscillators with a longer period. As for the effect of the post-yielding stiffness ratio, it was seen that this parameter only has a slight effect on the mean energy factor demand for very short period systems (i.e. $T = 0.1$ s), and increasing post-yielding stiffness ratio results in a reduction of the mean energy factor demand. To clarify the effect of the ductility factor, the energy factors of oscillators with varied $\mu_s$ were compared, and the mean energy factor of short-period systems (i.e. $T \leq 0.3$ s) and the counterparts with longer period with $\mu_s = 2$ were plotted against the counterparts with larger ductility factors, as shown in Fig. 3a and Fig. 3b, respectively, and the aforementioned observation can be demonstrated. To offer a direct comprehension of the influence of the post-yielding stiffness ratio, the mean energy factors with varied $\alpha$ were organised in Fig. 3c. It can be seen that the variation of the post-yielding stiffness only results in slight fluctuation of the energy factor of systems with $T$ = 0.1s. Comparing the energy factor spectra with varied ductility levels, it was also seen that increasing the ductility factor correspondingly increases the data scatteration considering all ground motion samples.

The energy factor spectra determined from the classical Newmark and Hall design spectra are also indicated in Fig. 2 for comparison. In general, the Newmark and Hall spectra produced reasonable estimates of the mean energy factor demand on the conservative side when the ductility factor was not significant, i.e. $\mu_s$ = 2. With the ductility factor increasing, the direct application of the Newmark and Hall spectra may underestimate the mean energy factor demand of the modified-Clough oscillators from the short-to-moderate period region.

*3.3. An alternative design model for estimating the energy factor demand of modified-Clough systems*

Recognising the potential limitation of the current equations mentioned in the previous section, an alternative design model for quantifying the mean energy factor demand of modified-Clough oscillators was proposed in this study based on data regression. To search for an appropriate empirical expression for quantifying the energy factor demand of the modified-Clough oscillator, the commercial software Tablecurve 3D [27] was utilised during the data regression procedure, and more than 25000 built-in equations based on linear and nonlinear models (e.g. linear equations, polynomial functions, and logarithmic functions) were fitted. The following design equation with practical simplicity and a high determination factor, i.e. $R^2$ = 0.98, was finalised.

$$\gamma_{\mathrm{MC}} = \frac{a + b\ln T + c(\ln T)^2 + d\ln(\mu_s - 1)}{1 + f\ln T + g\ln(\mu_s - 1) + h[\ln(\mu_s - 1)]^2} + 1 \qquad (4)$$

where $a$ =-0.2928, $b$ =-0.1419, $c$=0.0334, $d$ =0.0535, $f$ = 0.0919 and $g$ =-0.6698 and $h$ = 0.1611. Note that $a$, $b$, $c$, $d$, $f$, $g$ and $h$ are regressed coefficients and they were finalised based on 210 thousand analysis results of the energy factor using the least-square principle. It is worth mentioning that although the energy factor can be reduced to a function of the ductility factor and the strength reduction factor [10, 11], the objective of the regression analysis was to develop a preliminarily viable model for direct prediction of the energy factor, and hence only the following boundary condition was considered.

$$\gamma_{\mathrm{MC}} = 1 \quad (\mu_s \to 1) \qquad (5)$$

Note that the boundary condition is the classical Housner principle [12]. Being aware that the post-yielding stiffness ratio only slightly influences the mean energy factor demand, the effect of the parameter was not included in the model. To examine the effectiveness of the proposed design model for prescribing the seismic energy factor demand of the modified-Clough oscillators subjected to near-fault ground motions with velocity and displacement pulses, the mean energy factor demand from the analysis database was compared with the predictions by the proposed design equation, i.e. Eq. (4), and the predicted mean energy factor demand was plotted against the 'exact mean energy factor', as shown in Fig. 4a. In general, the proposed design equation quantifies the mean energy factor demand of the modified-Clough oscillators under the near-fault ground motion database with good accuracy, and the error is generally within 10%. The applicability of the Newmark and Hall design spectra for evaluating the energy factor demand of the modified-Clough SDOF systems was also revisited, and the predictions were compared with the exact values as shown in Fig. 4b. Thus, the improved accuracy of the design equation developed in this study is demonstrated.

## 4. Conclusions

In this study, the energy factor demand of oscillators with the modified Clough hysteretic model was explored using a constant-ductility-based procedure. The statistical investigation covering a practical range of hysteretic parameters and dynamic characteristics was conducted. In general, it was observed that the mean energy factor demand of the modified-Clough SDOF systems increases drastically for an oscillator in the short-period range (i.e. $T \leq 0.3$ s) with increasing ductility factors, while the reversed trend is evident when the oscillator falls into the longer period region (i.e. $T > 0.3$ s). Concurrently, the influence of the post-yielding stiffness ratio on the mean energy factor demand of the modified-Clough oscillators is only pronounced for oscillators within the very short-period region (i.e. $T < 0.2$ s), and the energy factor decreases slightly with an increasing post-yielding stiffness ratio.

The applicability of the widely used energy factor demand spectra based on the Newmark and Hall inelastic spectra for quantifying the energy factor demand of the modified-Clough oscillators subjected to pulse-like near-fault ground motions was evaluated, and it was found that the current design equations may produce unsafe estimates of the energy factor demand at significant ductility levels. To overcome the limitation, an alternative design equation motivated by nonlinear data regression was developed, and better agreement between the exact mean energy factor demands by inelastic spectral analyses and the predictions by the proposed equation was observed. The design model is believed to be helpful for developing full-fledged energy-factor-based seismic design procedures of structures, e.g. concrete systems governed by flexural behaviour, whose hysteretic performance could be reasonably idealised by the modified-Clough model. Nonetheless, further explorations are required to develop a direct design procedure relating the energy factor to structural parameters (e.g. member size and strength), which is currently being conducted by the authors. In addition, the research findings emphasise the influence of hysteretic parameters on the energy factor demand, and further works on the energy factor demand of SDOF systems governed by various hysteretic laws are also underway.

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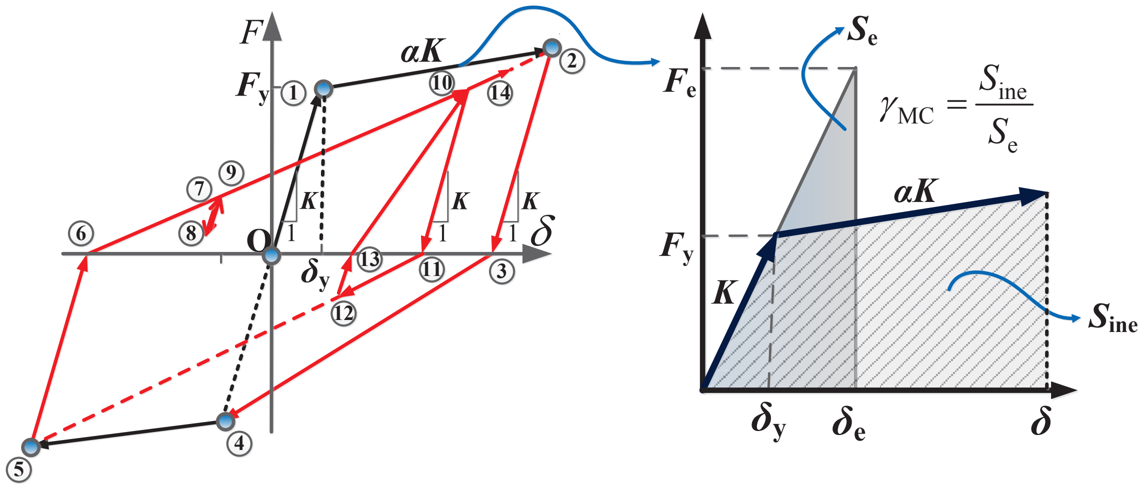



Fig. 1 Hysteretic behaviour and seismic energy balance of a modified-Clough oscillator [24, 25].

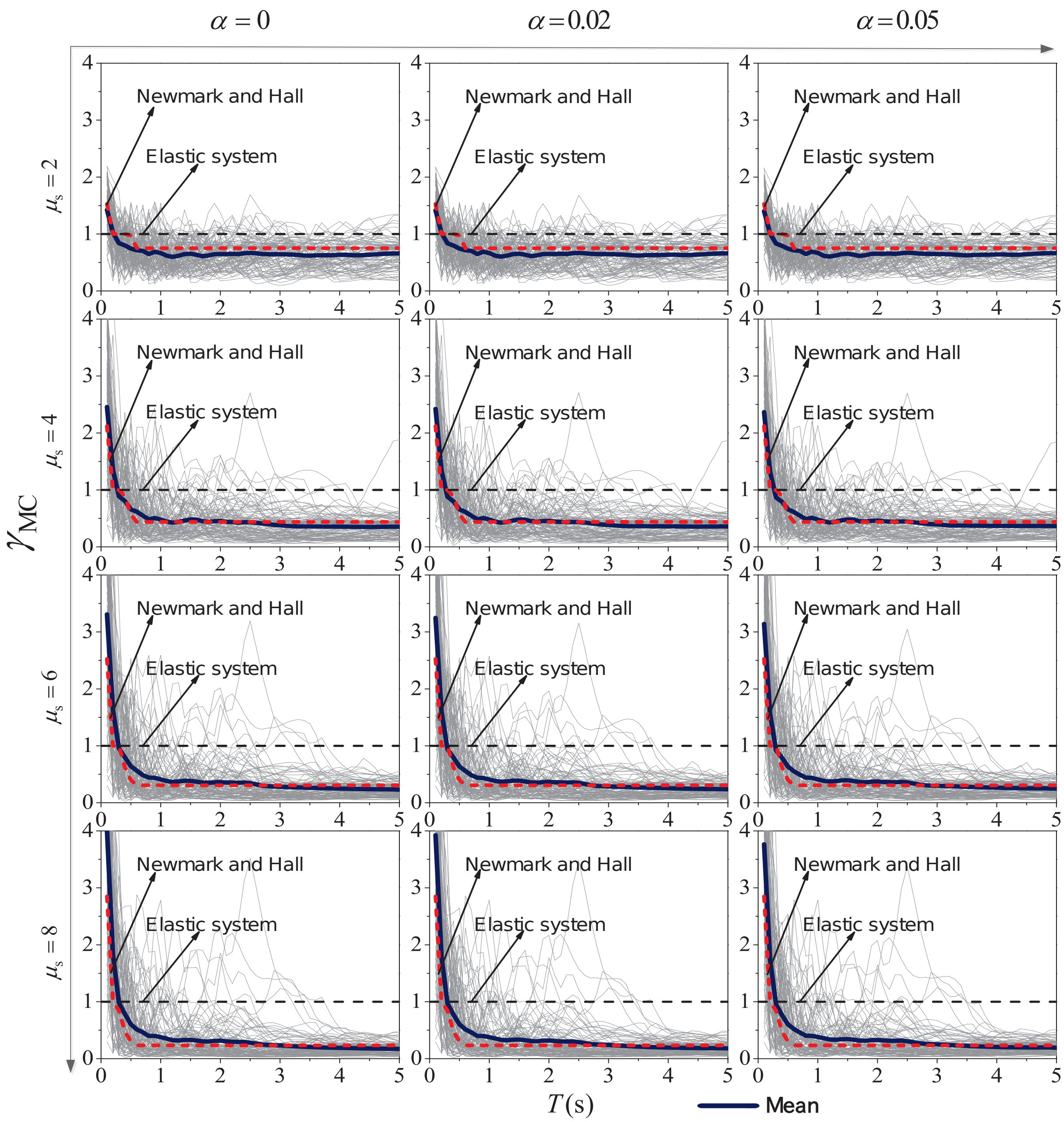



Fig. 2 Energy factor demand spectra of modified-Clough oscillators.

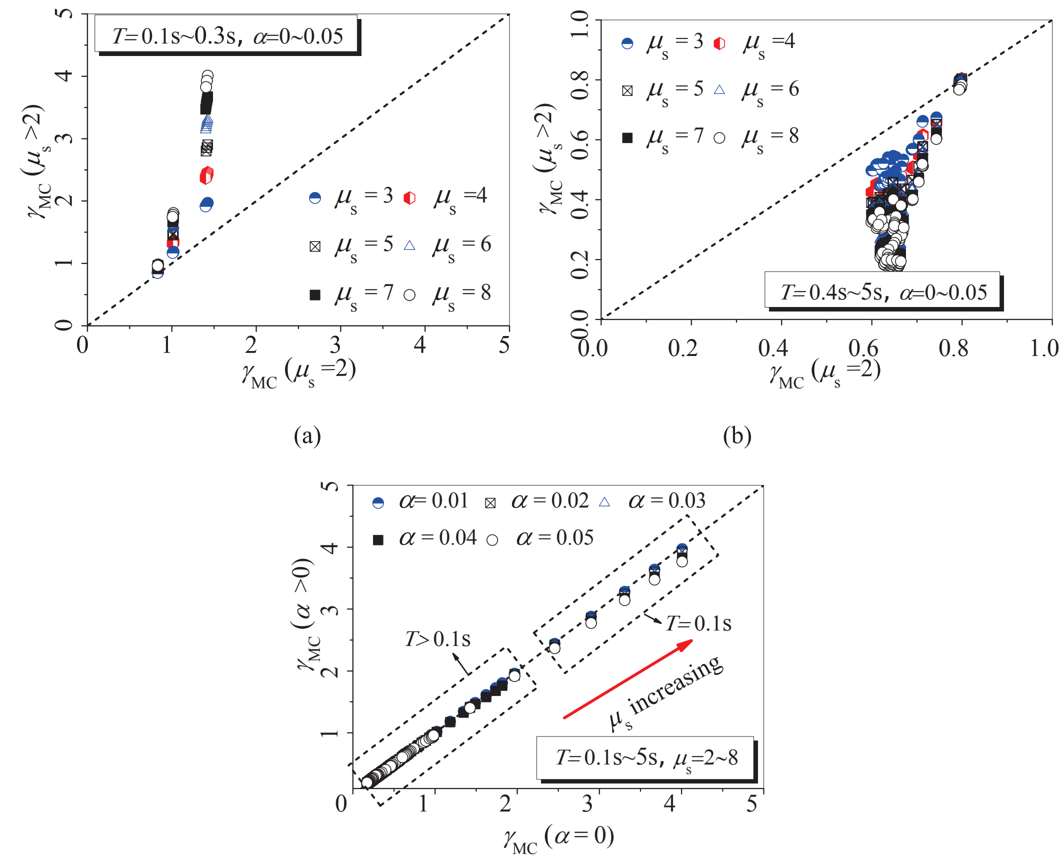
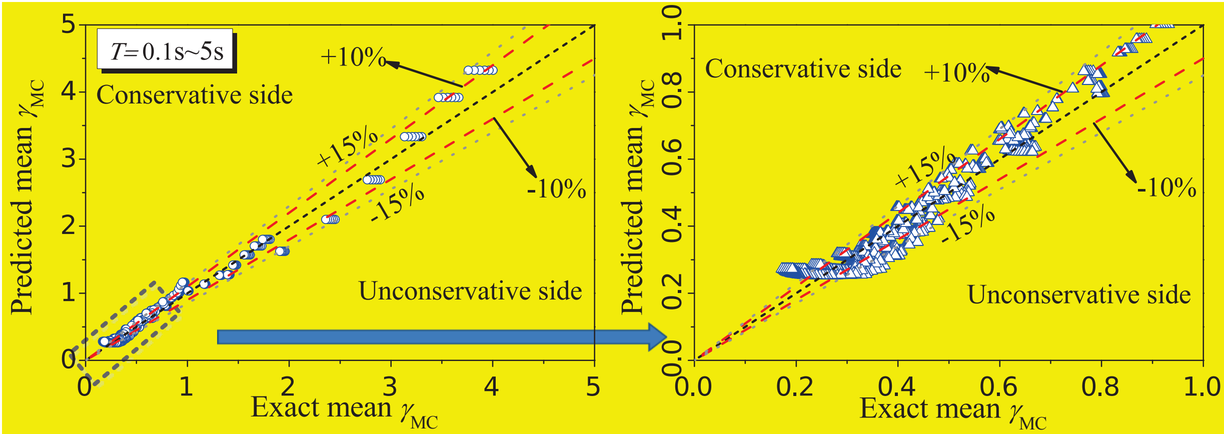
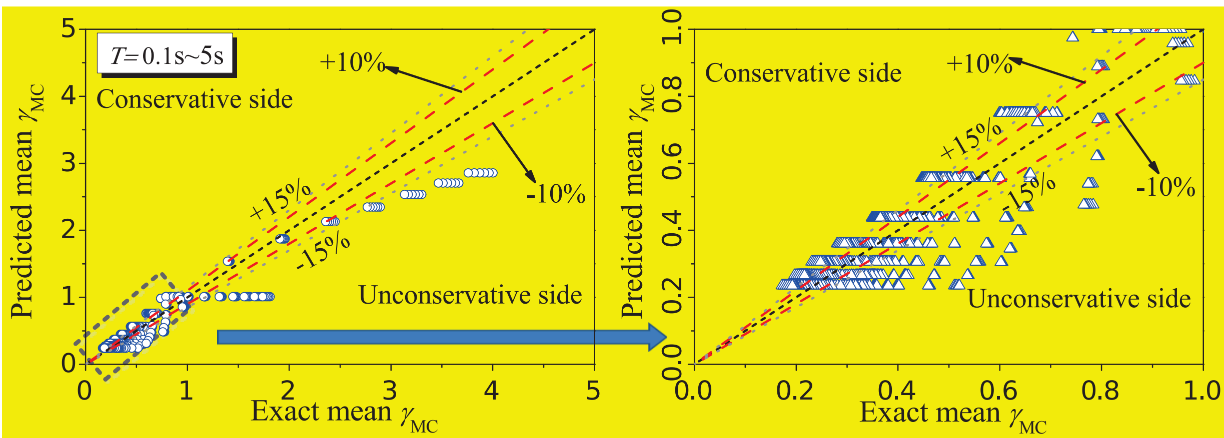



Fig. 3 Effect of hysteretic parameters: (a) effect of $\mu_s$ ($T \leq 0.3$ s), (b) effect of $\mu_s$ ($T > 0.3$ s) and (c) effect of $\alpha$.




(a)




(b)

Fig. 4 Comparison of different design equations: (a) proposed design equation and (b) design model from the Newmark and Hall spectra [13].