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Experiment and constitutive modeling on cyclic plasticity behavior of LYP100 under large strain range

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ABSTRACT: Low yield point (LYP) steel is a promising material for energy dissipation device resisting seismic actions, and is expected to experience large strain when subjected to strong earthquakes. In order to offer a better understanding of the cyclic behavior of LYP steel under large strain range, cyclic tests of eight (8) coupons made of LYP100 under the strain amplitude ranging from -10% to +12% are performed. Evident work-hardening, early re-yielding, and strain range dependence are characterized in the cyclic loading tests. To illustrate these characteristics, the peak stress of every cyclic loop and the elastic region of the unloading process are examined. Based on the analysis results of the peak stress and the elastic region, a modified Yoshida-Uemori model is proposed to quantify the cyclic behavior of LYP100, and the corresponding numerical algorithm is developed. In addition, a practical method based on the derivative-free optimization theory is proposed to calibrate the material parameters of the novel model. The proposed constitutive model shows a satisfactory accuracy for describing the cyclic behavior of LYP100 under large strain range.

Keywords: cyclic plasticity, constitutive model, LYP100, strain range dependence

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1. Introduction

1.1 Background

Owing to the noteworthy features including ultra-low yield stress, remarkable energy dissipation capacity and excellent ductility, low yield point (LYP) steel is a promising material for producing various energy dissipation devices (e.g. shear damper, buckling restrained brace) in seismic engineering [1–3]. In the recent decades, the research community has initiated extensive explorations on the cyclic plasticity behavior of LYP steel. For instance, Saeki et al. [4] conducted a series of cyclic tests on two LYP steels with the strain amplitude between -3% and +3%. In their study, the focus was given to the low-cycle fatigue performance of LYP steels to validate their potential in damping devices. Dusicka et al. [5] examined the cyclic response of two LYP steels under large inelastic strain with the strain amplitude ranging from -7% to +7. The test results showed that with increasing strain amplitude the cyclic stress increased up to 4.8 times of the initial yield strength, indicating that the work-hardening of LYP steel was evident. Wang et al. [6] studied the cyclic behavior of LYP100 and LYP160 under various types of loading protocols with the strain amplitude ranging from -3.5% to +3.5%. It was also observed that the LYP steel exhibited appreciable work-hardening behavior, and the cyclic response of the LYP steel was sensitive to the loading history. The test results also demonstrated that the work hardening characteristics of LYP steels were influenced by both plastic strain and strain amplitude. Later, Xu et al. [7] investigated the cyclic behavior of a LYP steel (i.e. steel grade BLY160) with the strain amplitude ranging from -3% to +3%. Notwithstanding the excellent ductile manner and the encouraging energy dissipation capacity of the material, the cyclic softening behavior of BLY160 was also observed.

In parallel with test explorations on the cyclic response of LYP steels, a number of constitutive models have been proposed to quantify the cyclic characteristics of LYP steels. Most of the constitutive models were based on the classical metal plasticity theory [8–10] and may be classified in a unified framework, where Eq. (1) shows the one-dimensional form of the framework.

$$f = \left|\sigma - \alpha\right| - R \leq 0 \tag{1}$$

Eq. (1) governs the yield surface for LYP steels, where σ is the stress, α is the backstress that represents the kinematic part, and R represents the isotropic part quantifying half of the elastic region of the unloading process.

In general, the evident work-hardening, strain range dependence, and cyclic softening are three major features of the cyclic behavior of LYP steels based on the previous test findings. The evident work-hardening effect can be quantified by modifying the mathematical formula of α and R. In particular, the modification of R was widely used in the literature when developing constitutive models of LYP steels. For example, Wang et al. [3] adopted a cyclic plasticity constitutive model, which was integrated into ABAQUS based on the model proposed by Chaboche [11], to characterize the cyclic behavior of LYP100. In the Chaboche model, it was assumed that the work-hardening is induced by the increase of the equivalent plastic strain, as shown in Eq. (2).

$$\sigma^{0} = \sigma\big|_{0} + Q_{\infty}\left(1 - \exp\left(-bp\right)\right) \tag{2}$$

Where σ^0 stands for the isotropic part, i.e. R in Eq. (1); $\sigma|_0$, Q_∞, and b are material parameters, and p represents the equivalent plastic strain. In the simulation results reported by Wang et al. [3], the Chaboche model produced a reasonable prediction of the cyclic response of the LYP100.

More recently, Wang et al. [6] adopted an alternative model, i.e. the Giuffré–Menegotto - Pinto model [12,13], to describe the cyclic behavior of LYP100 and LYP160. The ability of the model for predicting the test results was validated. Shi et al. [14] adopted a novel model proposed by Hu [15,16] to replicate the cyclic response of LYP steels including LYP100, LYP160, and LYP225. In Hu's model, a strain memory surface was adopted to reflect the strain range dependence effect of the LYP steels observed in the tests. Xu et al. [7] developed an innovative model for a LYP steel (i.e. BLY160) to describe the cyclic hardening and softening, and the strain range dependence of the material observed in the tests. A term Q_T was added to the kinematic part and the isotropic part to consider the cyclic softening in the case when the equivalent plastic stain was significant.

In conclusion, most of the constitutive models mentioned above are developed to describe the mechanical behavior of material obtained in the tests. The model should be adjusted to fit the mechanical characteristics, which is clear in the development of constitutive models of LYP100 as discussed above. At first, the excellent work-hardening of LYP100 was observed in the test, which could be described by Chaboche model [3,11]. With the accumulation of the test data, the strain range dependence is observed. Hence, a series of models have been developed to include this feature, such as Hu model [15,16] and Xu model [7]. In Hu model, a yield plateau is introduced, which is not necessary for LYP100, while in Xu model, a cyclic softening is included, which benefits much in predicting the low cyclic fatigue behavior. But the model should be adjusted if there are some new features observed in the test.

1.2 Motivation and objective

Energy dissipation devices are expected to experience large deformation under strong earthquakes, and hence it is essential to examine the hysteretic behavior of the material for the devices under large strain amplitude. As an example, **Fig. 1** shows the deformation pattern of a typical frame structure equipped with a shear damper. Under seismic events, the plastic deformation is expected to be concentrated in the shear damper, which leads to $\delta \approx \Delta$ (where δ is the shear deformation of the damper, and Δ is the story displacement of the frame structure, respectively). Assuming that the height of the shear damper (i.e. h) is 1/8~1/3 of the story height (i.e. H) as a rational design, the shear angle of the damper can reach 6%~16% when taking the story drift of 2% as the design threshold [17].

In order to have a better understanding of the material behavior under such a large shear deformation, the material tests under shear deformation should be conducted. However, the cyclic tests under shear deformation are hard to perform and the cyclic behavior under uniaxial loading could have a reasonable prediction of the damper's behavior under shear deformation [18]. Thus, it could be effective to obtain the cyclic behavior of LYP100 by performing the cyclic tests under uniaxial loading.

Eq. (3) shows the relationship between axial plastic strain, ε_p, and shear plastic strain, γ_p, derived from the presumption that the equivalent plastic strain of the axial deformation equals that of the shear deformation [19]. An approximate relation between the axial strain, ε, and the shear angle, γ, can be obtained from Eq. (3) when ignoring the minor elastic deformation, as shown in Eq. (4). Hence, the equivalent axial strain of the shear damper as shown in **Fig. 1** may reach 3.5%~9.2%.

$$\varepsilon_p = \frac{\gamma_p}{\sqrt{3}} \quad (3)$$

$$\varepsilon \approx \frac{\gamma}{\sqrt{3}} \tag{4}$$

Hence, to fully examine the hysteretic behavior of the LYP steels under cyclic loading, loading protocols covering a wider spectrum of strain amplitude more than ±10% may need to be included in test programs, and applicable constitutive models may also be required.

2. Experiment investigation

2.1 Test specimens and loading protocols

In the current work, a total of eight (8) hour-glass type specimens made of LYP100 with a nominal yield strength of 100 MPa are examined under cyclic loading. To eliminate early buckling, the specimens are carefully designed, and the rationale of the specimen configuration is supported by a recent work [20]. The configuration of the test specimens is provided in **Fig. 2**. For easy reference, the specimens are coded and listed in **Table 1**. The cyclic loading protocols include constant strain amplitude (CSA) loading protocols, which are used to examine the strain range dependence work-hardening and early re-yielding, and varying strain amplitude (VSA) loading protocols, which are used to verify the two features and to identify new features such as shrinkage of cyclic loops, as listed in **Table 1**. A strain control process at a strain rate of 0.002/s is included in the test program based on the previous study [5, 7]. **Fig.3**. shows the test setup of the cyclic tests.

Four specimens (i.e. specimen ZH-01~ZH-04) are examined under CSA loading protocols including various strain amplitudes ranging from ±2% to ±8%. The tests of the CSA loading protocols are terminated when the peak force of every loading cycle varies a little, which can be determined by Eq. (5).

$$\left|\frac{F_{n+1}-F_{n}}{F_{n}}\right| \leq 0.001 \qquad (5)$$

Where F_n and F_{n+1} represent the peak force of the n-th and the (n+1)-th loading cycle, respectively.

The other four specimens (i.e. specimen ZH-05~ZH-08) are examined under VSA loading protocols, where the number in the parentheses (i.e. 3) represents the number of the loading cycles. Specimens ZH-05 and ZH-06 are tested under increasing strain amplitude loading protocols. In particular, specimen ZH-07 is tested under a protocol with an ultra-large strain amplitude followed by an increasing strain amplitude sequence. Specimen ZH-08 is tested under a protocol with a decreasing strain amplitude sequence.

It should be underlined that only eight (8) loading protocols are adopted and only eight (8) specimens are designed, where the number of the specimens is less than that of other tests [6,7]. In order to verify the validity and rationality of the test, it is essential to clarify the principle for selection of the loading protocols. The principle adopted here is that loading protocols are designed to obtain the mechanical features. There are three major mechanical features of steels i.e. strain-range dependence work-hardening, early re-yielding and shrinkage of cyclic loops from the previous research [3-7,14-16,20], which could also be applicable to LYP100. The loading protocols adopted in the test are aimed to uncovering these features.

2.2 Cyclic behavior of specimens under CSA loading protocols and discussions

Cyclic responses of four specimens under the CSA loading protocols (i.e. specimen ZH-01~ZH-04) are shown in **Fig. 4**. The test results show that the work-hardening of LYP100 converges to a saturation stress for every CSA loading protocol, and the saturation stress increases with the strain amplitude increasing, which is often referred to the strain range dependence effect in previous works [7,21,22]. In particular, the saturation stress stagnates when the strain amplitude reaches ±6%, and the stagnation stress (i.e. f_s^t=363MPa for tension and f_s^c=367MPa for compression, as shown in **Fig. 4**) is almost 4.0 times of the initial yield stress (i.e. f_y^0=92MPa, as shown in **Fig. 4**). Thus, the remarkable work-hardening of the LYP100 is demonstrated.

In order to obtain a clear description of the effect of the strain range dependence, a detailed analysis of the test data based on the classical metal cyclic plasticity framework with the governing equation of Eq. (1) is performed. **Fig. 5** gives a global picture of the framework that defines the peak stress, the elastic region, and the backstress. The peak stress denoted by $\alpha_{max}+R$ is the maximum stress of every cycle. The elastic region defined by $2R$ is the magnitude of the linear elastic portion of the unloading process. The backstress represented by α is the center of the yield surface.

In the classical metal cyclic plasticity theory, the kinematic part and the isotropic part are often regarded as the function of the equivalent plastic strain [9,10,19]. In this context, the utilization of the equivalent plastic strain may shed lights on the mathematical structure of the evolution rule of the peak stress, the elastic region, and the backstress.

Fig. 6 shows the relationship between the peak stress and the equivalent plastic strain of four specimens under CSA loading protocols. It is evident that the peak stress increases with the equivalent plastic strain increasing under the same loading protocol, and it saturates at a certain level of the equivalent plastic strain. Moreover, the saturation values of the peak stress of specimens under different loading protocols increase with strain amplitude increasing and stagnate when the strain amplitude reaches ±6%. The saturation values of ZH-03 and ZH-04 are almost identical, which shows that there is an upper limit of work-hardening of LYP100 under cyclic loading.

Hence, the effects of strain range dependence of LYP100 can be concluded as follows:

i. The work-hardening of LYP100 steel is related to the loading strain amplitude and increases with strain amplitude increasing;

ii. The work-hardening of LYP100 steel converges to the saturation value with the increase of the equivalent plastic strain at a certain strain amplitude.

iii. The saturation value of the work-hardening of LYP100 steel reaches to an upper limit when the strain amplitude reaches a certain level;

Fig. 7 shows the relationship between the elastic region and the equivalent plastic strain of the four specimens under CSA loading protocols. It is clear that the elastic region increases with the equivalent plastic strain increasing under the same loading protocol, and it saturates at a certain level of the equivalent plastic strain. The difference of saturation values of the elastic region of LYP100 subjected to various CSA loading protocols is not evident. In particular, the saturation value of the elastic region of specimen ZH-01 varies due to the fluctuation of the test data of specimen ZH-01, which does not appear in the other loading protocols. Moreover, the saturation value of the elastic region is about 260 MPa, indicating an early re-yielding behavior of the material when the load is reversed. In addition, a close-up diagram of the elastic region of the first unloading process is given in **Fig. 7**, which demonstrates a rapid change of the elastic region of LYP100 when the equivalent plastic strain is not significant. Thus, it may be not appropriate to employ the equivalent plastic strain to reflect the evolution of the elastic region of LYP100. It should be noted that the equivalent plastic strain approximately equal to the strain amplitude to some extent as shown in the close-up diagram for there is no accumulation of the equivalent plastic strain until the loading is reversed. Moreover, the strain amplitude only evolves in the early stage of the loading process when subjected to CSA loading protocols, which shares the specify evolution feature of the elastic region. Thus, the strain amplitude may be more suitable to reflect the evolution of the elastic region.

In the analysis of the peak stress and the elastic region, the strain amplitude is used to illustrate the features observed in the tests. In the existing framework of the cyclic plasticity, a strain range memory surface given by Eq. (6) in one-dimensional form is introduced, and the strain range as an internal variable is widely used to reflect the influence of the strain amplitude:

$$g = \left|\varepsilon^{\mathrm{p}} - q\right| - r \le 0 \tag{6}$$

where ε^{p} is the plastic strain, q is the center of the strain range memory surface, and r represents the radius of the strain range memory surface.

Therefore, it is reasonable to quantify the elastic region (i.e. $2R$) as a function of the strain range from the previous analysis, which is shown in Eq. (7).

$$R = \phi(r) \tag{7}$$

The backstress α is another variable that needs to be determined in the classical framework of cyclic plasticity as shown in Eq. (1). The maximum backstress $\alpha_{\max}$ is selected to reflect the evolution of the backstress and can be obtained by Eq. (8). **Fig. 8** shows the relationship between the maximum backstress and the equivalent plastic strain of specimens under CSA loading protocols, which shows the same features as that of the peak stress that has been concluded above. Thus, the backstress α may be regarded as a function of the equivalent plastic strain and strain range as shown in Eq. (9).

$$\alpha_{max} = (\alpha_{max} + R) - R = \text{peak stress} - 1/2\,\text{elastic region} \tag{8}$$

$$\alpha = \varphi(r, p) \tag{9}$$

2.3 Cyclic behavior of specimens under VSA loading protocols and discussions

The results of two specimens subjected to different loading protocols of the increasing strain amplitude, i.e. specimen ZH-05 and ZH-06, are shown in **Fig. 9**(a) and **Fig. 9**(b), respectively. The original loading protocols were not completed since both coupons buckled after the strain amplitude reached ±6%. However, it can still be seen from **Fig. 9**(a) and **Fig. 9**(b) that the work-hardening of LYP100 is strain range dependent as the peak stress increases with increasing strain amplitude.

Fig. 10(a) shows the results of a specimen subjected to another VSA loading protocol, i.e. specimen ZH-07, with an ultra-large strain amplitude (i.e. +12%, -10%) followed by an increasing strain amplitude sequence. Hence, the first cyclic loop with the strain amplitude of ±2% of specimen ZH-07 may be influenced by the ultra-large strain amplitude, which can be regarded as the influence of the maximum strain history. **Fig. 10**(b) shows that the first loop with strain amplitude ±2% of specimen ZH-07 is larger than the stabilized loop of specimen ZH-01. It should be noted that the stabilized cyclic loop of specimen ZH-01 is due to the accumulation of the equivalent plastic strain. The difference between these two cyclic loops shows that the maximum strain history determines the upper limit of the work hardening, which is similar to the effect of the strain range mentioned above. Hence, the strain range is a promising candidate to be used to quantify the influence of the maximum strain history without introducing a new internal variable.

Fig. 11(a) shows the result of the loading protocol with a decreasing strain sequence i.e. ZH-08. The result indicates that the cyclic loops shrink with the decreasing strain amplitude. Translation of the cyclic loops under different strain amplitude is made and shown in **Fig. 11**(b). It is observed that the shape of these cyclic loops is similar. On the other hand, it should be noted that the shape of the cyclic loops can be determined by the backstress. Hence, an appropriate mathematical expression of the backstress can illustrate the feature of the shrinkage of the cyclic loops under the decreasing strain amplitude loading protocol.

3. Constitutive model

3.1 The guidelines of the constitutive model of LYP100

A careful analysis of the test results has been presented above and the features of the cyclic behavior of LYP100 have been discussed. The mathematical formulas of the backstress and the elastic region, as shown in Eq. (7) and Eq. (9), have been given in the previous analysis, which are the two guidelines of the constitutive model of LYP100. Moreover, the influence of the maximum strain amplitude as indicated by the specimen ZH-07 has been discussed and can be reflected by the strain range. In addition, the shrinkage of the cyclic loops as indicated by the specimen ZH-08 can be illustrated by an appropriate evolution rule of the backstress. Fortunately, the constitutive model proposed by Yoshida and Uemori [21], as shown in **Fig. 12**, can give a satisfactory reflection of these features. In his model, a term of the backstress is determined by Eq. (10), which is obtained from the theory of dislocation. However, the mathematical expression may be challenging to be implemented in the numerical algorithm. Therefore, an alternative formula shown in Eq. (11), which was proposed by Armstrong and Federick [23] and modified by Chaboche [11,22], is utilized.

$$\dot{\beta} = Ca\left(\dot{\varepsilon}^{\mathrm{p}} - \mathrm{sgn}(\beta)\sqrt{\frac{|\beta|}{a}}\left|\dot{\varepsilon}^{\mathrm{p}}\right| \right) \tag{10}$$

$$\beta = \sum_{i=1}^{n} \beta_i;\ \dot{\beta}_i = \frac{2}{3}C\dot{\varepsilon}_{\mathrm{p}} - \gamma\beta_i\dot{p} \tag{11}$$

Moreover, the elastic region was regarded as a constant in the constitutive model proposed by Yoshida and Uemori, which means that Y is a constant (i.e. $Y_1= Y_2=\cdots=Y_{\mathrm{n}}$) in the model. However, the test results in the current work indicate that the elastic region of LYP100 is a function of the strain range as shown in Eq. (7). Thus, it is essential to propose an applicable evolution rule of the elastic region of LYP100.

3.2 The basic framework of the constitutive model

The basic framework of the constitutive model is discussed in this section by following the guidelines summarized in section 3.1.

The total strain $\boldsymbol{\varepsilon}$ is decomposed into two parts including the elastic strain, $\boldsymbol{\varepsilon}^{\mathrm{e}}$, and the plastic strain, $\boldsymbol{\varepsilon}^{\mathrm{p}}$, where the bold-faced letter (i.e. $\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}^{\mathrm{e}}$, and $\boldsymbol{\varepsilon}^{\mathrm{p}}$) denotes a tensor quantity.

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{\mathrm{e}} + \boldsymbol{\varepsilon}^{\mathrm{p}} \tag{12}$$

Eq. (13) shows the yield rule of a three-dimensional form, where $\mathbf{s}$ is the tensor of stress deviator, $\boldsymbol{\alpha}$ is the center of the yield surface, and Y is the radius of the yield surface in the deviatoric stress space.

$$f=\sqrt{\frac{3}{2}(\mathbf{s}-\boldsymbol{\alpha}):(\mathbf{s}-\boldsymbol{\alpha})}-Y\le 0 \tag{13}$$

A boundary surface as shown in Eq. (14) is introduced to record the maximum stress state, particularly the 'peak stress' mentioned above. Note that $\boldsymbol{\beta}$ is the center of the boundary surface and R is the radius of the surface. Moreover, the yield surface cannot exceed the boundary surface.

$$F=\sqrt{\frac{3}{2}(\mathbf{s}-\boldsymbol{\beta}):(\mathbf{s}-\boldsymbol{\beta})}-R\le 0 \tag{14}$$

The flow rule is governed by Eq. (15), and the equivalent plastic strain defined in Eq. (16) can be simplified as λ that determines the magnitude of increment of the plastic strain. The $(\dot{\ })$ represents the time derivative and $\mathbf{n}=\frac{\mathbf{s}-\boldsymbol{\alpha}}{\|\mathbf{s}-\boldsymbol{\alpha}\|}$ represents the direction of the increment of the plastic strain.

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}=\lambda\frac{\partial f}{\partial\boldsymbol{\sigma}}=\sqrt{\frac{3}{2}}\lambda\frac{\mathbf{s}-\boldsymbol{\alpha}}{\|\mathbf{s}-\boldsymbol{\alpha}\|}=\sqrt{\frac{3}{2}}\lambda\mathbf{n} \tag{15}$$

$$\dot{p}=\sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}:\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}}=\lambda \tag{16}$$

The backstress $\boldsymbol{\alpha}$, which is used to characterize nonlinear kinematic hardening, is composed of two parts, i.e. $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$ as shown in Eq. (17), where $\boldsymbol{\beta}$ describes the motion of the center of the boundary surface, and $\boldsymbol{\theta}$ determines the relative motion of the center of the yield surface to that of the boundary surface. It is clear that $\boldsymbol{\beta}$ is the function of plastic strain. Comparatively, $\boldsymbol{\theta}$ is a function of strain range, equivalent plastic strain, and plastic strain from their definitions.

$$\boldsymbol{\alpha}=\boldsymbol{\beta}+\boldsymbol{\theta} \tag{17}$$

$$\boldsymbol{\beta}=\sum_{i=1}^{n_i}\boldsymbol{\beta}^{(i)},\quad \dot{\boldsymbol{\beta}}^{(i)}=m_{\beta}^{(i)}\left(\frac{2}{3}b^{(i)}\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}}-\boldsymbol{\beta}^{(i)}\dot{p}\right) \tag{18}$$

$$\boldsymbol{\theta}=\sum_{j=1}^{n_j}\boldsymbol{\theta}^{(j)},\quad \dot{\boldsymbol{\theta}}^{(j)}=m_{\theta}^{(j)}\left(\frac{2}{3}\omega^{(j)}\cdot c\cdot\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}}-\boldsymbol{\theta}^{(j)}\dot{p}\right)\ \left(\sum_{j=1}^{n_j}\omega^{(j)}=1\right) \tag{19}$$

where $c=R-Y\geq 0$. $m_{\beta}^{(i)}$, $b^{(i)}$, $m_{\theta}^{(j)}$ and $\omega^{(j)}$ are material parameters.

$$\dot{R}=m_{\mathrm{R}}\left(Q-R\right)\dot{p} \tag{20}$$

$$\dot{Q}=m_{\mathrm{Q}}\left(Q_{\infty}-Q\right)\dot{r} \tag{21}$$

In order to reflect the features of the backstress observed in the tests, two intermediate variables R and Q are introduced. Eq. (20) shows the evolution rule of R that is determined by equivalent plastic strain p with an upper limit of Q. Eq. (21) gives the evolution rule of Q that is determined by the strain range r with an upper limit Q_{∞}. Note that m_{R}, m_{Q}, and Q_{∞} are material parameters.

Eq. (22) shows the mathematical expression of Y, where Y is determined by the strain range r with an upper limit Y_{∞}. Note that m_{Y} and Y_{∞} are material parameters.

$$\dot{Y}=m_{\mathrm{Y}}\left(Y_{\infty}-Y\right)\dot{r} \tag{22}$$

The strain range memory surface used to reflect the strain range dependence effect is defined by Eq. (23) in the plastic strain space by introducing the concept of non-hardening region developed by Ohno [23]. The evolution rules of the center of the surface $\mathbf{q}$ and the radius of the surface r are given by Eq. (24) and Eq. (25), respectively.

$$g=\sqrt{\frac{2}{3}\left(\boldsymbol{\varepsilon}^{\mathrm{p}}-\mathbf{q}\right):\left(\boldsymbol{\varepsilon}^{\mathrm{p}}-\mathbf{q}\right)}-r\leq 0 \tag{23}$$

$$\dot{\mathbf{q}}=\left(1-h\right)\eta\frac{\boldsymbol{\varepsilon}^{\mathrm{p}}-\mathbf{q}}{\left\|\boldsymbol{\varepsilon}^{\mathrm{p}}-\mathbf{q}\right\|}=\left(1-h\right)\eta\mathbf{n}^{*} \tag{24}$$

$$\dot{r}=h\left\langle\mathbf{n}:\mathbf{n}^{*}\right\rangle\dot{p} \tag{25}$$

where h is a material parameter, $\mathbf{n}^* = \dfrac{\boldsymbol{\varepsilon}^{\mathrm{p}} - \mathbf{q}}{\left\|\boldsymbol{\varepsilon}^{\mathrm{p}} - \mathbf{q}\right\|}$ is the direction of the evolution of **q,** and $\langle x \rangle \equiv \dfrac{x + |x|}{2}$.

The Kuhn-Tucker complementarity condition and consistency condition are defined in Eq. (26) and Eq. (27), respectively.

$$\lambda \geq 0,\ f \leq 0,\ \lambda f = 0;\ \eta \geq 0,\ g \leq 0,\ \eta g = 0;\ \dot{r} \geq 0,\ g \leq 0,\ \dot{r} g = 0 \tag{26}$$

$$\lambda \dot{f} = 0\ (\text{if}\ f = 0);\ \eta \dot{g} = 0\ (\text{if}\ g = 0) \tag{27}$$

3.3 A few notes about the model

There are many material parameters to be calibrated in the modified constitutive model given in section 3.2. Therefore, it is essential to clarify the physical meaning of every parameter to give insights into the process of parameters calibration. It should be noted that the mathematical structure of all the evolution rules of the proposed constitutive model can be regarded as the form of Eq. (28), where A and m are material parameters, and x is a generalized quantity representing the plastic strain, the equivalent plastic strain, and the strain range.

$$\dot{y} = m(A - y)\dot{x} \tag{28}$$

A closed-form as shown in Eq. (29) can be obtained by integrating Eq. (28). **Fig. 13**(a) shows the picture of Eq. (29), which indicates the physical meaning of these three parameters. In particular, y_0 represents the initial value, A represents the upper limit of the evolution and m indicates the evolution rate. In general, the initial value y_0 and upper limit value A are easy to be obtained from the hysteretic curves. However, it is vague to expound what the evolution rate (i.e. m) refers to.

$$\int_{y_0}^{y}\frac{dy}{A-y}=m\cdot\int_0^x dx\Rightarrow y=A-\left(A-y_0\right)e^{-mx} \tag{29}$$

In order to clarify the exact meaning of *m*, a transformation of Eq. (29) is given in Eq. (30) by substituting $\bar{y}$ for $A-y$ that represents the gap from the initial value to the upper limit, substituting $\bar{x}$ for *mx*, and setting $A-y_0=1$.

$$y=A-\left(A-y_0\right)e^{-mx}\xrightarrow[A-y_0=1]{\bar{y}=A-y,\bar{x}=mx}\bar{y}=e^{-\bar{x}} \tag{30}$$

Fig. 13(b) graphically shows Eq. (30), where all of these three parameters are normalized. It is evident that $\bar{y}$ evolves rapidly at the very beginning and $\bar{y}_{(3)}=0.05$, which is small enough. Thus, a conclusion is obtained that the influence of *mx* can be limited in an interval (0,3). Moreover, it should be noted that *mx* is a non-dimensional variable, which leads to the fact that 1/*m* has the same physical meaning of *x*. Hence, 1/*m* represents the range of the plastic strain, the range of the strain range, or the range of equivalent plastic strain. In this context, the evolution rate *m* could be interpreted as a reciprocal of an appropriate value of the plastic strain, of the strain range and of the equivalent plastic strain.

4. Numerical implementation

4.1 Update plastic strain and state variables

The numerical method used herein is the general closet point projection as shown in **Fig. 14** based on the backward Euler algorithm [25]. Nevertheless, solving the equivalent linearized problem simultaneously is ineffective according to the general closet point projection as there are many state variables listed in Eq. (31) to be updated in every incremental step. Coincidentally, the linearized problem can be simplified to only one nonlinear equation and only $\Delta\lambda$ is unknown owing to the mathematical structure of the evolution rule.

$$\left\{ \boldsymbol{\varepsilon}^{\mathrm{p}},\ \boldsymbol{\beta}^{(i)},\ \boldsymbol{\theta}^{(j)},\ \mathbf{q}, p, c, Y, R, Q, r \right\} \tag{31}$$

Eq. (32)-Eq. (39) present the linearization of all the differential equations of the evolution rule, where all the state variables of the (n+1)-th step are expressed with that of the n-th step and $\Delta\lambda$.

$$\boldsymbol{\varepsilon}^{\mathrm{p}}_{\mathrm{n+1}} = \boldsymbol{\varepsilon}^{\mathrm{p}}_{\mathrm{n}} + \sqrt{\frac{3}{2}}\Delta\lambda \mathbf{n}_{\mathrm{n+1}} \quad (\Delta\lambda = \lambda_{\mathrm{n+1}}\Delta t) \tag{32}$$

$$p_{\mathrm{n+1}} = p_{\mathrm{n}} + \sqrt{\frac{2}{3} \times \sqrt{\frac{3}{2}}\Delta\lambda \mathbf{n}_{\mathrm{n+1}} : \sqrt{\frac{3}{2}}\Delta\lambda \mathbf{n}_{\mathrm{n+1}}} = p_{\mathrm{n}} + \Delta\lambda \tag{33}$$

$$\boldsymbol{\beta}^{(i)}_{\mathrm{n+1}} = \boldsymbol{\beta}^{(i)}_{\mathrm{n}} + m^{(i)}_{\beta}\left(\frac{2}{3} b^{(i)} \times \sqrt{\frac{3}{2}}\Delta\lambda \mathbf{n}_{\mathrm{n+1}} - \boldsymbol{\beta}^{(i)}_{\mathrm{n+1}}\Delta\lambda \right) \Rightarrow \boldsymbol{\beta}^{(i)}_{\mathrm{n+1}} = \frac{\boldsymbol{\beta}^{(i)}_{\mathrm{n}} + \sqrt{\frac{2}{3}} m^{(i)}_{\beta} b^{(i)} \Delta\lambda \mathbf{n}_{\mathrm{n+1}}}{1 + m^{(i)}_{\beta}\Delta\lambda} \tag{34}$$

$$\boldsymbol{\theta}^{(j)}_{\mathrm{n+1}} = \boldsymbol{\theta}^{(j)}_{\mathrm{n}} + m^{(j)}_{\theta}\left(\frac{2}{3} \omega^{(j)} \cdot c_{\mathrm{n+1}} \cdot \sqrt{\frac{3}{2}}\Delta\lambda \mathbf{n}_{\mathrm{n+1}} - \boldsymbol{\theta}^{(j)}_{\mathrm{n+1}}\Delta\lambda \right) \Rightarrow \boldsymbol{\theta}^{(j)}_{\mathrm{n+1}} = \frac{\boldsymbol{\theta}^{(j)}_{\mathrm{n}} + \sqrt{\frac{2}{3}} m^{(j)}_{\theta} \omega^{(j)} c_{\mathrm{n+1}} \Delta\lambda \mathbf{n}_{\mathrm{n+1}}}{1 + m^{(j)}_{\theta}\Delta\lambda} \tag{35}$$

$$\boldsymbol{\alpha}_{\mathrm{n+1}} = \boldsymbol{\beta}_{\mathrm{n+1}} + \boldsymbol{\theta}_{\mathrm{n+1}} = \sum_{i=1}^{n_i} \boldsymbol{\beta}^{(i)} + \sum_{j=1}^{n_j} \boldsymbol{\theta}^{(j)} = \sum_{i=1}^{n_i} \frac{\boldsymbol{\beta}^{(i)}_{\mathrm{n}} + \sqrt{\frac{2}{3}} m^{(i)}_{\beta} b^{(i)} \Delta\lambda \mathbf{n}_{\mathrm{n+1}}}{1 + m^{(i)}_{\beta}\Delta\lambda} + \sum_{j=1}^{n_j} \frac{\boldsymbol{\theta}^{(j)}_{\mathrm{n}} + \sqrt{\frac{2}{3}} m^{(j)}_{\theta} \omega^{(j)} c_{\mathrm{n+1}} \Delta\lambda \mathbf{n}_{\mathrm{n+1}}}{1 + m^{(j)}_{\theta}\Delta\lambda} \tag{36}$$

Where $c_{\mathrm{n+1}} = R_{\mathrm{n+1}} - Y_{\mathrm{n+1}} \geq 0$.

$$R_{\mathrm{n+1}} = R_{\mathrm{n}} + m_{\mathrm{R}}\left(Q_{\mathrm{n+1}} - R_{\mathrm{n+1}} \right)\Delta\lambda \Rightarrow R_{\mathrm{n+1}} = \frac{R_{\mathrm{n}} + m_{\mathrm{R}} Q_{\mathrm{n+1}}\Delta\lambda}{1 + m_{\mathrm{R}}\Delta\lambda} \tag{37}$$

$$Q_{\mathrm{n+1}} = Q_{\mathrm{n}} + m_{\mathrm{Q}}\left(Q_{\infty} - Q_{\mathrm{n+1}} \right)\Delta r \Rightarrow Q_{\mathrm{n+1}} = \frac{Q_{\mathrm{n}} + m_{\mathrm{Q}} Q_{\infty}\Delta r}{1 + m_{\mathrm{Q}}\Delta r} \tag{38}$$

$$Y_{\mathrm{n+1}} = Y_{\mathrm{n}} + m_{\mathrm{Y}}\left(Y_{\infty} - Y_{\mathrm{n+1}} \right)\Delta r \Rightarrow Y_{\mathrm{n+1}} = \frac{Y_{\mathrm{n}} + m_{\mathrm{Y}} Y_{\infty}\Delta r}{1 + m_{\mathrm{Y}}\Delta r} \tag{39}$$

The strain of (n+1)-th step $\boldsymbol{\varepsilon}_{\mathrm{n+1}}$ is determined by Eq. (40), where $\Delta\boldsymbol{\varepsilon}_{\mathrm{n}}$ is a variable that is already known. A trial state (i.e. $f^{\mathrm{trial}}_{\mathrm{n+1}}$) is evaluated to determine $\Delta\lambda$ at first, where $\sigma^{\mathrm{trial}}_{\mathrm{n+1}}$ is the trial stress; $\mathbf{s}^{\mathrm{trial}}_{\mathrm{n+1}}$ is the trial stress deviator; **C** is elasticity tensor; $\mathbf{I} = 1/2\left(\delta_{\mathrm{ik}}\delta_{\mathrm{jl}} + \delta_{\mathrm{il}}\delta_{\mathrm{jk}} \right)$ is the fourth-order symmetric identity tensor, and $\mathbf{1} = \delta_{\mathrm{ij}}$ is the second-order identity tensor.

$$\boldsymbol{\varepsilon}_{\mathrm{n+1}} = \boldsymbol{\varepsilon}_{\mathrm{n}} + \Delta\boldsymbol{\varepsilon}_{\mathrm{n}} \tag{40}$$

$$\sigma_{\mathrm{n+1}}^{\mathrm{trial}} = \mathbf{C}:\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}}\right) = \mathbf{C}:\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n}} + \boldsymbol{\varepsilon}_{\mathrm{n}} - \boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}}\right) = \boldsymbol{\sigma}_{\mathrm{n}} + \mathbf{C}:\Delta\boldsymbol{\varepsilon}_{\mathrm{n}} \tag{41}$$

$$\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} = \boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}\prime} = \left(\boldsymbol{\sigma}_{\mathrm{n}} + \mathbf{C}:\Delta\boldsymbol{\varepsilon}_{\mathrm{n}}\right)' = \mathbf{s}_{\mathrm{n}} + 2G\Delta\mathbf{e}_{\mathrm{n}} \tag{42}$$

$$\Delta\mathbf{e}_{\mathrm{n}} = \mathrm{dev}\left[\Delta\boldsymbol{\varepsilon}_{\mathrm{n}}\right] = \left(\mathbf{I} - \frac{\mathbf{1}\otimes\mathbf{1}}{3}\right):\Delta\boldsymbol{\varepsilon}_{\mathrm{n}} = \left(\mathbf{I} - \frac{\mathbf{1}\otimes\mathbf{1}}{3}\right):\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n}}\right) \tag{43}$$

$$f_{\mathrm{n+1}}^{\mathrm{trial}} = \sqrt{\frac{3}{2}\left(\boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}\prime} - \boldsymbol{\alpha}_{\mathrm{n}}\right):\left(\boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}\prime} - \boldsymbol{\alpha}_{\mathrm{n}}\right)} - Y_{\mathrm{n}} = \sqrt{\frac{3}{2}\left(\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \boldsymbol{\alpha}_{\mathrm{n}}\right):\left(\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \boldsymbol{\alpha}_{\mathrm{n}}\right)} - Y_{\mathrm{n}} \tag{44}$$

If $f_{\mathrm{n+1}}^{\mathrm{trial}} \le 0$, no plastic flow occurs and $\Delta\lambda = 0$. All the state variables remain unchanged. If $f_{\mathrm{n+1}}^{\mathrm{trial}} > 0$, plastic flow occurs and $\Delta\lambda > 0$. Then, a nonlinear equation of $\Delta\lambda$ as shown in Eq. (47) can be obtained by substituting all the variables in the yield condition with Eq. (32)-Eq. (39).

$$\boldsymbol{\sigma}_{\mathrm{n+1}} = \mathbf{C}:\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}\right) = \mathbf{C}:\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}} + \boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}} - \boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}\right) = \boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}} - \mathbf{C}:\sqrt{\frac{3}{2}}\Delta\lambda\mathbf{n}_{\mathrm{n+1}} = \boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}} - \sqrt{6}G\Delta\lambda\mathbf{n}_{\mathrm{n+1}} \tag{45}$$

$$\mathbf{s}_{\mathrm{n+1}} = \left(\boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}} - \sqrt{6}G\Delta\lambda\mathbf{n}_{\mathrm{n+1}}\right)' = \mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \sqrt{6}G\Delta\lambda\mathbf{n}_{\mathrm{n+1}} \tag{46}$$

$$f_{\mathrm{n+1}} = \sqrt{\frac{3}{2}\left(\mathbf{s}_{\mathrm{n+1}} - \boldsymbol{\alpha}_{\mathrm{n+1}}\right):\left(\mathbf{s}_{\mathrm{n+1}} - \boldsymbol{\alpha}_{\mathrm{n+1}}\right)} - Y_{\mathrm{n+1}} = \sqrt{\frac{3}{2}}\left\|\mathbf{s}_{\mathrm{n+1}} - \boldsymbol{\alpha}_{\mathrm{n+1}}\right\| - Y_{\mathrm{n+1}} = 0 \tag{47}$$

In order to solve the nonlinear equation of Eq. (47), some equivalent transformations need to be performed to simplify the nonlinear equation. Setting $\boldsymbol{\xi}_{\mathrm{n+1}} = \mathbf{s}_{\mathrm{n+1}} - \boldsymbol{\alpha}_{\mathrm{n+1}}$, Eq. (48) can be obtained.

$$\begin{aligned}
\boldsymbol{\xi}_{\mathrm{n+1}} &= \mathbf{s}_{\mathrm{n+1}} - \boldsymbol{\alpha}_{\mathrm{n+1}} \\
&= \mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \sqrt{6}G\Delta\lambda\mathbf{n}_{\mathrm{n+1}} - \sum_{i=1}^{n_i}\frac{\boldsymbol{\beta}_{\mathrm{n}}^{(i)} + \sqrt{\frac{2}{3}}m_{\beta}^{(i)}b^{(i)}\Delta\lambda\mathbf{n}_{\mathrm{n+1}}}{1 + m_{\beta}^{(i)}\Delta\lambda} - \sum_{j=1}^{n_j}\frac{\boldsymbol{\theta}_{\mathrm{n}}^{(j)} + \sqrt{\frac{2}{3}}m_{\theta}^{(j)}\omega^{(j)}c_{\mathrm{n+1}}\Delta\lambda\mathbf{n}_{\mathrm{n+1}}}{1 + m_{\theta}^{(j)}\Delta\lambda} \\
&= \left(\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \sum_{i=1}^{n_i}\frac{\boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{1 + m_{\beta}^{(i)}\Delta\lambda} - \sum_{j=1}^{n_j}\frac{\boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{1 + m_{\theta}^{(j)}\Delta\lambda}\right) - \sqrt{\frac{2}{3}}\left(3G + \sum_{i=1}^{n_i}\frac{m_{\beta}^{(i)}b^{(i)}}{1 + m_{\beta}^{(i)}\Delta\lambda} + \sum_{j=1}^{n_j}\frac{m_{\theta}^{(j)}\omega^{(j)}c_{\mathrm{n+1}}}{1 + m_{\theta}^{(j)}\Delta\lambda}\right)\Delta\lambda\mathbf{n}_{\mathrm{n+1}}
\end{aligned} \tag{48}$$

Setting $\overline{\boldsymbol{\xi}}_{\mathrm{n+1}} = \mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \sum_{i=1}^{n_i}\frac{\boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{1 + m_{\beta}^{(i)}\Delta\lambda} - \sum_{j=1}^{n_j}\frac{\boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{1 + m_{\theta}^{(j)}\Delta\lambda}$ and accepting $\mathbf{n}_{\mathrm{n+1}} = \frac{\mathbf{s}_{\mathrm{n+1}} - \boldsymbol{\alpha}_{\mathrm{n+1}}}{\left\|\mathbf{s}_{\mathrm{n+1}} - \boldsymbol{\alpha}_{\mathrm{n+1}}\right\|} = \frac{\boldsymbol{\xi}_{\mathrm{n+1}}}{\left\|\boldsymbol{\xi}_{\mathrm{n+1}}\right\|}$, the direction of $\mathbf{n}_{\mathrm{n+1}}$ is the same as that of $\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}$ according to Eq. (48). Thus, $\mathbf{n}_{\mathrm{n+1}}$ can be determined exclusively in terms of the trial state $\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}$ as shown in Eq. (49), which simplifies Eq. (48) to Eq. (50).

$$\mathbf{n}_{\mathrm{n+1}}=\frac{\boldsymbol{\xi}_{\mathrm{n+1}}}{\left\|\boldsymbol{\xi}_{\mathrm{n+1}}\right\|}=\frac{\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}}{\left\|\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}\right\|} \tag{49}$$

$$\left\|\boldsymbol{\xi}_{\mathrm{n+1}}\right\|=\left\|\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}\right\|-\sqrt{\frac{2}{3}}\left(3G+\sum_{i=1}^{n_i}\frac{m_{\beta}^{(i)}b^{(i)}}{1+m_{\beta}^{(i)}\Delta\lambda}+\sum_{j=1}^{n_j}\frac{m_{\theta}^{(j)}\omega^{(j)}c_{\mathrm{n+1}}}{1+m_{\theta}^{(j)}\Delta\lambda}\right)\Delta\lambda \tag{50}$$

Substituting all the simplified form of the linearized expression into the original nonlinear equation Eq. (47), a new nonlinear equation can be obtained as shown in Eq. (51). However, there are two unknown variables $\Delta\lambda$ and Δr included in this equation.

$$f_{\mathrm{n+1}}=\sqrt{\frac{3}{2}}\left\|\boldsymbol{\xi}_{\mathrm{n+1}}\right\|-\mathrm{Y}_{\mathrm{n}}=\sqrt{\frac{3}{2}}\left\|\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}\right\|-\left(3G+\sum_{i=1}^{n_i}\frac{m_{\beta}^{(i)}b^{(i)}}{1+m_{\beta}^{(i)}\Delta\lambda}+\sum_{j=1}^{n_j}\frac{m_{\theta}^{(j)}\omega^{(j)}c_{\mathrm{n+1}}}{1+m_{\theta}^{(j)}\Delta\lambda}\right)\Delta\lambda-Y_{\mathrm{n+1}}=0 \tag{51}$$

A trail state $g_{\mathrm{n+1}}^{\mathrm{trial}}$ is shown in Eq. (52), which is introduced to determine whether the strain memory surface evolves or not. The memory surface enlarges when $g_{\mathrm{n+1}}^{\mathrm{trial}}>0$ while keeps unchanged when $g_{\mathrm{n+1}}^{\mathrm{trial}}\leq 0$.

$$g_{\mathrm{n+1}}^{\mathrm{trial}}=\sqrt{\frac{2}{3}\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}-\mathbf{q}_{\mathrm{n}}\right):\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}-\mathbf{q}_{\mathrm{n}}\right)}-r_{\mathrm{n}}\leq 0 \tag{52}$$

Especially, the evolution of strain memory surface occurs only when $g_{\mathrm{n+1}}^{\mathrm{trial}}>0$, which means that Δr is determined by $\Delta\lambda$ as $g_{\mathrm{n+1}}^{\mathrm{trial}}$ is a function of $\Delta\lambda$. In this way, the nonlinear equation Eq. (51) can only include one unknown $\Delta\lambda$ if the relationship between $\Delta\lambda$ and Δr is determined.

Defining a variable $\Delta\lambda_0$ that satisfies $g_{\mathrm{n+1}}^{\mathrm{trial}}=0$, Δr can be obtained by integration of Eq. (25) form $\Delta\lambda_0$ to $\Delta\lambda$ as shown in Eq. (53). All of these variables are shown in **Fig. 15**.

$$\Delta r=\int_{r_{\mathrm{n}}}^{r_{\mathrm{n+1}}}dr=\int_{\Delta\lambda_0}^{\Delta\lambda}h\left\langle\mathbf{n}:\mathbf{n}^{*}\right\rangle d\left(\Delta\lambda\right) \tag{53}$$

When the evolution occurs, $\mathbf{n}:\mathbf{n}^{*}>0$, Eq. (53) transforms into Eq. (54).

$$\Delta r=\int_{r_{\mathrm{n}}}^{r_{\mathrm{n+1}}} dr=\int_{\Delta\lambda_0}^{\Delta\lambda} h\left(\mathbf{n}:\mathbf{n}^{*}\right) d\left(\Delta\lambda\right) \tag{54}$$

Setting $\boldsymbol{\zeta}_{\mathrm{n+1}}=\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}-\mathbf{q}_{\mathrm{n+1}}$ and linearizing the evolution rule of Eq. (24) and Eq. (25), Eq. (55) and Eq. (56) can be obtained.

$$\mathbf{q}_{\mathrm{n+1}}=\mathbf{q}_{\mathrm{n}}+\left(1-h\right)\Delta\eta\mathbf{n}_{\mathrm{n+1}}^{*} \tag{55}$$

$$\boldsymbol{\zeta}_{\mathrm{n+1}}=\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}-\mathbf{q}_{\mathrm{n+1}}=\boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}}+\sqrt{\frac{3}{2}}\Delta\lambda\mathbf{n}_{\mathrm{n+1}}-\mathbf{q}_{\mathrm{n}}-\left(1-h\right)\Delta\eta\mathbf{n}_{\mathrm{n+1}}^{*} \tag{56}$$

Setting $\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}=\boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}}+\sqrt{\frac{3}{2}}\Delta\lambda\mathbf{n}_{\mathrm{n+1}}-\mathbf{q}_{\mathrm{n}}$, then we get $\boldsymbol{\zeta}_{\mathrm{n+1}}=\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}-\left(1-h\right)\Delta\eta\mathbf{n}_{\mathrm{n+1}}^{*}$. It should be noted that $\mathbf{n}_{\mathrm{n+1}}^{*}=\dfrac{\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}-\mathbf{q}_{\mathrm{n+1}}}{\left\|\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}-\mathbf{q}_{\mathrm{n+1}}\right\|}=\dfrac{\boldsymbol{\zeta}_{\mathrm{n+1}}}{\left\|\boldsymbol{\zeta}_{\mathrm{n+1}}\right\|}$, and hence the direction of $\mathbf{n}_{\mathrm{n+1}}^{*}$ is the same as that of $\boldsymbol{\zeta}_{\mathrm{n+1}}$. Thus, it can be obtained that $\mathbf{n}_{\mathrm{n+1}}^{*}=\dfrac{\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}}{\left\|\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|}$. Then Eq. (56) can be simplified to Eq. (57). Moreover, by substituting $\mathbf{n}_{\mathrm{n+1}}^{*}=\dfrac{\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}}{\left\|\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|}$ into Eq. (53), Δr can be obtained by integration as shown in Eq. (58).

$$\left\|\boldsymbol{\zeta}_{\mathrm{n+1}}\right\|=\left\|\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|-\left(1-h\right)\Delta\eta \tag{57}$$

$$\Delta r=\int_{r_{\mathrm{n}}}^{r_{\mathrm{n+1}}} dr=\int_{\Delta\lambda_0}^{\Delta\lambda} h\left(\mathbf{n}:\mathbf{n}^{*}\right) d\left(\Delta\lambda\right)=\sqrt{\frac{2}{3}}h\left(\left\|\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|-\left\|\overline{\boldsymbol{\zeta}}_{0}\right\|\right) \tag{58}$$

It should be noted that $g_{\mathrm{n+1}}^{\mathrm{trial}}\Big|_{\Delta\lambda_0}=\sqrt{\frac{2}{3}}\left\|\overline{\boldsymbol{\zeta}}_{0}\right\|-r_{\mathrm{n}}=0$ has been defined before, thus we can get $\left\|\overline{\boldsymbol{\zeta}}_{0}\right\|=\sqrt{\frac{3}{2}}r_{\mathrm{n}}$. By substituting it into Eq. (58), Eq. (59) can be obtained and can be used to calculate Δr.

$$\Delta r=h\left(\sqrt{\frac{2}{3}}\left\|\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|-\sqrt{\frac{2}{3}}\left\|\overline{\boldsymbol{\zeta}}_{0}\right\|\right)=h\left(\sqrt{\frac{2}{3}}\left\|\overline{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|-r_{\mathrm{n}}\right)=h\cdot g_{\mathrm{n+1}}^{\mathrm{trial}} \tag{59}$$

In addition, $\langle\cdot\rangle$ is introduced to take into account that $\Delta r = 0$ when $g_{\mathrm{n+1}}^{\mathrm{trial}} \leq 0$, then Eq. (59) converts to Eq.(60).

$$\Delta r = h\left\langle g_{\mathrm{n+1}}^{\mathrm{trial}} \right\rangle \tag{60}$$

Then, by substituting Eq. (60) into Eq. (51), a new nonlinear equation with one unknown $\Delta\lambda$ is obtained, which can be solved by using the classical Newton method.

The iteration formula is shown in Eq. (61) and the initial condition is $\Delta^{(0)}\lambda = 0$. The termination condition of iteration is $\left\| f_{\mathrm{n+1}}^{(\mathrm{k})} \right\| \leq \mathrm{TOL}$, where the TOL is the error determined by the user. All of the iteration processes are shown in **Fig. 16**.

$$\Delta^{(\mathrm{k+1})}\lambda = \Delta^{(\mathrm{k})}\lambda - \frac{f_{\mathrm{n+1}}^{(\mathrm{k})}}{f_{\mathrm{n+1}}^{\prime(\mathrm{k})}} \tag{61}$$

$$f'_{\mathrm{n+1}} = \sqrt{\frac{3}{2}}\frac{d\left\|\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}\right\|}{d\Delta\lambda} - 3G - \sum_{i=1}^{n_i}\frac{m_\beta^{(i)} b^{(i)}}{\left(1+m_\beta^{(i)}\Delta\lambda\right)^2} - \sum_{j=1}^{n_j}\frac{m_\theta^{(j)}\omega^{(j)} c_{\mathrm{n+1}}}{\left(1+m_\theta^{(j)}\Delta\lambda\right)^2} - \sum_{j=1}^{n_j}\frac{m_\theta^{(j)}\omega^{(j)}\Delta\lambda}{1+m_\theta^{(j)}\Delta\lambda}\frac{dc_{\mathrm{n+1}}}{d\Delta\lambda} - \frac{dY_{\mathrm{n+1}}}{d\Delta\lambda} \tag{62}$$

Where

$$\frac{d\left\|\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}\right\|}{d\Delta\lambda} = \frac{d\left\|\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}\right\|}{d\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}} : \frac{d\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}}{d\Delta\lambda} = \frac{\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}}{\left\|\bar{\boldsymbol{\xi}}_{\mathrm{n+1}}\right\|} : \left(\sum_{i=1}^{n_i}\frac{m_\beta^{(i)}\boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{\left(1+m_\beta^{(i)}\Delta\lambda\right)^2} + \sum_{j=1}^{n_j}\frac{m_\theta^{(j)}\boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{\left(1+m_\theta^{(j)}\Delta\lambda\right)^2}\right) \tag{63}$$

$$\frac{d\mathrm{c}_{\mathrm{n+1}}}{d\Delta\lambda} = \frac{d\left(\mathrm{R}_{\mathrm{n+1}} - \mathrm{Y}_{\mathrm{n+1}}\right)}{d\Delta\lambda} = \frac{d\mathrm{R}_{\mathrm{n+1}}}{d\Delta\lambda} - \frac{d\mathrm{Y}_{\mathrm{n+1}}}{d\Delta\lambda} \tag{64}$$

$$\frac{dR_{\mathrm{n+1}}}{d\Delta\lambda} = \frac{m_{\mathrm{R}}\left(Q_{\mathrm{n+1}} - R_{\mathrm{n}}\right)}{\left(1+m_{\mathrm{R}}\Delta\lambda\right)^2} + \frac{m_{\mathrm{R}}\Delta\lambda}{1+m_{\mathrm{R}}\Delta\lambda}\frac{dQ_{\mathrm{n+1}}}{d\Delta\lambda} \tag{65}$$

$$\frac{dY_{\mathrm{n+1}}}{d\Delta\lambda} = \frac{m_{\mathrm{Y}}\left(Y_\infty - Y_{\mathrm{n}}\right)}{\left(1+m_{\mathrm{Y}}\Delta r\right)^2}\frac{d\Delta r}{d\Delta\lambda} \tag{66}$$

$$\frac{dQ_{\mathrm{n+1}}}{d\Delta\lambda} = \frac{m_{\mathrm{Q}}\left(Q_\infty - Q_{\mathrm{n}}\right)}{\left(1+m_{\mathrm{Q}}\Delta r\right)^2}\frac{d\Delta r}{d\Delta\lambda} \tag{67}$$

$$\frac{d\Delta r}{d\Delta\lambda} = h\cdot\sqrt{\frac{2}{3}}\frac{d\left\|\bar{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|}{d\Delta\lambda} = h\cdot\sqrt{\frac{2}{3}}\frac{\bar{\boldsymbol{\zeta}}_{\mathrm{n+1}}}{\left\|\bar{\boldsymbol{\zeta}}_{\mathrm{n+1}}\right\|} : \frac{d\bar{\boldsymbol{\zeta}}_{\mathrm{n+1}}}{d\Delta\lambda} = h\cdot\mathbf{n}_{\mathrm{n+1}}^{*} : \left(\mathbf{n}_{\mathrm{n+1}} + \Delta\lambda\frac{d\mathbf{n}_{\mathrm{n+1}}}{d\Delta\lambda}\right) \tag{68}$$

$$\frac{d\mathbf{n}_{\mathrm{n+1}}}{d\Delta\lambda}=\frac{d\mathbf{n}_{\mathrm{n+1}}}{d\boldsymbol{\xi}_{\mathrm{n+1}}}:\frac{d\boldsymbol{\xi}_{\mathrm{n+1}}}{d\Delta\lambda}=\frac{\mathbf{I}-\mathbf{n}_{\mathrm{n+1}}\otimes\mathbf{n}_{\mathrm{n+1}}}{\left\|\boldsymbol{\xi}_{\mathrm{n+1}}\right\|}:\left(\sum_{i=1}^{n_i}\frac{m_\beta^{(i)}\boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{\left(1+m_\beta^{(i)}\Delta\lambda\right)^2}+\sum_{j=1}^{n_j}\frac{m_\theta^{(j)}\boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{\left(1+m_\theta^{(j)}\Delta\lambda\right)^2}\right) \tag{69}$$

4.2 Consistent elastoplastic tangent moduli

The consistent elastoplastic tangent modulus $\mathbf{J}$ as shown in Eq. (72) can be obtained by differentiating Eq. (45) as shown in Eq. (70) and Eq. (71). The simplified form of Eq. (72) is shown in Eq. (75).

$$\boldsymbol{\sigma}_{\mathrm{n+1}}=\mathbf{C}:\left(\boldsymbol{\varepsilon}_{\mathrm{n+1}}-\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}\right)=\mathbf{C}:\boldsymbol{\varepsilon}_{\mathrm{n+1}}-2G\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}}=\mathbf{C}:\boldsymbol{\varepsilon}_{\mathrm{n+1}}-2G\left(\boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}}+\sqrt{\frac{3}{2}}\Delta\lambda\mathbf{n}_{\mathrm{n+1}}\right) \tag{70}$$

$$d\boldsymbol{\sigma}_{\mathrm{n+1}}=\left(\mathbf{C}-\sqrt{6}G\Delta\lambda\frac{d\mathbf{n}_{\mathrm{n+1}}}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}}-\sqrt{6}G\mathbf{n}_{\mathrm{n+1}}\otimes\frac{d\Delta\lambda}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}}\right):d\boldsymbol{\varepsilon}_{\mathrm{n+1}}=\mathbf{J}:d\boldsymbol{\varepsilon}_{\mathrm{n+1}} \tag{71}$$

Where

$$\mathbf{J}=\mathbf{C}-\sqrt{6}G\Delta\lambda\frac{d\mathbf{n}_{\mathrm{n+1}}}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}}-\sqrt{6}G\mathbf{n}_{\mathrm{n+1}}\otimes\frac{d\Delta\lambda}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}} \tag{72}$$

$$\frac{d\mathbf{n}_{\mathrm{n+1}}}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}}=\frac{\mathbf{I}-\mathbf{n}_{\mathrm{n+1}}\otimes\mathbf{n}_{\mathrm{n+1}}}{\left\|\boldsymbol{\xi}_{\mathrm{n+1}}\right\|}:\left(\frac{d\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}}}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}}+\left(\sum_{i=1}^{n_i}\frac{m_\beta^{(i)}\boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{\left(1+m_\beta^{(i)}\Delta\lambda\right)^2}+\sum_{j=1}^{n_j}\frac{m_\theta^{(j)}\boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{\left(1+m_\theta^{(j)}\Delta\lambda\right)^2}\right)\otimes\frac{d\Delta\lambda}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}}\right) \tag{73}$$

$$\frac{d\Delta\lambda}{d\boldsymbol{\varepsilon}_{\mathrm{n+1}}}=-\frac{\partial f_{\mathrm{n+1}}/\partial\boldsymbol{\varepsilon}_{\mathrm{n+1}}}{\partial f_{\mathrm{n+1}}/\partial\Delta\lambda}=-\frac{\sqrt{6}G\mathbf{n}_{\mathrm{n+1}}}{\partial f_{\mathrm{n+1}}/\partial\Delta\lambda} \tag{74}$$

$$\mathbf{J}=\mathbf{C}-2\sqrt{6}G^2\Delta\lambda\frac{\mathbf{I}-\mathbf{n}_{\mathrm{n+1}}\otimes\mathbf{n}_{\mathrm{n+1}}}{\left\|\boldsymbol{\xi}_{\mathrm{n+1}}\right\|}:\mathbf{N}_{\mathrm{n+1}}+6G^2\frac{\mathbf{n}_{\mathrm{n+1}}\otimes\mathbf{n}_{\mathrm{n+1}}}{\partial f_{\mathrm{n+1}}/\partial\Delta\lambda} \tag{75}$$

Where

$$\mathbf{N}_{\mathrm{n+1}}=\left(\mathbf{I}-\frac{\mathbf{1}\otimes\mathbf{1}}{3}\right)-\sqrt{\frac{3}{2}}\left(\sum_{i=1}^{n_i}\frac{m_\beta^{(i)}\boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{\left(1+m_\beta^{(i)}\Delta\lambda\right)^2}+\sum_{j=1}^{n_j}\frac{m_\theta^{(j)}\boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{\left(1+m_\theta^{(j)}\Delta\lambda\right)^2}\right)\otimes\frac{\mathbf{n}_{\mathrm{n+1}}}{\partial f_{\mathrm{n+1}}/\partial\Delta\lambda} \tag{76}$$

All of the numerical procedures are summarized, as shown in **Appendix 1.**

5. Parameters calibration

5.1 One-dimensional plasticity

In the one-dimensional cyclic test, the uniaxial state is defined by Eq. (77), Eq. (78), and Eq. (79). These equations can be substituted into the three-dimensional constitutive model to obtain the one-dimensional form as shown in **Table 2**.

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{77}$$

$$\mathbf{s} = \begin{pmatrix} \frac{2\sigma}{3} & 0 & 0 \\ 0 & -\frac{\sigma}{3} & 0 \\ 0 & 0 & -\frac{\sigma}{3} \end{pmatrix} \tag{78}$$

$$\boldsymbol{\varepsilon}^{\mathrm{p}} = \begin{pmatrix} \varepsilon^{\mathrm{p}} & 0 & 0 \\ 0 & -\frac{1}{2}\varepsilon^{\mathrm{p}} & 0 \\ 0 & 0 & -\frac{1}{2}\varepsilon^{\mathrm{p}} \end{pmatrix} \tag{79}$$

5.2 Optimization method

The proposed cyclic plasticity constitutive model contains many parameters that need to be calibrated. A new method based on the derivative-free optimization theory is introduced in this paper. The basic mathematical form of the optimization problem is listed in Eq. (80) with the constrained conditions of the proposed constitutive model.

$$\text{Minimize} \quad f(x)$$

$$\text{Subject to} \quad \begin{cases} x_L \le x \le x_U \\ Ax = b \end{cases} \tag{80}$$

Where x represents the material parameters vector as shown in Eq. (81) by setting n_i=n_j=3; $f(x)$ is an objective function defined as Eq. (82); x_L and x_U are the lower boundary and the upper boundary of these material parameters that can be determined approximately from test results, which are listed in Eq. (83) and Eq. (84); $Ax = b$ is a constraint condition of material parameters refers to $\sum_{j=1}^{n_j} \omega^{(j)} = 1$ in the modified model.

$$x = \left(\bar{m}_{\beta}^{(1)}, \bar{m}_{\beta}^{(2)}, \bar{m}_{\beta}^{(3)}, b_1, b_2, b_3, \bar{m}_{\theta}^{(1)}, \bar{m}_{\theta}^{(2)}, \bar{m}_{\theta}^{(3)}, \omega_1, \omega_2, \omega_3, m_{\mathrm{R}}, m_Q, Q_0, Q_\infty, m_{\mathrm{Y}}, Y_0, Y_\infty, h, r_0\right) \quad (81)$$

$$f(x) = \sum_{i=1}^{n} \left(\frac{\hat{\sigma}_i - \sigma_i}{\sigma_i} \right)^2 \quad (82)$$

$$x_L = \left(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 50\ 200\ 0\ 40\ 100\ 0\ 0\right) \quad (83)$$

$$x_U = \left(0.01\ 0.02\ 0.05\ 50\ 50\ 50\ 0.01\ 0.02\ 0.03\ 1\ 1\ 1\ 100\ 100\ 150\ 350\ 100\ 90\ 200\ 1\ 0.001\right) \quad (84)$$

The start point of optimization is $x_0 = \frac{1}{2}\left(x_L + x_U\right)$ and the optimization algorithm is based on the interior point method. The optimized material parameters are listed in **Table 3**. It should be noted that $\bar{m}$, which represents the reciprocal of m (i.e. $m_{\beta}^{(i)}$, $m_{\theta}^{(j)}$), is used in the optimization process as shown in **Fig. 17**, to give the boundary of the parameters conveniently.

5.3 Validation of the calibrated parameters

Fig. 18. shows the comparison between the results of the tests and the simulation of the proposed model. In the comparison of the test results and the simulation results under CSA loading protocols (i.e. specimen ZH-01~ZH-04), the hysteretic loops of the experiments and that of the simulations are almost identical. While in the comparison of the tests and the simulations of the specimen ZH-05 and specimen ZH-06, the simulation results show a little difference with the test results after the buckling of the specimens. In the last two loading protocols (i.e. ZH-07~ZH-08), the comparison of the test results and the simulations indicates

that the model gives an appropriate description of the effect of the maximum strain history and the influence of the decreasing strain amplitude.

In general, the simulations of the proposed model show a good agreement with the test results, demonstrating the ability of the proposed model for describing the cyclic behavior of LYP100.

6. Conclusion

This paper presents an experimental investigation of cyclic plasticity behavior of low yield point (LYP) steel LYP100 under cyclic loading of large strain amplitude and proposes a modified constitutive model of LYP100 for practical application.

Cyclic test results show that there are three important mechanical features of LYP100 including remarkable work-hardening, early re-yielding, and strain range dependence. A detailed analysis is made in this paper to quantify the peak stress and the elastic region of LYP100, which indicates the influence of the strain range. In addition, the influence of the maximum strain history and the shrinkage of the cyclic loops under the decreasing strain sequence are also observed in the tests.

A modified constitutive model is proposed based on the features observed in the tests and the corresponding numerical algorithm is presented. The modification of the evolution rule of the backstress leads to only one nonlinear equation with one unknown variable to be solved. The modification of the isotropic part gives a reasonable quantification of the elastic region.

Moreover, an efficient method of parameter calibration based on the derivative-free optimization theory is proposed, which helps avoid tedious data processing. The comparison between the results of the test and the simulation of the model shows that the constitutive model proposed in this paper is able to predict the cyclic behavior of LYP100 reasonably.

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Reference

[1] N. Masayoshi, Strain-Hardening Behavior of Shear Panels Made of Low-Yield Steel. I: Test, J. Struct. Eng. 121 (1995) 1742–1749. doi:10.1061/(ASCE)0733-9445(1995)121:12(1742).

[2] Y.-H. Wang, Q. Wu, G.-L. Xiong, Test study on mechanical properties of shear panel energy-dissipation components using low yield point steel with different parameters, Gongcheng Lixue/Engineering Mech. 34 (2017) 21–29. doi:10.6052/j.issn.1000-4750.2016.07.0576. [in Chinese]

[3] J. Wang, Y. Shi, Y. Wang, Constitutive Model of Low-Yield Point Steel and Its Application in Numerical Simulation of Buckling-Restrained Braces, J. Mater. Civ. Eng. 28 (2015) 4015142.

[4] E. Saeki, M. Sugisawa, T. Yamaguchi, A. Wada, Mechanical Properties of Low Yield Point Steels, J. Mater. Civ. Eng. 10 (1998) 143–152. doi:10.1061/(ASCE)0899-1561(1998)10:3(143).

[5] P. Dusicka, A.M. Itani, I.G. Buckle, Cyclic response of plate steels under large inelastic strains, J. Constr. Steel Res. 63 (2007) 156–164. doi:10.1016/j.jcsr.2006.03. 006.

[6] M. Wang, L.A. Fahnestock, F. Qian, W. Yang, Experimental cyclic behavior and constitutive modeling of low yield point steels, Constr. Build. Mater. 131 (2017) 696–712. doi:10.1016/j.conbuildmat.2016.11.035.

[7] L. Xu, X. Nie, J. Fan, M. Tao, R. Ding, Cyclic hardening and softening behavior of the low yield point steel BLY160: Experimental response and constitutive modeling, Int. J. Plast. 78 (2016) 44–63. doi:10.1016/j.ijplas.2015.10.009.

[8] J. Lubliner, Plasticity theory, Dover Publications, 2006.

[9] J.L. Chaboche, A review of some plasticity and viscoplasticity constitutive theories, Int. J. Plast. 24 (2008) 1642–1693. doi:10.1016/j.ijplas.2008.03.009.

[10] J.L. Chaboche, Constitutive equations for cyclic plasticity and cyclic viscoplasticity, Int. J. Plast. 5 (1989) 247–302. doi:http://dx.doi.org/10.1016/0749-6419(89)90015-6.

[11] J.L. Chaboche, G. Rousselier, On the Plastic and Viscoplastic Constitutive Equations—Part I: Rules Developed With Internal Variable Concept, J. Press. Vessel Technol. 105 (1983) 153. doi:10.1115/1.3264257.

[12] M. Menegotto, P.E. Pinto, Method of Analysis for Cyclically Loaded Reinforced Concrete Plane Frames Including Changes in Geometry and Non-elastic behavior of Elements under Combined Normal Force and Bending, in: Proceedings, IABSE Symposium on Resistance Ultimate Deformability of Structures Acted on by Well Defined Repeated Loads, Lisbon, 1973, 15–22.

[13] F.C. Filippou, E.P. Popov, V.V. Bertero, Effect of Bond Deterioration on Cyclic Behavior of Reinforced Concrete Joints: EERC Report 83-19, Earthquake Engineering Research Center, Berkeley, 1983.

[14] G. Shi, Y. Gao, X. Wang, Y. Zhang, Mechanical properties and constitutive models of low yield point steels, Constr. Build. Mater. 175 (2018) 570–587. doi:10.1016/j.conbuildmat.2018.04.219.

[15] F. Hu, G. Shi, Y. Shi, Constitutive model for full-range elasto-plastic behavior of structural steels with yield plateau: Formulation and implementation, Eng. Struct. doi:10.1016/j.engstruct. 2016.02.037.[in press]

[16] F. Hu, G. Shi, Y. Shi, Constitutive model for full-range elasto-plastic behavior of structural steels with yield plateau: Calibration and validation, Eng. Struct. 118 (2016) 210–227. doi:10.1016/j.engstruct.2016.03.060.

[17] GB50011-2012, Code for Seismic Design of Buildings, (2014) 47–49.

[18] L.Y. Xu, M.X. Tao, X. Nie, J.S. Fan, E. Taciroglu, Modeling Techniques for Strain-Range-Dependent Hardening Behavior of Low-Yield-Point Steel Shear Panel Dampers, J. Struct. Eng. 143 (2017). doi: 10.1061/(ASCE)ST.1943-541X.0001896

[19] J. Lemaitre, J.-L. Chaboche, Mechanics of solid materials, Cambridge University Press, Cambridge, 1990.

[20] F. Zhou, Y. Chen, Q. Wu, Dependence of the cyclic response of structural steel on loading history under large inelastic strains, J. Constr. Steel Res. 104 (2015) 64–73. doi:10.1016/J.JCSR.2014.09.019.

[21] F. Yoshida, T. Uemori, A model of large-strain cyclic plasticity describing the Bauschinger effect and workhardening stagnation, Int. J. Plast. 18 (2002) 661–686. doi:10.1016/S0749-6419(01)00050-X.

[22] J.L. Chaboche, K. Dang Van, G. Cordier, Modelizaion of the Strain Memory Effect on

the Cyclic Hardening of 316 Stainless Steel, Int. Assoc. Struct. Mech. React. Technol. (1979) L11/3.

[23] P.J. Armstrong, C.O. Frederick, A Mathematical Representation of the Multi-axial Baushinger Effect, Tech. Report C.E.G.B. Report RD/B/N731, (1965).

[24] N. Ohno, Y. Takahashi, Cyclic Plasticity: Modeling Based on Non-hardening Region, Springer Berlin Heidelberg, 1988.

[25] J.C. Simo, T.J.R. Hughes, Computational inelasticity, Springer, 1999. doi:10.1016/S0898-1221(99)90413-3.

Appendix 1: Numerical algorithm of the constitutive model

1. Compute trail state

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{n}} = \boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n}} \tag{A1}$$

$$\boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}} = \mathbf{C} : \left(\boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}}\right) = \boldsymbol{\sigma}_{\mathrm{n}} + \mathbf{C} : \Delta\boldsymbol{\varepsilon}_{\mathrm{n}} \tag{A2}$$

$$\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} = \boldsymbol{\sigma}_{\mathrm{n+1}}^{\mathrm{trial}\prime} \tag{A3}$$

2. Check yield condition

$$f_{\mathrm{n+1}}^{\mathrm{trial}} = \sqrt{\frac{3}{2}\left(\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \boldsymbol{\alpha}_{\mathrm{n}}\right) : \left(\mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \boldsymbol{\alpha}_{\mathrm{n}}\right)} - Y_{\mathrm{n}} \tag{A4}$$

IF $f_{\mathrm{n+1}}^{\mathrm{trial}} \leq 0$ THEN

Set $(\square)_{\mathrm{n+1}} = (\square)_{\mathrm{n}}$ & EXIT

ELSE GO TO STEP 3

3. Compute $\Delta\lambda$ by iterating from $\Delta\lambda^{(0)}=0$

DO UNTIL $\left|f_{\mathrm{n+1}}^{(\mathrm{k})}\right| \leq$ TOL1

$$\boldsymbol{\xi}_{\mathrm{n+1}}^{(\mathrm{k})} = \mathbf{s}_{\mathrm{n+1}}^{\mathrm{trial}} - \sum_{i=1}^{n_i} \frac{\boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{1 + m_{\beta}^{(i)}\Delta\lambda^{(\mathrm{k})}} - \sum_{j=1}^{n_j} \frac{\boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{1 + m_{\theta}^{(j)}\Delta\lambda^{(\mathrm{k})}} \tag{A5}$$

$$\mathbf{n}_{\mathrm{n+1}}^{(\mathrm{k})} = \frac{\boldsymbol{\xi}_{\mathrm{n+1}}^{(\mathrm{k})}}{\left\|\boldsymbol{\xi}_{\mathrm{n+1}}^{(\mathrm{k})}\right\|} \tag{A6}$$

$$\boldsymbol{\zeta}_{\mathrm{n+1}}^{(\mathrm{k})} = \boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}} + \sqrt{\frac{3}{2}}\Delta\lambda^{(\mathrm{k})}\mathbf{n}_{\mathrm{n+1}} - \mathbf{q}_{\mathrm{n}} \tag{A7}$$

$$\mathbf{n}_{\mathrm{n+1}}^{*(\mathrm{k})} = \frac{\boldsymbol{\zeta}_{\mathrm{n+1}}^{(\mathrm{k})}}{\left\|\boldsymbol{\zeta}_{\mathrm{n+1}}^{(\mathrm{k})}\right\|} \tag{A8}$$

$$g_{\mathrm{n+1}}^{\mathrm{trial}(\mathrm{k})} = \sqrt{\frac{2}{3}}\left\|\boldsymbol{\zeta}_{\mathrm{n+1}}^{(\mathrm{k})}\right\| - r_{\mathrm{n}} \tag{A9}$$

$$\Delta^{(\mathrm{k})} r = h\left\langle g_{\mathrm{n+1}}^{\mathrm{trial(k)}}\right\rangle \tag{A10}$$

$$Q_{\mathrm{n+1}}^{(\mathrm{k})} = \frac{Q_{\mathrm{n}} + m_{\mathrm{Q}} Q_{\infty} \Delta^{(\mathrm{k})} r}{1 + m_{\mathrm{Q}} \Delta^{(\mathrm{k})} r} \tag{A11}$$

$$R_{\mathrm{n+1}}^{(\mathrm{k})} = \frac{R_{\mathrm{n}} + m_{\mathrm{R}} Q_{\mathrm{n+1}} \Delta^{(\mathrm{k})} \lambda}{1 + m_{\mathrm{R}} \Delta^{(\mathrm{k})} \lambda} \tag{A12}$$

$$Y_{\mathrm{n+1}}^{(\mathrm{k})} = \frac{Y_{\mathrm{n}} + m_{\mathrm{Y}} Y_{\infty} \Delta^{(\mathrm{k})} r}{1 + m_{\mathrm{Y}} \Delta^{(\mathrm{k})} r} \tag{A13}$$

$$c_{\mathrm{n+1}}^{(\mathrm{k})} = R_{\mathrm{n+1}}^{(\mathrm{k})} - Y_{\mathrm{n+1}}^{(\mathrm{k})} \tag{A14}$$

$$f_{\mathrm{n+1}}^{(\mathrm{k})} = \sqrt{\frac{3}{2}}\left\|\overline{\boldsymbol{\xi}}_{\mathrm{n+1}}^{(\mathrm{k})}\right\| - \left(3G + \sum_{i=1}^{n_i} \frac{m_{\beta}^{(i)} b^{(i)}}{1 + m_{\beta}^{(i)} \Delta^{(\mathrm{k})} \lambda} + \sum_{j=1}^{n_j} \frac{m_{\theta}^{(j)} \omega^{(j)} c_{\mathrm{n+1}}}{1 + m_{\theta}^{(j)} \Delta^{(\mathrm{k})} \lambda}\right) \Delta^{(\mathrm{k})} \lambda - Y_{\mathrm{n+1}}^{(\mathrm{k})} \tag{A15}$$

$$\frac{d\mathbf{n}_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} = \frac{\mathbf{I} - \mathbf{n}_{\mathrm{n+1}}^{(\mathrm{k})} \otimes \mathbf{n}_{\mathrm{n+1}}^{(\mathrm{k})}}{\left\|\overline{\boldsymbol{\xi}}_{\mathrm{n+1}}^{(\mathrm{k})}\right\|} : \left(\sum_{i=1}^{n_i} \frac{m_{\beta}^{(i)} \boldsymbol{\beta}_{\mathrm{n}}^{(i)}}{\left(1 + m_{\beta}^{(i)} \Delta^{(\mathrm{k})} \lambda\right)^2} + \sum_{j=1}^{n_j} \frac{m_{\theta}^{(j)} \boldsymbol{\theta}_{\mathrm{n}}^{(j)}}{\left(1 + m_{\theta}^{(j)} \Delta^{(\mathrm{k})} \lambda\right)^2}\right) \tag{A16}$$

$$\frac{d\Delta^{(\mathrm{k})} r}{d\Delta\lambda} = \begin{cases} h \cdot \mathbf{n}_{\mathrm{n+1}}^{*(\mathrm{k})} : \left(\mathbf{n}_{\mathrm{n+1}}^{(\mathrm{k})} + \Delta^{(\mathrm{k})} \lambda \dfrac{d\mathbf{n}_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda}\right) & \left|g_{\mathrm{n+1}}^{\mathrm{trial}}\right| > \mathrm{TOL2} \\ 0 & \left|g_{\mathrm{n+1}}^{\mathrm{trial}}\right| \le \mathrm{TOL2} \end{cases} \tag{A17}$$

$$\frac{dQ_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} = \frac{m_{\mathrm{Q}}\left(Q_{\infty} - Q_{\mathrm{n}}\right)}{\left(1 + m_{\mathrm{Q}} \Delta r_{\mathrm{n}}\right)^2} \frac{d\Delta^{(\mathrm{k})} r}{d\Delta\lambda} \tag{A18}$$

$$\frac{dY_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} = \frac{m_{\mathrm{Y}}\left(Y_{\infty} - Y_{\mathrm{n}}\right)}{\left(1 + m_{\mathrm{Y}} \Delta r_{\mathrm{n}}\right)^2} \frac{d\Delta^{(\mathrm{k})} r}{d\Delta\lambda} \tag{A19}$$

$$\frac{dR_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} = \frac{m_{\mathrm{R}}\left(Q_{\mathrm{n+1}} - R_{\mathrm{n}}\right)}{\left(1 + m_{\mathrm{R}} \Delta\lambda\right)^2} + \frac{m_{\mathrm{R}} \Delta\lambda}{1 + m_{\mathrm{R}} \Delta\lambda} \frac{dQ_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} \tag{A20}$$

$$\frac{dc_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} = \frac{d\left(R_{\mathrm{n+1}} - Y_{\mathrm{n+1}}\right)}{d\Delta\lambda} = \frac{dR_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} - \frac{dY_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} \tag{A21}$$

$$\begin{aligned} f_{\mathrm{n+1}}^{(\mathrm{k})\prime} = {} & \sqrt{\frac{3}{2}} \frac{d\left\|\overline{\boldsymbol{\xi}}_{\mathrm{n+1}}^{(\mathrm{k})}\right\|}{d\Delta\lambda} - 3G - \sum_{i=1}^{n_i} \frac{m_{\beta}^{(i)} b^{(i)}}{\left(1 + m_{\beta}^{(i)} \Delta^{(\mathrm{k})} \lambda\right)^2} - \sum_{j=1}^{n_j} \frac{m_{\theta}^{(j)} \omega^{(j)} c_{\mathrm{n+1}}}{\left(1 + m_{\theta}^{(j)} \Delta^{(\mathrm{k})} \lambda\right)^2} \\ & - \sum_{j=1}^{n_j} \frac{m_{\theta}^{(j)} \omega^{(j)} \Delta^{(\mathrm{k})} \lambda}{1 + m_{\theta}^{(j)} \Delta^{(\mathrm{k})} \lambda} \frac{dc_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} - \frac{dY_{\mathrm{n+1}}^{(\mathrm{k})}}{d\Delta\lambda} \end{aligned} \tag{A22}$$

$$\Delta^{(\mathrm{k+1})} \lambda = \Delta^{(\mathrm{k})} \lambda - \frac{f_{\mathrm{n+1}}^{(\mathrm{k})}}{f_{\mathrm{n+1}}^{(\mathrm{k})\prime}} \tag{A23}$$

4. Updating the stress and state variables by using $\Delta\lambda$ from STEP 3

$$\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}} = \boldsymbol{\varepsilon}_{\mathrm{n}}^{\mathrm{p}} + \sqrt{\frac{3}{2}} \Delta\lambda \mathbf{n}_{\mathrm{n+1}} \tag{A24}$$

$$\boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{e}} = \boldsymbol{\varepsilon}_{\mathrm{n+1}} - \boldsymbol{\varepsilon}_{\mathrm{n+1}}^{\mathrm{p}} \tag{A25}$$

$$p_{\mathrm{n+1}} = p_{\mathrm{n}} + \Delta\lambda \tag{A26}$$

$$\boldsymbol{\beta}^{(i)}_{\text{n+1}} = \frac{\boldsymbol{\beta}^{(i)}_{\text{n}} + \sqrt{\frac{2}{3}} m^{(i)}_{\beta} b^{(i)} \Delta\lambda \mathbf{n}_{\text{n+1}}}{1 + m^{(i)}_{\beta} \Delta\lambda} \tag{A27}$$

$$\boldsymbol{\theta}^{(j)}_{\text{n+1}} = \frac{\boldsymbol{\theta}^{(j)}_{\text{n}} + \sqrt{\frac{2}{3}} m^{(j)}_{\theta} \omega^{(j)} c_{\text{n+1}} \Delta\lambda \mathbf{n}_{\text{n+1}}}{1 + m^{(j)}_{\theta} \Delta\lambda} \tag{A28}$$

$$r_{\text{n+1}} = r_{\text{n}} + \Delta r \tag{A29}$$

$$\Delta\eta = \sqrt{\frac{3}{2}} \left\langle g^{\text{trial}}_{\text{n+1}} \right\rangle \tag{A30}$$

$$\mathbf{q}_{\text{n+1}} = \mathbf{q}_{\text{n}} + (1-h)\Delta\eta \mathbf{n}^{*}_{\text{n+1}} \tag{A31}$$

5. Compute consistent tangent moduli

$$\mathbf{N}_{\text{n+1}} = \left(\mathbf{I} - \frac{\mathbf{1}\otimes\mathbf{1}}{3}\right) - \sqrt{\frac{3}{2}} \left(\sum_{i=1}^{n_i} \frac{m^{(i)}_{\beta} \boldsymbol{\beta}^{(i)}_{\text{n}}}{\left(1 + m^{(i)}_{\beta}\Delta\lambda\right)^2} + \sum_{j=1}^{n_j} \frac{m^{(j)}_{\theta} \boldsymbol{\theta}^{(j)}_{\text{n}}}{\left(1 + m^{(j)}_{\theta}\Delta\lambda\right)^2} \right) \otimes \frac{\mathbf{n}_{\text{n+1}}}{f'_{\text{n+1}}} \tag{A32}$$

$$\mathbf{J} = \mathbf{C} - 2\sqrt{6} G^2 \Delta\lambda \frac{\mathbf{I} - \mathbf{n}_{\text{n+1}} \otimes \mathbf{n}_{\text{n+1}}}{\left\| \overline{\boldsymbol{\xi}}_{\text{n+1}} \right\|} : \mathbf{N}_{\text{n+1}} + 6G^2 \frac{\mathbf{n}_{\text{n+1}} \otimes \mathbf{n}_{\text{n+1}}}{f'_{\text{n+1}}} \tag{A33}$$