

# A hybrid system using Brayton cycle to harvest the waste heat from a direct carbon solid oxide fuel cell

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**Abstract:** A new hybrid system model is developed to evaluate the potentials of a Brayton cycle heat engine for waste heat recovery from a direct carbon solid oxide fuel cell (DC-SOFC). The maximum power density of the proposed system is up to  $0.8675 \text{ Wcm}^{-2}$ , which is approximately 1.8 times as large as that of the single DC-SOFC. Numerical calculations also indicate that the proposed hybrid system is an efficient approach to boost the fuel utilization, and the maximum power density of the proposed system is markedly better than that of the DC-SOFC/thermophotovoltaic cell, DC-SOFC/thermionic generator, and DC-SOFC/Otto heat engine hybrid systems except for the DC-SOFC/Stirling engine hybrid system. The optimum regions for power density, efficiency and operating current density of the proposed system are determined. The higher operating temperature and lower gap between the anode and carbon layer increase the power density and efficiency of the proposed system. Moreover, the higher heat transfer coefficient boosts the power density and efficiency at high current density. The compression efficiency, expansion efficiency and recuperator coefficient significantly affect the power density and efficiency.

**Keywords:** Brayton cycle, Direct carbon solid oxide fuel cell, Hybrid system, Optimum working region, maximum power density

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## 1. Introduction

Fuel cells are electrochemical devices that directly convert chemical energy of fuels into electrical energy [1], and they are one of the most promising energy conversion technologies owing to their unique merits such as high efficiency, cleanliness and silent operation [2, 3]. Based on the types of electrolytes, fuel cells can be classified into the following categories [1, 4]: polymer electrolyte membrane fuel cell (PEMFC), high temperature PEMFC, direct methanol fuel cell (DMFC), molten carbonate fuel cell (MCFC), phosphoric acid fuel cell (PAFC), solid oxide fuel cell (SOFC), and alkaline fuel cell (AFC). Most of these kinds of fuel cells are fed with hydrogen. However, the generation and storage of hydrogen still suffer from several technology problems [5]. Therefore, people resort to find some cheap and abundant alternative fuels such as carbon, methane and natural gas [6].

A direct carbon fuel cell (DCFC) is a kind of fuel cells that employs carbon as fuel [5, 7], which provide advantages such as high theoretical thermal efficiency, abundant and cheap fuels and convenient CO<sub>2</sub> sequestration [8, 9]. DCFCs can be mainly classified into three types based on the electrolyte categories including molten hydroxide, molten carbonate, and solid oxide DCFCs [10]. Compared with other types, direct carbon solid oxide fuel cells (DC-SOFCs) are whole-solid state devices that avoid electrolyte degradation and leakage problems and offer significant advantages, such as fast electrochemical processes, concentrated CO<sub>2</sub> product streams and high-quality waste heat for combined heat and power (CHP) applications [11]. To enhance the DC-SOFC performance, most of the endeavors are made on electrolytes [12], fuels [13], catalysts [14], physical/chemical processes [15], electrodes [16], thermal management [17], and prototypes [18, 19]. Alternatively, harvesting the waste heat for additional power generation is also an alternative approach to improve the DC-SOFC performance [20, 21]. A number of thermal devices can serve as the bottoming cycles for the topping fuel cells, including the thermoelectric generator (TEG) [22], vacuum thermionic generator (VTIG) [23],

thermophotovoltaic cell (TPVC) [24], Brayton cycle [25], Stirling cycle [26], Braysson cycle [27, 28], heat engine [29], thermally regenerative electrochemical cycle (TREC) [30] and Rankine cycle [31]. Amongst, the Brayton cycles are much closer to practical applications owing to their compactness, maturity and lower cost [32]. The Brayton cycles have been extensively utilized in many areas such as waste heat recovery [33], nuclear energy [34], concentrated solar power [35], and coal-fired power plants. The Brayton cycles have been widely used as the bottoming or intermediate cycles for SOFCs and MCFCs. For example, Huang et al. [36] proposed a hybrid system consisting of the SOFC, Carnot cycle, and Brayton cycle and analyzed the effects of the operating temperature, fuel flow rate, and hydrogen utilization on the power density and efficiency of the hybrid system. Pirkandi et al. [37] presented the thermo-economic analyses for four different configurations of a solid oxide fuel cell/gas turbine system and determined the optimum configuration. Sánchez et al. [38] compared the MCFC combined with the supercritical carbon dioxide and air Brayton cycles. Obviously, the Brayton cycles can be readily used to harvest the waste heat generated by the DC-SOFCs and thus the advantages for the DC-SOFCs could be further reinforced. However, few studies regarding DC-SOFC/Brayton cycle hybrid system have been reported in the current literatures. Compared with the previous studies on the fuel cells/Brayton cycle hybrid systems, a more accurate model of the topping DC-SOFC is adopted, in which the electrochemical/chemical reactions, ionic/electronic charge transport, mass/momentum transport, and heat transfer are considered. Moreover, the maximum power densities for the available DC-SOFC-based hybrid systems are compared.

In this study, a Brayton cycle heat engine is proposed to recover the possible waste heat from a DC-SOFC. The thermal characteristics of the DC-SOFC are solved using the finite element method, from which the integration and assessment of the hybrid system will be demonstrated. The effectiveness and feasibility of this hybrid system will be shown by

comprehensive comparisons. Comparisons between the proposed hybrid system and the stand-alone DC-SOFC will be performed to evaluate the effectiveness of the proposed hybrid system. The feasibility will be justified by comparing the performance improvements of several existing waste heat recovery technologies. Meanwhile, the dependence of the hybrid system performance on some design parameters and operating conditions will be revealed through comprehensive parametric studies.

## 2. Model description

Figure 1 illustrated the energy-flow sketch of the DC-SOFC/ Brayton cycle heat engine hybrid system. It is seen that the hybrid system is mainly composed of a DC-SOFC, a Brayton cycle heat engine, an external heat source, and a regenerator.  $q_E$  is the heat flow provided to the DC-SOFC from the external heat source if necessary;  $q_h$  is the heat flow generated in the DC-SOFC and transferred to the Brayton cycle heat engine at large operating current densities;  $q_{Loss}$  is the heat loss from the DC-SOFC to the environment;  $q_l$  is the heat flow from the Brayton cycle heat engine to the environment;  $P_f$  and  $P_b$  are, respectively, the power outputs of the DC-SOFC and the Brayton cycle, and  $T_0$  is the ambient temperature. When the DC-SOFC works at a low current density, the heat resulted from the electrochemical reaction and overpotential losses is less than the heat demand by the Boudouard reaction. In this situation, the external heat source should provide an amount of heat  $q_E$  to the DC-SOFC for ensuring its normal operation. When the DC-SOFC runs at a high current density, the heat released in the cell could be larger than the heat demand by the Boudouard reaction. In this situation, the excessive heat  $q_h$  is collected and transferred to drive the Brayton cycle for additional power generation. As a result, the performance of the hybrid system can be improved. The regenerator is used to preheat the input reactants to reach the working temperature of the DC-SOFC with the assistance of the high-temperature exhaust gases. The DC-SOFC and Brayton cycle heat

engine are connected indirectly, and there is only heat exchange instead of mass exchange between the DC-SOFC and the Brayton cycle heat engine.

#### a) The DC-SOFC

The DC-SOFC model is adopted from Refs. [15, 39]. As illustrated in Fig. 2, the 2D tubular DC-SOFC mainly includes two electrodes, an electrolyte and gas channels, where  $d$  is the distance between the anode and the carbon layer. The DC-SOFC operates as a carbon internal dry reforming SOFC as per the “CO shuttle” mechanism. The solid carbon (C) reacts with initial oxygen ( $O_2$ ) to produce carbon dioxide ( $CO_2$ ) in the anode chamber, and the produced  $CO_2$  reacts with the solid carbon to generate carbon monoxide (CO) via the Boudouard reaction (i.e.,  $C+CO_2 \rightarrow 2CO$ ). The generated CO is electrochemically oxidized by the  $O^{2-}$  at the triple phase boundaries (TPBs) to produce  $CO_2$  and electrons. The produced  $CO_2$  flows back into the anode chamber and reacts with the carbon to produce CO via the Boudouard reaction. The generated electrons arrive the cathode via the external electric circuit and react with  $O_2$  to produce  $O^{2-}$ . The electrochemical reactions can be summarized as follows.



The power output  $P_f$  and the efficiency  $\eta_f$  of the DC-SOFC are, respectively, given by

$$P_f = i_f A_f V_f, \quad (4)$$

and

$$\eta_f = \begin{cases} \frac{P_f}{q_E + P_f}, & i_f < i_q \\ \frac{P_f}{q_f + P_f}, & i_f > i_q \end{cases}, \quad (5)$$

where  $i_f$  is the current density,  $A_f$  is the electrode area, and  $V_f$  is the output voltage,  $i_q$  is the critical operating current density that the heat demand by the Boudouard reaction is satisfied

by the heat released in the SOFC. The heat rejection rate from the DC-SOFC,  $q_r$ , is the difference between  $q_t$  resulted from the electrochemical reaction and overpotential losses and  $q_b$  required by the Boudouard reaction [39], i.e.,

$$q_r = q_t - q_b, \quad (6)$$

where  $q_t = [-T_f \Delta S / (2F) + (V_{act} + V_{ohm})] i_f A_f$ ,  $F$  is the Faraday's constant,  $T_f$  is the operating temperature of the DC-SOFC, and  $\Delta S$  is the molar entropy change of the electrochemical reactions.

The governing equations for the DC-SOFC model are listed in Table 1 [15] and can be solved by the finite element method using commercial COMSOL MULTIPHYSICS® based on the parameters summarized in Table 2 [15, 40]. The heat flux  $q_r^* = q_r / A_f$  from the DC-SOFC under different operating temperatures are shown in Fig. 3, where  $i_q$  is the operating current density at which  $q_t$  equals  $q_b$ , i.e.,  $q_r^* = 0$ . As seen in Fig. 3, when  $i_f < i_q$  (i.e.,  $q_t < q_b$ ), some heat should be provided to the DC-SOFC from the external heat source to ensure the DC-SOFC normal operation. When  $i_f > i_q$  (i.e.,  $q_t > q_b$ ), the excessive waste heat released from the DC-SOFC is transferred to drive the Brayton cycle for additional power generation. It is also seen that  $q_r^*$  is increased as  $i_f$  increases. Moreover,  $q_r^*$  decreases while  $i_q$  increases as the fuel cell operating temperature  $T_f$  increases.

#### b) The Brayton cycle

The Brayton cycle model is adopted from Refs. [36, 42]. As shown in Fig. 1, the Brayton cycle heat engine consists of a compressor (C), a turbine (T), a recuperator (REC), and two heat exchangers (i.e., HE1 and HE2). Working substance with low pressure and temperature enters the compressor at state 1 and is compressed. The compressed working substance flows into the recuperator and is heated to a temperature of  $T_5$ . The heated working substance is further heated by absorbing the heat  $q_h$  generated in the DC-SOFC through the HE1.

Afterwards, the high-pressure and high-temperature working substance enters the turbine and expands to produce power. The exhaust gas of the turbine is used to preheat the inlet working substance through the recuperator and further cooled down after passing through HE2. The leaving working substance with low pressure and temperature enters the compressor to complete the cycle. The  $T-S$  diagram of the Brayton cycle is shown in Fig. 4, where  $T_i$  ( $i=1, 2, 2s, 3, 4, 4s, 5$  or  $6$ ) is the temperature of the working substance at operating state  $i$ ,  $1 \rightarrow 2$  and  $3 \rightarrow 4$  are two irreversible adiabatic processes,  $1 \rightarrow 2s$  and  $3 \rightarrow 4s$  are, respectively, the corresponding reversible adiabatic processes.

Assuming that the heat transfer obeys Newton's law, the heat flow from the DC-SOFC to the Brayton cycle is given by [42]

$$q_h = C \dot{m} \varepsilon_h (T - T_5) = C \dot{m} (T_3 - T_5) = h_h A_h (T_3 - T_5) \left/ \left( \ln \frac{T - T_5}{T - T_3} \right) \right., \quad (12)$$

where  $T$  is the temperature of the heat source,  $h_h$  and  $A_h$  are, respectively, the heat transfer coefficient and heat transfer area between the heat source and working substance, and  $\varepsilon_h$  is the hot side coefficient. The heat flow from the Brayton cycle to the environment is given by

$$q_l = C \dot{m} \varepsilon_L (T_6 - T_0) = C \dot{m} (T_6 - T_1), \quad (13)$$

where  $C$  is the heat capacity,  $\dot{m}$  is the mass flow rate,  $\varepsilon_L$  is the cold side coefficient. The compression and expansion efficiencies are introduced to describe the irreversibility of these processes, i.e.,

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}, \quad (14)$$

and

$$\eta_e = \frac{T_3 - T_4}{T_3 - T_{4s}}, \quad (15)$$

and both  $\eta_c$  and  $\eta_e$  are ranged from 0 to 1. When  $\eta_c = \eta_e = 1$ , it means that the two adiabatic processes are reversible [43]. According to the characteristics of the reversible adiabatic

processes, we have [36, 42]

$$\left(\frac{T_{2s}}{T_1}\right) = \left(\frac{T_3}{T_{4s}}\right) = \left(\frac{p_{B1}}{p_{B2}}\right)^{(k-1)/k} = x, \quad (16)$$

where  $p_{B1}$  and  $p_{B2}$  are, respectively, the pressures at the processes of  $2 \rightarrow 3$  and  $1 \rightarrow 4$ ,  $k$  is the ratio of specific heats. Based on Eqs. (12)-(16),  $T_2$  and  $T_4$  can be, respectively, expressed as

$$T_2 = T_1 \left( \frac{x-1}{\eta_c} + 1 \right), \quad (17)$$

and

$$T_4 = T_3 \left[ 1 - \eta_e \left( 1 - \frac{1}{x} \right) \right]. \quad (18)$$

The temperatures  $T_1$ ,  $T_3$ ,  $T_5$ , and  $T_6$  can be, respectively, expressed as [36, 44]

$$T_1 = \varepsilon_L T_0 + (1 - \varepsilon_L) T_2 \varepsilon_{rc} + (1 - \varepsilon_L)(1 - \varepsilon_{rc}) T_4, \quad (19)$$

$$T_3 = \varepsilon_h T + (1 - \varepsilon_{rc}) T_4 \varepsilon_{rc} + (1 - \varepsilon_{rc})(1 - \varepsilon_{rc}) T_2, \quad (20)$$

$$T_5 = T_4 \varepsilon_{rc} + (1 - \varepsilon_{rc}) T_2, \quad \square\square\square\square(21)$$

and

$$T_6 = T_2 \varepsilon_{rc} + (1 - \varepsilon_{rc}) T_4, \quad (22)$$

where  $\varepsilon_{rc}$  is the recuperator coefficient.

Combining Eqs. (17), (19) and (22), we can obtain the relationship between  $T_4$  and  $T_1$ , i.e.,

$$T_4 = \frac{T_1 - \left[ \varepsilon_L T_0 + \varepsilon_{rc} T_1 (1 - \varepsilon_L) \left( \frac{x-1}{\eta_c} + 1 \right) \right]}{(1 - \varepsilon_{rc})(1 - \varepsilon_L)}. \quad (23)$$

Substituting Eqs. (17) and (23) into Eqs. (20) and (21), the relationship between  $T_3$  and  $T_1$  as well as the relationship between  $T_5$  and  $T_1$  can be, respectively, given by



$$T_3 = \varepsilon_h T + T_1 \left( \frac{x-1}{\eta_c} + 1 \right) (1 - \varepsilon_{rc}) (1 - \varepsilon_h) + \varepsilon_{rc} (1 - \varepsilon_h) \left\{ \frac{T_1 - \left[ \varepsilon_L T_0 + \varepsilon_{rc} T_1 (1 - \varepsilon_L) \left( \frac{x-1}{\eta_c} + 1 \right) \right]}{(1 - \varepsilon_{rc}) (1 - \varepsilon_L)} \right\}, \quad (24)$$

and

$$T_5 = \left[ 1 - \eta_e \left( 1 - \frac{1}{x} \right) \right] \varepsilon_{rc} \varepsilon_h T + T_1 \left( \frac{x-1}{\eta_c} + 1 \right) \left[ 1 - \eta_e \left( 1 - \frac{1}{x} \right) \right] (1 - \varepsilon_{rc}) (1 - \varepsilon_h) \varepsilon_{rc} + \left\{ \frac{T_1 - \left[ \varepsilon_L T_0 + \varepsilon_{rc} T_1 (1 - \varepsilon_L) \left( \frac{x-1}{\eta_c} + 1 \right) \right]}{(1 - \varepsilon_{rc}) (1 - \varepsilon_L)} \right\} \left[ 1 - \eta_e \left( 1 - \frac{1}{x} \right) \right] \varepsilon_{rc} \varepsilon_{rc} (1 - \varepsilon_h) + (1 - \varepsilon_{rc}) \left( \frac{x-1}{\eta_c} + 1 \right) T_1. \quad (25)$$

Combining Eqs. (16) and (18),  $T_{4s}$  can be expressed as

$$T_{4s} = T_3 - \frac{T_3 - T_4}{\eta_e}. \quad (26)$$

Substituting Eqs. (24) and (26) into  $T_3/T_{4s} = x$ , the relationship between  $T_1$  and  $x$  is determined by

$$x = \frac{\varepsilon_h \eta_e T + \eta_e T_1 \left( \frac{x-1}{\eta_c} + 1 \right) (1 - \varepsilon_{rc}) (1 - \varepsilon_h) + \varepsilon_{rc} \eta_e (1 - \varepsilon_h) f_2}{(\eta_e - 1) \left\{ \varepsilon_h T + T_1 \left( \frac{x-1}{\eta_c} + 1 \right) (1 - \varepsilon_{rc}) (1 - \varepsilon_h) + \varepsilon_{rc} (1 - \varepsilon_h) f_2 \right\} + f_2}, \quad (27)$$

$$\text{where } f_1 = \left[ \varepsilon_L T_0 + \varepsilon_{rc} T_1 (1 - \varepsilon_L) \left( \frac{x-1}{\eta_c} + 1 \right) \right], \text{ and } f_2 = \frac{T_1 - f_1}{(1 - \varepsilon_{rc}) (1 - \varepsilon_L)}.$$

Thus, the efficiency  $\eta_B$  and the power output  $P_B$  of the Brayton cycle can be, respectively, given by

$$\eta_B = 1 - \frac{q_l}{q_h} = 1 - \frac{T_6 - T_1}{T_3 - T_5} = 1 - \frac{T_1 \left( \frac{x-1}{\eta_c} + 1 \right) \varepsilon_{rc} + (1 - \varepsilon_{rc}) \left[ 1 - \eta_e \left( 1 - \frac{1}{x} \right) \right] [T_5 (1 - \varepsilon_{rc}) + \varepsilon_h T] - T_1}{\varepsilon_h T - \varepsilon_{rc} T_5}, \quad (28)$$

and

$$P_B = q_h - q_L = q_h \eta_B. \quad (29)$$

### c) The performance of the hybrid system

Assuming the heat dissipated from the DC-SOFC to the environment obeys radiative and

convective laws, the net heat flow transferred to the Brayton cycle heat engine is given by

$$q_n = q_r - q_{\text{Loss}}, \quad (30)$$

where  $q_{\text{Loss}} = (A_f - A_E)[h_{\text{Loss}}(T_f - T_0) + \zeta\delta(T_f^4 - T_0^4)]$  is heat flow rejected from the DC-SOFC into the ambience,  $h_{\text{Loss}}$  is the convention heat transfer coefficient,  $A_E$  is the front area of the heat exchanger 1,  $\delta$  is the Stefan-Boltzmann constant, and  $\zeta$  is the emissivity of the DC-SOFC.

The DC-SOFC acts as the heat source of the Brayton cycle with the assumption that the temperature of the heat transferred to the Brayton cycle is the same as the operating temperature of the fuel cell i.e.,  $T = T_f$ . Combining Eq. (12) and Eq. (30), the mathematical relationship between  $x$  and  $i_f$  is given by the following formula:

$$\frac{q_n}{h_h A_h} = \frac{\varepsilon_h T - \varepsilon_{rc} T_5}{\ln \frac{T - T_5}{T - T_5(1 - \varepsilon_{rc}) - \varepsilon_h T}}. \quad (31)$$

Eq. (31) can be explicitly rewritten as

$$q_r^* - (1 - b)[h_{\text{Loss}}(T_f - T_0) + \zeta\delta(T_f^4 - T_0^4)] = \frac{\varepsilon_h T - \varepsilon_{rc} T_5}{\ln \frac{T - T_5}{T - T_5(1 - \varepsilon_{rc}) - \varepsilon_h T}} a, \quad (32)$$

where  $q_r^* = q_r / A_f$ ,  $a = k_h A_h / A_f$ ,  $b = A_E / A_f$ , and  $\zeta\delta T_0^3 / h_{\text{Loss}} = 1.5$  [39].

Considering the contribution of the bottoming Brayton cycle heat engine, the power output  $P$  and efficiency  $\eta$  of the hybrid system can be, respectively, expressed as

$$P = P_f + P_B, \quad (33)$$

and

$$\eta = \frac{P}{q_r + P_f}. \quad (34)$$

#### d) Model validation

An up-to-date literature survey shows that there is no experimental investigation reported on this specific hybrid system. Based on the parameters used in the DC-SOFC and Brayton cycle as summarized in Tables 2 and 3, the DC-SOFC and Brayton cycle models can be

validated. The DC-SOFC model has been validated in Refs. [15] and [39]. Ref. [39] had shown that the modeling results were in good agreement with the experimental data from Ref. [45], suggesting that the DC-SOFC model could accurately predict the performance of the DC-SOFC. Based on Refs. [25], [42] and [44], the expressions of the efficiency and power output for the Brayton cycle can be derived. To validate the Brayton cycle model, the efficiency of the Brayton cycle is compared with that derived from Sánchez et al. [38], as shown in Fig. 5. Fig. 5 shows that the efficiencies difference between the present Brayton cycle model and the referred model is negligible small, indicating that the present Brayton cycle model is valid and reliable.

### 3. Results and Discussion

According to the parameter summarized in Table 2 and 3, the numerical study is carried out to predict the performance of the hybrid system. It is observed from Fig. 6 that  $x$  and  $T_1$  monotonically increase as  $i_f$  increases. This is because the heat generated by the DC-SOFC increases as  $i_f$  increases, which results in a variation of  $T_5$ , as shown by Eq. (32).

The curves of the efficiencies ( $\eta_f$ ,  $\eta_B$ , and  $\eta$ ) and the power densities ( $P_f^* = P_f/A_f$ ,  $P_B^* = P_B/A_f$ , and  $P^* = P/A_f$ ) for the DC-SOFC, Brayton cycle, and hybrid system are shown in Fig. 7, where  $i_s$  is the starting operating current density from which the Brayton cycle begins to work,  $P_m^*$  and  $P_{f,m}^*$  are, respectively, the maximum power densities for the hybrid system and the DC-SOFC,  $i_p$  and  $i_{f,p}$  are, respectively, the current densities at  $P_m^*$  and  $P_{f,m}^*$ ,  $\eta_p$  and  $\eta_{f,p}$  are, respectively, the efficiencies of the hybrid system and the DC-SOFC at  $P_m^*$  and  $P_{f,m}^*$ . Fig. 7 shows that when  $i_f$  is larger than  $i_s$ , both  $\eta_f$  and  $\eta$  monotonically decrease with increasing  $i_f$ , while  $P_f^*$  and  $P^*$  first increase and then decrease with optimum operating current densities at which  $P_m^*$  and  $P_{f,m}^*$  can be achieved. Moreover,  $\eta$  and  $P^*$  are markedly higher than  $\eta_f$  and  $P_f^*$ , especially at high current densities. It shows that the

Brayton cycle is effective to recover the waste heat from the DC-SOFC for additional power generation. For the parameters given in Table 1, numerical calculations show that  $P_m^*$  with value of  $0.8675 \text{ W cm}^{-2}$  is attained at a  $i_f$  of  $3.275 \text{ A cm}^{-2}$ , while  $P_{f,m}^*$  with value of  $0.4773 \text{ W cm}^{-2}$  is achieved at a  $i_f$  of  $2.06 \text{ A cm}^{-2}$ .  $P_m^*$  is approximately 81.8% larger than  $P_{f,m}^*$ . Furthermore,  $\eta_p$  and  $\eta_{f,p}$  are 65.29% and 50.61%, respectively.

Fig. 8 shows that  $P^*$  continuously decreases as  $\eta$  is increased from  $\eta_p$ , and vice versa.  $P^*$  and  $\eta$  are two objective functions that conflict with each other. How to make trade-offs between them is an important problem for optimizing the whole system. A simple and available method is to introduce a new objective function that simultaneously considers  $P^*$  and  $\eta$ , such as efficient power [46, 47]. The efficient power is defined as the product of the power output and efficiency, i.e.,  $P \times \eta$ . The point D on the  $P^* \sim \eta$  curve corresponds to the special case that the efficient power ( $P \times \eta$ ) achieves its maximum value, where  $P_D^*$  and  $\eta_D$  are  $P^*$  and  $\eta$  at the point D, respectively. When  $\eta < \eta_p$ ,  $P^*$  increases as  $\eta$  increases. When  $\eta > \eta_D$ , both  $\eta P^*$  and  $P^*$  decrease as  $\eta$  increases. Therefore, the operating regions for  $P^*$ ,  $\eta$  and  $i_f$  can be, respectively, determined as

$$P_D^* \leq P^* \leq P_m^*, \quad (35)$$

$$\eta_D \geq \eta \geq \eta_p, \quad (36)$$

and

$$i_{f,D} \leq i_f \leq i_{f,p}, \quad (37)$$

where  $i_{f,D}$  is the operating current density at the point D.

$T_f$  not only affects the DC-SOFC performance but also affects the amount of the heat released from the DC-SOFC. The DC-SOFC performance is improved as  $T_f$  is increased since the Boudouard reaction kinetics, electrochemical reaction kinetics and ionic conduction are all improved at a higher  $T_f$ . In addition, the amount of the heat released from the DC-SOFC is

reduced as  $T_f$  is increased. The effects of  $T_f$  on  $P^*$  and  $\eta$  are displayed in Fig. 9, where  $d = 59 \mu\text{m}$  and  $a = 60 \text{ W m}^{-2} \text{ K}^{-1}$ . Fig. 9 shows that both  $P^*$  and  $\eta$  are improved as  $T_f$  increases. Moreover, the critical current density  $i_q$  and the starting current density  $i_s$  shift rightward with an increase in  $T_f$  because the heat generated at the higher temperature is less than that at the lower temperature. However,  $P_f^*$  significantly increases as  $T_f$  increases. As a result, both  $P^*$  and  $\eta$  are obviously promoted with an increase in  $T_f$ .

The fuel carbon monoxide (CO) concentration and the solid carbon consumption rate of the DC-SOFC are closely related with  $d$ . CO is produced from the Boudouard reaction and then consumed by the electrochemical reaction in the porous anode. When  $d$  is small, the CO fraction in the porous anode is high. As  $d$  increases, the transport of CO from the carbon chamber to the porous anode becomes more difficult, and therefore, the CO fraction in the porous anode decreases as  $d$  increases [15]. As seen from Fig. 10, both  $P^*$  and  $\eta$  are improved as  $d$  decreases because the DC-SOFC performs better at a smaller distance which benefits the CO concentration. The effects of  $d$  is initiated from  $i_s$  and become strong as  $i_f$  increases. Moreover, the values of  $i_q$  and  $i_s$  get larger as  $d$  decreases.

Fig. 11 shows both  $P^*$  and  $\eta$  increase with the increase of  $a$  in a wide range of high current densities. A higher  $a$  results in a smaller thermal resistance in the heat-transfer process between DC-SOFC and Brayton cycle heat engine, which in return yields a higher power output for the bottoming Brayton cycle heat engine.

To further reveal the influence of the Brayton cycle performance on the overall performance of the hybrid system, the effects of the compression efficiency  $\eta_c$  and expansion efficiency  $\eta_e$  as well as the recuperator coefficient  $\varepsilon_{rc}$  on  $P^*$  and  $\eta$  are discussed. The performance of the DC-SOFC/ Brayton cycle hybrid systems with different  $\eta_c$ ,  $\eta_e$  and  $\varepsilon_{rc}$  are shown in Fig. 12. It is shown that both  $P^*$  and  $\eta$  would be enhanced if a higher  $\eta_c$ ,  $\eta_e$  or  $\varepsilon_{rc}$  is

chosen. This result suggests that the performance of the DC-SOFC/ Brayton cycle hybrid system has the potential to be further improved. Fig. 13 compares the DC-SOFC-based hybrid systems available in literatures. It is seen that  $P_m^*$  of the DC-SOFC/ Brayton cycle hybrid system is smaller than that of the DC-SOFC/Stirling cycle hybrid system, but it is higher than that of the DC-SOFC/TPVC, DC-SOFC/VTIG, and DC-SOFC/Otto heat engine hybrid systems in a wide temperature span. Due to the disadvantages of the low power/weight ratio, unchanging torque and power swiftly, and high cost of Stirling cycle [48], the Brayton cycle can be still deemed as a competitive alternative for waste heat recovery from the DC-SOFCs. In an actual case, the economic feasibility of the DC-SOFC/Brayton cycle hybrid system should be carefully considered with respect to that of the DC-SOFC. The introduction of Brayton cycle would lead to the increase of investment and running costs. However, the cost increases can be easily recovered by the efficiency gain for a long running time.

#### 4. Conclusions

The hybrid system consisting of the DC-SOFC and the Brayton cycle heat engine is proposed to utilize the waste heat released from the 2D tubular DC-SOFC. The overall heat released in the DC-SOFC can be lower than, equal to or higher than the heat demand by the Boudouard reaction, and consequently, how to operate and evaluate the hybrid system are stated in detail. Compared with the stand-alone DC-SOFC with a maximum power density of  $0.4773 \text{ Wcm}^{-2}$ , the hybrid system can achieve a higher maximum power density of  $0.8675 \text{ Wcm}^{-2}$ . The maximum attainable power density of the DC-SOFC/ Brayton cycle hybrid system is higher than that of the DC-SOFC/TPVC, DC-SOFC/VTIG, and DC-SOFC/Otto heat engine hybrid systems except for the DC-SOFC/Stirling cycle hybrid system. The Brayton cycle can be treated as a competitive alternative waste heat recovery technology for DC-SOFCs. The optimal operating regions of the power density, efficiency and current density are determined through introducing a new objective function that simultaneously considers the power density

and efficiency. In addition, the influences of the working temperature of the DC-SOFC, distance between the anode and carbon layer, heat transfer coefficient, compression efficiency, expansion efficiency and recuperator coefficient on the performance of the hybrid system are discussed in detail. A higher operating temperature of the DC-SOFC or a lower distance between the anode and the carbon layer induces a better performance of the combined system. A higher compression efficiency, a larger expansion efficiency or a greater recuperator coefficient can also enhance the combined system performance. Moreover, the maximum power density of the hybrid system can be improved by designing a higher parameter  $a$ . The results obtained are helpful for the design and operation of such a DC-SOFC/ Brayton cycle hybrid system.

337    **Acknowledgments**

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341



$a$	Heat transfer coefficient between DC-SOFC and Brayton cycle heat engine, $\text{W m}^{-2}\text{K}^{-1}$
$A_E$	Front area of the heat exchanger, $\text{m}^2$
$A_f$	Electrode area, $\text{m}^2$
$A_h$	Heat transfer area between the heat source and working substance, $\text{m}^2$
$B_0$	Permeability, $\text{m}^2$
$C$	Heat capacity, $\text{J Kg}^{-1} \text{K}^{-1}$
$C_p$	Heat capacity of fluid, $\text{J K}^{-1}$
$C_{\text{CO}_2}$	Molar concentration of $\text{CO}_2$ , $\text{Kg mol m}^{-3}$
$d$	Distance between the anode and the carbon layer, $\text{m}$
$D_j^{\text{eff}}$	Effective diffusion coefficient of species $j$ , $\text{m}^2 \text{s}^{-1}$
$E_{\text{CO}}$	Equilibrium potential, $\text{V}$
$E_{\text{CO}}^0$	Standard potential, $\text{V}$
$E_{rb}$	Activation energy of Boudouard reaction, $\text{kJ mol}^{-1}$
$F$	Faraday's constant, $\text{J mol}^{-1}$
$h_{\text{LOSS}}$	Heat conduction/convection coefficient, $\text{W m}^{-2}\text{K}^{-1}$
$h_h$	Heat transfer coefficient between the heat source and working substance, $\text{W m}^{-2}\text{K}^{-1}$
$i_0$	Exchange current density, $\text{A m}^{-2}$
$i_f$	Current density of DC-SOFC, $\text{A m}^{-2}$
$i_s$	Starting operating current density, $\text{A m}^{-2}$
$i_q$	Critical operating current density, $\text{A m}^{-2}$
$k$	Ratio of specific heats
$k_{rb}$	Equilibrium constant of Boudouard reaction, $\text{s}^{-1}$
$\dot{m}$	Mass flow rate, $\text{Kg s}^{-1}$
$n$	Number of electrons transferred
$N_i$	Flux of mass transport, $\text{Kg m}^{-3}\text{s}^{-1}$
$P$	Power output of the hybrid system, $\text{W}$
$P^*$	Power density of the hybrid system, $\text{W m}^{-2}$
$P_B$	Power output of the Brayton cycle, $\text{W}$
$P_B^*$	Power density of the Brayton cycle, $\text{W m}^{-2}$
$P_f$	Power output of the DC-SOFC, $\text{W}$
$P_f^*$	Power density of the DC-SOFC, $\text{W m}^{-2}$
$p$	Partial pressure, $\text{Pa}$
$p_{B1}$	Pressure at the processes of $2 \rightarrow 3$ in Brayton cycle, $\text{Pa}$
$p_{B2}$	Pressure at the processes of $1 \rightarrow 4$ in Brayton cycle, $\text{Pa}$
$\dot{q}$	Heat consumption or generation, $\text{J s}^{-1}$
$q_l$	Heat flow resulted from the electrochemical reaction and overpotential losses, $\text{W}$
$q_b$	Heat flow required by the Boudouard reaction, $\text{W}$
$q_E$	Heat flow provided to the DC-SOFC from the external heat source, $\text{W}$
$q_h$	Heat flow generated in the DC-SOFC and transferred to the Brayton cycle heat engine, $\text{W}$
$q_l$	Heat flow from the Brayton cycle heat engine to the environment, $\text{W}$
$q_r$	Heat flow rejected from the DC-SOFC, $\text{W}$

$q_r^*$	Heat flux from the DC-SOFC $q_r / A_f$ , $\text{W m}^{-2}$
$q_{\text{Loss}}$	Heat loss from the DC-SOFC to the environment, $\text{W}$
$R$	Gas constant, $\text{J mol}^{-1}\text{K}^{-1}$
$R_{rb}$	Reaction rate of Boudouard reaction, $\text{mol m}^{-3}\text{s}^{-1}$
$\Delta S$	Molar entropy change of the electrochemical reactions, $\text{J mol}^{-1}$
$T_0$	Ambient temperature, $\text{K}$
$T_f$	Operating temperature of the DC-SOFC, $\text{K}$
$T_i$ ( $i=1,2,3,4,5,6$ )	Temperatures of the working substance at operating state $i$ , $\text{K}$
$T$	Temperature, $\text{K}$
$u$	Velocity vector, $\text{ms}^{-1}$
$V_f$	Output voltage, $\text{V}$
$V_{\text{act}}$	Activation overpotential, $\text{V}$
$V_{\text{ohm}}$	Ohmic overpotential, $\text{V}$
$y_j$	Mole fraction of component $j$
<i>Greek letters</i>	
$\alpha$	Electron transfer coefficient,
$\delta$	Stefan-Boltzmann constant, $\text{W m}^{-2}\text{K}^{-4}$
$\varepsilon_{rc}$	Recuperator coefficient
$\varepsilon_h$	Hot side coefficient
$\varepsilon_L$	Cold side coefficient
$\zeta$	Emissivity of the DC-SOFC
$\eta$	Efficiency of the hybrid system
$\eta_c$	Compression efficiency
$\eta_e$	Expansion efficiency
$\eta_f$	Efficiency of the DC-SOFC
$\lambda_{\text{eff}}$	Effective heat conductivity, $\text{W m}^{-1}\text{K}^{-1}$
$\mu$	Dynamic viscosity of fluid, $\text{Pa s}$
$\rho$	Density, $\text{Kg m}^{-3}$
$\sigma^{\text{eff}}$	Effective conductivity, $\text{S m}^{-1}$
$\phi$	Potential, $\text{V}$
<i>Subscripts</i>	
D	Point D
f	Fuel cell
j	Species ( $\text{CO}_2$ , $\text{CO}$ , $\text{O}_2$ )
P	Power
m	Maximum
<i>Abbreviations</i>	
DC-SOFC	Direct carbon solid oxide fuel cell
DCFC	Direct carbon fuel cell

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463    **List of Table**

464    Table 1. Governing equations in DC-SOFC [15].

465    Table 2. The parameters used in the DC-SOFC model [15, 40].

466    Table 3. The parameters used in the Brayton cycle model [33, 38].

467



Table 1.

<b>Chemical reaction</b>	
Boudouard reaction	$C + CO_2 = 2CO$
Reaction rate of Boudouard reaction ( $\text{mol m}^{-3} \text{s}^{-1}$ )	$R_{rb} = k_{rb} C_{CO_2} \exp(-E_{rb} / RT)$
<b>Electrochemical reaction</b>	
Anode electrochemical reaction	$2CO + 2O^{2-} \rightarrow 2CO_2 + 4e^-$
Cathode electrochemical reaction	$O_2 + 4e^- \rightarrow 2O^{2-}$
Output voltage (V)	$V_f = E_{CO} - V_{act} - V_{ohm}$
Equilibrium potential (V)	$E_{CO} = E_{CO}^0 + \frac{RT}{2F} \ln \left[ \frac{p_{CO}^L (p_{O_2}^L)^{0.5}}{p_{CO_2}^L} \right]$
Standard potential (V)	$E_{CO}^0 = 1.46713 - 0.000452T$
Butler-Volmer equation	$i_f = i_0 \left\{ \exp\left(\frac{\alpha n F V_{act}}{RT}\right) - \exp\left[\frac{(1-\alpha) n F V_{act}}{RT}\right] \right\}$
Ohmic potential (V)	$\nabla(\phi) = \frac{\sigma^{eff}}{i}$
<b>Species transport</b>	
Extended Fick's model	$N_i = -\frac{1}{RT} \left( \frac{B_0 y_i p}{\mu} \frac{\partial p}{\partial z} - D_j^{eff} \frac{\partial (y_i p)}{\partial z} \right) \quad (j = 1, \dots, m)$
<b>Momentum conservation</b>	
Navier-Stokes equation	$\rho \frac{\partial u}{\partial t} + \rho u \nabla u = -\nabla p + \nabla [\mu (\nabla u + (\nabla u)^T) - \frac{2}{3} \mu \nabla u]$
<b>Heat transfer</b>	
General heat balance equation	$\rho C_p u \cdot \nabla T + \nabla \cdot (-\lambda_{eff} \nabla T) = \dot{q}$

472 Table 2.

Parameters	Value or expression
<i>Ionic conductivity, <math>\text{S m}^{-1}</math></i>	
Gadolinium doped ceria (GDC)	$(100/T)10^{(6.66071-5322.92/T)}$
Yttrium stabilized zirconium (YSZ)	$33400\exp(-10300/T)$
<i>Electronic conductivity, <math>\text{S m}^{-1}</math></i>	
Ag	$\frac{1.59e^8}{0.0038T - 0.1134}$
Porosity	
Cathode	0.46
Anode	0.46
<i>Electrode volume fraction</i>	
GDC	0.21
Ag	0.79
STPB	
Cathode layer,	$2.14 \times 10^5$
Anode layer,	$2.14 \times 10^5$
<i>Tortuosity</i>	
Cathode	3
Anode	3
<i>Exchanger current density, <math>\text{A m}^{-2}</math></i>	
CO	450
O <sub>2</sub>	400
<i>Charge transfer coefficient,</i>	
CO	0.5
O <sub>2</sub>	0.5
Equilibrium constant of Boudouard reaction $k_{rb}, \text{ s}^{-1}$	$6 \times 10^{13}$
Activation energy of Boudouard reaction $E_{rb}, \text{ kJ mol}^{-1}$	248

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474

475      Table 3

Parameters	Value
Compression efficiency, $\eta_c$	0.91
Expansion efficiency, $\eta_e$	0.94
Recuperator coefficient, $\varepsilon_{rc}$	0.98
Cold side coefficient, $\varepsilon_L$	0.90
Hot side coefficient, $\varepsilon_h$	0.90
Environment temperature, $T_0$ (K)	300

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**Fig. 4.** Temperature-Entropy diagram of a Brayton cycle.

**Fig. 5.** Validation of the Brayton cycle model with the Sánchez et al. [38], where  $x=1.355$ ,  $\eta_c = 0.80$ ,  $\eta_e = 0.85$ ,  $\varepsilon_L = 0.90$ ,  $\varepsilon_h = 0.90$ ,  $T = 923\text{K}$ , and  $T_0 = 300\text{K}$ .

**Fig. 6.** The curves of  $x$  and  $T_1$  versus the current density  $i_f$ , where  $T_f = 1123\text{K}$ ,  $d = 59\mu\text{m}$ , and  $a = 60\text{W m}^{-2}\text{K}^{-1}$ .

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**Fig. 8.** The curve of the power density varying with the efficiency of the hybrid system.

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**Fig. 10.** The effects of the distance between the anode and carbon layer on the (a) efficiency and the (b) power density of the hybrid system, where  $T_f = 1123\text{K}$  and  $a = 60\text{W m}^{-2}\text{K}^{-1}$ .

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**Fig. 12.** The (a) efficiency and (b) the power density of the DC-SOFC/ Brayton cycle hybrid system with different compression  $\eta_c$  and expansion  $\eta_e$  efficiencies as well as the recuperator coefficient  $\varepsilon_{rc}$ , where  $\eta_c = 0.85$ ,  $\eta_e = 0.90$  and  $\varepsilon_{rc} = 0.92$  [33, 38].

**Fig. 13.** The maximum power densities of the available DC-SOFC-based hybrid systems.

Fig. 1.

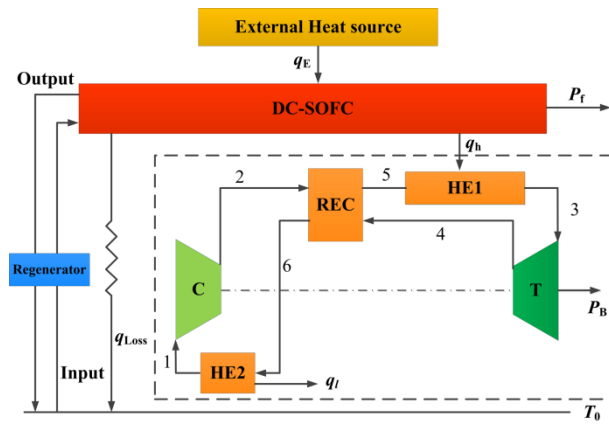
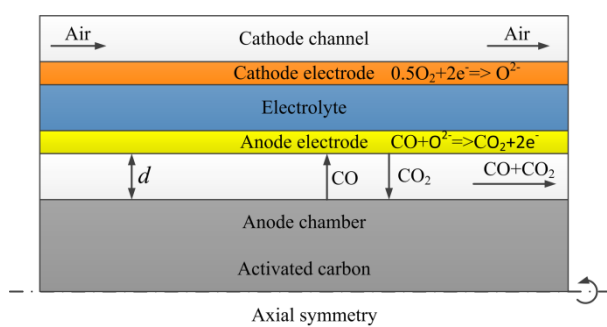
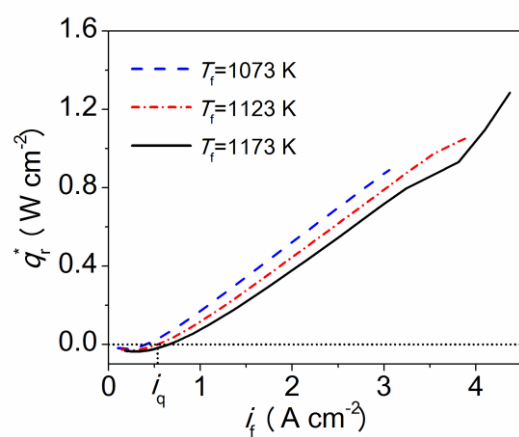


Fig. 2.

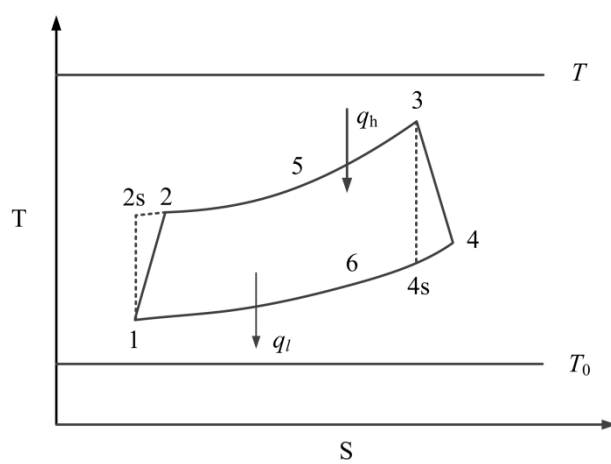


508 Fig. 3.



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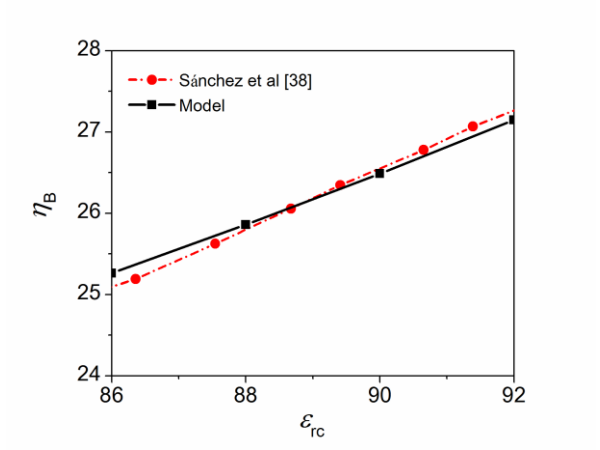
511 Fig. 4.



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514 Fig. 5.



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517 Fig. 6.

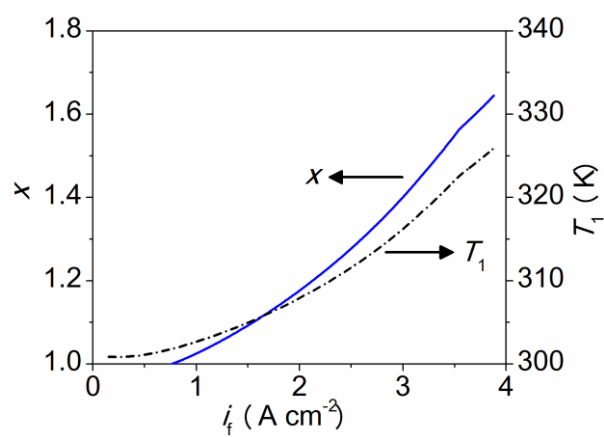
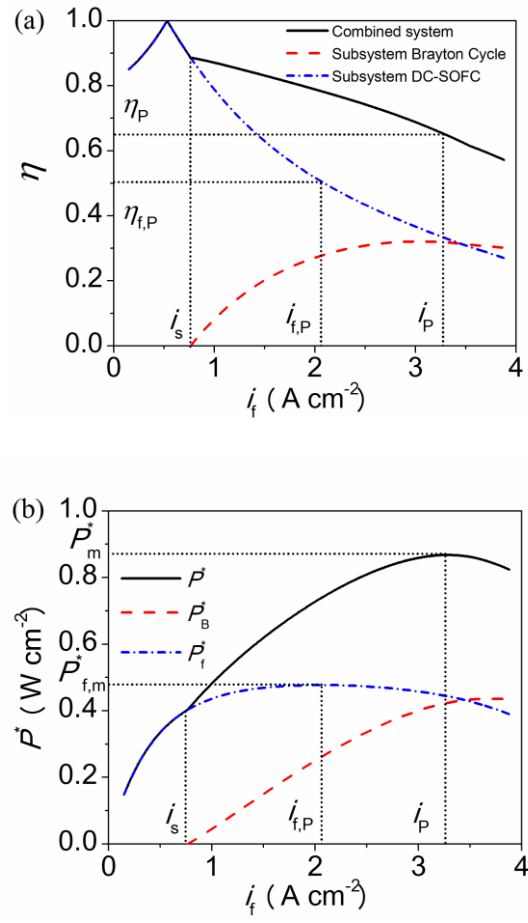
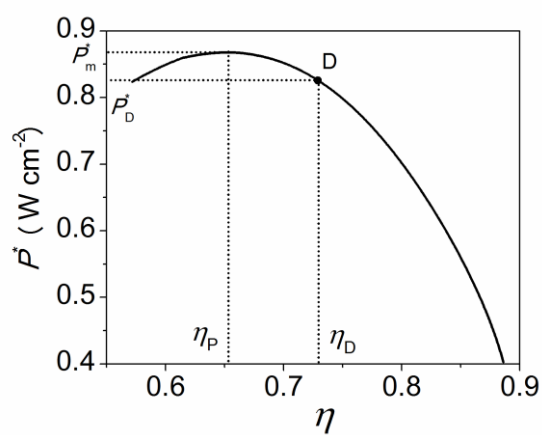


Fig. 7.

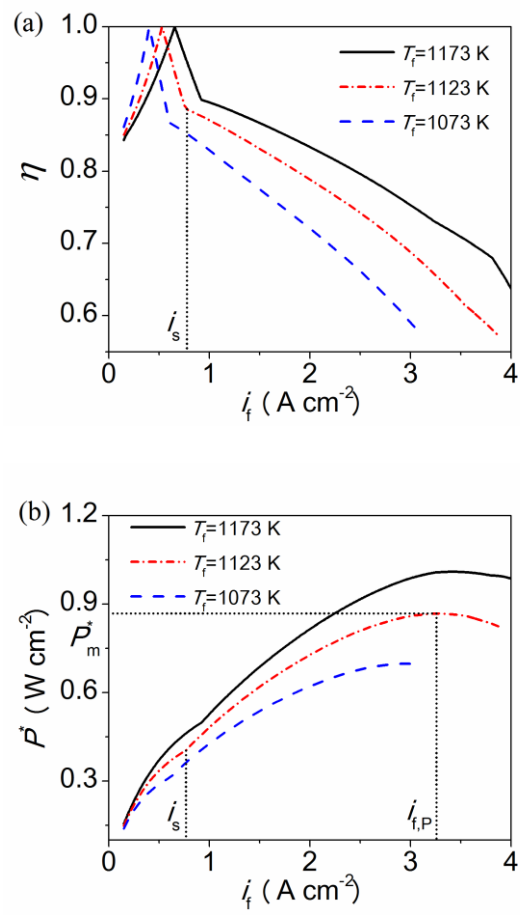


524 Fig. 8.



526

Fig. 9.



531 Fig. 10.

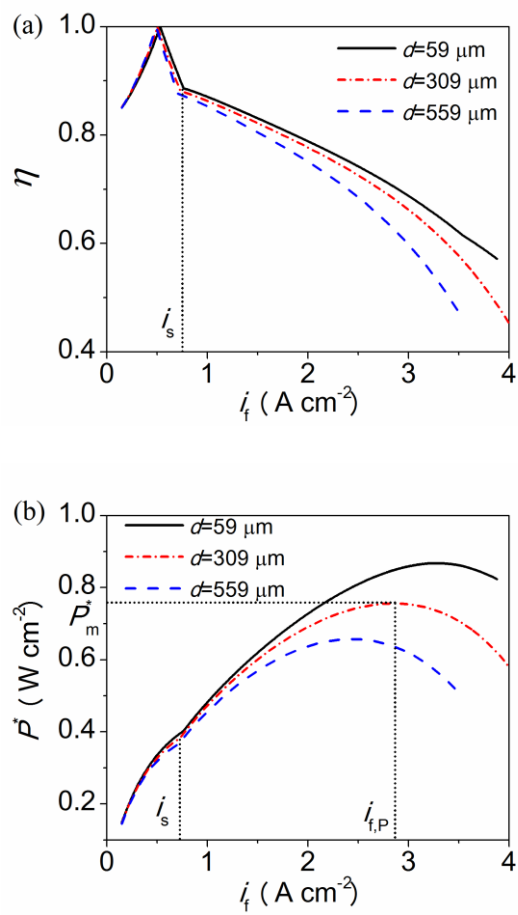


Fig. 11.

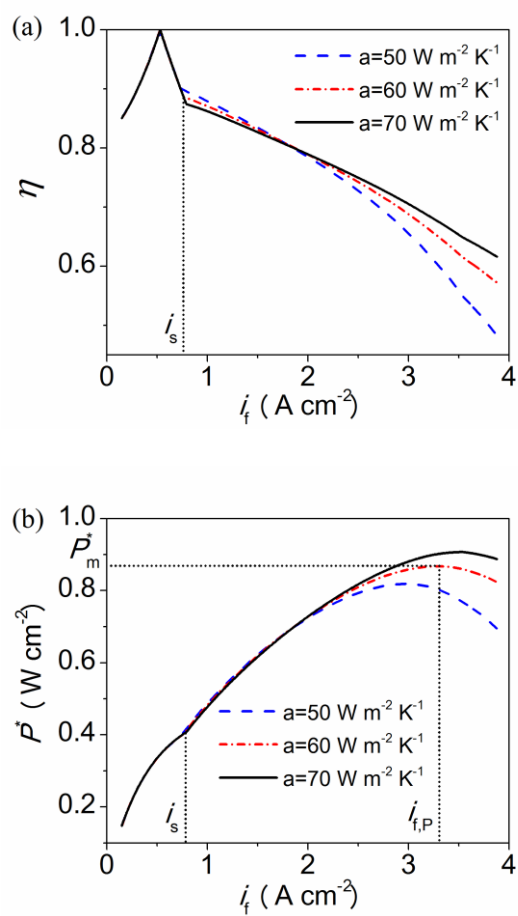
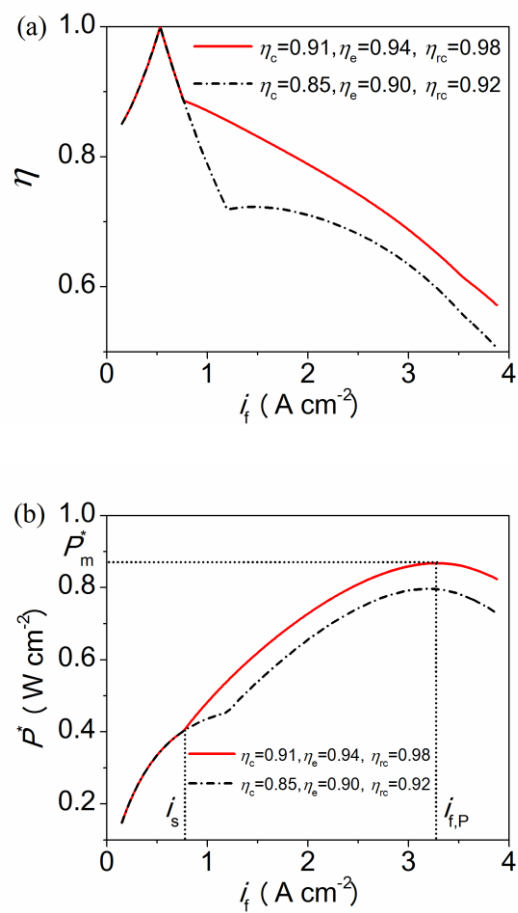


Fig. 12.





543 Fig. 13.

544

