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House Hedging Model – which income group is more affected by risk?

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a b s t r a c t

This paper investigates the housing market dynamics based on the individual choices of tenants and homeowners. We integrate Han (2008)'s individual homeowners' transaction choice model with tenants' housing tenure choice into a single, new model, and take tenants' and homeowners' decision into account jointly. As a result, our new model can capture the interaction between the transaction and rental markets and hence is more realistic. The model is applicable to Hong Kong and other countries of which the rental market is a significant proportion of the whole property market. We solve the new model in an analytical way. In addition, we conduct an agent-based simulation on the model. Compared with Han (2008)'s hump-shaped life-cycle paths, our results show that the life-cycle paths are irregular. Nevertheless, the middle-income households are least affected by housing price risk. Moreover, the average Time on Market (TOM) is significantly positively affected by housing vacancy rate under the exogenous price assumption. Finally, the trend of homeownership rate in a relatively closed market is not significantly affected by the specific housing price or rent process. Policy makers can make use of our model to implement more appropriate housing policies to regulate the housing market.

Keywords:
Homeowner
Tenant
Housing tenure choice
Dynamic optimization
Agent-based simulation

1. Introduction

Residential consumption is probably one of the most important items in a household's budget. This consumption has different forms for households — they may either rent or own a house. Housing tenure choice, which refers to household behavior of switching between owning and renting a house, has always drawn large attention. This topic involves different realms in economics, such as household income, consumption and savings during life-cycle, rental and selling price fluctuation, homeowner tax policy, etc. In general, the methods of investigating this topic can be categorized in two ways: one is to look at the factors affecting the choices of a single household such as tax policy [1,2] and life cycle [3], the other is to solve for market equilibrium based on composite factors [4].

In this study, our model is based on Han [5]'s individual homeowners' transaction choice model, and we extend it to the transaction–tenure model. The intuition behind our model is as follows: the tenants can buy a house to hedge against rental risk, but they have to pay a fixed mortgage payment. Hence they have to take a risk of income fluctuation and probably lower future consumptions; on the other hand, if they keep renting a house, they not only face the income risk (homeowners save the rental payment which can be used to pay the mortgage loan), but also have to bear the rental risk, and more importantly, the risk that they will never be afford to buy a house in the future (losing the house hedging value). This risk is more significant for investors who plan to consume more housing service in the future, e.g., young couple. Therefore, this study is particularly useful for young people. The purchasing decisions of the households are affected by their income levels. Hong Kong's income gap is getting wider and wider in recent years. In 2015, its Gini Index reached a 40-year high of 0.537. For different income groups, households behave differently since their income level restricts their choice. Therefore, their housing purchasing decisions would change differently when the housing price risk changes. Hence the main objective of this study is to explore how housing purchasing decisions for different income groups are affected by housing price risk. Therefore, we divide all households into three different income groups, and investigate how housing price risk affects households of the three income groups differently.

Since our new model takes tenants' and homeowners' decision into account jointly, it can show how homeowner transaction choice and housing tenure choice affects the housing price dynamics. This can help policy makers in regulating the housing market. If they know homeowners' and tenures' choices and behavior, they can understand the housing price dynamics more thoroughly and hence implement more appropriate housing policies. Since tenure choice decision is a critical element for understanding not only home price dynamics, but also development of housing bubbles, future studies can make use of our model to investigate housing bubbles.

Previous studies never investigate the models of homeowners' transaction choice and tenants' housing tenure choice together (see Section 2 for details). In order to bridge this gap, this study integrates both models and investigates the market dynamics based on the individual behavior pattern. The analysis consists of two parts: (1) to integrate and expand the models of housing transaction choice with housing tenure choice; (2) to investigate the housing market dynamics via agent-based simulation. The key novel feature of this study is that we integrate Han [5]'s individual homeowners' transaction choice model with tenants' housing tenure choice into a single, new model, and take tenants' and homeowners' decision into account jointly. In this way, the new model can capture the interaction between the transaction and rental markets and hence is more realistic, so the model is more applicable to Hong Kong's situation. Our new model is also applicable to other countries of which the rental market takes up a significant portion of the whole real estate market (e.g. the Netherlands and some cities in the U.S.). We investigate how different price risks affect housing purchasing decisions for different income groups by agent-based simulation. This paper proceeds as follows: Section 2 reviews related works. Section 3 describes our model. Section 4 conducts an agent-based simulation and discusses the results and key findings. Section 5 concludes the paper.

2. Literature review

This study is closely related to price risk hedging approach. Early discussions on housing price hedging mainly deal with investment allocation problems involving housing. Households hedge housing price risk with stocks [6,7] and pursuit a high investment return. Although such approaches are quite empirical, there has not been any common hedging tactics because strategies can often vary from time to time and from region to region. Later studies on housing price risk hedging gradually consider households' choices from both tenants and homeowners.

Another approach to study housing tenure choice is generalized optimization [8]. This method uses utility as the objective function, but still, it only concerns the wealth and return aspects of a house and ignores its consumption feature. Sinai and Souleles [9] study the hedging and tenure choice problem simultaneously. Although they fail to capture other factors which affect tenure choice, they successfully utilize the risk hedging mechanism to explain tenure choice decision. Ortalo-Magne and Rady [10] also apply dynamic utility optimization to indicate that the likelihood of home-owning is negatively related with the covariance of housing rent and household earning.

Han [5] applies housing price risk hedging approach to solve homeowner transaction choice with large transaction cost. He reveals that real estate cannot be treated as a common financial asset because of transaction cost. He considers both wealth and consumption aspects of real estate simultaneously, enabling the homeowner choice problem to be closely linked with tenure choice problem.

Still, there are certain factors involved in the tenure choice problem when the whole market is under study. For instance, income [11] and consumption [12] are always treated as constant or are ignored in the household-level study. Davis and Ortalo-Magné [13] consider that the housing and non-housing consumptions are heavily affected by the income level of the resides. Goodman [14] models, as equilibrium behavior, buyers' behaviors under transactions costs and market failure constraints. He finds that If transactions costs are greater than the economic costs of the constraints, then the household remains in a residence. When observing the market dynamics, these factors will heavily affect the decision of each agent in the market. Thus, although income and consumption will be regarded as exogenous factors, they are stochastically generated in our simulation.

This study conducts agent-based simulation on the model. Agent-based models are applied by a number of previous studies to investigate the equity market. For example, Zhang and Li [15] study credit trading in financial market by an agent-based model. They find a continuous phase transition as the leverage threshold increases, which may provide a further prospective of credit trading. Wei et al. [16] propose a simple agent-based model of trading incorporating momentum investors. They show that real market data can be used to constrain the model parameters, which in turn provide information on the behavior of momentum investors in different markets. Some studies apply agent-based models to examine the housing market. For example, Gilbert et al. [17] introduce an agent-based model to investigate shocks to the English housing market. Axtell et al. [18] construct an agent-based computational model to examine housing market bubbles in Washington, D. C. They find that tighter interest rate policies would have done little to attenuate the price bubble, while limiting household leverage would have had a larger effect.

The above summarizes previous studies on related topics. As seen, none of the studies investigate homeowners' transaction choice and tenants' housing tenure choice together. In order to fill in this gap, based on Han [5]'s framework, we incorporate homeowners' transaction choice and tenants' housing tenure choice into a single model. Then an agentbased simulation is conducted to model homeowners' and tenants' behavior. The model will be described in the next section.

3. Theory and model

Housing tenure choice does not complete the story since it only concerns about the demand side of a city's housing market circulation. It is worthwhile noting that housing supply is always assumed to be inexhaustible in traditional tenure choice studies [19]. Although this assumption is suitable for individual choice model, it conflicts with empirical results in a larger environment. In a fully developed metropolis, there should not be much free land for newly constructed houses unless redeveloping the existing built areas. In this case, second hand transactions should occupy a large proportion of all housing transactions. Taking Hong Kong as an example, this proportion is 90% on average in the recent 18 months.[1] The supply of second hand housing is decided by homeowners and is rather limited. On the other hand, after owning a house, households are unlikely to switch back to renting [20], i.e., housing tenure choice problem is only

[1] Data come from Rating and Valuation Department of the Government of HKSAR.

encountered by tenants. Once tenants settle in their first house, they might choose to stay in this house, or move to another. So the problem will change to whether housing-occupiers should transact to a new house [5,21]. Homeowners' transaction behavior largely explains the supply of second hand housing. These two behaviors constitute a rough housing market circulation of a city. Since supply and demand sides are mostly included in the market, this market can be viewed as a relatively closed market.

Our model proposed in this section is the generalized case. It can be used in any other cities. We use Hong Kong data to calibrate the parameters of housing price and income in Section 4, so that we can study how habitants behave in a city with little new housing property entering the market, which is representative for most large cities with well-developed real estate markets. The tradeoff is described in introduction. Another reason why tenants continue to rent is that they always have anticipation for how long they have to stay in the city. If they stay less than 5 years, which occupies a considerable proportion in metropolis population, owning a house is not a good choice.

The incentives and constraints for different income groups are as follows: the low income group may choose between staying in old place and renting a better house; the middle income group may choose between renting and owning a house. Since the transaction cost is significant for this group, they are not likely to improve their housing service frequently; the high income group frequently improve their housing service, but they are still restricted by limited housing supply. Their choice depends on the available housing in the market, which in turn stimulate the fluctuation of housing service in this group. The households may choose to rent a house at first, then buy a small house and move to a better house later. This forms a house ladder. All the household decisions are affected by house price risk, but the behavior discrepancy between different income groups comes from the constraints of their income, which was not discussed in the model of Han [5].

In fact, our model has the house hedging effect, but we cannot carefully study it in the model part. In our setting, people can only choose houses which are available, while the house hedging value is substantial only when households can choose housing service freely. In this study, the housing service option is restricted, people may not find proper house to hedge the price risk when they need. This case is more realistic than that of Han [5] and the households' action may simply be motivated by the tradeoff between buying and renting, moving and staying. However, these settings cannot be totally covered by the theoretical model, so we need simulation to make a further investigation.

The model in this paper includes two behaviors, housing tenure choice and housing transaction choice, in the framework established by Han [5]. Since different considerations are taken in respective behaviors, separate studies for each of them shall be made. Han [5] considers existing homeowners only. However, the property market in Hong Kong is different. According to the statistics of Hong Kong Housing Authority, 44.8% of Hong Kong's total population lived in public permanent housing in 2016, with about two-third of them living in public rental housing (PRH) (29.1% of total population) [22]. This shows that the rental market plays a significant role in Hong Kong's housing market. Therefore, we extend Han [5]'s model from homeowners' transaction problem to tenants' housing tenure choice problem. In this way, the rental market is also included and our model is more applicable to Hong Kong's situation.

In this section, we extend Han [5]'s model from homeowners' transaction problem to tenants' housing tenure choice problem. The model is discrete and the life span of a household contains T+1 periods. The exit of households can either be passing away or moving to another market. Two consumptions are taken into consideration in this model: housing services consumption (H_t) and non-durable goods consumption (C_t). Housing services are kept constant for each household. To alter households' consumption on housing services, they may either own a new house if they are tenants originally, or sell their current house and purchase a new one if they are homeowners. Transaction cost is modeled by a fixed part F plus a part proportional (δ) to the value of the sold house. Exogenous house price $\boldsymbol{P} = (\boldsymbol{p}_0 \ldots \boldsymbol{p}_T)$ and rent $\boldsymbol{R} = (\boldsymbol{r}_0 \ldots \boldsymbol{r}_T)$ are regarded as first order Markov processes with conditional densities $\boldsymbol{\Phi}(\boldsymbol{p}_t | \boldsymbol{p}_{t-1})$ and $\boldsymbol{\Psi}(\boldsymbol{r}_t | \boldsymbol{r}_{t-1})$ respectively. Each $\boldsymbol{p}_t$ and $\boldsymbol{r}_t$ are k-dimensional vectors $(p^1_t, p^2_t, \ldots, p^K_t)$ and $(r_t^1, r_t^2, \ldots, r_t^K)$ respectively, of which the subscripts denote time and the superscripts denote the corresponding location to housing price and rent. Housing prices and rents vary across locations and over time.

In each period t, households can borrow and lend at a positive interest rate i_t. Meanwhile, each household is assumed to receive income as a stream $(y_1, y_2 \ldots y_T)$ and feature housing taste as a profile $(\vartheta_1, \vartheta_2 \ldots \vartheta_T)$.

In each period t, the utility function consists of a deterministic component $\tilde{u}(H_t, C_t; \vartheta_t)$ and a stochastic utility shifter s_t. *For households' tenure choice* $(d_{t-1}^1 = 0)$:

$$u(H_t, C_t; \vartheta_t, \boldsymbol{s_t}) = \begin{cases} \tilde{u}(H_t, C_t; \vartheta_t) + s^O_t, & \text{where } H_t = H_t^j \text{ if } d^1_t = 1 \\ \tilde{u}(H_t, C_t; \vartheta_t) + s^R_t, & \text{where } H_t = H_t^{j-1} \text{ if } d^1_t = 0 \end{cases} \tag{1}$$

If a household keeps the ownership from period t-1 till period t, the utility function for housing transaction choice is $(d_{t-1}^1 = 1)$:

$$u(H_t, C_t; \vartheta_t, \boldsymbol{s_t}) = \begin{cases} \tilde{u}(H_t, C_t; \vartheta_t) + s^N_t, & \text{where } H_t = H_t^j \text{ if } d^2_t = 1 \\ \tilde{u}(H_t, C_t; \vartheta_t) + s^T_t, & \text{where } H_t = H_t^{j-1} \text{ if } d^2_t = 0 \end{cases} \tag{2}$$

where $\tilde{u}(H_t, C_t;\vartheta_t)$ refers to the utility from the consumptions H_t and C_t subject to the condition on a given housing taste ϑ_t, and $\boldsymbol{s}_t$ is a utility shifter, which is determined only at period t. The superscripts O and R indicate choices that household makes when it does not own a house: either to own a house ($d^1_t = 1$) or continue renting ($d^1_t = 0$). Since homeowners can enjoy capital gains earned by selling their housing units, but tenants cannot, tenants may choose to stop renting and shift to homeowners, but it is unlikely that homeowners shift back to tenants. Smith and Smith [20] also explained that due to high transaction costs, the possibility for a homeowner to shift back to a tenant is quite low. Therefore, this paper shall not discuss this situation. Thus, once the household has decided to become a homeowner at time t, it will be considered as homeowner for the rest of its lifetime and d^1_τ will be set to 1 for all $\tau \geq t$. The superscripts T and N indicate choices that the household makes when it is already homeowner: to transact ($d^2_t = 1$) or not ($d^2_t = 0$). We may assume that

(s^O_t, s^R_t) and (s^T_t, s^N_t) are independent identically distributed as type-I extreme value distributions $g_1(s^O_t, s^R_t)$ and $g_2(s^T_t, s^N_t)$ respectively.

With finite horizon, households may realize their terminal wealth by selling their house, if they own one, at the previous period. For each period t prior to the previous period, households observe the previous housing service H^{j-1}_{t-1}, financial wealth W_{t-1}, house prices and rents up to time t and current utility shifters (s^O_t, s^R_t) or (s^T_t, s^N_t). They then determine non-durable consumption C_t, whether to own (d^1_t, for renters) or transact (d^2_t, for owners), and the optimal housing service H^j_t of the target house in order to maximize the expected remaining lifetime utility through owning or transacting:

$$\underset{(C_t, d^i_t, H^j_t)}{\text{Max}} \; E_t \sum_{\tau=t}^{T} \beta^{\tau-t} u(H_\tau, C_\tau; \vartheta_\tau, \boldsymbol{s}_\tau) + \beta^{T+1} \frac{W_{T+1}^{\gamma}}{1-\gamma} \quad i = 1 \text{ or } 2$$

Subject to:

$$W_t = y_t + (1+i_t) W_{t-1} - C_t - \left(1 - d^1_t\right) r^{j}_{t} H^{j-1}_{t-1} - \left(d^1_t - d^1_{t-1}\right)\left(p^j_t H^j_t + F\right)$$

$$+ d^1_t d^2_t \left[(1-\delta) p^{j-1}_t H^{j-1}_{t-1} - p^j_t H^j_t - F\right] \tag{3}$$

$$W_{T+1} = (1 + i_{T+1}) W_{t-1} + d^1_{T+1}\left[(1-\delta) p^J_{T+1} H^J_T - F\right] \geq 0 \tag{4}$$

where J indicates the location of the last house bought in household horizon and E_t indicates the expectation for remaining lifetime subject to information got on time t. Eq. (2) shows the household budget restriction at time t prior to the last period, while Eq. (3) is the terminal household budget restriction which rules out the situation that households leave with debt.

To solve the model, we need to establish several Bellman equations in order to derive the result recursively. First, for tenant households which decide to continue renting the old house at period t, the value function will be defined as:

$$V^R_t\left(W_{t-1}, H^{j-1}_{t-1}; s^R_t, \boldsymbol{r}_t\right) = \underset{(C_t)}{\text{Max}}\; u\left(H^{j-1}_{t-1}, C_t; \theta_\tau, s^R_t\right) + \beta E_t V_{t+1}\left(W^R_t, H^{j-1}_{t-1}; \boldsymbol{s}_{t+1}, \boldsymbol{r}_{t+1}\right) \tag{5}$$

Subject to:

$$W^R_t = y_t + (1 + i_t) W_{t-1} - C_t - r^j_{t-1} H^{j-1}_{t-1}$$

Then, for tenant households which decide to own a new house in period t, the value function should be:

$$V^O_t\left(W_{t-1}, H^{j-1}_{t-1}; s^O_t, \boldsymbol{p}_t\right) = \underset{H^j_t, C_t}{\text{Max}}\; u\left(H^j_t, C_t; \vartheta_\tau, s^O_t\right) + \beta E_t V_{t+1}\left(W^O_t, H^j_t; \boldsymbol{s}_{t+1}, \boldsymbol{p}_{t+1}\right) \tag{6}$$

Subject to:

$W^O_t = y_t + (1 + i_t) W_{t-1} - C_t - p^j_t H^j_t - F$ where F indicates the fixed part in the transaction cost. Since there is no house for sale for tenant households, the proportional part in the transaction cost will be 0. Next, considering that households have already owned a house, if they decide to stay in the old house for period t, the value function will be:

$$V^N_t\left(W_{t-1}, H^{j-1}_{t-1}; s^N_t, \boldsymbol{p}_t\right) = \underset{(C_t)}{\text{Max}}\; u\left(H^{j-1}_{t-1}, C_t; \vartheta_\tau, s^N_t\right) + \beta E_t V_{t+1}\left(W^N_t, H^{j-1}_{t-1}; \boldsymbol{s}_{t+1}, \boldsymbol{p}_{t+1}\right) \tag{7}$$

Subject to:

$$W_t^N = y_t + (1 + i_t)\, W_{t-1} - C_t$$

Finally, define:

$$V_t^T\left(W_{t-1}, H_{t-1}^{j-1}; s_t^T, \boldsymbol{p}_t\right) = \max_{H_t^j, C_t} u\left(H_t^j, C_t; \vartheta_t, s_t^T\right) + \beta E_t V_{t+1}\left(W_t^T, H_t^j; \boldsymbol{s}_{t+1}, \boldsymbol{p}_{t+1}\right) \tag{8}$$

Subject to:

$$W_t^T = y_t + (1 + i_t)\, W_{t-1} - C_t + \left[(1 - \delta)\, p_{t-1}^j H_{t-1}^{j-1} - p_t^j H_t^j - F\right]$$

to denote the expected utility of households if they transact in period t and have already owned a house prior to t.

In period T+1, the households sell their houses or just leave the market if they are still tenants. Prior to the previous period, the expected value function in period t is based on the housing status of period t-1, i.e. the value for d^1_{t-1}. When $d^1_{t-1} = 0$, i.e. households are tenants and making tenure choice, the expected value function is:

$$V_t\left(W_{t-1}, H_{t-1}^{j-1}; \boldsymbol{s}_t, \boldsymbol{r}_t, \boldsymbol{p}_t\right) = \max_{\left(d_t^1, H_t^j, C_t\right)} \left[V_t^R\left(W_{t-1}, H_{t-1}^{j-1}; s_t^R, \boldsymbol{r}_t\right), V_t^O\left(W_{t-1}, H_{t-1}^{j-1}; s_t^O, \boldsymbol{p}_t\right)\right] \tag{9}$$

Given the conditional distributions for future housing prices, rents and utility shifters, the expected future utility term in the next period is:

$$E_t V_{t+1}\left(W_t, H_t^j; \boldsymbol{s}_{t+1}, \boldsymbol{p}_{t+1}, \boldsymbol{r}_t\right) = \iiint V_{t+1}(W_t, H_t)\, g_1(\boldsymbol{s}_{t+1})\, d\boldsymbol{s}_{t+1}\, \varphi(\boldsymbol{p}_{t+1} | \boldsymbol{p}_t)\, d(\boldsymbol{p}_{t+1} | \boldsymbol{p}_t)\, \psi(\boldsymbol{r}_{t+1} | \boldsymbol{r}_t)\, d(\boldsymbol{r}_{t+1} | \boldsymbol{r}_t) \tag{10}$$

where the expectation is taken over future utility shifters, house rents and prices. The optimal decision about whether to own a house in period t depends on the differences in the expected utility for two choices:

$$d_t^1 = \begin{cases} 1 & \text{if } \left(s_t^R - s_t^O\right) \le \Gamma_1\left(W_{t-1}, H_{t-1}^{j-1}; \boldsymbol{r}_t, \boldsymbol{p}_t\right) \\ 0 & \text{if } \left(s_t^R - s_t^O\right) > \Gamma_1\left(W_{t-1}, H_{t-1}^{j-1}; \boldsymbol{r}_t, \boldsymbol{p}_t\right) \end{cases} \tag{11}$$

with:

$$\Gamma_1\left(W_{t-1}, H_{t-1}^{j-1}; \boldsymbol{r}_t, \boldsymbol{p}_t\right) \equiv \tilde{u}\left(H_t^j, C_t\right) - \tilde{u}\left(H_{t-1}^{j-1}, C_t\right) + \beta E_t \left[V_{t+1}\left(W_t^O, H_t^j; s_{t+1}^O, \boldsymbol{p}_{t+1}\right) - V_{t+1}\left(W_t^R, H_{t-1}^{j-1}; s_{t+1}^R, \boldsymbol{r}_{t+1}\right)\right] \tag{12}$$

Analogously, the expected value function for homeowners ($d^1_{t-1} = 1$) is:

$$V_t\left(W_{t-1}, H_{t-1}^{j-1}; \boldsymbol{s}_t, \boldsymbol{p}_t\right) = \max_{\left(d_t^1, H_t, C_t\right)} \left[V_t^N\left(W_{t-1}, H_{t-1}^{j-1}; s_t^R, \boldsymbol{p}_t\right), V_t^T\left(W_{t-1}, H_{t-1}^{j-1}; s_t^O, \boldsymbol{p}_t\right)\right] \tag{13}$$

Next period's expected future utility term is:

$$E_t V_{t+1}\left(W_t, H_t^j; \boldsymbol{s}_{t+1}, \boldsymbol{p}_{t+1}\right) = \iint V_{t+1}(W_t, H_t)\, g_2(\boldsymbol{s}_{t+1})\, d\boldsymbol{s}_{t+1}\, \varphi(\boldsymbol{p}_{t+1} | \boldsymbol{p}_t)\, d(\boldsymbol{p}_{t+1} | \boldsymbol{p}_t) \tag{14}$$

Since Eqs. (11) and (12) are for the case of homeowners, the information of rents is excluded from the equations. Then, for the optimal decision of whether or not to sell the current house and purchase a new one depends on:

$$\ _{t}^{2} = \begin{cases} & \left(s_t^N - s_t^T\right) \le \Gamma_2\left(W_{t-1}, H_{t-1}^{j-1}; \boldsymbol{p_t}\right) \\ 0 & \left(s_t^N - s_t^T\right) > \Gamma_2 \left(\quad 1 \quad if \quad _{-1}, H_{t-1}^{j-1}; \boldsymbol{p_t}\right) \end{cases}$$

d(15)

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with:

$$\Gamma_2\left(W_{t-1}, H_{t-1}^{j-1} \quad) \quad ;r_t, p_t \quad \equiv \tilde{u}\ H_t, C \quad [\,(^{j}) \quad _t \quad - \tilde{u}\left(H_{t-1}^{j-1}, C_t\right)\right] \quad _{-1}^{-1}; s_{t+1}^N, \boldsymbol{p_{t+1}}\Big)\Big]$$

$$+ \beta E_t \left[V_{t+1} \left(W_t^T, H_t^j; s_{t+1}^T, \boldsymbol{p_{t+1}}\right) - V_{t+1}\left(W_t^N, H_t^j\right.\right. \quad (16)$$

4. Simulation and key findings

We have integrated the behaviors of tenants and homeowners into one framework and the model can trace household behavior from entering to leaving the market. Considering that historical data is unable to capture such behaviors thoroughly, we conduct an agent-based simulation to fully utilize our model.

4.1. Simulation design

Numerical solution can give a housing and non-housing consumption pair for the model at each period. Han [5] also gives a two-location simulation for homeowner choice. Obviously, both homeowner and tenant might affect each other with their decisions. For example, a homeowner might sell an old house and buy a new one; the old house can thus be bought by a former tenant. Therefore, a tenant's decision to own a house and a homeowner's decision to move to a new house must be taken into account jointly in the simulation (Han [5] considers the transaction market only and hence does not take this into account). This section shall simulate such interaction between homeowner and tenant by an agent-based method.

Firstly, we need to get a stationary price and rental process randomly. Based on Edelstein and Tsang [23], the long-run price is defined as the long-run rent divided by the capitalization rate. Thus the unconditional mean for price process can be set as a constant multiple of unconditional mean for rent process. To make things easier, we use index process instead of real price and rental processes. The price and rental indices can share the same process with only one difference in the variance of error term. Based on the assumption of our model, we utilize a VAR process with one lag:

$$\begin{pmatrix} I_t^1 \\ I_t^2 \\ \vdots \\ I_t^k \end{pmatrix} = \mu + A \begin{pmatrix} I_{t-1}^1 \\ I_{t-1}^2 \\ \vdots \\ I_{t-1}^k \end{pmatrix} + e_t$$

where $e_t \sim N\left(\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_k^2 \end{pmatrix}\right)$, k indicates k different locations, A is the k∗k transition matrix. In this way, we extend Han [5]'s two-location simulation to a k-location simulation. The standard deviation may also be different between price and rent. To get a relatively realistic result, we use the Hong Kong average rental level in 1999, which is \$1920/m^2, as one unit rental index. With 4% capitalization rate, one unit price index implies that \$49,800[2] is needed to buy one unit housing service.

Secondly, we generate different households which behave under the regulation of our model. All households entering the market are of different ages, horizons and average incomes. The horizon may be considered as the expected date they leave the market. The average households' income in Hone Kong is generated with reference to the 1999 data. The households' behaviors are based on our model for each period.

It should be noted that in the actual market, a transaction may last for a long time so that households can hardly sell their houses instantly. This may violate one of our assumptions that all households need to exit the market with positive wealth. As households cannot

2 The unit of currency is HK dollar here and below. One unit of area refers to 1 square meter.

sell their houses immediately at the leaving time t+1, they may unlikely to have a positive terminal wealth based on the solution to our model. As a result, we design a housing inventory to which the households sell and from which they rent and purchase residential houses, ensuring that they do not have any liquidity risk for their wealth. The housing inventory has other benefits: it provides information about vacancy houses and thereby restricts the available solutions to our model within a limited domain.

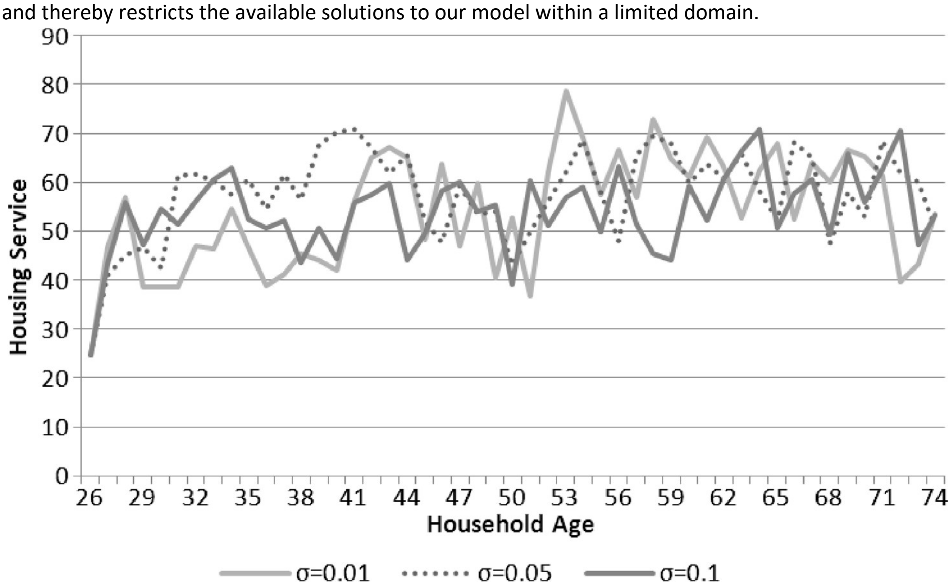

Fig. 1. Average optimal housing service for low-income group.

Thirdly, in the simulation, we specify the utility function as follows:

$$u(H_\tau, C_\tau; \theta_\tau) = \frac{\left(H_\tau^{\theta_\tau} C_\tau^{1-\theta_\tau}\right)^{1-\gamma}}{1-\gamma} \tag{17}$$

where H_τ refers to the housing service at time τ; C_τ refers to the non-housing consumption at the same period; γ , as aforementioned, refers to the risk aversion factor; ϑ_τ, as a housing taste profile, provides the relative substitution rate between housing and non-housing consumption at household age τ.

To further simplify the simulation, we set the utility discount factor β to be 0.9 per year, interest rate r to be 0.03 per year (for simplicity, we assume the interest rate to be constant), $\gamma = 3$, $\delta = 0.06$, T = 1000. For simplicity, we assume that in repeated simulations, the economy converges to similar dynamic patterns. ϑ_τ is defined as follows:

$$\theta_\tau = \begin{cases} \frac{1}{800}\left[500-(\tau-45)^2\right], \tau \leq 45 \\ 0.125+0.4exp\left[-0.16\pi\,(\tau-45)^2\right], \tau > 45 \end{cases}$$

4.2. Simulation result and key findings

This paper aims at investigating the whole market dynamic based on the individual choice model. We find out how different variance price processes affect housing service choice along lifetime. In order to make a comparison, we generate the market with the following parameters: the market consists of 100 houses and 25 households; all households start their choices at age 25 and end at 75; the households are divided into three groups by their average income. The highand low-income groups make up the top and bottom 20% of all households respectively, while the remaining 60% are regarded as the middle-income group. We assume no new housing supply, so the houses in the market are all secondhanded. Hence if most households leave the city for some reason, e.g. a large negative income shock or unemployment, then oversupply of houses may exist. If the initial market is undersupplying, the excess households may leave the market and the remaining households can hardly make a transaction, since there are no houses left to purchase. We use the same price process for simulations. The only difference among the three simulations is the variance of the price process.

Figs. 1 to 3 plot the average life-cycle path of optimal housing service for different price variance and income groups.

In Figs. 1–3, if life-cycle paths representing different values of σ are clustered together, this indicates that the housing service choice of the group remains stable even when the housing price risk changes, i.e. the group is less affected by housing price risk. On the other hand, if the life-cycle paths moves by a larger extent when σ varies, this reflects that the housing service choice of the group changes significantly when the housing price risk varies, i.e. the group is more affected by housing price risk.

Han [5] shows that the life-cycle path of optimal conditional housing demand attains a pronounced hump shape, with the optimal housing size attaining a maximum when the household age is around 44–45. However, our result is different. From Figs. 1–3, the life-cycle paths are irregular. No clear maximum of optimal housing service can be seen. This is mainly because Han [5]'s model considers the housing transaction market only, but the model in this study includes both

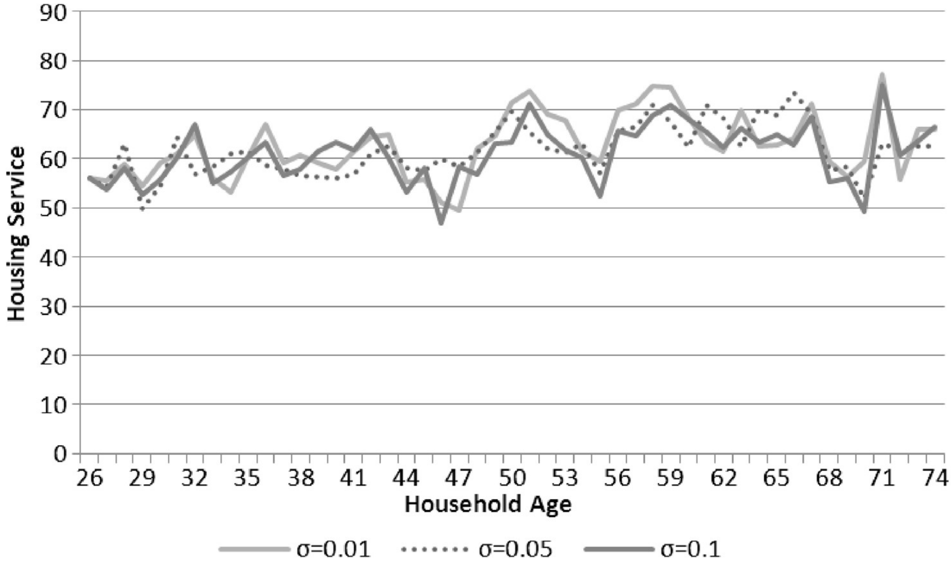

Fig. 2. Average optimal housing service for middle-income group.

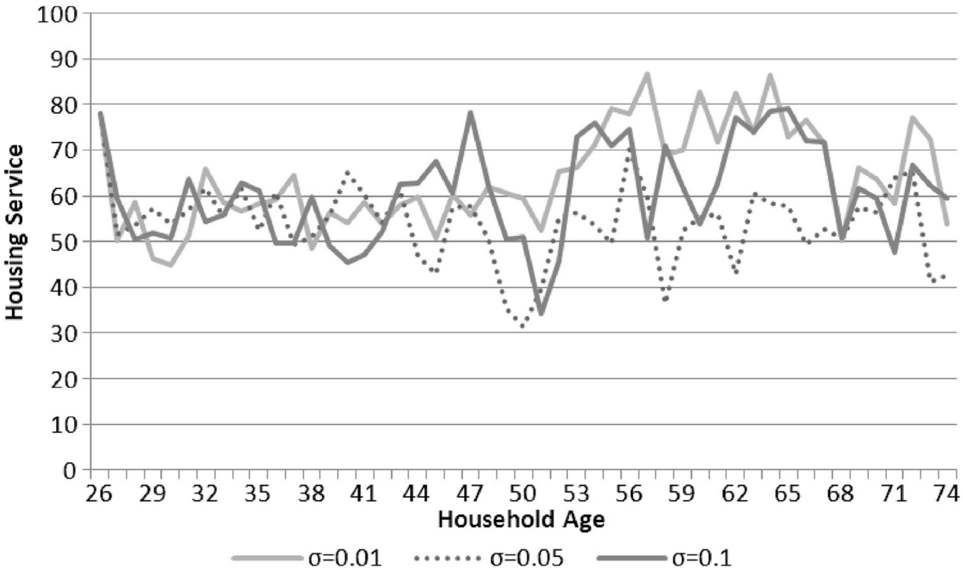

Fig. 3. Average optimal housing service for high-income group.

transaction and rental markets. Unlike Han [5]'s model, our simulation model takes tenants' and homeowners' decision into account jointly. Moreover, the housing service provided in our simulated market is quite restricted and discrete. Furthermore, the households need to compete with each other and make a relatively beneficial choice. Although the life-cycle paths are irregular, some patterns can still be seen. From Fig. 2, the three life-cycle paths representing different values of σ are clustered together. This shows that when the housing price risk changes, the housing service choice for the middle-income group remains stable, i.e. this group is less affected by housing price risk. In comparison, Figs. 1 and 3 shows that for the low-income and high-income groups, the life-cycle paths moves by a larger extent when σ changes in value, reflecting that these two groups are more affected by housing price risk. To further illustrate this result, the standard deviation of housing service as pricing risk varies for the three income groups is plotted in Fig. 4:

Fig. 4 shows that the standard deviation for the middle-income group is significantly smaller than the other two groups along the whole life span, i.e. the housing service choice is the most stable for this group based on different pricing risk, i.e. the least affected by housing price risk. Furthermore, it is found that if we draw a vertical line in the age 45, the standard deviation of the low-income group is generally larger than that of the high-income for age below 45, while the situation is reversed for age above 45.

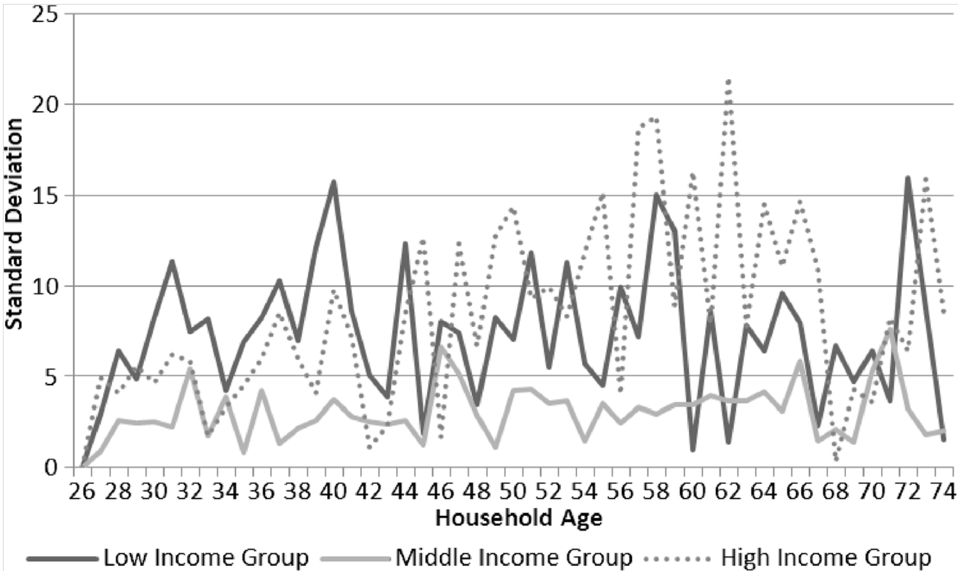

Fig. 4. Standard deviation of housing service as pricing risk varies.

The above results can be explained as follows. Most of the middle-income group have stable jobs and live in stable accommodations, so their purchasing decisions are not significantly affected by changes in housing price risk. Moreover, any further improvement of the housing service will not compensate the utility loss in non-durable consumption of the middle-income group because of the transaction cost of purchasing a new house. This cost will neither affect the behavior of low-income group nor that of high-income group since renting another house does not need to pay the transaction cost and the cost will not affect much on the utility of households in high-income group. Hence the standard deviation among different price volatilities is the lowest for the middle-income group. Meanwhile, the low- and high-income groups are more affected by housing price risk, especially for low-income group of age below 45 and high-income group of age above 45. Most of the people in the low-income group of age below 45 are youngsters at their initial stage of work. Many of them wish to buy a house. Since they do not have much disposal income, a change in housing price risk will significantly affect their purchasing decisions. While the high-income group of age above 45 tends to have multiple housing units for investment. If they expect the housing price to rise, they will tend to buy houses to earn capital gain. On the other hand, if they expect a fall in housing price, they will tend to sell houses to reduce their losses. If the housing price risk varies, their investment values will change sharply and hence their purchasing decisions will be significantly affected. This explains why they are greatly affected by housing price risk.

After investigating the life-cycle path of housing service, we study the market dynamics in a simulated environment: we fix the rent and price variance at 0.05. The ages of households in the market are still limited within the range 25–75. Then we vary the age that households enter the market such that 40% of the households stay in the market for the whole lifetime, 30% stay for 7–20 years, and 30% stay for less than 7 years. Once a household leaves the market, a new household will be generated and added to the market so that the market can be kept in the same magnitude. Still, there are 100 houses in the market. Since the pricing process is exogenously generated, the only way to study the demand–supply relationship is through investigating the average time on market for all houses available in housing inventory. We set the vacancy rate to be 50% and 70% because if a small vacancy rate is used, then the households' choices will be very limited, so the life-cycle effect and the relationship between the vacancy rate and TOM will not be observable. Furthermore, the vacancy rate is exogenous. We do not involve the rational vacancy rate generation process in the equilibrium market. Note that Hong Kong has never recorded such high vacancy rates, so this simulation is for illustration only. The result is shown in Fig. 5.

For 70% housing vacancy rate, i.e. there are 30 households in the market, the line representing average time on market fluctuates around 1.5 years despite the huge variation for the first several years. For 50% housing vacancy rate, the line is undulated around 0.15 year. The time on market is positively affected by housing vacancy rate as expected. Nevertheless, this effect might not be observed in reality because there will never be such a large vacancy rate and the time on market is also affected by many other factors [24].

We further perform a simulation with vacancy rate of 5% so as to compare with the actual market data in Hong Kong. In this case, housing transaction will be much more frequent than that in the markets with 50% and 70% vacancy rates. This simulation is conducted such that homeowners and tenants make monthly choices instead of yearly choices as in the former simulations. Since most transactions occur between the 1st and 2nd months, it is difficult to show the result clearly in a figure. Therefore, we show the percentile of the transaction time in almost 5000 housing transactions in Table 1:

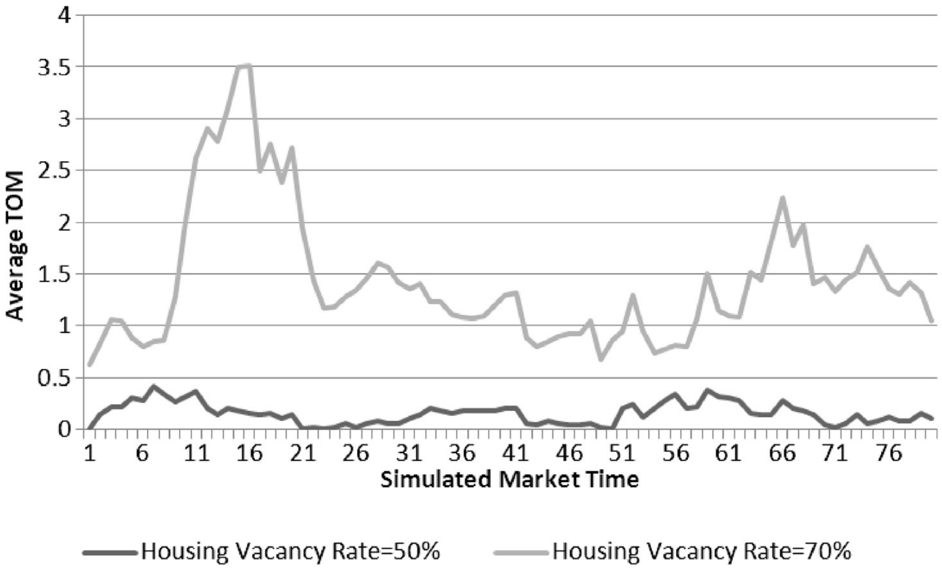

Fig. 5. Average time on market (TOM) for different housing vacancy rate.

Table 1
Time on market percentile for 5% housing vacancy rate.

TOM	<1	<2	<3	<4	<5	<6	<7	<8	<9	<10
Prob	0.197	0.945	0.974	0.987	0.992	0.995	0.996	0.997	0.997	0.999

The probabilities in Table 1 for each TOM interval are slightly larger than that shown in [24]'s paper. Two possible explanations are that: (1) we ignore the down-payment for each housing purchase, which makes households occupy houses more easily; and (2) we assume perfect information in our simulation, which shortens households' duration on house hunting. However, the two results are still quite close, which means that the individual household choice (both for tenants and homeowners) based on our model is quite reasonable from this aspect.

Finally, we compare the period of homeownership in our simulation with the relevant real market data in Hong Kong to test explanatory power of our model.

In Fig. 6, the two lines fit well although the simulated homeownership rate is slightly higher than the actual one. A possible reason is that households in our simulated market do not need to pay down payment when they begin to mortgage a house. We can say that this homeownership trend might be independent with the specific price or rental process because we do not use specific processes from real market in the simulation. That is to say, with the fixed distribution of household income, age, living duration and housing market information, the trend of homeownership rate should be fixed base on the individual choice strategy modeled in this paper.

5. Conclusion

The primary aim of this paper is to investigate how housing choices affect housing market dynamics of different income groups in Hong Kong. We integrate Han [5]'s individual homeowners' transaction choice model with tenants' housing tenure choice into a single framework, and hence develop our new model. In particular, our simulation model takes tenants' and homeowners' decision into account jointly. Therefore, our new model is applicable to Hong Kong and other countries of which the rental market is a significant proportion of the whole property market. Due to Hong Kong's wide income gap and people of different income levels have different housing conditions and hence response to changes in housing price risk differently, we investigate how housing price risk affects housing purchasing decisions for different income groups. Furthermore, we develop a new agent-based method by utilizing such model to identify individual behavior and simulate large amount of such individuals to constitute a small market. The agent-based method enables combined study of individual living status choice behavior and market dynamic, of which their correlation has seldom been studied.

The main results of this study are summarized as follows:

(1) Unlike Han's [5] hump-shaped optimal life-cycle consumption path, the life-cycle paths in this study are irregular. No clear maximum of optimal housing service can be seen.

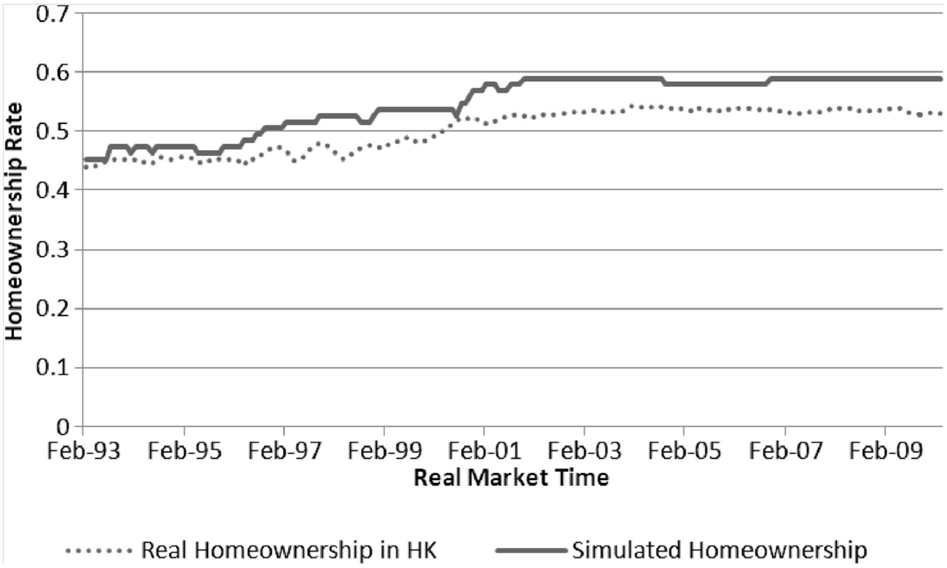

Fig. 6. Simulated homeownership vs. actual homeownership.

(2) The standard deviation among different price volatilities is the lowest for the middle-income group, showing that the middle-income households are least likely to be affected by housing price risk. Meanwhile, the standard deviation is higher for low-income group of age below 45 and high-income group of age above 45, reflecting that households belonging to these two groups are more likely to be affected by housing price risk.
(3) The average time on market is positively related to the vacancy rate.
(4) The simulated homeownership rate fits well with the actual rate (but slightly higher).

The models in previous studies consider the transaction market only and result in hump-shaped optimal life-cycle paths. However, the model in this study incorporates both transaction and rental markets, and takes tenants' and homeowners' decision into account jointly. As a result, the life-cycle paths generated in the simulation become irregular. No regular patterns can be seen. This is a limitation of our findings. Hence we see that changing the nature of the market may yield a totally different result. Therefore, if scholars would like to conduct similar research on real estate markets of other countries, they should pay attention to the nature of markets as this may affect the results.

Since Hong Kong's wide income gap and people of different income levels have different housing conditions and hence response to changes in housing price risk differently, we investigate how housing price risk affects housing purchasing decisions for different income groups. The results in this study really show that housing purchasing decisions for different income groups are affected by housing price risk, while the low- and high-income groups are more affected by housing price risk, especially for low-income group of age below 45 and high-income group of age above 45. The reasons for this result are explained in Section 4.

This study has practical implications to policy makers in regulating the housing market. For example, since young lowincome households and old high-income households are more likely to be affected by housing price risk, the government should particularly help these two groups of people when housing price risk increases, e.g. when a financial crisis occurs. For instance, the government can increase the priority of low-income youths in the waiting list for public rental housing (PRH) so that they need not wait for a long time to move into PRH. Furthermore, as the average time on market is positively related to housing vacancy rate, in order to shorten the average time on market, the government should lower the housing vacancy rate, for example, converting unused factory buildings into residential units, or simplifying the transaction procedures to shorten the time of housing transactions. In this way, the government can implement more appropriate housing policies to meet the needs of different groups of people. A suggestion for future research is to extend our model to property markets of other countries. This can help their government to improve the housing policies.

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