A Computerized Hybrid Bayesian-Based Approach for Modeling the Deterioration of Concrete Bridge Decks

Eslam Mohammed Abdelkader[1,2], Tarek Zayed[3], and Mohamed Marzouk[4]

ABSTRACT

Bridges are aging and deteriorating. Reliable deterioration modeling is regarded as one of the vital components of Bridge management systems. This paper presents an automated defect-based tool to predict the future condition of the bridge decks by calibrating the Markovian model based on a hybrid Bayesian-optimization approach. The in-state probabilities are demonstrated in the form of posterior distributions, whereas the transition from a condition state to the next lower state is a function of the severities of five types of bridge defects. In the present study, the Bayesian belief network is employed to construct the likelihood function by modeling the dependencies between the bridge defects. The maximum entropy optimization is incorporated to compute the missing conditional probabilities. The proposed approach utilizes Markov chain Monte Carlo Metropolis-Hastings algorithm to derive the posterior distributions. Finally, a stochastic optimization model is designed to build a variable transition probability matrix for each five-year zone via genetic algorithm. An automated tool is programmed using C#.net programming language to facilitate the implementation of the developed deterioration model by the users. Results show that the proposed model outperformed some commonly-utilized deterioration models as per three performance indicators which are: root-mean squared error, mean absolute error, chi-squared statistic.

Keywords: Bridge decks, deterioration modeling, Bayesian belief network, Metropolis-Hastings algorithm, genetic algorithm, C#.net.

[1] *Ph.D. student, Department of Building, Civil, and Environmental Engineering, Concordia University, Montreal, QC, Canada. Corresponding author, E-mail: eslam_ahmed1990@hotmail.com.*

[2] *Assistant lecturer, Structural Engineering Department, Faculty of Engineering, Cairo University, Egypt.*

[3] *Professor, Department of Building and Real Estate, the Hong Kong Polytechnic University, Hong Kong. E-mail: tarek.zayed@polyu.edu.hk.*

[4] *Professor of Construction Engineering and Management, Structural Engineering Department, Faculty of Engineering, Cairo University, Egypt. E-mail: mm_marzouk@yahoo.com.*

1. INTRODUCTION

Bridges are vital links in transportation networks. Nevertheless, large portion of them are deteriorating constantly because of several degradation agents such as extreme weather conditions, cycles of freeze and thaw, exposure to de-icing salt, deferred maintenance, design and construction errors, poor quality of workmanship, high traffic volumes, etc. Bridges in Canada experience significant deterioration levels. One-third of Canada's bridges suffer from structural or functional deficiencies as well as short remaining service life, whereas 750,000 trucks and 15,000 public transit buses utilize the Canadian bridges annually. As per statistics Canada, the average age of bridges in Canada was 24.5 years in 2007 compared to a mean service life of 43.3 years. Thus, 57% of the estimated service life has already been consumed (Statistics Canada, 2009a).

There are two main reasons for the significant deterioration of bridges which are: the decrease in the public investment, and the high age of bridges. The investment in bridges is below the required level to maintain the age of bridges constant, whereas the age of bridges increased by 3.2 years from 21.3 years in 1985 to 24.5 years in 2007. Bridges in Quebec have the highest average age followed by Nova Scotia. On the other hand, bridges in Prince Edward Island have the smallest average age, whereas the average age of bridges on Quebec, Nova Scotia, and Prince Edward Island are 31, 24.5, and 15.6, respectively (Viami International Inc. and The Technology Strategies Group, 2013).

Transportation agencies established Bridge Management Systems to maximize the functionality, safety and serviceability of transportation networks by developing cost-effective intervention actions. A cost-effective maintenance, repair and rehabilitation (MR&R) activity is highly dependent on the capability of the deterioration model to predict the future time-dependent performance of the bridge element, whereas a reliable deterioration curve is needed in order to obtain exact information about the need and timing of maintenance activities for a certain planning horizon. A deterioration model can be defined as a relationship between the condition of the bridge element and a vector of explanatory variables, which represent a group of variables that affect the performance of the bridge element such as age, environmental conditions, applied loads, material properties, etc.

The deterioration models can be divided into two main categories which are: deterministic and stochastic models. Deterministic models assume that the future performance of the bridge elements is certain over time based on mathematical and statistical approaches such as linear and non-linear regression, artificial neural network, support vector machines, straight-line extrapolation, and curve-fitting. The main limitation of the deterministic models is that they fail to consider the vagueness and uncertainty of the deterioration process of bridges due to the existence of un-observed explanatory variables and in-accurate inspection procedures (Agrawal, 2010). Stochastic models define the deterioration process in the form of one or more random variables, which can be modeled using probability density functions. Markov-based model is considered one of the most common stochastic models. Stochastic models can be classified into: state-based models and time-based models.

Thus, the objectives of the present study are as follows:

1- Review the previously-developed deterioration models.
2- Model the deterioration of bridge decks based on a hybrid Bayesian-based optimization approach.
3- Compare the developed model with the regression-based optimization, weibull, and gamma models.

4- Compare between different architectures of the Bayesian belief networks.

5- Develop a computerized tool to facilitate the implementation of the developed model.

2. LITERATURE REVIEW

Recently, several studies have been performed to model the deterioration of the concrete bridges. Lee et al. (2011) utilized statistical regression to model the time-dependent performance of bridge decks, prestressed girders, and piers in Korea. An equation is constructed for each one of the bridge components, whereas the condition grade is a function of the elapsed time. Shim and Lee (2016) developed a Markovian deterioration model to predict the future condition rating based on the national bridge inventory (NBI) condition rating. The transition probabilities were calculated based on the median duration years.

Le and Andrews (2015) modeled the deterioration of the bridge elements based on the two-parameter weibull distribution. Anderson Darling test is used to compare between a group of probability distributions. The parameters of the weibull distribution were defined based on the rank regression. Muñoz et al. (2016) presented a methodology to predict the deterioration of the bridges using both Markov chain and regression analysis in the case of small sample size. They illustrated that the proposed methodology provided conservative estimates for the future condition ratings as well as similar estimates to the traditional methods in calibrating the Markovian models and regression analysis.

Agrawal et al. (2010) compared between the Markov chain and weibull distribution in modeling the deterioration of bridge elements based on historical inspection records from the New York State Department of Transportation (NYSDOT) starting from 1981. They concluded that the weibull-based approach performed better than the Markov chain in terms of the prediction accuracy. Mašović and Hajdin (2014) modeled the deterioration of the bridge elements in Serbia based on the Markov chain. They employed expectation maximization (EM) algorithm to estimate the transition probabilities. They highlighted that the EM algorithm provided reasonable deterioration curve even if the inspection records were limited.

Bu et al. (2015) incorporated both state-based model and time-based model to predict the future bridge condition ratings. The state-based model was based on both Elman neural network and backward prediction model. For the time-based model, the transition probabilities were calculated based on the Kaplan and Meier (K-M) method. They compared between the proposed model and the traditional regression-based optimization method to calibrate the Markov model. The proposed model provided better performance than the traditional method, whereas 464 inspection records were used as an input for the model.

Zambon et al. (2017) compared between homogenous and non-homogenous Markov chain with waiting times of weibull and exponential distributions in addition to the gamma process. The developed models were based on 1100 concrete bridge decks from the Portuguese inventory. They concluded that the prediction capability of the gamma process outperformed the Markov chain model. Ranjith et al. (2013) compared between three stochastic models based on their capability to predict the future performance of timber bridge elements. Three methods were used to calculate the transition probabilities which are: 1) percentage prediction, 2) regression-based optimization, and 3) non-linear optimization. They concluded that the non-linear optimization method outperformed other models based on goodness of fit and reliability tests. Based on the previous literature, the present study introduces a method that is based on time-based stochastic modelling to overcome the drawbacks of the deterministic models and state-based models.

3. RESEARCH METHODOLOGY

The framework of the proposed research methodology is shown in Figure 1. The proposed methodology is a defect-based model which is concerned with bridge elements because they are regarded as the elements that are vulnerable to the highest levels of deterioration. The proposed model is a five-stage methodology, whereas it is divided into five main modules which are: 1) data pre-processing module, 2) conditional probabilities module, 3) Bayesian belief network (BBN) module, 4) Metropolis-Hastings module, and 5) stochastic optimization module. The input and output of every stage are depicted in Figure 1. The first step of the data-preprocessing module is the definition of the condition states. The deterioration model is constructed based on a historical data of the element-level bridge inspections of concrete bridge decks. The output of the data processing stage is a group of censored events

The element-level inspection obtained from the Ministry of Transportation in Quebec (MTQ) defines the status (extent of damage) of the bridge elements based on four condition states which are: 1) condition state 1 (good), condition state 2 (fair), 3) condition state 3 (poor), and 4) condition state 4 (very poor). The bridge inspection data provides the infrastructure managers with information about the current status of the bridge inventory. Moreover, they are used to define future maintenance requirements.

The second step in the first stage is to design the architecture of the BBN. The BBN is composed of nodes and direct links. The nodes are the concrete bridge defects as well as the in-state probabilities. The term “in-state probability” refers to the probability that a certain element remains in a certain condition state i within a certain period of time. The direct links denote the dependencies between the bridge defects in addition to the dependency between the bridge defect and the in-state probabilities.

The proposed model is concerned with five types of bridge defects which are: corrosion, delamination, cracking, spalling, and pop-out. The in-state probabilities include: P_{11}, P_{22}, and P_{33}. The next step is to define a set of mutually exclusive states for each node. For each one of the five bridge defects, there are four condition states which are: “Good”, “Medium”, “Poor”, and “Very poor”. For the in-state probability, there are two states which are: “Yes”, and “No”. Then, a condition rating index is calculated based on the five bridge defects, and for each event, which helps in specifying whether or not, the bridge element will remain in its condition state.

The marginal probabilities are computed based on the frequency of occurrence of the condition state for a certain bridge defect such as the probability that the corrosion is in a good condition, or the probability that the spalling is in a poor condition. The proposed model is concerned with three transition events (TEs) due to the existence of four condition states. The transition events are: the transition from condition state 1 to condition state 2 ($TE(1,2)$), the transition from condition state 2 to condition state 3 ($TE(2,3)$), and the transition from condition state 3 to condition state 4 ($TE(3,4)$). There are two types of events which are: transition events and censored events. Censored events mean that the observed event which is the sequential change in condition state does not occur during the observation period (Morcous and Lounis, 2010). Transition time is the time that the facility takes to deteriorate from a certain condition state to the next lower condition state. The transition event may not be observed within the analysis period for two main reasons (Destefano and Grivas, 1998):

1. The element may be replaced while it is in its initial condition state, therefore it will not transit to the next condition state, and

2. The analysis period may be not long enough to allow the transition to the lower condition state.

Therefore, the events that are included in the latter stages are only the transition events. Within the proposed model, the following set of assumptions is incorporated.

1- The transition time is a random variable, whereas it is modeled based on probability distribution.
2- Distribution of the transition time is equivalent to the survival function. Survival function is sometimes called "reliability function" where it can be defined as the probability that a bridge element remains in its condition state for at least time (t).
3- The transition is assumed to occur in the middle of the inspection period.
4- A bridge element deteriorates one stage in unit step (one year), whereas no multi-stage transition is encountered.

The second stage is the conditional probabilities module. The purpose of this module is to calculate the conditional probabilities. The conditional probabilities can be either known or unknown. The conditional probabilities are computed based on the transition time in order to overcome the limitations of the state-based models. An example of the conditional probability is the probability that delamination becomes in a severe condition given corrosion is in a very severe condition within one year. For the known conditional probabilities, the Anderson-Darling test is performed to select the best-fit distribution of the transition time. The best-fit distribution is the one associated with the smallest Anderson-Darling statistic. Then, the parameters of the probability distribution of the transition time are defined using the maximum likelihood estimation (MLE) algorithm. Subsequently, the cumulative distributions are obtained, which enables the computation of the probability that a bridge defect x becomes in a condition state i given another bridge defect y is in a condition state j within one year. The unknown conditional probabilities are calculated based on the maximum entropy (ME) principle. The conditional probabilities are calculated based on a single objective function that maximizes entropy of the conditional probabilities. The decision variables are the conditional probabilities, whereas the optimization problem is solved via genetic algorithm. Genetic algorithm is a method that is used to solve problems based on genetic processes of biological organisms, whereas it is mainly based on two operators which are: mutation and crossover to search for the optimum solutions.

The third stage is the Bayesian belief network module. The proposed model deals with two sources of uncertainties which are: uncertainty associated with the transition time as mentioned before, in addition to the uncertainty associated with the transition probability. The transition times and in-state probabilities are dealt with as random variables that follow probability distributions, which enables the model to capture the randomness and uncertainties of the deterioration process. This provides more robustness to the stochastic modeling of the presented hybrid Bayesian-based optimization model, which aids in constructing robust and reliable deterioration curves. As such, the conditional probabilities and the marginal probabilities are expressed in the form of probability distributions rather than discrete values. The computerized tool enables the user to select the number of samples, the type and parameters of both conditional and marginal probabilities. The probability distributions are generated via stratified sampling technique called "Latin hypercube sampling" in order to overcome the limitations of the Monte Carlo sampling technique.

BBN is employed to investigate the relationship between the extent of severity of each of the bridge effect and their effect on the transition probability. The joint probability distribution is constructed based on the conditional and marginal probabilities. Finally, the distribution of the in-state probability is generated, and subsequently the type and the parameters of the probability distribution are defined. The constructed distribution represents the likelihood distribution of the unknown parameters (P_{11}, P_{22}, and P_{33}), which are the in-state probabilities in the present model. The parameters: P_{11}, P_{22}, and P_{33} indicate the probability that the bridge element remains in condition state 1, probability that the probability that the bridge element remains in condition state 2, the probability that the bridge element remains in condition state 3 within one year, respectively.

The fourth stage is the Metropolis-Hastings algorithm module. The Bayesian inference of the posterior distribution is performed via Metropolis-Hastings algorithm. Metropolis-Hastings algorithm is employed to generate the posterior distribution of the in-state probability by integrating two sources of information, which are the prior distribution and the likelihood function obtained from the Bayesian belief network model. The computerized tool enables the user to define the following parameters to calculate the posterior probabilities: 1) number of samples, 2) number of burn-in samples, 3) optimum acceptance rate, 4) parameters of the proposal distribution, and 5) the lag of the autocorrelation function. The previous modules are repeated for each of the three transition events $TE(1,2)$, $TE(2,3)$, and $TE(3,4)$, whereas the output of the fourth stage is the three posterior distributions for P_{11}, P_{22}, and P_{33}.

The fifth stage is the stochastic optimization model module. To this point, the in-state probabilities are demonstrated in the form of distributions. Thus, the primary objective of the stochastic optimization model is to compute the transition probabilities based on the posterior distributions obtained from the previous module. The stochastic optimization model is designed in order to address the stochastic nature of the decision variables. The transition process of the condition of the bridge element is assumed to be non-homogenous. A variable transition matrix is employed because it is not reasonable to assume the same deterioration pattern for the whole service life.

The service life of the bridge element is divided into a group of zones. In order to fulfill the requirements of the homogeneity of the Markov chain, zoning concept is implemented, whereas a transition probability matrix is calculated for each zone. Within each zone, the Markov chain model and the transition probability matrix are assumed to be homogenous. The decision variables of the stochastic optimization model are the transition probabilities for each zone, whereas they are calculated based on a single objective function that maximizes the joint probability distribution. The transition probabilities are computed using genetic algorithm by sampling from the posterior distributions. Via the transition probabilities, the future performance of the bridge element can be forecasted.

Two different architectures of the BBNs are investigated in order to analyze the influence of considering the dependencies on the accuracy of the performance prediction. The proposed model utilizes three performance indicators to compare between the four deterioration models. The three performance indicators are: root mean-squared error ($RMSE$), mean absolute error (MAE), and chi-squared statistic (x^2). The proposed model is then compared with the regression-based optimization method to calculate the transition probabilities. The previous method utilizes non-linear optimization to calibrate the Markovian model. In addition to that, the proposed model is compared with the gamma and weibull distributions. The deterioration prediction software is developed in Microsoft visual studio 2010, whereas the computerized tool incorporates a combination of both Matlab and C#.net programming languages.

INSERT FIGURE ١

4. MODEL DEVELOPMENT

4.1 Bayesian Belief Networks

Bayesian belief networks are probabilistic models that are based on directed acyclic graphs (DAG), which allows the modeling of probabilistic relationships between a set of variables. BBNs have been widely employed to develop decision support systems in several domains including: project management, risk management, environmental management, industrial processes, and medical diagnosis (Nguyen et al., 2016). BBN is usually formulated as follows $BBN = (P, G)$, whereas P indicates the parameters of the marginal probabilities while G indicates the model structure. G is formulated as follows $G = (V, A)$, and it represents the DAGs, which is composed of a finite set of nodes as well as directed arcs between pairs of nodes (Liang and Ghazel, 2017).

DAGs are graphs, where the links have directions and don't form cycles, whereas the nodes represent the random variables of interest, and each node is associated with several possible states (Siraj et al., 2014). The relationships between the nodes in the BBN are demonstrated in the form of family relationships, whereas X and Y are regarded as parents of Z, and Z is considered as the child of both X and Y if a link goes from X to Z, and Y to Z. The steps of formulating the BBN are as follows (Siraj et al., 2014):

1- Identifying the random variables that are needed to model the problem of interest such as X, Y and Z.

2- Establishing the casual interrelationships between the nodes, whereas Z is dependent on both X and Y as shown in Figure 2. A link between two variables indicates that there is direct dependence between the two variables, whereas one of them is called a "parent" while the other is called a "child".

3- Assigning a set of mutually exclusive states to each variable as well as the probability of each state such as very poor, poor, medium, and good. For instance, if random variable x is associated with four states, the sum of the probabilities equals to one.

4- Quantifying the conditional probability, which is the probability of occurrence of a certain event given that another event has occurred. The conditional probability allows the quantification the influence of the parent node on the desired node. For the parent nodes, there are no conditional probabilities and they are only given marginal probabilities.

INSERT FIGURE 2

Assume a domain $S = \{x_1, x_2, x_3, \dots \dots \dots \dots x_n\}$ is a set of random variables. The joint probability distribution can be calculated using the chain rule and it expressed as follows.

$$f(x_1, x_2, x_3, \dots \dots \dots . x_n) = \prod_{i=1}^{n} f(x_i | pa(x_i)) \quad (1)$$

Where;

$f(x_i|pa(x_i))$ represents the joint probability distribution associated with node x_i given its parent $pa(x_i)$. The random variables are assumed to be conditionally independent of its non-descendants given their parents.

BBN is based on Bayes' theorem, which has proven to be an efficient method to represent the conditional probabilities between a set of random variables. In a BBN, for a set of mutually exclusive events $\{A_1, A_2, A_3, \dots \dots \dots \dots . A_m\}$, and a given observed event B, the updated probability can be computed as follows (Kabir et al., 2016).

$$P(A_j|B) = \frac{P(B|A_j) \times P(A_j)}{\sum_{i=1}^{m} P(B|A_i) \times P(A_i)} \quad (2)$$

Where;

$P(A_j|B)$ represents the posterior occurrence probability of A given that the condition B has occurred. $P(B)$ indicates the marginal probability. $P(A)$ indicates the prior probability. $P(B|A)$ is the conditional probability of occurrence of B given that the even A has occurred. The implemented approach to compute the conditional probabilities are discussed in the following sections.

The developed Bayesian network is depicted in Figure 3. In order to provide a sound platform for the logic of the architecture of the BBN, the description of the bridge defects and the sequence of the deterioration process are described first. As mentioned before, the developed model considers five bridge defects: corrosion, delamination, cracking, spalling, and pop-out. Concrete bridges are vulnerable to several deterioration agents, which promote their deterioration along their service life. The deterioration agents include: freeze-thaw cycles, variable traffic overload, chloride-induced corrosion of the steel reinforcement, sulfates, deferred maintenance etc. Corrosion of the steel reinforcement is now recognized as the predominant and major cause of the degradation of the concrete structures. When the steel reinforcement corrodes, the resulting rust (iron oxides) expands to occupy a greater volume than the steel reinforcement. Delamination is the separation of concrete planes parallel to the surface of concrete, which occur because of the tensile failures (Sohangphurwala, 2006).

Cracking, spalling and pop-out are surface defects. A crack is a linear fracture, which extends partly or completely through the concrete member because of the tensile stresses. The tensile stresses are initially carried out by the steel reinforcement and concrete. When the tensile stresses exceed the structural capacity of the concrete, the concrete starts to crack and the tensile stresses are transformed completely to the steel reinforcement (Department of Transport and Main Roads in State of Queensland, 2016). A spall is a fragment of concrete, which has been detached from a larger concrete mass, whereas spalling occurs because of corrosion, overloading, fire, among others. (Department of Transport and Main Roads in State of Queensland, 2016). A pop-out is a conical fragment that breaks out of the concrete surface leaving a hole that varies in size from a diameter of 5 mm to 50 mm. and it can reach in some cases to 300 mm. Fractured aggregates are usually found at the bottom of the hole (Champiri et al., 2012).

The bridge defects are initially induced as a result of corrosion. Therefore, at the initial stages of the corrosion of the reinforcement, corrosion appears as rust-staining on the surface of concrete. In the latter stages, concrete cover above the reinforcement delaminates, cracks, and finally spalls off exposing the heavily corroded reinforcement rebars (Department of Transport and Main Roads in State of Queensland, 2016).

The same Bayesian network is repeated for P_{22} as well as P_{33}. Thus, the developed model incorporates three BBNs for P_{11}, P_{22}, and P_{33}. The marginal probability is the probability that a bridge defect x is in a condition state i such as the probability that delamination is in a severe condition or the probability that cracking is in a good condition. Besides, the Parent node "corrosion", Bayesian belief network is divided into three levels: level 1, level 2, and level 3. Level 1 describes the relationship between corrosion, and delamination. Level 2 describes the relationship between delamination, and surface defects. Finally, level 3 describe the relationship between the surface defects and the transition in the condition state.

The presented BBN takes into consideration the dependency between the bridge defects, which finally leads to the transition in the condition state. An example of the conditional probability in the first level is the probability that delamination becomes in a good condition given corrosion is in a medium condition within one year. The probability that spalling becomes in a very severe condition given delamination is in a severe condition within one year is an example of the condition probabilities in the second level. An example of the conditional probabilities in the third level is the probability that the bridge deck remains in a good condition state (bridge deck doesn't transit from the good state to the medium to the medium condition) given spalling is in a severe condition within one year. The resultant of the Bayesian belief network model is the likelihood function of the in-state probabilities P_{11}, P_{22}, and P_{33}. The described model is compared with another model that assumes that the bridge defects are independent to study the implication of modelling the dependencies on the prediction of the condition of the bridge deck. The architecture of the Bayesian belief network in the second hybrid Bayesian-based optimization model is depicted in Figure 4. Inference of the Bayesian belief network is conducted using Microsoft MSBNx.

INSERT FIGURE 3

INSERT FIGURE 4

4.1.1 Known conditional probabilities

As mentioned before, the conditional probabilities are divided into two categories which are: known conditional probabilities and unknown conditional probabilities. Known conditional probabilities are the conditional probabilities that can be computed from the available dataset. On the other hand, unknown conditional probabilities are the conditional probabilities that cannot be computed from the available dataset. Thus, they are calculated using the maximum entropy optimization approach. The conditional probabilities are calculated based on the transition time.

The transition time differs from one facility to another because of the stochastic nature of the deterioration process (Morcous and Lounis, 2007). Therefore, the transition time is expressed in the form of distribution in order to capture the uncertainties and randomness associated with the deterioration process, whereas there is no certainty that an element will enter a certain condition state within a certain interval of time. The developed automated tool selects the best-fit distribution from a set of five commonly used probability distributions, which are: weibull, lognormal, exponential, normal and extreme value. The best-fit distribution of the transition time is determined based on Anderson Darling test. Anderson Darling is one of the goodness of fit tests that can evaluate the compatibility of fitting a set of data to a certain probability distribution.

Anderson Darling test (A^2) is utilized to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function. Anderson darling provides more weight to the distributions' tail than the Kolmogorov-Smirnov test. Anderson Darling Statistic can be calculated as follows (Love at al., 2013).

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2\mathrm{i} - 1) \times (\ln \mathrm{F}(\mathrm{x_i}) + \ln(1 - \mathrm{F}(\mathrm{X_{n-i+1}}))) \quad (3)$$

Maximum likelihood estimation (MLE) algorithm is used to compute the parameters of the probability distributions. MLE is based on finding the parameters that maximize the likelihood function where the observations are assumed to be independent (Nielsen, 2011). The MLE is characterized by being is asymptotically efficient where larger the sample size the more likely the parameters converge to precise values (Datta and Datta, 2013).

Assume $\mathrm{y}_1, y_2, y_3, \dots \dots \dots, y_n$ are independent and identical distributed random variables (iid). The probability density function of a random variable y conditioned on a set of parameters θ is denoted as $f(y_i/\theta)$. The joint density is the product of the probability density functions and it is at the same time likelihood function. The joint density is depicted as follows.

$$f(y_1, y_2, \dots \dots . y_n|\theta) = \prod_{i=1}^{n} f(y_i|\,\theta) = \mathrm{L}(\theta/\mathrm{y}) \tag{4}$$

The main idea behind the MLE is to find the unknown parameter θ that maximizes the $\log \mathrm{L}(\theta/\mathrm{y})$ because it is often easier to maximize the log-likelihood function than the likelihood function itself.

$$\log \mathrm{L}(\theta/\mathrm{y}) = \sum_{i=1}^{n} \log\,(f(y_i|\theta)) \tag{5}$$

4.1.2 Unknown conditional probabilities

The terms unknown and missing conditional probabilities are used interchangeably within the context of the paper. Estimation of the missing probabilities is a computationally complex task with multiple solutions (Paris, 2005). There are several approaches to estimate the missing conditional probabilities such as assigning an equal probability equals to $1/n$ to each state, whereas n indicates the number of states. Another approach is to assume a uniform distribution for the unknown conditional probabilities (Siraj et al., 2014). Pendharkar (2008) addressed the problem of filling-in missing conditional probabilities using the maximum entropy approach. Entropy can be defined as the expected amount of information produced by a stochastic model, whereas entropy can be used as a measure of disorder or uncertainty in a random experiment. The principal of maximum entropy states that subject to precisely stated prior data, the probability distribution which best describes the current state of knowledge, is the one with the largest entropy. The maximum entropy optimization problem can be formulated as follows (Pendharkar, 2008). The following optimization problem is solved using the genetic algorithm.

$$\xi = -\sum_{p_{ij}\epsilon M} (\mathrm{p_{ij}} * \ln(p_{ij})) - \sum_{p_{ij}\epsilon N} \mathrm{p_{ij}} \frac{p_{ij}}{o_{ij}} - \sum_{p_{ij}\epsilon Z} \mathrm{p_{ij}} \quad , p_{ij} \geq 0, \forall\, i, j \tag{6}$$

Where;

ξ denotes the entropy. p_{ij} represents the predicted conditional probabilities. o_{ij} represents the observed conditional probabilities. The predicted conditional probabilities are divided into three main sets which are: missing set (M), non-missing set (N), and zero set (Z). M is a set where the predicted conditional probabilities do not have corresponding observed probabilities. N is a set where the predicted conditional probabilities are associated with their corresponding observed probabilities. Z represents a set where the predicted conditional probabilities have corresponding observed probabilities of zero value. More details about the genetic algorithm can be in Mohammed Abdelkader et al. (2019).

4.2 Bayesian Inference

Statistical inference can be defined as "theory, methods, and practices of building judgments about the parameters of a population commonly. Random sampling is a commonly-used approach, which helps in drawing conclusions from the data that are subjected to random variation (Garfield and Ben-Zvi, 2008). There are two broad categories of interpreting the probabilities in the statistical inference which are: Bayesian inference and frequentist inference. The main significant difference between the Bayesian inference and frequentist inference is the capability of the Bayesian inference to include additional information in the form of prior distribution (Rudas, 2008). Kelly et al. (2010) illustrated that the main distinctive feature of the Bayesian inference is its capability to consider information from different sources into the inference model. Thus, the Bayesian inference integrates the old knowledge and the new knowledge into an evidence-based state of knowledge distribution. Bayesian inference is based on interpreting the probability as "a rational, and conditional measure of uncertainty", which nearly matches the sense of the word probability in the ordinary language (Bernardo, 2011).

Bayesian inference provides a complete paradigm for both statistical inference and decision-making under uncertainty. In addition to that, Bayesian inference helps to incorporate scientific hypothesis in the analysis using prior knowledge. Thus, Bayesian inference can be applied to model complex problems, which conventional approaches are incapable to deal with (Bernardo, 2011). In the Bayesian inference, there are two sources of information about the unknown parameter which are: prior distribution and likelihood function. The prior distribution is mainly based on previous studies. Consequently, the prior distribution and the likelihood function are used to generate the posterior distribution. Prior probability is the probability of the parameter of interest before the current data is examined while the likelihood function represents the likelihood of the parameter of interest given the current data is observed. Finally, posterior probability is the probability of the after the current data is examined. In other words, the Bayesian inference provides a compromise between the likelihood data and prior knowledge.

Assume $Y = \{Y_1, Y_2, Y_3, \dots \dots \dots \dots Y_n\}$ represents a set of condition ratings for a group of bridge decks. Bayesian inference is used to estimate the unknown parameter θ of the probabilistic model M as follows (Micevski et al., 2002).

$$\varphi(\theta_1, \theta_2 | Y, M) = \frac{f(Y \,|\, \theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2, M)}{\int f(Y|\theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2, M) \; d\theta} = \frac{f(Y \,|\, \theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2, M)}{f(Y|M)} \quad (7)$$

Where;

$\theta_1$and θ_2 denote the unknown parameters of the model which are the transition probabilities in the present study. $\varphi(\theta_1, \theta_2 | Y, M)$ denotes the posterior distribution. $f(Y \,|\, \theta_1, \theta_2, M)$ indicates the likelihood function and it is obtained as a result of a combination of BBN, Latin hypercube sampling, and genetic algorithm. $f(Y|M)$ is the marginal likelihood function, whereas it is independent of θ. Therefore, the posterior distribution can be expressed as follows. $f(Y|M)$ is a normalizing constant to ensure that the posterior distribution is integrated to one over all its possible values.

$$\varphi(\theta_1, \theta_2 | Y, M) \propto f(Y \,|\, \theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2\theta_3, M)) \quad (8)$$

Where;

The posterior distribution is proportional to the likelihood function multiplied by the prior distribution. The definition of prior probabilities requires special attention, whereas it is based on knowledge and feedback of the experts. Prior distribution is assumed non-informative distribution. Consequently, the posterior distribution is proportional to the likelihood distribution. As the prior distribution becomes more informative, it will have a greater influence on the posterior distribution. Non-informative distribution is chosen because there is a lack of available information about the unknown parameters, whereas all the values of the unknown parameters are assumed to be of equal probability.

Several researchers compared between a set of commonly-utilized non-informative priors in the Bayesian analysis and Bayesian estimation such as Jeffreys prior and uniform prior. Ataf et al. (2013), Gilani and Abbas (2008), and Tahir and Hussain (2008) compared between uniform and Jeffreys priors. They concluded that they provided similar results. As such, the uniform prior presents a better alternative due to its simplicity to be programmed and modeled. Aslam et al. (2018) compared between the uniform and Jeffreys priors in the Bayesian estimation of the mixture of exponentiated inverted weibull distribution. They highlighted that the uniform prior provided better results than the Jeffreys prior as per some performance metrics. Thus, the prior distribution is assumed to be non-informative uniform distribution within the interval $[0, 1]$.

4.2.1 Metropolis-Hastings algorithm

Metropolis-Hastings algorithm (MHA) is a Markov chain Monte Carlo (MCMC) method that is used to draw samples from a posterior distribution of the model parameters through a Bayesian chain. MHA is capable of simulating the posterior distribution by calculating the acceptance probability. MHA was selected as one of the ten algorithms that had the greatest influence on the development and practice of science and engineering in the twentieth century (Connor, 2003). Metropolis-Hastings is a modified version of the Metropolis algorithm (Metropolis et al., 1953; Hastings, 1970). MHA is a universal algorithm for deriving the Markov kernel in the MCMC, which is theoretically valid for sampling from any target probability density function (Robert and Casella, 2009). In the present study, the parameter uncertainties are addressed by estimating the posterior probability distributions by the Metropolis-Hastings algorithm.

The basic idea about the Metropolis-Hastings algorithm is to draw samples from an arbitrary distribution called the "proposal distribution", and then correcting those samples to better approximate and converge the proposal distribution to the target distribution. One of the key components of the MHA is the proposal distribution, whereas given a current state, a candidate state. Thus, the proposal distribution is used to determine the probability of moving from the current state to the next state (Mehrez et al., 2014). The proposal distribution is sometimes called "candidate-generating distribution", "probing distribution", or "jumping distribution" (Wu and Chen, 2009). The parameters of the proposal distribution play a very important role in the convergence speed of the MHA. The implementation of MHA incorporates a Markov chain, whereas the distribution of the current sample depends on the previous sample (Dilip and Babu, 2013).

The process of drawing samples from a posterior distribution of parameter θ can be performed as follows (Dilip and Babu, 2013; Hsein Juang et al., 2013):

1- For the first iteration (k=1), the first sample can be drawn randomly from the prior distribution, or the first sample can be the mean value of the prior distribution.

2- For the next iterations (starting from k=2), a new candidate sample θ^* is drawn from the proposal distribution $q(\theta^*|\theta^k)$. The multivariate normal distribution is selected to be the proposal distribution because it is symmetric, easy to sample from in addition to its fast convergence in the Bayesian inference. The proposal distribution is selected to be a symmetric. In other words, the MHA provides symmetric random-walk, whereas $q(\theta^*|\theta^{k-1}) = q(\theta^{k-1}|\theta^*)$.

3- Generate a random number u from the uniform distribution $U\ [0, 1]$ within each iteration.

4- Compute the acceptance probability (transition kernel) as follows.

$$\alpha = \min\left[1, \frac{\varphi(\theta^*|Y) \times q(\theta^{k-1}|\theta^*)}{\varphi(\theta^{k-1}|Y) \times q(\theta^*|\theta^{k-1})}\right] = \min\left[1, \frac{f(Y|\theta^*) \times \pi(\theta^*) \times q(\theta^{k-1}|\theta^*)}{f(Y|\theta^{k-1}) \times \pi(\theta^{k-1}) \times q(\theta^*|\theta^{k-1})}\right] \quad (9)$$

Where;

If $u \leq \alpha$, set $\theta^k = \theta^*$, otherwise set $\theta^k = \theta^{k-1}$

5- Repeat the steps from two to five until the target number of samples is reached, i.e., Markov chain length is reached.

The parameters of the proposal distribution should be carefully specified because they significantly control the convergence speed. For instance, small values of the covariance matrix result in high acceptance rate and slow convergence of the MCMC sampler, and consequently slow exploration of the target distribution as well as high-auto-correlated samples. On the contrary, high values of covariance matrix result in small acceptance rate. Thus, for a large number of iterations, the MCMC sampler will stick to the same values, which results in poor exploration of the target distribution and high auto-correlated samples. The acceptance rate represents the number of accepted samples in the simulation process.

Because of the Markovian nature of the MCMC algorithms, the first values of the samples are highly dependent on the starting value at the first iteration. Therefore, the initial values are discarded as burn-in samples. Burn-in samples aid in minimizing the influence of the initial values on the inference of the posterior distribution. Roberts et al. (1997) stated that an acceptance rate of 0.234 provides a good indication of the Markov chain, and therefore convenient candidate solutions. As such, the number of iterations, number of burn-in samples, and the parameters of the proposal distribution such as the mean and covariance matrix are tuned to ensure that the acceptance rate is close to 23.4%.

Geweke test is one of the convergence diagnostics, which provides a robust assessment for the convergence of the MCMC algorithms that are based on a single chain (Geweke, 1992). Geweke test compares the sample mean of an early segment of the Markov chain to the mean of a latter segment of the Markov chain. The means of two non-overlapping segments should be selected. In the present study, the first 10% of the chain and the last 50% of the chain are utilized. Geweke test is based on a null hypothesis that the values of the mean are sampled from a stationary distribution. If the value of the Z-score is less than 1.96, therefore the null hypothesis is accepted, i.e., convergence has been reached. Otherwise, the null hypothesis is rejected (Wang and Wang, 2016). The Z-score is equal to the difference between the two means divided by the asymptotic standard error of their difference (Furrer and Molinaro, 2016). Geweke test is a two-sided test, whereas large absolute values of Z-score indicate convergence problems. The Z-score can be calculated as follows.

$$Z = \frac{|\theta_1^- - \theta_2^-|}{\sqrt{\frac{S_1(0)}{n_1} + \frac{S_2(0)}{n_2}}} \tag{10}$$

Where;

θ_1^- and θ_2^- represent the means of the first segment, and second segment, respectively. $S_1(0)$ and $S_2(0)$ denote the variance of the first segment, and second segment, respectively. The variances are calculated as the spectral density at frequency zero.

Autocorrelation function of each parameter is examined in order to ensure that the chain has converged (Bai et al., 2011). The autocorrelation coefficient is bounded between -1 and 1. When the chains have high autocorrelation, this means that the current chain is inefficient in simulating the posterior distribution. Consequently, there is less precision in the parameter estimate and the chain has not converged. On the other hand, small values of autocorrelation indicate that the Markov chain is efficient in simulating the posterior distribution and the chain has converged. Moreover, the parameter space is well explored.

4.2.2 Latin hypercube sampling

Latin hypercube sampling (LHS) was initially proposed by Mckay et al. (1979) and it was later improved by Iman and Conover (1985). As mentioned before, the values of marginal probabilities, conditional probabilities and subsequently likelihood function probabilities are demonstrated in the form of probability distributions to model the uncertainties of the deterioration process. Latin hypercube sampling is utilized to draw samples from the probability distributions of the marginal and conditional probabilities. LHS is a modified stratified sampling of Monte Carlo simulation. LHS is an efficient sampling technique that accurately constructs probability distributions using sampling within less number of iterations and less sampling error. One of the main advantages of the LHS is that it converges faster than the Monte Carlo sampling (less computational effort), whereas it provides a more accurate and efficient method to estimate the statistical parameters of a model. The steps of the LHS can be performed as follows (Li et al., 2013):

1. The probability distribution of the input factor is partitioned into M non-overlapping and mutually exclusive bins of equal marginal probability $1/M$.
2. One of the bins is selected randomly within the first iteration and the first sample is selected from the first bin.
3. Until the remaining N iterations, one of the bins that has not been chosen in the previous iterations is selected for sampling where step 2 is again repeated. This process ensures all the N bins will be sampled from once.
4. The previous steps are repeated for all the input probability distribution.

4.3 Stochastic Optimization Model

Markov chain is considered as a special case of the Markov process and it can be considered as a series of transitions between condition states, whereas it assumes discrete condition states. A stochastic process is regarded as a first-order Markov chain if the probability in the future state depends on the present state not on the past state. Markov chain for a discrete parameter stochastic process (X_t) with discrete space can be expresses using Equation (11) (Bu et al., 2015).

$$P(X_{t+1} = i_{t+1} | X_t = i_t, X_{t-1} = i_{t-1}, \dots \dots .., X_1 = i_1, X_0 = i_0) = P(X_{t+1} = i_{t+1} | X_t = i_t) \qquad (11)$$

Where;

i_t represents the state of the process at time t. P indicates the conditional probability at any future event. $|$ indicates given condition.

The transition probabilities are represented by a matrix of order $n \times n$. The proposed model assumes only a single state transition within one-year period. A homogenous transition probability matrix is utilized within each zone. The transition probability matrix can be defined as follows.

$$P^{t,t+1} = \begin{bmatrix} P_{11}^{t,t+1} & P_{12}^{t,t+1} & 0 & \dots & 0 \\ 0 & P_{22}^{t,t+1} & P_{23}^{t,t+1} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & P_{n-1,n-1}^{t,t+1} & P_{n-1,n}^{t,t+1} \\ 0 & 0 & \dots & 0 & P_{n,n}^{t,t+1} \end{bmatrix} \qquad (12)$$

Where;

$P^{t,t+1}$ represents the transition probability matrix. Each element in the transition probability matrix represents the probability that an element will transit from a condition state i to condition state i within a certain time interval.

The condition state vector at any time ($Q(t+1)$) is obtained by multiplying the previous condition state vector ($Q(t)$) by the transition probability matrix ($P^{t,t+1}$). The $Q(t+1)$ can be calculated as follows (Bu et al., 2015).

$$Q(t+1) = Q(t) \times P^{t,t+1} \qquad (13)$$

The condition ratings using the Markov decision process can be calculated using Equation (14).

$$E(t) = Q(t) \times R' \qquad (14)$$

Where;

$E(t)$ represents the estimated condition rating using Markovian-chain method. $Q(0)$ represents the initial state vector where $Q(0) = [100\% \quad 0 \quad 0 \quad 0]$. R' represents the transpose of a vector of condition ratings where $R = [100\% \quad 71.71\% \quad 64.04\% \quad 43.49\%]$.

One of the main challenges of the Markov chain is the computation of the transition probabilities. For each in-state probability (P_{11}, P_{22}, and P_{33}), a posterior distribution is constructed for each one of the three transition events from the previous steps. The stochastic optimization model provides a solution to deal with the uncertainties associated with the transition probabilities of the bridge decks. The goal now is to compute the transition probabilities for each zone. Zoning concept is applied because of the variable deterioration of an infrastructure system. Thus, the entire service life is divided into zones and a transition probability matrix is associated with each zone. The transition probabilities are calculated based on the joint probability theory by maximizing the log-likelihood function. The optimization problem is a single objective function, whereas it is solved using the genetic algorithm. The optimum solutions are obtained by sampling from the posterior distribution of the three transition probabilities. The objective function can be expressed as follows. As shown in Equation (15), the likelihood function is maximized in order to estimate the unknown parameters given the current condition ratings of the bridge elements in the available dataset.

$$f\left(Y \middle| x_{iz}, y_{iz}, z_{iz}\right) = \prod_{z=1}^{zones} [[(CS_t \times P_z)[1]]^{N_{1t}} \times [(CS_t \times P_z)[2]]^{N_{2t}} \times [(CS_t \times P_z)[3]]^{N_{3t}} \times [(CS_t \times P_z)[4]]^{N_{4t}}] \quad (15)$$

$$log\left(f(Y|x_{iz}, y_{iz}, z_{iz})\right) = \sum_{z=1}^{Zones} \left[[(CS_t \times P_z)[1]] \times N_{1t}\right] + \left[[(CS_t \times P_z)[2]] \times N_{2t}\right] + \left[[(CS_t \times P_z)[3]] \times N_{3t}\right] + \left[[(CS_t \times P_z)[4]] \times N_{4t}\right] \quad (16)$$

$P_z = F(X_1, Y_2, Z_3)$

$X_1, \sim Normal(\mu_1, \sigma_1{}^2)$ and For 1≤ $\boldsymbol{i}$ ≤58, $\boldsymbol{i}$= $\boldsymbol{i}$+3

$Y_2 \sim Normal(\mu_2, \sigma_2{}^2)$ and For 2≤ j ≤59, j= j+3

$Z_3 \sim Normal(\mu_3, \sigma_3{}^2)$ and For 3≤ l ≤60, l= l+3

$f(Y|x_{iz}, y_{iz}, z_{iz})$ denotes the likelihood function, whereas it represents the probability of observing a set of Y condition ratings of bridge decks given unknown parameters x_{iz}, y_{iz} and z_{iz}. In other words, it can be expressed as the likelihood of observing unknown parameters x_{iz}, y_{iz} and z_{iz} given a set of condition ratings. P_z represents the transition probability of each zone z. $X_1, Y_2,$ and Z_3 represent the posterior distributions of the in-state probabilities of P_{11}, P_{22}, and P_{33}, respectively. These random variables follow the stated normal distributions. μ_1, μ_2, and μ_3 represent the mean values of the posterior distributions of P_{11}, P_{22}, and P_{33}, respectively. $\sigma_1{}^2$, $\sigma_2{}^2$, and $\sigma_3{}^2$ represent the variances of the posterior distributions of P_{11}, P_{22}, and P_{33}, respectively. x_{iz} represents the in-state probability P_{11} of the zone z. y_{iz} represents the in-state probability P_{22} of the zone z. z_{iz} represents the in-state probability P_{33} of the zone z. The variables i, j and l depict the index of the decision variable. The optimization model contains a constraint to ensure that the sum of the in-state probabilities and the transition probabilities equals to one for each transition event and for each zone.

4.4 Automated Platform

The prediction model is developed in Microsoft Visual Studio 2010, Microsoft SQL server 2010, visual C#.net, and Matlab. C#.net is a simple, modern, and object-oriented language, which is derived from C and C++. C#.net language is utilized due to its effectiveness, and flexibility of integration with other modules (Qu et al., 2011). The developed model incorporates a programming language, which is C#.net in addition to a scripting language, which is Matlab. The windows application is designed using Microsoft Visual Studio 2010, which helps to integrate the developed model with the Matlab scripts. Three references were added in order to be able to communicate with other programs:

1- "MSBNx" which is a COM reference used to communicate with the Microsoft belief networks software.
2- "MLApp" which is a COM reference used to communicate with Matlab scripts.
3- "Microsoft.Office.Interlop.Excel" which is a .net reference used to communicate with Microsoft Excel files.

The windows application is composed of 26 forms (from form "0" to form "25) in addition to three classes. The three classes are for the three performance indicators, which are $RMSE$, MAE, and x^2. Each class is composed of one method, whereas each method takes two parameters which are: training and testing datasets. A sample of the C#.net code is shown in Figure 5. The shown code enables the developed platform to access the inference engine and the nodes of the Bayesian belief network.

INSERT FIGURE 5

5. Current practices of Modeling Deterioration

The proposed model is compared with the regression-based optimization technique to calibration the Markovian model in addition to the weibull and gamma distributions. The previous methods are discussed in the following lines.

5.1 Regression-based optimization

Regression-based optimization method is the most common method to calibrate the Markovian model. The transition probabilities are calculated using non-linear optimization by minimizing the absolute differences between the condition ratings obtained from the Markovian model and the condition ratings obtained from the regression model. The transition probabilities are obtained using the following equation (Bu et al., 2015).

$$z = \sum_{i=1}^{N} \left| \left(A(t) - E(t, p_{ij}) \right) \right| \quad subject\ to \quad 0 \leq p_{ij} \leq 1\ , i = 1,2\ \ and\ 3, j = 1,2\ and\ 3 \qquad (17)$$

Where;

z denotes sum of the absolute differences. $A(t)$ represents the condition rating obtained from the regression model. $E(t, p_{ij})$ represents the condition rating obtained from the Markovian model. N represents number of years. The active set algorithm is used for non-linear optimization.

5.2 Stochastic distributions

The proposed model is compared with two stochastic distributions which are commonly used to model the deterioration of bridge elements which are: weibull and gamma distributions, whereas the conditions of the bridge elements are assumed to follow the weibull and gamma distributions. Semaan (2011) and Grussing et al. (2006) adopted the weibull distribution in modeling the deterioration process. The parameters of the two-previously mentioned stochastic distributions are computed based on the maximum likelihood estimation algorithm. The condition index at a certain time t is computed based on the derivation of cumulative distribution function (CDF) of the weibull distribution as follows.

$$C(t) = [1 - e^{-\left(\frac{t}{\beta}\right)^{\alpha}}] \times a \qquad (18)$$

Where;

$C(t)$ represents the condition index of the bridge element at a certain time t. β and α represent the scale parameter and shape parameter of the weibull distribution, respectively. a represents the initial condition of the bridge deck, which is 100 in the present study.

The condition index based on the cumulative distribution function of the gamma distribution can be mathematically defined using Equation (19) as per shape and scale parameters greater than zero (Pandey et al., 2005).

$$C(t) = \left[1 - \frac{1}{\beta^{\alpha}\Pi(\alpha)}\int_0^t \times y^{\alpha-1} \times e^{\frac{-y}{\beta}}\, dy\right] \times a \tag{19}$$

Such that;

$\Pi(\alpha)$ for any positive real number is calculated as follows.

$$\Pi(\alpha) = \int_0^{\infty} x^{\alpha-1} \times e^{-x}\, dx \quad for\ \alpha > 0 \tag{20}$$

Where;

α and β represent the shape parameter and scale parameter of the gamma distribution, respectively.

6. MODEL IMPLEMENTATION

The model implementation section is divided into two main parts. The firsts section describes how the transition probabilities are calculated. The second section describes a comparison between the developed model and other deterioration models.

6.1 Computation of transition probabilities

The proposed methodology utilizes 181 inspection records from the Ministry of Transportation in Quebec (MTQ), Canada. One hundred fifty six are used for training the model, while the remaining twenty five records are used for testing the model. Out of the 156 inspection records, there are 104 transition events and 52 censored events, whereas the number of events for the $TE(1,2)$, $TE(2,3)$, and $TE(3,4)$ are 55, 19 and 30, respectively. As mentioned before, there are four condition states covering the three transition events. The four condition states are: Good (1), medium (2), poor (3), and very poor (4). There are 18 known conditional probabilities (level 3) of P_{11} based on a four condition state model. The automated tool enables the user to calculate the known conditional probabilities. As shown in Figure 6, the “Import” button enables the user to enter the events and the transition time associated with each event in a Microsoft Excel sheet. Then, by clicking “View” button, the automated tool calculates the conditional probability, defines the type of the probability distribution of the transition time in addition to the parameters of the probability distributions.

INSERT FIGURE 6

For the unknown conditional probabilities, they are calculated based on the maximum entropy approach. As shown in Figure 7, the user is asked to specify the following parameters: population size, number of iterations, crossover rate, mutation rate, number of elites, type of parents’ selection strategy, and the tournament size in the case of the tournament selection strategy. The computerized tool enables the user to choose between three parents’ selection strategies which are: roulette wheel strategy, tournament selection strategy, and uniform selection strategy. The population size, number of iterations, crossover rate, mutation rate, and number of elites are assumed 50, 50, 0.8, 0.05, and 10, respectively. A tournament selection strategy is selected with a tournament size equals to two.

The output of the model is the values of the conditional probabilities, optimum fitness function value, and the convergence curve. The values of the calculated conditional probabilities are shown on the right side of the interface in the data-grid view. The convergence graph is shown in Figure 8. “Best” indicates the best fitness function value within the last population in the last iteration while “Mean” indicates the average fitness function values of all chromosomes in the final iteration. The optimum fitness function value is -0.7732. As shown in Figure 8, the objective function starts to stabilize after iteration 40.

INSERT FIGURE 7

INSERT FIGURE 8

The next module is the Bayesian network module, after the calculation of the marginal and conditional probabilities, the probability distributions for both of them are generated. The probability distributions are generated via Latin hypercube sampling. The user can select between five probability distributions: weibull, normal, lognormal, exponential, and extreme value. For the present study, the probability distributions are obtained based on 150 samples from a normal distribution, whereas the mean equals to the previously calculated probability and a small standard deviation. The next step is to generate the probability distribution of the likelihood function for the in-state probabilities, which is done by accessing the inference engine of the MSBNx.

The posterior distributions for each of the three in-state probabilities are computed using the Metropolis-Hastings algorithm interface, whereas the user is able to define the number of samples, number of burn-in samples, optimum acceptance rate, parameters of the proposal distribution, and the lag of the autocorrelation function. The proposed model utilizes a multi-variate normal distribution as a proposal distribution and a uniform distribution as a prior distribution. The model output is the posterior probabilities of the in-state probabilities, values of the convergence diagnostics, and a set of plots. The interface of the Metropolis-Hastings algorithm module is shown in Figure 9. The parameters of the Metropolis-Hastings algorithm are described in Table 1. By clicking the "View" button, three types of output are provided. First, the values of the samples of the in-state probability P_{11} are then shown in the data-grid view.

INSERT TABLE 1

INSERT FIGURE 9

The second output of the Metropolis-Hastings module is a group of trace plots for the in-state probabilities. The trace plots are as follows: posterior distributions, sampling process, convergence of the mean, and autocorrelation function. The posterior distributions of P_{11}, P_{22}, and P_{33} are shown in Figures 10 and 11. As shown in Figures 10 and 11, the posterior distributions of the in-state probabilities are normal distributions. The values of the mean of three posterior distributions of P_{11}, P_{22}, and P_{33} are: 0.9552, 0.9597 and 0.9211, respectively. The values of the standard deviation of the three posterior distributions are small, whereas the values of the standard deviation of the three posterior distributions of P_{11}, P_{22}, and P_{33} are: 0.01348, 0.01328 and 0.01361, respectively.

INSERT FIGURE 10

INSERT FIGURE 11

The trace plots for the 5,000 samples of P_{11}, P_{22}, and P_{33} are depicted in Figures 12 and 13. After setting 1,000 burn-in samples, the present study generated 5,000 samples for each one of the three in-state probabilities. These figures provide a simulation of the three in-state probabilities within each iteration. The trace plots of the mean convergence of P_{11}, P_{22}, and P_{33} are shown in Figures 14 and 15. Figures 14 and 15 describe the variation of the mean within iterations. The mean of the posterior distribution almost stabilizes within the first 500 iterations, which proves that the Markov chain has converged. The autocorrelation function of P_{11}, P_{22}, and $P_{33}$are depicted in Figures 16 and 17. Trace plots of the autocorrelation function depict how the autocorrelation coefficient decays. As shown in Figures 16 and 17, the values of the correlation coefficient for the posterior distributions of P_{11}, P_{22}, and P_{33} are very small within the last iteration, which proves that the chains have converged.

INSERT FIGURE 12

INSERT FIGURE 13

INSERT FIGURE 14

INSERT FIGURE 15

INSERT FIGURE 16

INSERT FIGURE 17

In addition to the trace plots, three convergence diagnostics are presented acceptance rate, Z-score of Geweke test, and the final autocorrelation coefficient. As shown in Figure 9, the acceptance rate is 0.2306, Z-score is 1.7706, and final autocorrelation coefficient is -0.00114 for the in-state probability P_{11}. If the calculated probabilities satisfy the three tests, a message box will appear indicating that the current chain has converged. Otherwise, a message box will appear indicating that the current chain did not converge. As shown in Figure 9, the constructed chain of

P_{11} satisfies the three convergence diagnostics, which means that the current chain has converged. Since, the Markov chains fulfill the convergence diagnostics. Thus, the type and parameters of the prior and proposal distributions are correctly defined.

The final module is the stochastic optimization module, whereas the output of the optimization module is the transition probabilities for each zone. The interface of the stochastic optimization model is illustrated in Figure 18. The stochastic optimization module enables the user to define the following: 1) number of elements in each condition state, 2) study period, and 3) parameters of the genetic algorithm. The user first clicks the "Import" button, which enables he/she to enter the number of elements in each condition state in a Microsoft Excel sheet. The study period is assumed 100 years. The population size, number of generations, crossover rate, mutation rate and number of elites are assumed 200, 100, 0.8, 0.05 and 10, respectively. A tournament selection strategy is utilized with a tournament size equals to two.

INSERT FIGURE 18

By clicking the "View" button, three types of outputs are provided. The outputs of the model are the following: 1) optimum fitness function value, 2) transition probability matrix for each zone, and 3) convergence graph. The convergence curve of the stochastic optimization model is depicted in Figure 19, whereas the optimum fitness function value equals to 275.1863. The calculated transition probability matrices are then shown in the data-grid view. All values of the transition probabilities are depicted in Table 2. The deterioration of the bridge deck does not follow the same pattern along the study period whereas the transition probabilities of each zone are different from each other. The convergence graph of the stochastic optimization model is shown in Figure 19. The fitness function starts to stabilize starting from iteration 39.

The automated tool provides plots for the deterioration curve and condition states distribution (see Figure 20). As shown in Figure 20.a, condition state 1 is the dominant condition state. Then, condition state 4 becomes the dominant condition state. The deterioration curve of the hybrid Bayesian-based optimization model is shown in Figure 20.b. The performance metrics interface is depicted in Figure 21. The developed tool enables the user to define the number of training and testing cases. The “Import” button enables the user to enter the age and condition rating for both the training and testing cases in a Microsoft Excel sheet. The automated tool incorporates a database for the different values of thee chis-squared statistic at different degrees of freedom at a significance level of 5%. By clicking the “Calculate” button, the values of $RMSE$, MAE, and x^2 for both training and testing cases. Moreover, a message box will pop-up indicating that the null hypothesis is rejected. Otherwise, the null hypothesis is accepted. In the present study, the value of the chi-squared statistic is smaller the chi-squared critical value. Therefore, the null hypothesis is accepted. In other words, the observed condition ratings of the bridge decks are consistent with the predicted condition ratings.

INSERT TABLE 2

INSERT FIGURE 19

INSERT FIGURE 20

INSERT FIGURE 21

6.2 Comparison between Deterioration Models

The developed hybrid Bayesian-based model is denoted as hybrid Bayesian-based model-1 ($H - B1$), which is based on the Bayesian belief network depicted in Figure 3. Another model is developed following the same procedures of the developed model except for the architecture of the Bayesian belief network. For the second model, the bridge defects are assumed to be independent on each other as shown in Figure 4. The second model is denoted as hybrid Bayesian-based model-2 ($H-B2$). The comparison between the two models enables to investigate the effect of dependencies on the prediction accuracy of the condition of bridge decks (deterioration mechanism). The regression-based optimization model is denoted as RBO. As such, the transition probabilities obtained using the regression-based optimization method is shown in Equation (21). The parameters of the weibull and gamma distributions are calculated using the maximum likelihood estimation algorithm. The scale and shape parameters of the weibull distribution are 84.2378 and 5.0643, respectively. The scale and shape parameters of the gamma distribution are 21.767 and 3.5521, respectively.

$$P^{t,t+1} = \begin{bmatrix} 85.5\% & 2.25\% & 0 & 0 \\ 0 & 91.8\% & 15.15\% & 0 \\ 0 & 0 & 91.2\% & 7.8\% \\ 0 & 0 & 0 & 100\% \end{bmatrix} \quad (21)$$

A comparison between different deterioration models is shown in Table 3. The chi-squared critical values at 180 degrees of freedom and a significance level of 5% equals to 212.304. In terms of $RMSE$, $H-B1$ achieved the lowest $RMSE$ ($RMSE = 0.7716$). On the other hand, gamma distribution achieved the highest $RMSE$ ($RMSE = 1.4584$). Thus, $H-B1$ achieved the best performance based on $RMSE$. For MAE, $H-B1$ provided the lowest MAE ($MAE = 0.5401$). On the other hand, gamma distribution achieved the highest MAE ($MAE = 0.9899$). Thus, $H-B1$ provided the best performance according to MAE. The gamma and weibull distributions fail to pass the chi-squared test because the chi-squared critical value is larger than the chi-squared statistic. $H-B1$ provided the best performance according to x^2 ($x^2 = 46.0583$) followed by $H-$

$B2$ ($x^2 = 62.5$), and finally RBO ($x^2 = 69$). Based on the previous statistics, $H - B1$ outperformed other models in terms of $RMSE$, MAE, and x^2.

It is worth mentioning that $H - B1$ outperformed the model $H - B2$ for all the performance metrics, which proves that modeling the interaction between the bridge defects provides more accurate results. Moreover, $H - B1$ outperformed the weibull distribution and gamma distribution models which proves the superiority of the time-based models over the state-based models in modeling the deteriorations process of the bridge elements, which proves the conclusion derived by Ravirala and Grivas (1995) that it is more reasonable to model the deterioration process as a function of time. As per the previous comparison, the proposed model surpassed other deterioration models for both training and testing datasets, which the infrastructure managers can benefit from in deciding the optimal intervention actions. Deterioration prediction is one of the most crucial parameters in maintenance optimization models. Thus, infrastructure managers need reliable prediction models such as the hybrid Bayesian-based optimization model to forecast the condition of the bridge elements, whereas the early diagnosis of the deterioration scenarios helps in optimizing maintenance, repair and rehabilitation activities for both project and network levels.

INSERT TABLE 3

7. CONCLUSIONS

This paper presented an automated defect-based model that predicts the future performance of concrete bridge based on a hybrid Bayesian-based optimization approach. The proposed methodology utilizes historical inspection records of the Ministry of Transportation in Quebec. The Bayesian belief network enables the investigation of the degree of influence of the bridge defects on the condition. Five bridge defects are considered which are: corrosion, delamination, cracking, spalling and pop-out. The proposed model takes into consideration the uncertainty associated with the transition time and transition probabilities, whereas the marginal and conditional are demonstrated in the form of probability distributions rather than discrete values. The conditional probabilities are either known or missing. For the known conditional probabilities, the best-fit distribution of the transition time is determined based on Anderson-Darling statistic and the parameters of the transition time distribution are defined maximum likelihood algorithm. For the missing conditional probabilities, maximum entropy approach is utilized to calculate their values using the genetic algorithm.

Metropolis-Hastings is then employed to calculate the posterior distribution of the in-state probabilities by integrating likelihood and prior probabilities. The automated tool enables the user to update the posterior distributions of the in-state probabilities as per the available inspection records. A stochastic optimization model is introduced in order to deal with the stochastic nature of the in-state probabilities. The stochastic optimization model employs genetic algorithm to calculate the transition probability matrix for each zone. A computerized tool is developed in order to facilitate the implementation all the features of the deterioration model for the users. The computerized tool incorporates both a programing language and a scripting language. The programming language is C#.net while the scripting language is the Matlab. Two different architectures of the Bayesian belief networks are compared in order to study the influence of considering the deterioration mechanism of bridge decks. The proposed model outperformed other models based on three performance indicators whereas the hybrid Bayesian model predicts the future condition ratings of the bridge decks with $RMSE, MAE$, and x^2 equal to 0.7716, 0.540 and 46.0583, respectively.

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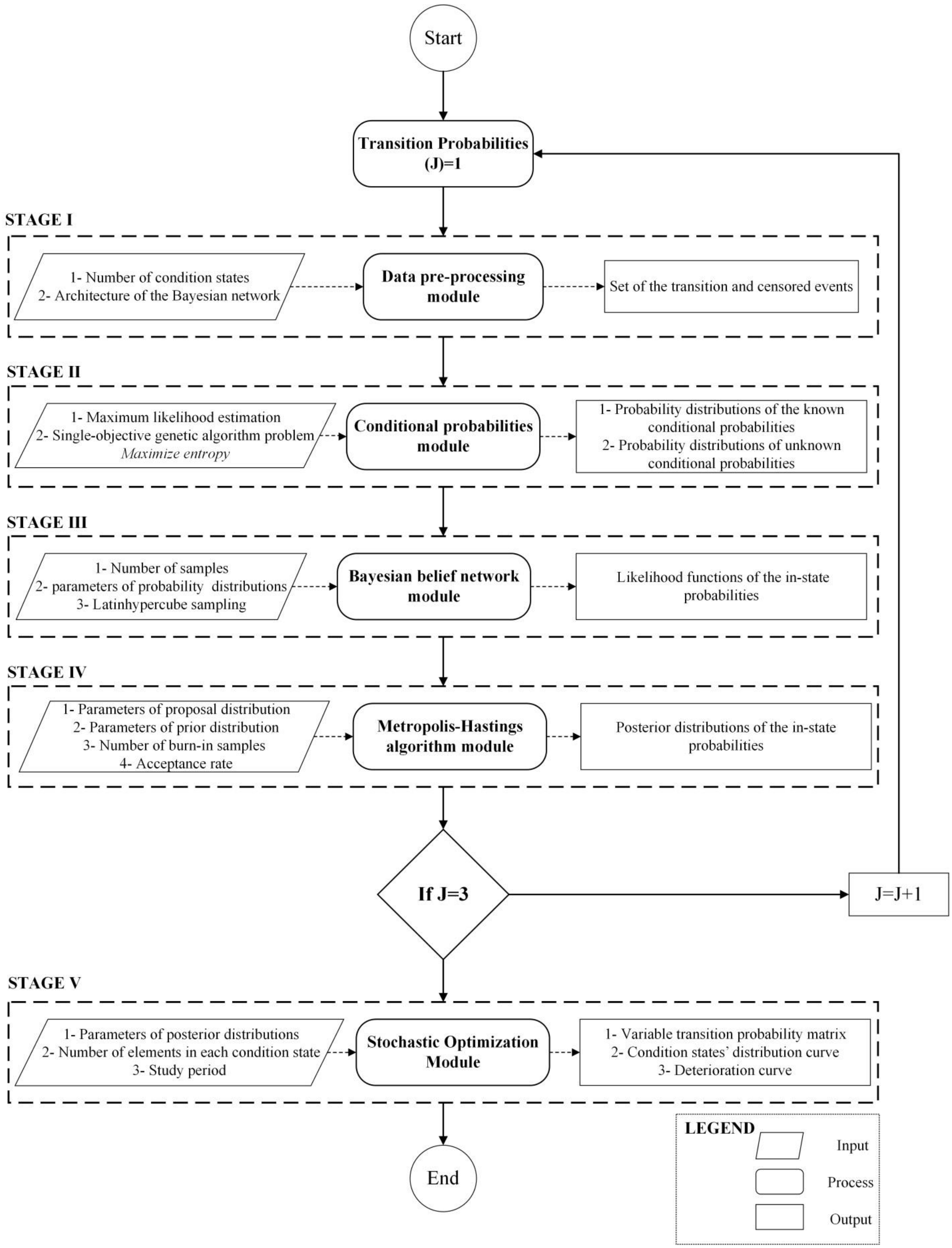
List of Figures

Figure 18: Interface of the stochastic optimization model

Figure 19: Convergence curve of the negative log likelihood of the unknown transition probabilities

Figure 20: Condition states' distribution and deterioration curve of the hybrid Bayesian model

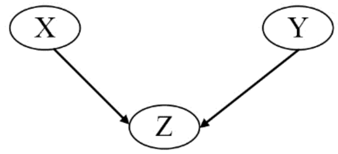
Figure 21: Interface of the performance metrics computation module


Start
Transition Probabilities
(J)=1
STAGE I
1- Number of condition states
2- Architecture of the Bayesian network
Data pre-processing module
Set of the transition and censored events
STAGE II
1- Maximum likelihood estimation
2- Single-objective genetic algorithm problem
Maximize entropy
Conditional probabilities module
1- Probability distributions of the known conditional probabilities
2- Probability distributions of unknown conditional probabilities
STAGE III
1- Number of samples
2- parameters of probability distributions
3- Latinhypercube sampling
Bayesian belief network module
Likelihood functions of the in-state probabilities
STAGE IV
1- Parameters of proposal distribution
2- Parameters of prior distribution
3- Number of burn-in samples
4- Acceptance rate
Metropolis-Hastings algorithm module
Posterior distributions of the in-state probabilities
If J=3
J=J+1
STAGE V
1- Parameters of posterior distributions
2- Number of elements in each condition state
3- Study period
Stochastic Optimization Module
1- Variable transition probability matrix
2- Condition states' distribution curve
3- Deterioration curve
End
LEGEND
Input
Process
Output

Marginal probabilities

State	Probability
Very poor (VP)	P(X=VP)
Poor (P)	P(X=P)
Medium (M)	P(X=M)
Good (G)	P(X=G)

Parent node **Parent node**



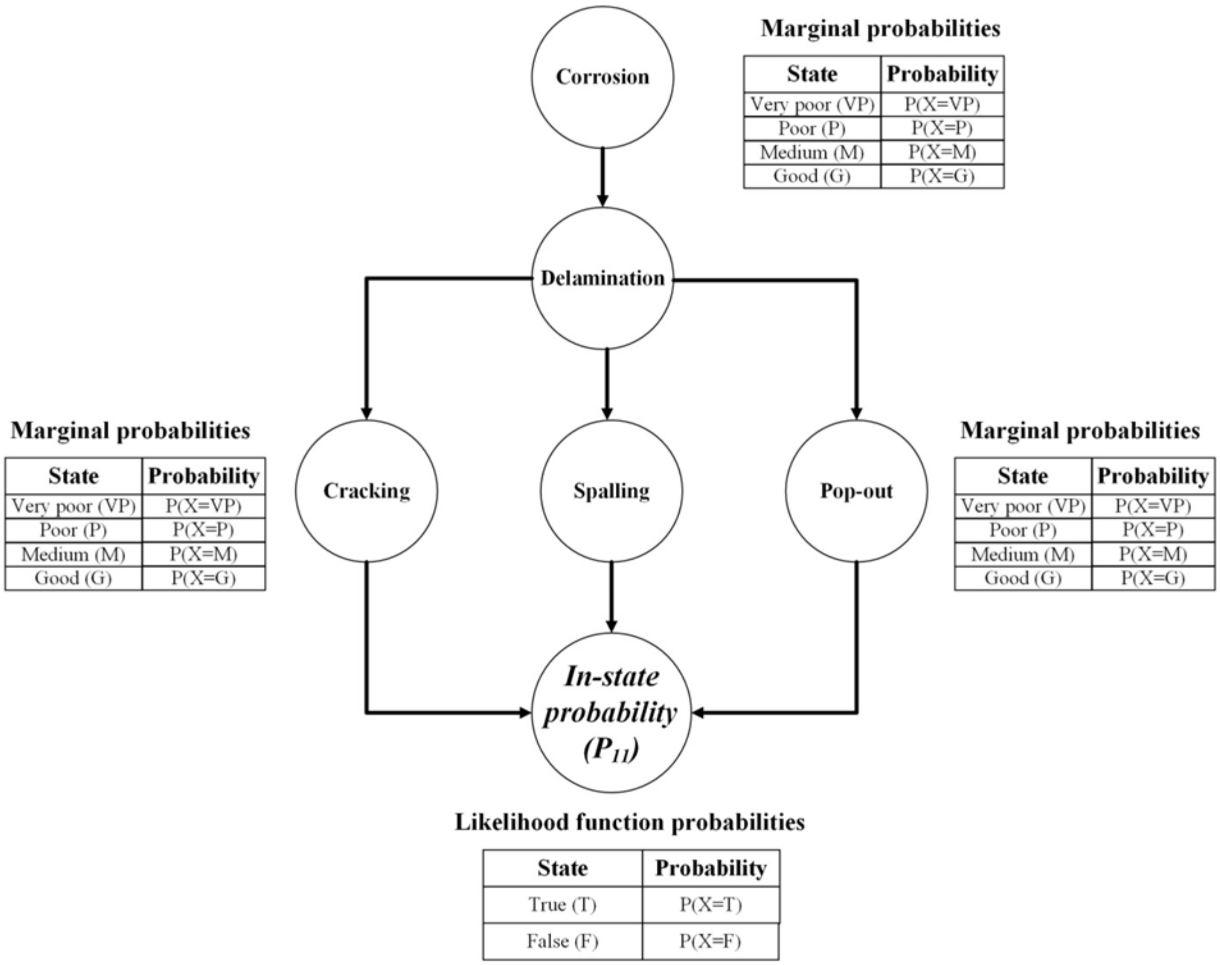
Child node

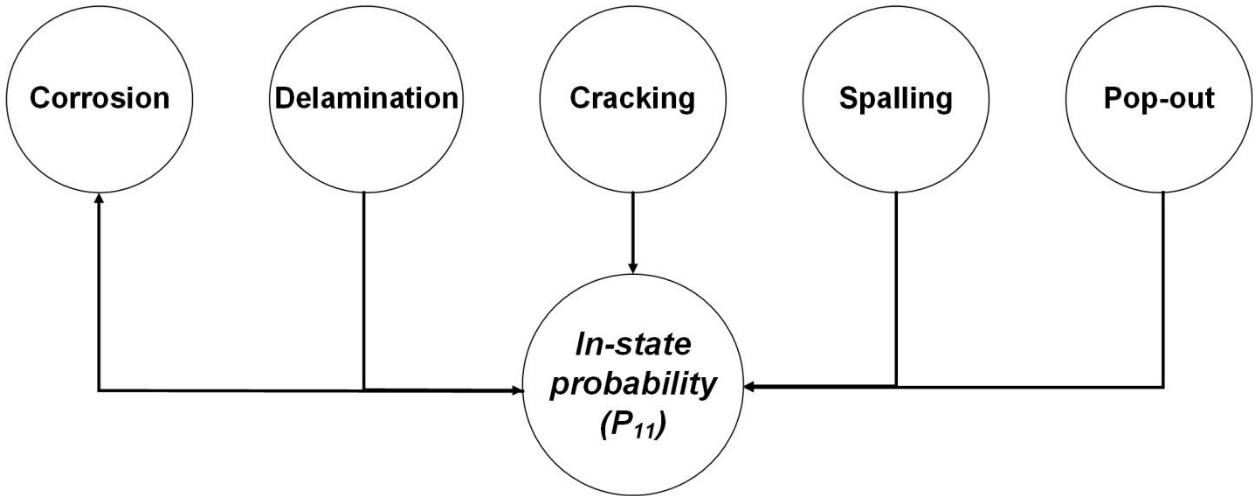
Marginal probabilities

State	Probability
Very poor (VP)	P(Y=VP)
Poor (P)	P(Y=P)
Medium (M)	P(Y=M)
Good (G)	P(Y=G)

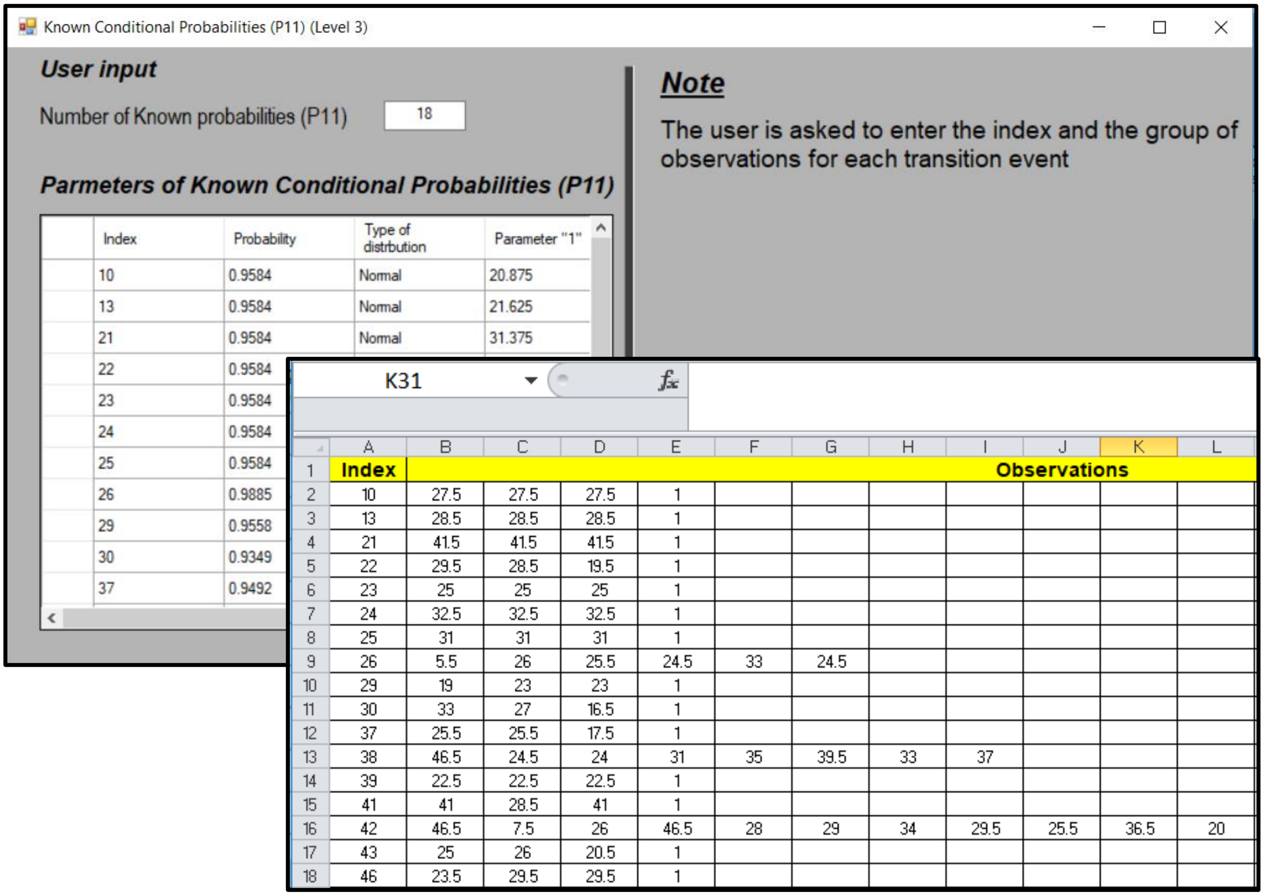
Conditional probabilities

Variable X	Variable Y	Conditional probability of variable Z			
		Very poor	**Poor**	**Medium**	**Good**
Very poor	Very poor	P(Z=VP|X=VP,Y=VP)	P(Z=P|X=VP,Y=VP)	P(Z=M|X=VP,Y=VP)	P(Z=G|X=VP,Y=VP)
Poor	Very poor	P(Z=VP|X=P,Y=VP)	P(Z=P|X=P,Y=VP)	P(Z=M|X=P,Y=VP)	P(Z=G|X=P,Y=VP)
Medium	Very poor	P(Z=VP|X=M,Y=VP)	P(Z=P|X=M,Y=VP)	P(Z=M|X=M,Y=VP)	P(Z=G|X=M,Y=VP)
Good	Very poor	P(Z=VP|X=G,Y=VP)	P(Z=P|X=G,Y=VP)	P(Z=M|X=G,Y=VP)	P(Z=G|X=G,Y=VP)
Very poor	Poor	P(Z=VP|X=VP,Y=P)	P(Z=P|X=VP,Y=VP)	P(Z=M|X=VP,Y=VP)	P(Z=G|X=VP,Y=VP)
Poor	Poor	P(Z=VP|X=P,Y=P)	P(Z=P|X=P,Y=P)	P(Z=M|X=P,Y=P)	P(Z=G|X=P,Y=P)
Medium	Poor	P(Z=VP|X=M,Y=P)	P(Z=P|X=M,Y=P)	P(Z=M|X=M,Y=P)	P(Z=G|X=M,Y=P)
Good	Poor	P(Z=VP|X=G,Y=P)	P(Z=P|X=G,Y=P)	P(Z=M|X=G,Y=P)	P(Z=G|X=G,Y=P)
Very poor	Medium	P(Z=VP|X=VP,Y=M)	P(Z=P|X=VP,Y=M)	P(Z=M|X=VP,Y=M)	P(Z=G|X=VP,Y=M)
Poor	Medium	P(Z=VP|X=P,Y=M)	P(Z=P|X=P,Y=M)	P(Z=M|X=P,Y=M)	P(Z=G|X=P,Y=M)
Medium	Medium	P(Z=VP|X=M,Y=M)	P(Z=P|X=M,Y=M)	P(Z=M|X=M,Y=M)	P(Z=G|X=M,Y=M)
Good	Medium	P(Z=VP|X=G,Y=M)	P(Z=P|X=G,Y=M)	P(Z=M|X=G,Y=M)	P(Z=G|X=G,Y=M)
Very poor	Good	P(Z=VP|X=VP,Y=G)	P(Z=P|X=VP,Y=G)	P(Z=M|X=VP,Y=G)	P(Z=G|X=VP,Y=G)
Poor	Good	P(Z=VP|X=P,Y=G)	P(Z=P|X=P,Y=G)	P(Z=M|X=P,Y=G)	P(Z=G|X=P,Y=G)
Medium	Good	P(Z=VP|X=M,Y=G)	P(Z=P|X=M,Y=G)	P(Z=M|X=M,Y=G)	P(Z=G|X=M,Y=G)
Good	Good	P(Z=VP|X=G,Y=G)	P(Z=P|X=G,Y=G)	P(Z=M|X=M,Y=G)	P(Z=G|X=M,Y=G)


Corrosion
Marginal probabilities
State
Probability
Very poor (VP)
P(X=VP)
Poor (P)
P(X=P)
Medium (M)
P(X=M)
Good (G)
P(X=G)
Delamination
Cracking
Spalling
Pop-out
Marginal probabilities
State
Probability
Very poor (VP)
P(X=VP)
Poor (P)
P(X=P)
Medium (M)
P(X=M)
Good (G)
P(X=G)
Marginal probabilities
State
Probability
Very poor (VP)
P(X=VP)
Poor (P)
P(X=P)
Medium (M)
P(X=M)
Good (G)
P(X=G)
In-state probability (P11)
Likelihood function probabilities
State
Probability
True (T)
P(X=T)
False (F)
P(X=F)

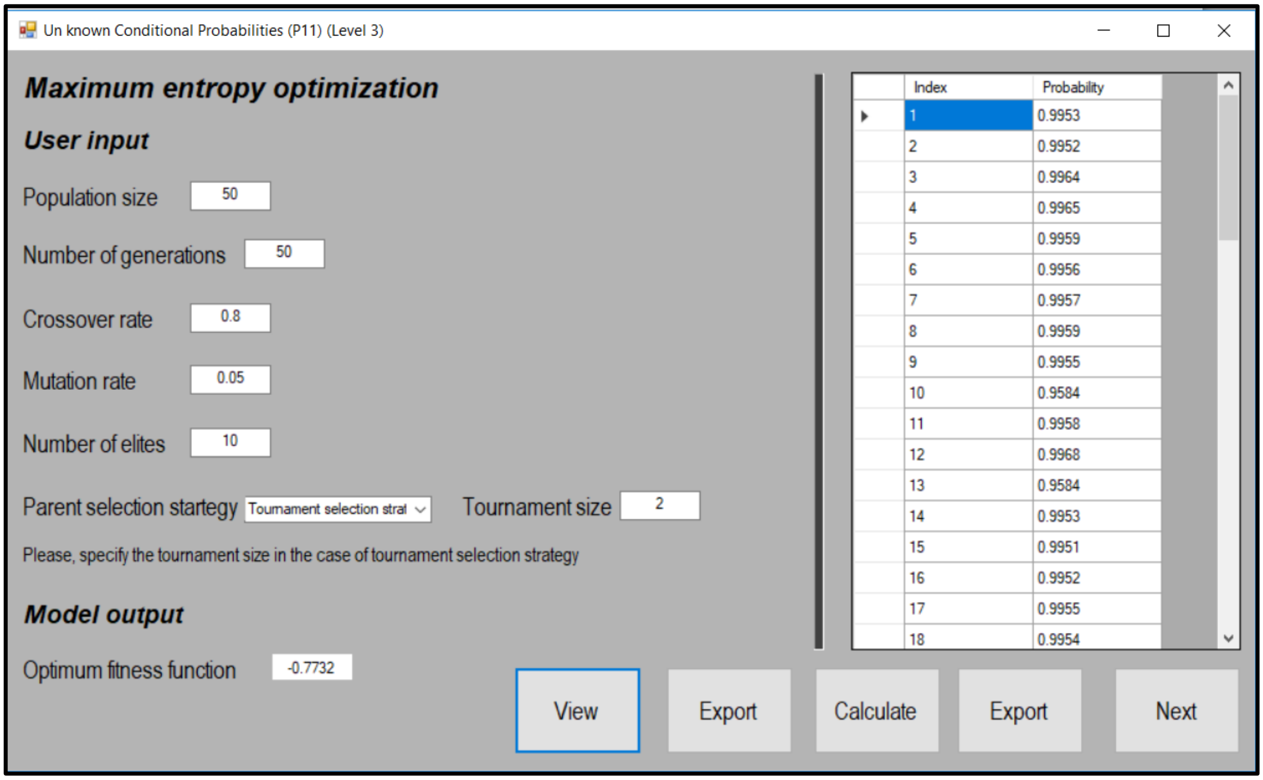

Corrosion
Delamination
Cracking
Spalling
Pop-out
In-state probability (P_{11})

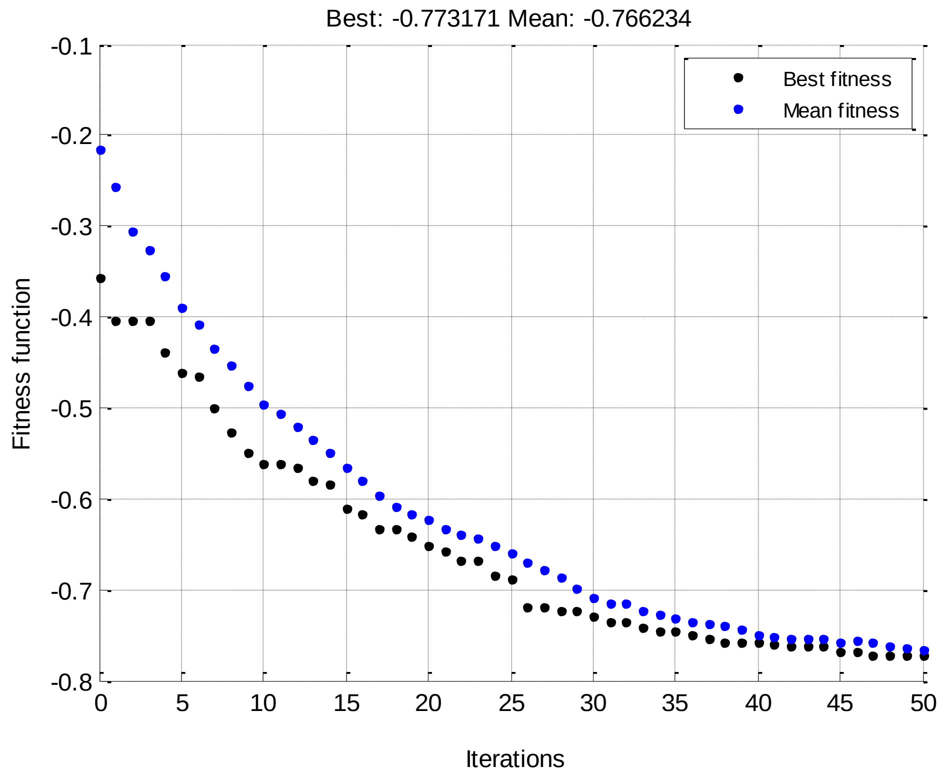
```
Microsoft.Office.Interop.Excel.Range mycells;

myexcel.Workbooks.Open(@"C:\Users\abdelkader\Desktop\Automation\LHS1.xlsx");

myexcel.Visible = true;

MYWORKSSET = myexcel.Worksheets.Item[1];

mycells = MYWORKSSET.Cells;
MSBN aMSBN = new MSBN();

Model modelCat = aMSBN.Models.Add("Cat", Directory.GetCurrentDirectory() + @"\..\..\..\P12.dsc",
Directory.GetCurrentDirectory() + @"\..\..\..\loaderror.log");


Node nodeTransition = modelCat.ModelNodes["Transition"];
Node nodeCorrosion = modelCat.ModelNodes["Corrosion"];
Node nodeDelamination = modelCat.ModelNodes["Delamination"];
Node nodeCracking = modelCat.ModelNodes["Cracking"];
Node nodeSpalling = modelCat.ModelNodes["Spalling"];
Node nodePopout = modelCat.ModelNodes["Popout"];

Dist aDist = nodeTransition.get_Dist();
```

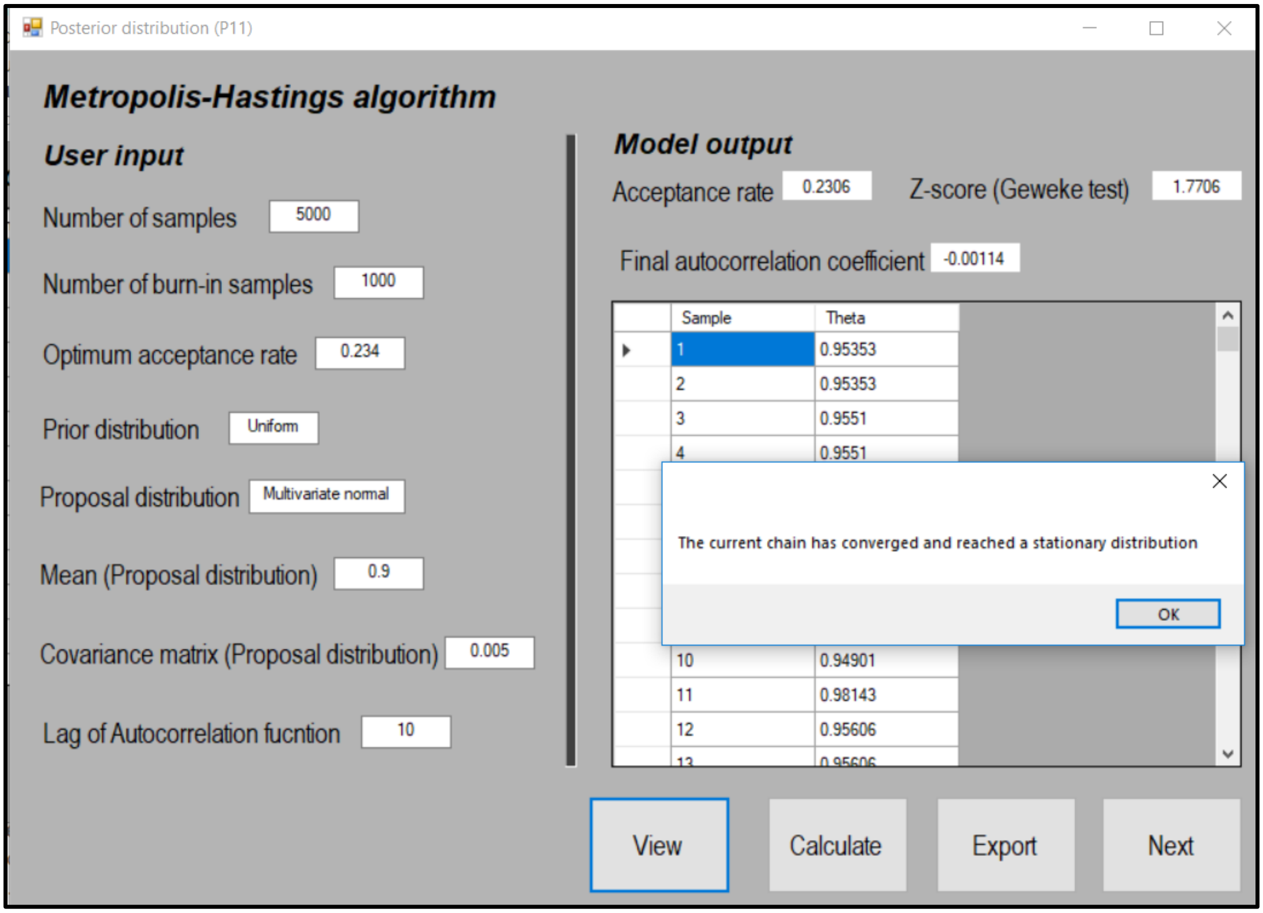


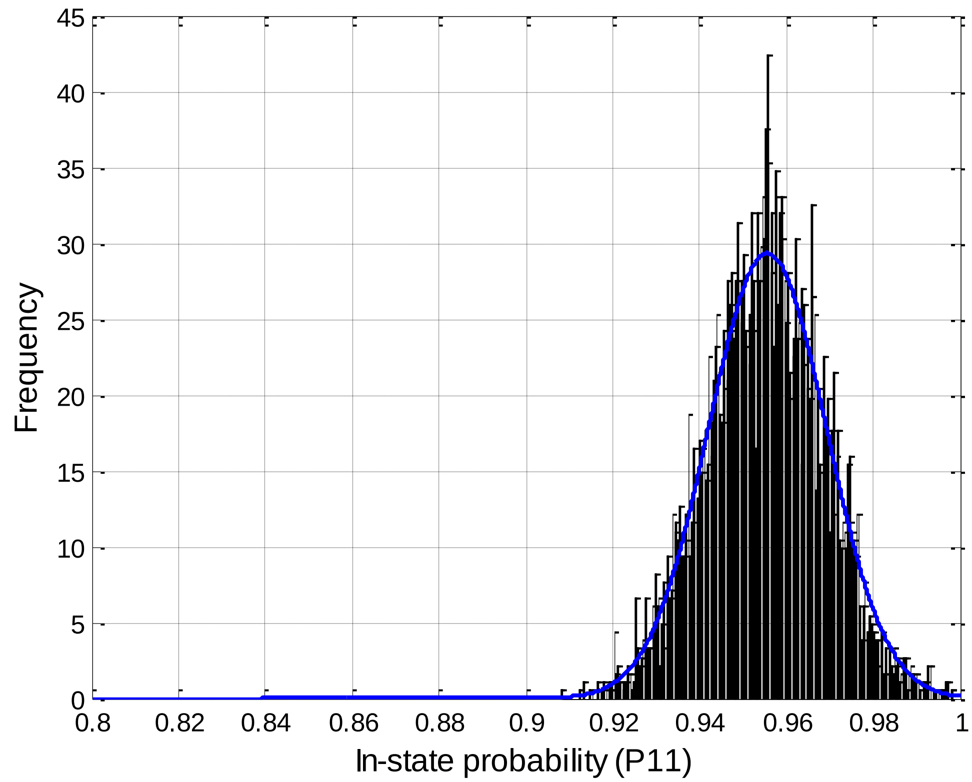
Index	Probability	Type of distrbution	Parameter "1"
10	0.9584	Normal	20.875
13	0.9584	Normal	21.625
21	0.9584	Normal	31.375
22	0.9584		
23	0.9584		
24	0.9584		
25	0.9584		
26	0.9885		
29	0.9558		
30	0.9349		
37	0.9492		

	A	B	C	D	E	F	G	H	I	J	K	L
1	Index									Observations		
2	10	27.5	27.5	27.5	1							
3	13	28.5	28.5	28.5	1							
4	21	41.5	41.5	41.5	1							
5	22	29.5	28.5	19.5	1							
6	23	25	25	25	1							
7	24	32.5	32.5	32.5	1							
8	25	31	31	31	1							
9	26	5.5	26	25.5	24.5	33	24.5					
10	29	19	23	23	1							
11	30	33	27	16.5	1							
12	37	25.5	25.5	17.5	1							
13	38	46.5	24.5	24	31	35	39.5	33	37			
14	39	22.5	22.5	22.5	1							
15	41	41	28.5	41	1							
16	42	46.5	7.5	26	46.5	28	29	34	29.5	25.5	36.5	20
17	43	25	26	20.5	1							
18	46	23.5	29.5	29.5	1							

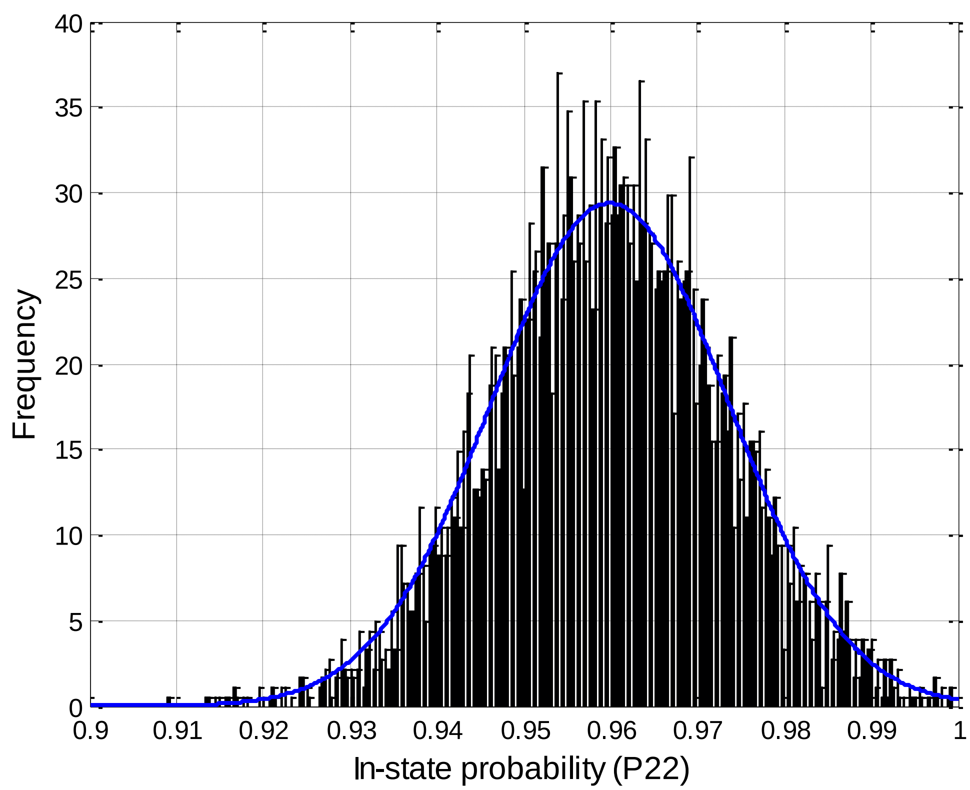

Un known Conditional Probabilities (P11) (Level 3)
Maximum entropy optimization
User input
Population size
50
Number of generations
50
Crossover rate
0.8
Mutation rate
0.05
Number of elites
10
Parent selection startegy
Tournament selection strat
Tournament size
2
Please, specify the tournament size in the case of tournament selection strategy
Model output
Optimum fitness function
-0.7732
Index
Probability
1
0.9953
2
0.9952
3
0.9964
4
0.9965
5
0.9959
6
0.9956
7
0.9957
8
0.9959
9
0.9955
10
0.9584
11
0.9958
12
0.9968
13
0.9584
14
0.9953
15
0.9951
16
0.9952
17
0.9955
18
0.9954
View
Export
Calculate
Export
Next


Best: -0.773171 Mean: -0.766234
Best fitness
Mean fitness
Fitness function
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
0
5
10
15
20
25
30
35
40
45
50
Iterations

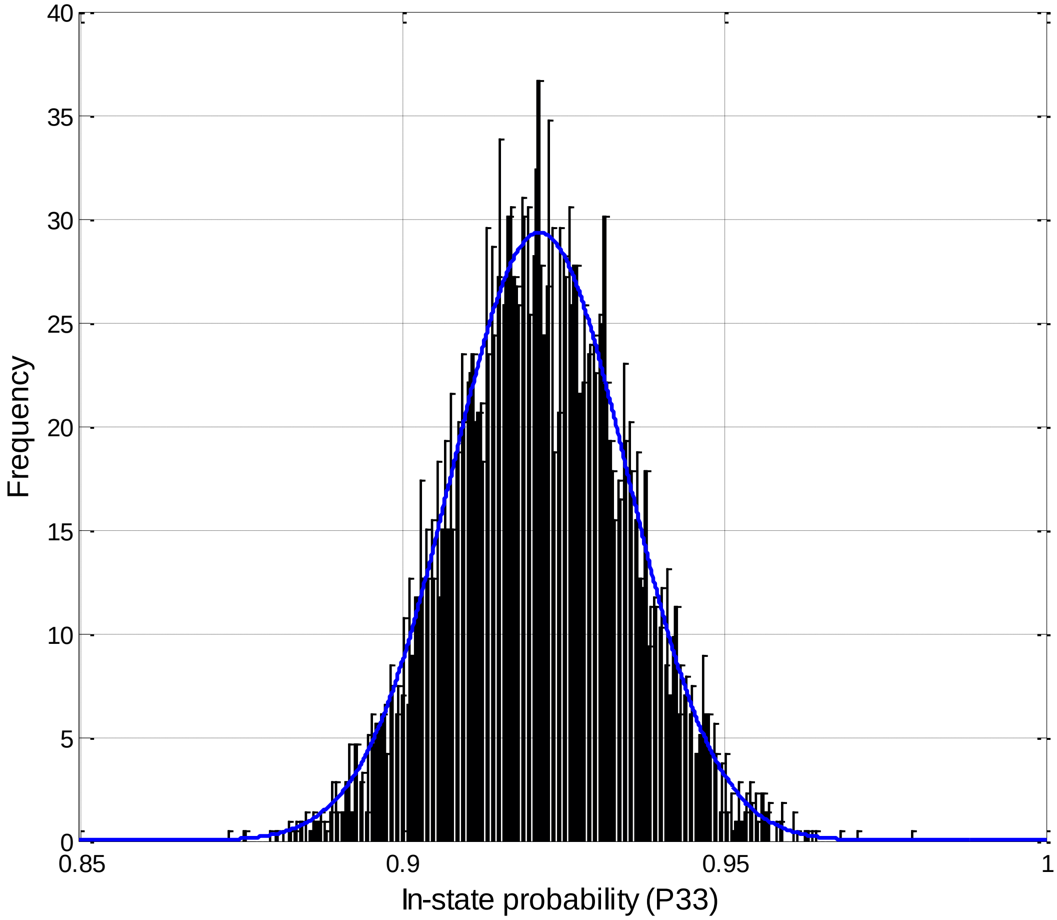

Posterior distribution (P11)
Metropolis-Hastings algorithm
User input
Number of samples
5000
Number of burn-in samples
1000
Optimum acceptance rate
0.234
Prior distribution
Uniform
Proposal distribution
Multivariate normal
Mean (Proposal distribution)
0.9
Covariance matrix (Proposal distribution)
0.005
Lag of Autocorrelation fucntion
10
Model output
Acceptance rate
0.2306
Z-score (Geweke test)
1.7706
Final autocorrelation coefficient
-0.00114
Sample
Theta
1
0.95353
2
0.95353
3
0.9551
4
0.9551
The current chain has converged and reached a stationary distribution
OK
10
0.94901
11
0.98143
12
0.95606
View
Calculate
Export
Next



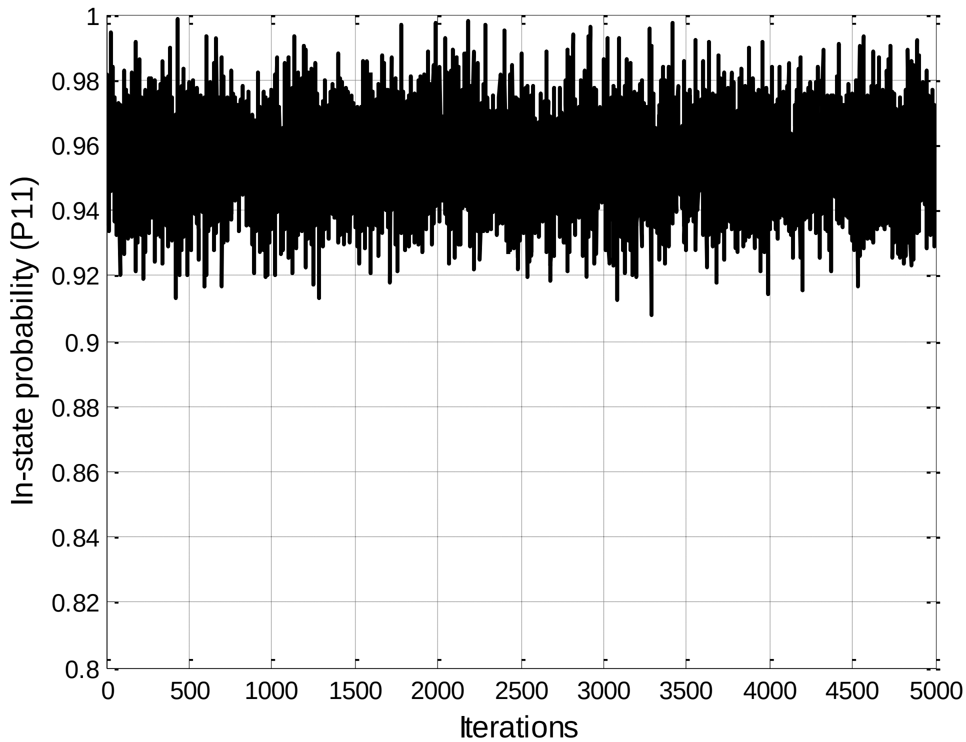
(a) Posterior distribution of the in-state probability P_{11}



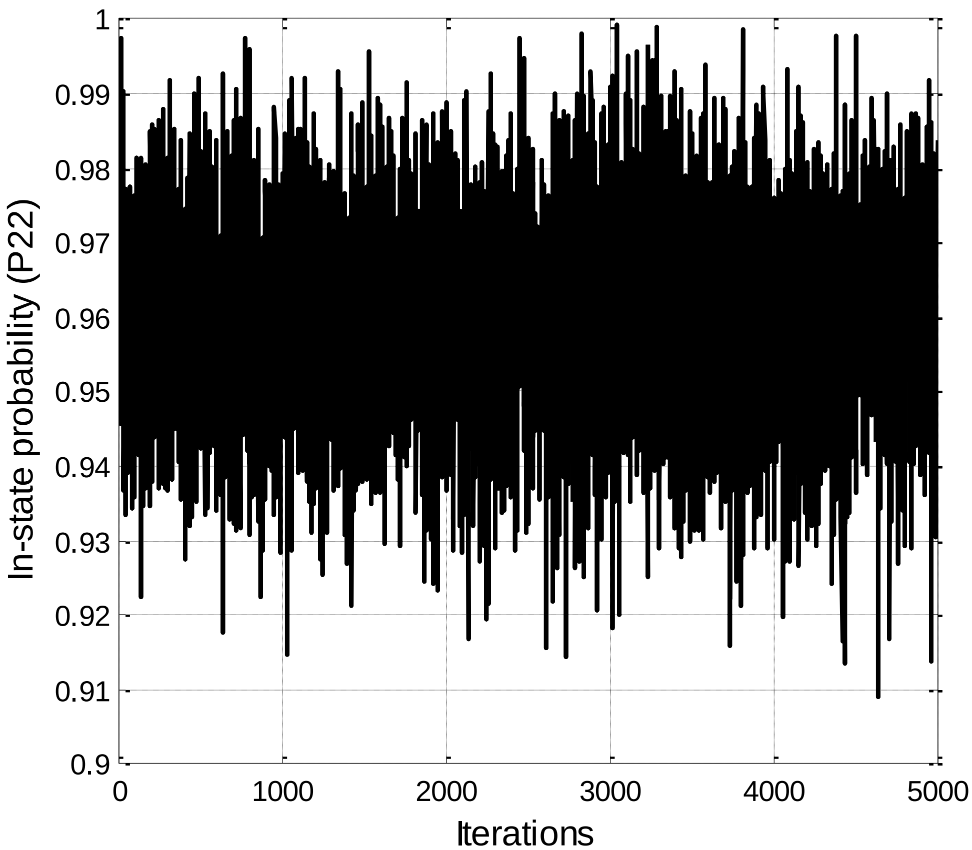
(b) Posterior distribution of the in-state probability P_{22}



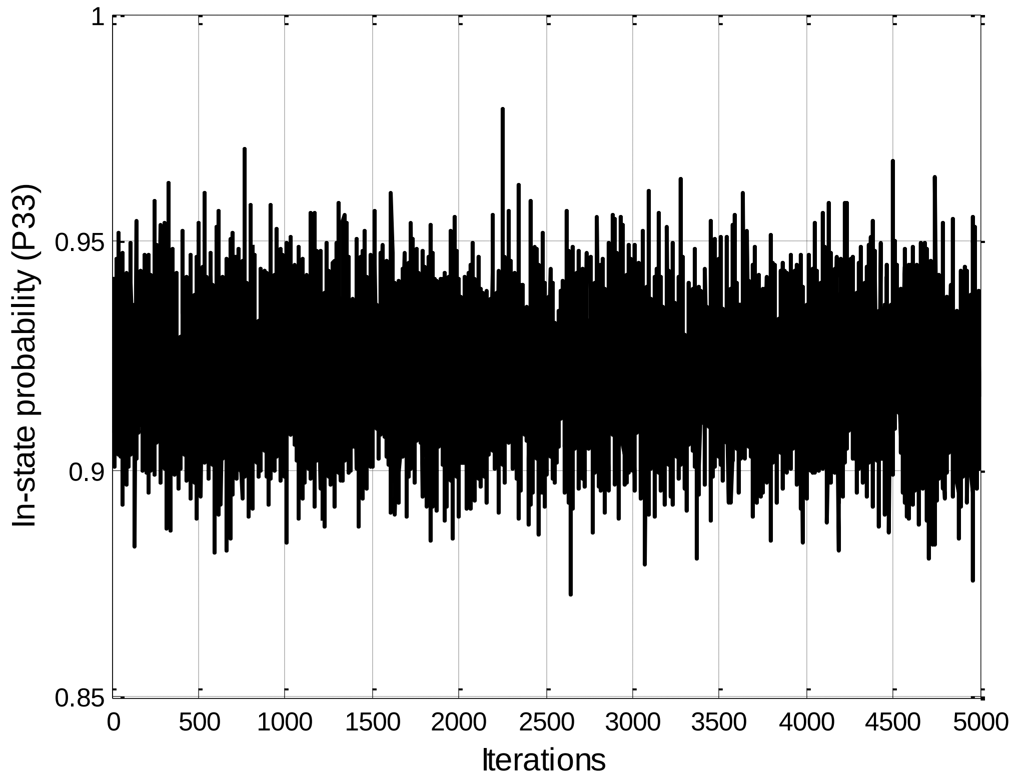
Frequency
40
35
30
25
20
15
10
5
0
0.85
0.9
0.95
1
In-state probability (P33)



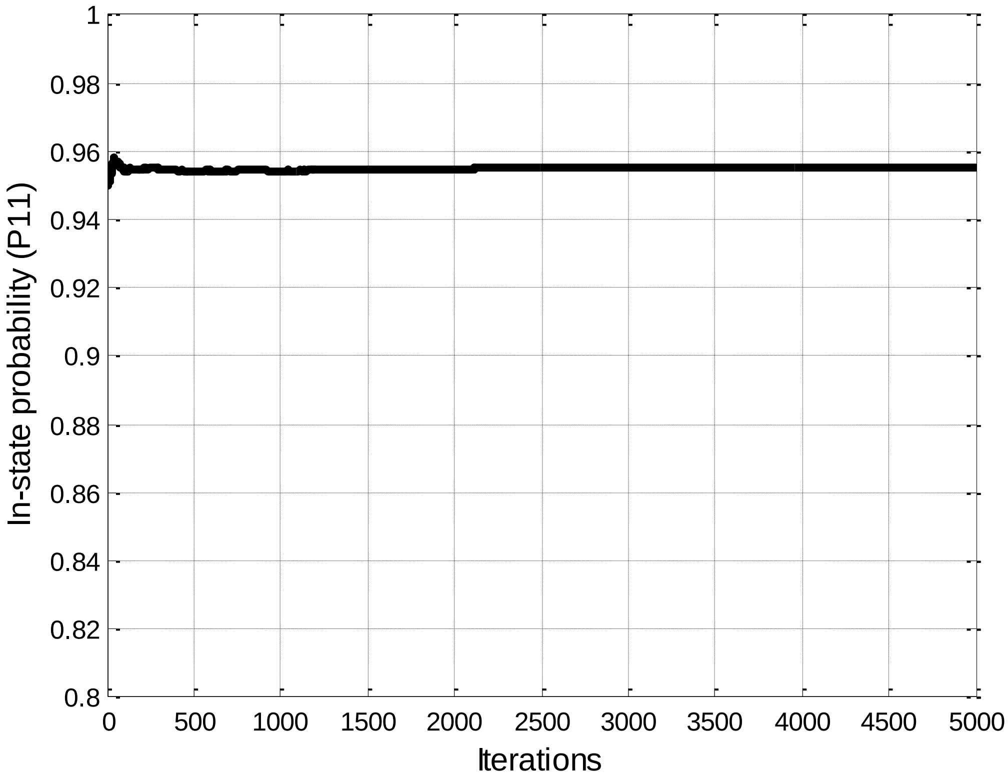
(a) Trace plot of the in-state probability P_{11}



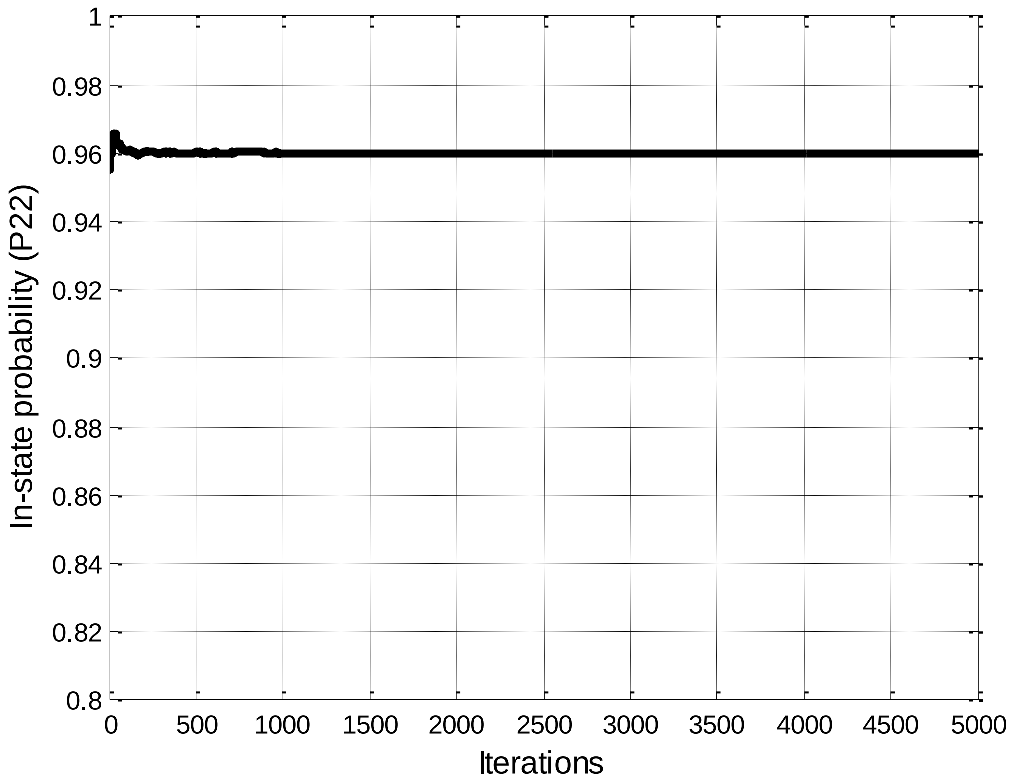
(b) Trace plot of the in-state probability P_{22}



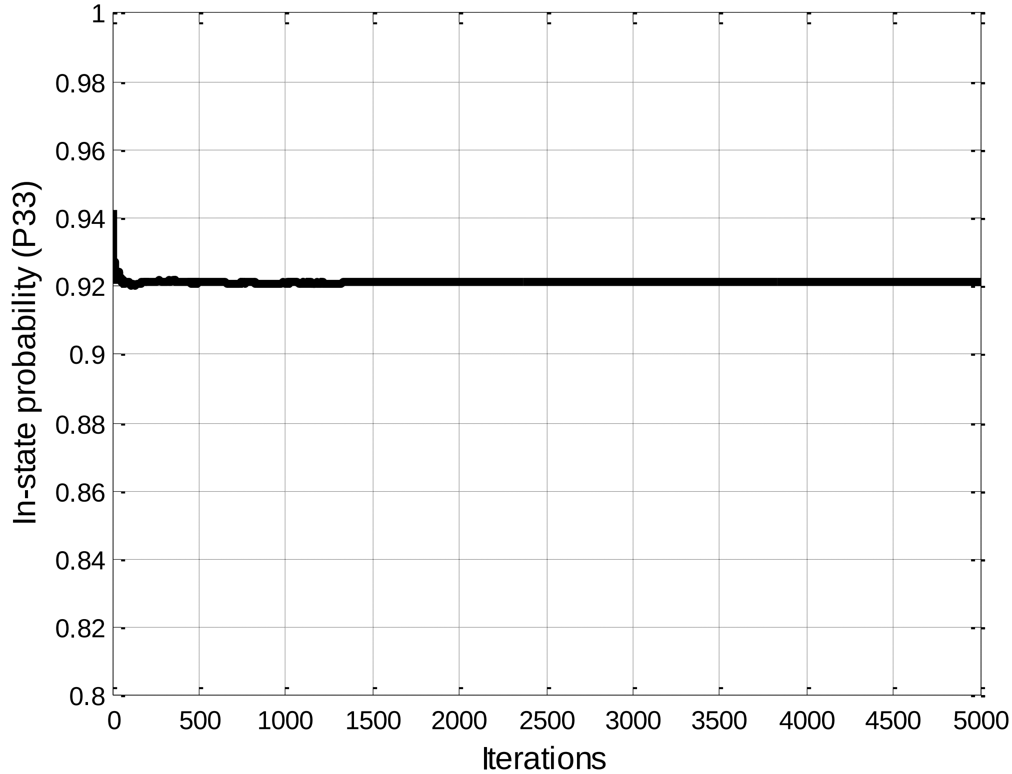
1
0.95
0.9
0.85
In-state probability (P33)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Iterations



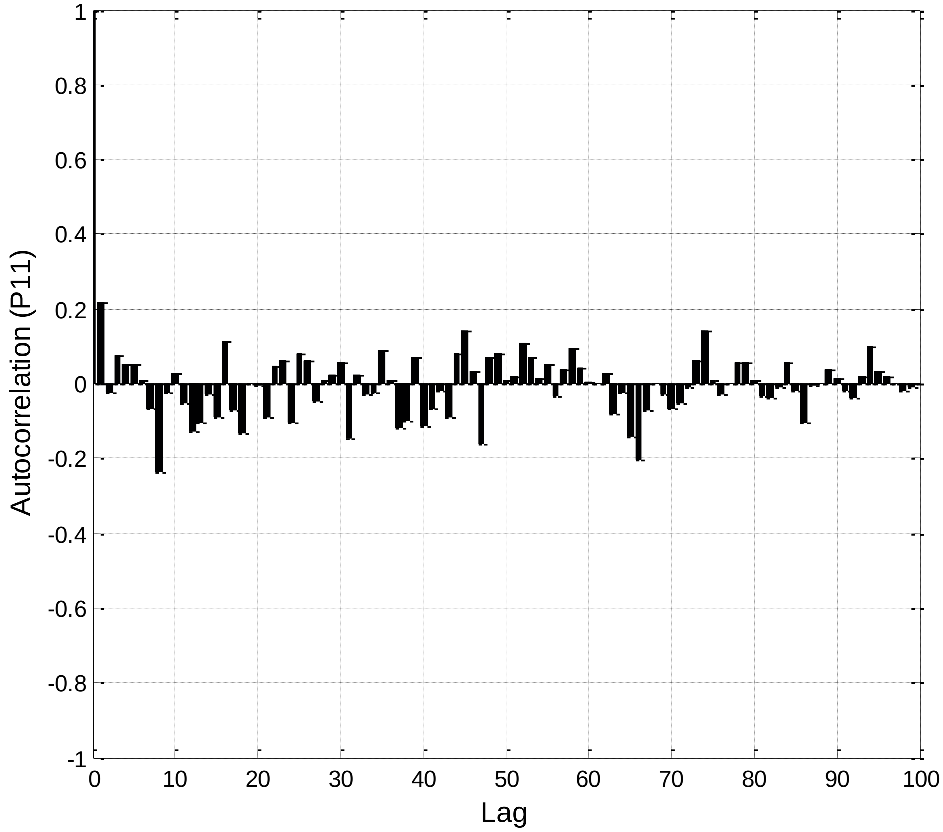
(a) Trace plot of the mean convergence of the in-state probability P_{11}



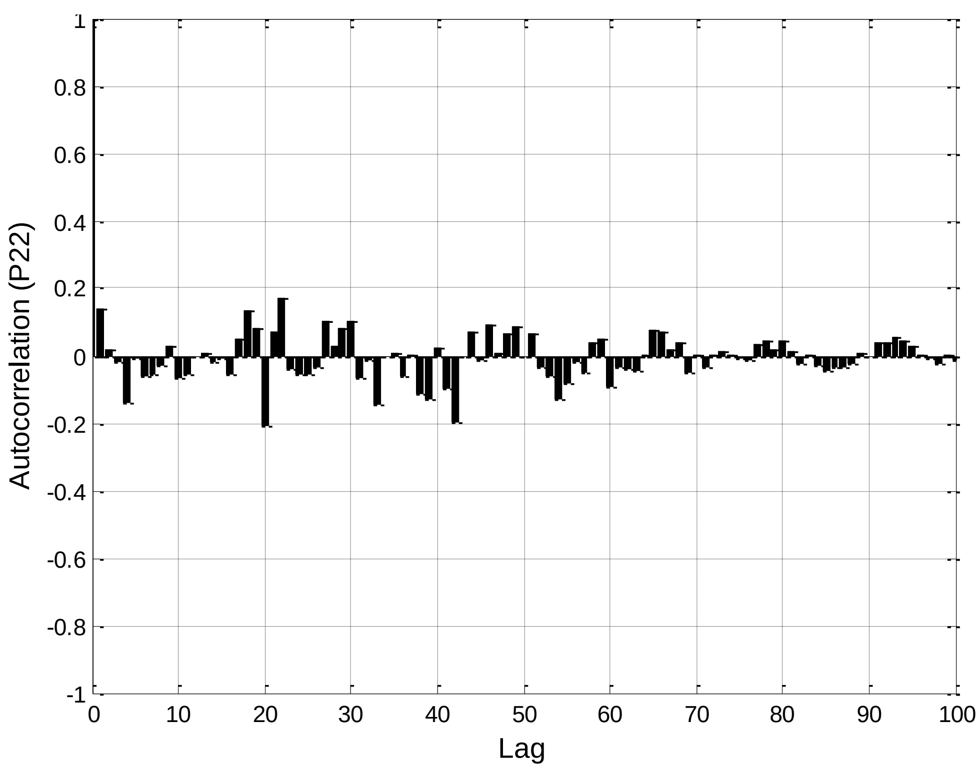
(b) Trace plot of the mean convergence of the in-state probability P_{22}



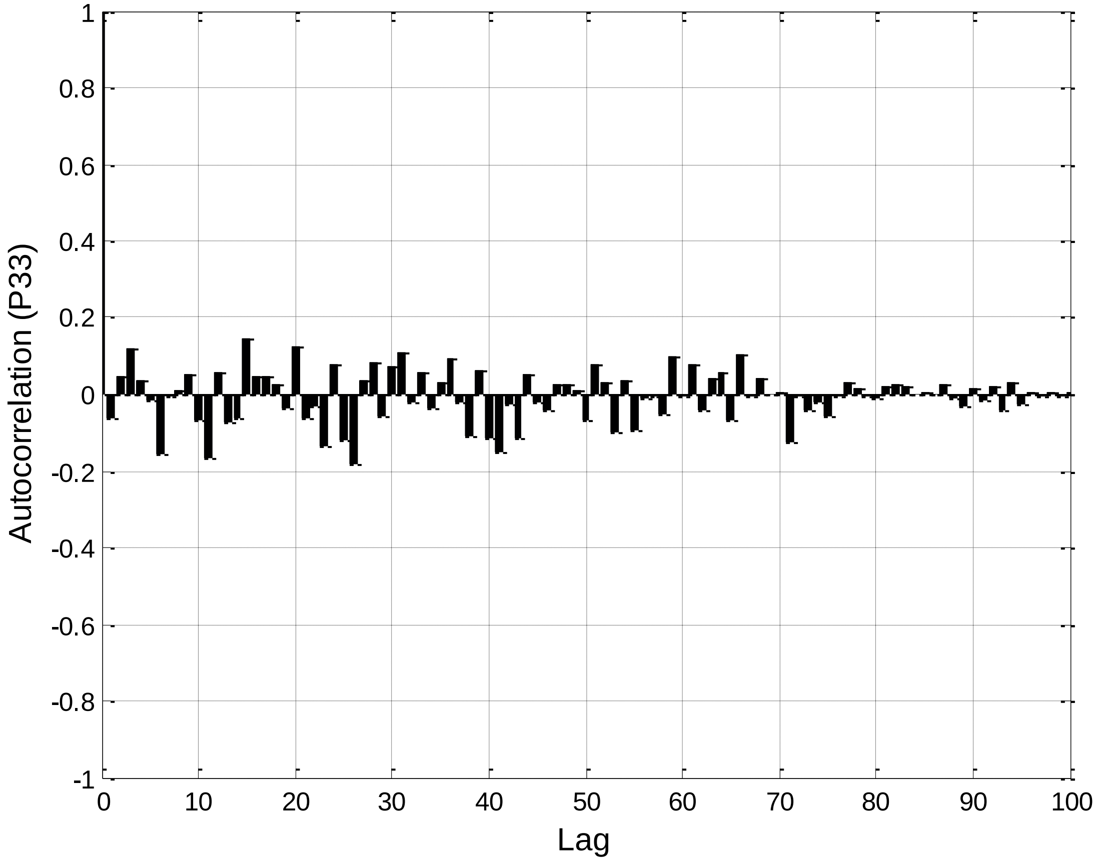
In-state probability (P33)
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Iterations

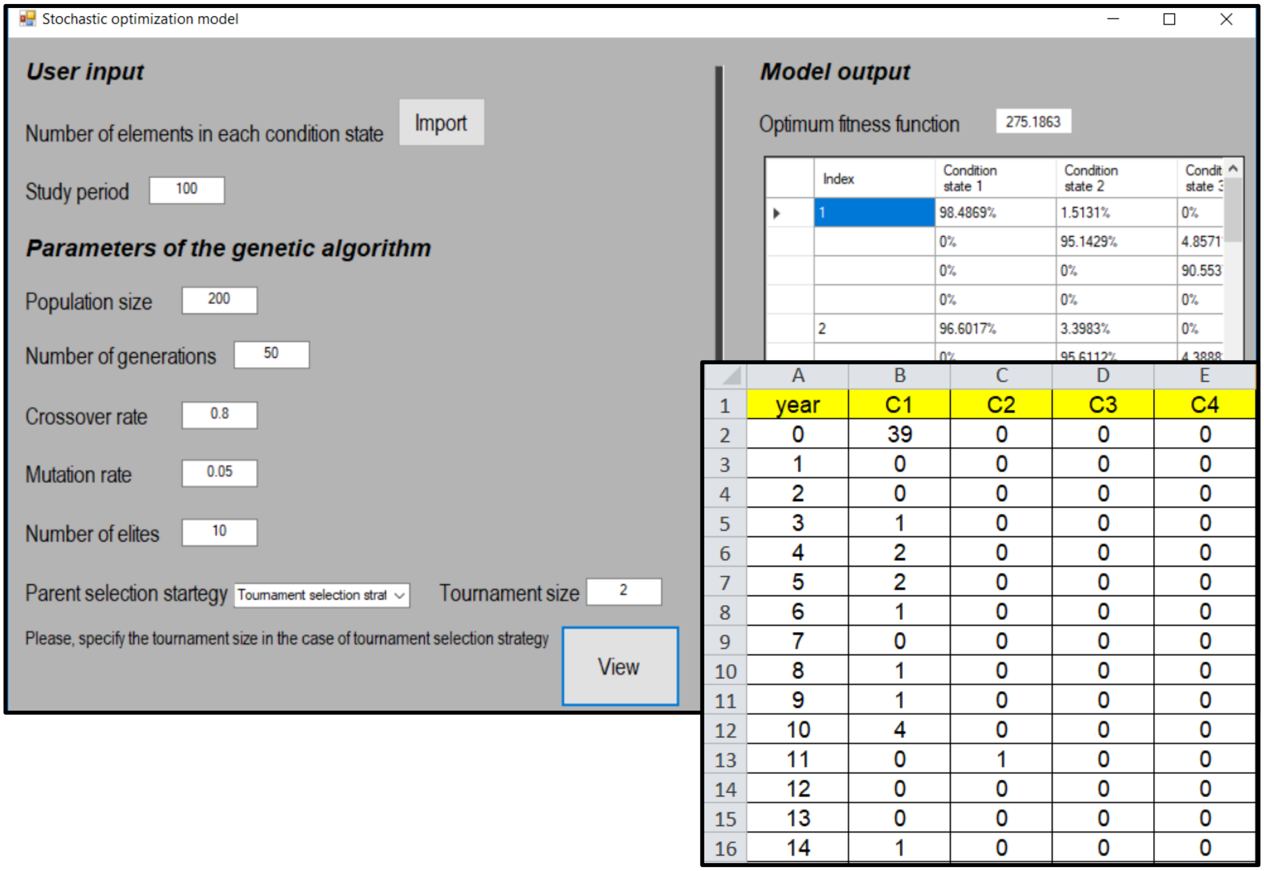


(a) Trace plot of autocorrelation function of in-state probability P_{11}

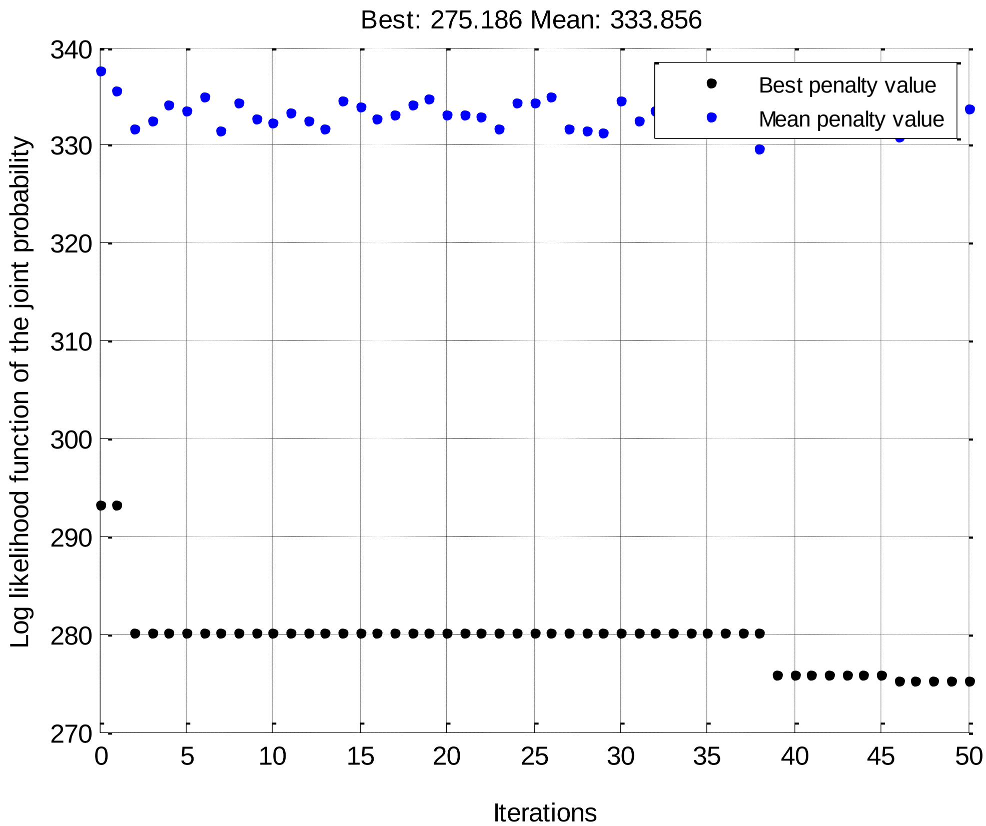


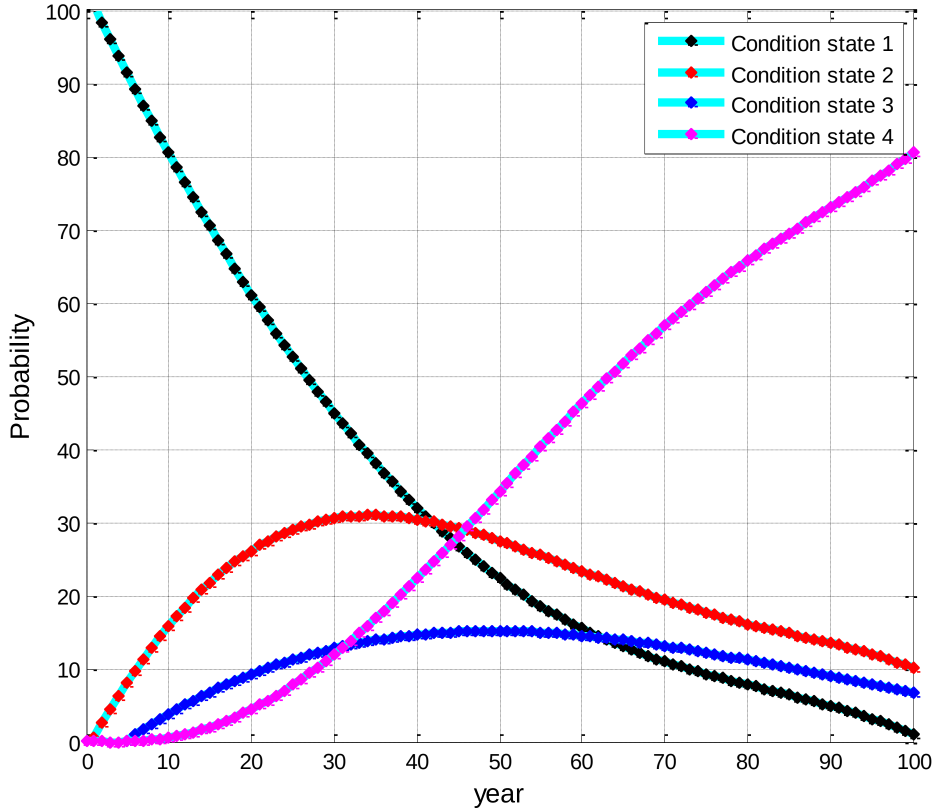
(b) Trace plot of autocorrelation function of in-state probability P_{22}


Autocorrelation (P33)
Lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
10
20
30
40
50
60
70
80
90
100

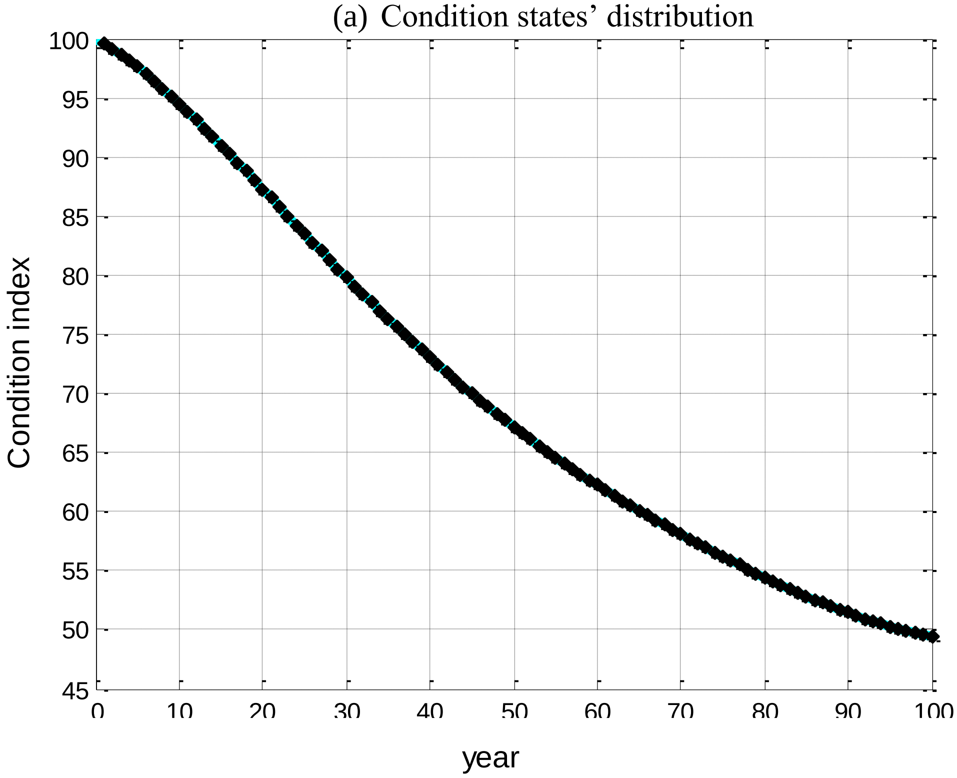


	A	B	C	D	E
1	year	C1	C2	C3	C4
2	0	39	0	0	0
3	1	0	0	0	0
4	2	0	0	0	0
5	3	1	0	0	0
6	4	2	0	0	0
7	5	2	0	0	0
8	6	1	0	0	0
9	7	0	0	0	0
10	8	1	0	0	0
11	9	1	0	0	0
12	10	4	0	0	0
13	11	0	1	0	0
14	12	0	0	0	0
15	13	0	0	0	0
16	14	1	0	0	0

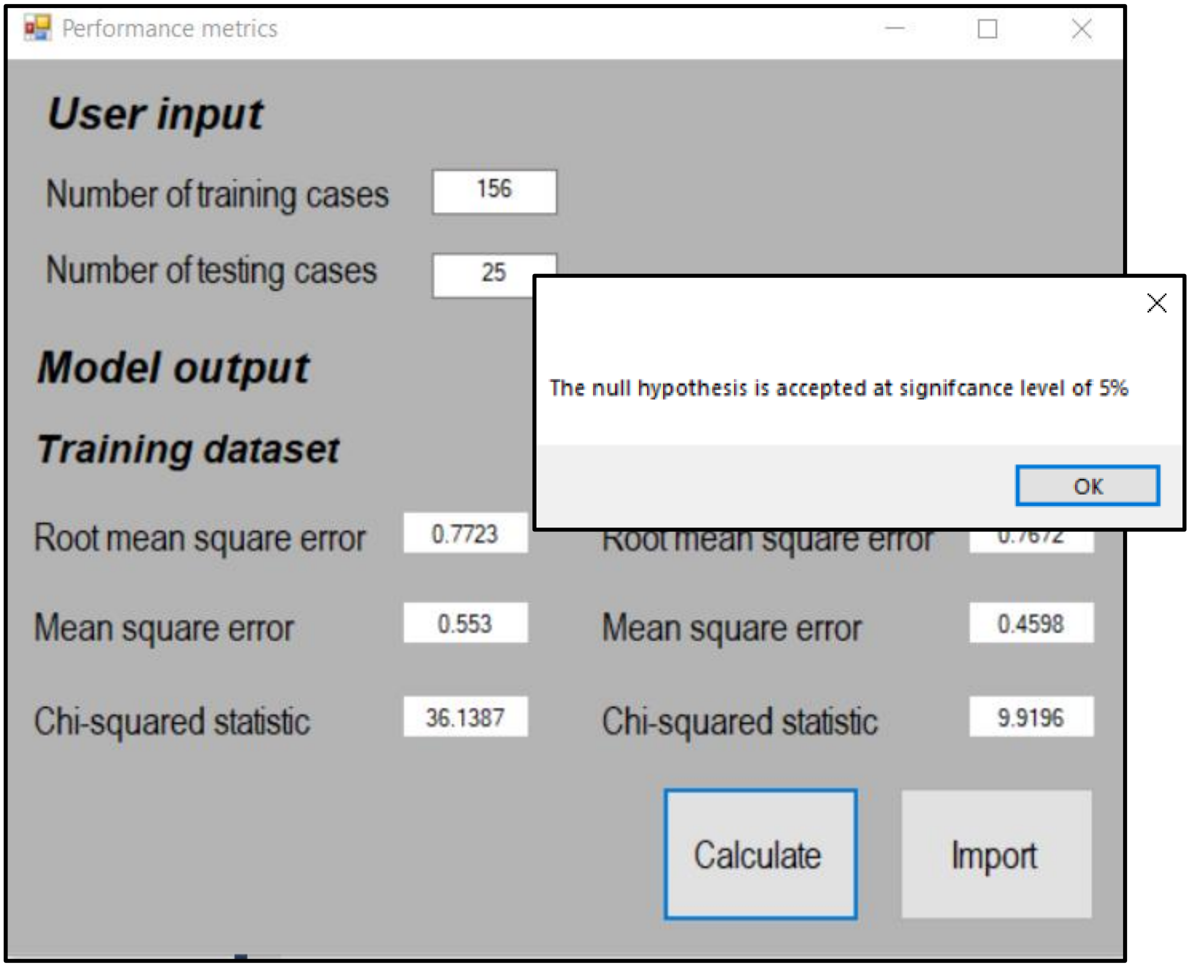

Best: 275.186 Mean: 333.856
Best penalty value
Mean penalty value
Log likelihood function of the joint probability
Iterations
340
330
320
310
300
290
280
270
0
5
10
15
20
25
30
35
40
45
50



(a) Condition states' distribution



(b) Deterioration curve


Performance metrics
User input
Number of training cases
156
Number of testing cases
25
Model output
Training dataset
Root mean square error
0.7723
Mean square error
0.553
Chi-squared statistic
36.1387
Mean square error
0.4598
Chi-squared statistic
9.9196
Calculate
Import
The null hypothesis is accepted at signifcance level of 5%
OK

List of Tables

Table 1: Parameters of the Metropolis-Hastings algorithm

Parameter	**value**
Number of samples	5000
Number of burn-in samples	1000
Type of proposal distribution	Multivariate normal distribution
Type of prior distribution	Uniform distribution
Optimum acceptance rate	0.234
Lag of autocorrelation function	10

Table 2: Calculated transition probabilities for each zone

Zone	Year	In-state probabilities $P11$	In-state probabilities $P22$	In-state probabilities $P33$	Zone	Year	In-state probabilities $P11$	In-state probabilities $P22$	In-state probabilities $P33$
Zone 1	1-5	98.487%	95.134%	90.554%	Zone 11	51-55	97.503%	95.497%	90.57%
Zone 2	6-10	96.602%	95.611%	92.113%	Zone 12	56-60	94.689%	95.985%	92.642%
Zone 3	11-15	99.225%	96.193%	91.571%	Zone 13	61-65	97.425%	96.274%	92.593%
Zone 4	16-20	95.666%	95.288%	91.012%	Zone 14	66-70	95.231%	95.053%	93.418%
Zone 5	21-25	95.751%	95.848%	91.093%	Zone 15	71-75	96.95%	97.065%	91.7%
Zone 6	26-30	97.402%	94.552%	93.658%	Zone 16	76-80	95.131%	94.76%	91.5%
Zone 7	31-35	98.914%	96.444%	94.323%	Zone 17	81-85	94.038%	96.155%	92.265%
Zone 8	36-40	94.086%	94.397%	93.44%	Zone 18	86-90	95.428%	96.324%	93.755%
Zone 9	41-45	95.846%	94.397%	93.44%	Zone 19	91-95	93.816%	96.075%	91.411%
Zone 10	46-50	99.33%	96.096%	91.63%	Zone 20	96-100	95.224%	96.399%	91.191%

Table 3: Comparison between the different deterioration models

Model	$RMSE$	MAE	x^2
Hybrid Bayesian-based model- 1 $(H-B1)$	0.7716	0.5401	46.0583
Hybrid Bayesian-based model- 2 $(H-B2)$	0.8572	0.542	62.5
Regression-based optimization model (RBO)	1.1489	0.8066	69
Weibull	1.4527	0.9834	356
Gamma	1.4584	0.9889	356.6667