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Finance-Based Scheduling Multi-Objective Optimization: Benchmarking NSGA-II against SPEA

By

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Abstract: Many contractors fail every year to execute their projects within planned budget, time, and quality. One of the main reasons for such failure is improper cash flow management where contractors require a credit amount exceeding their credit limits at certain periods of the project. In such cases, contractors become unable to follow their planned schedule leading to additional costs. This problem escalates when contractors execute multiple projects simultaneously. Thus, construction projects' scheduling and financing should be thought of together during the planning phase. Many studies have been carried to consider both financing and scheduling of construction projects simultaneously using different optimization techniques. However, no studies were found to evaluate and assess the outputs and performance of the techniques used in this area. Thus, this study first presents the development and implementation of an adjusted finance-based scheduling multi-objective optimization model for multiple projects using the elitist non-dominated sorting genetic algorithm (NSGA-II). Then, the model's results obtained using the NSGA-II technique were compared with a previously developed similar model using the strength Pareto evolutionary algorithm (SPEA) technique. Both techniques were compared with respect to the quality of obtained solutions as well as their relative performance. It was found that the NSGA-II technique outperformed SPEA in most aspects with improvements varying from 1.7% to 98.2%. This study is expected to help researchers to identify the strength and weakness of these two techniques in solving finance-based scheduling problems.

Keywords: Finance-Based Scheduling, Multi-objective Optimization, Multiple Projects, Evolutionary Algorithms

1. Introduction

Cash flow forecasting is one of the important construction management tools a contractor utilizes to determine the company's cash requirements during different periods of the construction project. Thus, cash is always considered as a critical resource in which contractors prioritize securing its availability throughout construction. In case of insufficient cash during any project period, the contractor becomes unable to commit to the planned project schedule leading to duration extension. Such duration extension can eventually lead to additional overheads and, or liquidated damages, and hence, less profit. Moreover, inability of financing may lead to contracting companies' failure. Based on a survey carried by a marketing research firm named BizMiner, the U.S. contractors' failure rate between 2014 and 2016 reached to nearly 30% [1]. The U.S. Surety Information Office has listed the factors that cause contractor's failure of which financial issues were a major factor [2]. Such financial factors intensify when contractors manage and execute more than one project simultaneously. According to Payne [3], more than 90% of all construction projects are executed in a multi-project environment. Moreover, nearly 84% of construction companies manage and execute multiple projects simultaneously [4].

As the execution of construction projects requires huge investments, the common practice for contractors is to seek out external sources of financing to cover the cash requirements of each period of the construction project [5]. Such external financing sources can be in the form of line of credit, loans, leases, trade financing, or even credit cards. The most common practice is the line of credit where contractors are allowed to take out cash during any project period without exceeding an agreed top credit limit [6]. Developing a project cash flow forecast based on the initial planned schedule will usually results in required cash exceeding the credit limit. Thus, more realistic projects' schedules should be devised to comply with the cash constraints of the credit limit (i.e. finance-based scheduling). In addition, such schedules should also aim to minimize the projects' durations and the contractor's financing cost and hence maximize his/her profit. These different conflicting objectives requires an optimization tool that creates a balance between them while fulfilling the projects' needs.

Different algorithms were implemented to solve finance-based scheduling problems. These algorithms can be classified into: (1) exact methods (e.g. linear programming, integer programming, and constraint programming); (2) heuristic methods; and (3) meta-heuristic methods (e.g. genetic algorithms, fuzzy sets with genetic algorithms, frog-leaping algorithm, ant colony optimization algorithm, strength Pareto evolutionary algorithm, and simulated annealing algorithm). The meta-heuristic methods usually result in different set of optimal or near optimal schedules that optimizes the conflicting objectives. Thus, there is a need to study and compare between the results of the finance-based scheduling problems solved using the different meta-heuristic optimization techniques in order to assess their performance. Such performance assessment will help practitioners and, or researchers to identify the best optimization technique for solving finance-based scheduling problems. Accordingly, the main objective of this study is to first introduce the development and implementation of an adjusted finance-based scheduling multi-objective optimization model for multiple projects using the elitist non-dominated sorting genetic algorithm (NSGA-II). Consequently, the assessment and comparison between the results of the NSGA-II and a previously utilized strength Pareto evolutionary algorithm (SPEA) for the same problem is presented.

2. Literature Review

Construction scheduling optimization problems were solved using different exact, heuristic, and meta-heuristic methods. Exact methods such as linear or dynamic programming fail to produce good quality optimal solution when solving large-sized construction projects [7]. Heuristic methods depend mainly on rules of thumb making them problem-dependent, i.e. their effectiveness varies with different construction projects. Moreover, they face algorithmic limits to optimize multi-objectives at the same time [8]. On the other hand, meta-heuristic methods such as Genetic algorithms (GAs) have widely been applied in construction scheduling optimization [5,8,9]. In fact, GAs suites construction scheduling optimization very well, as they are robust and do not experience combinational explosion or rely on heuristic rules [7].

Over the last two decades, several efforts were made to optimize construction project scheduling in terms of time and cost trade-off, resource leveling, and resource-constrained scheduling optimization [10-18]. However, these studies did not consider cash as a constraint. Therefore, other studies considered cash constraints by developing finance-based scheduling optimization models.

Finance-based scheduling was introduced by Elazouni and Gab-Allah [19] using integer programming to produce schedules that do not exceed the set credit limit. Consequently, GAs were applied to develop finance-based scheduling models [5,20-22]. Liu and Wang [23] developed a scheduling model by incorporating the cash flow to maximize the project's profit under constrained resources using constraint programming. Elazouni [24] solved the finance-based scheduling problem for multiple projects using heuristic rules. Such heuristic rules were later improved to reduce the search space before reaching an optimal solution [25]. The ant colony optimization algorithm was implemented by Al-Shihabi and AlDurgam [26] to minimize the project's duration under a maximum credit limit.

These studies focused on single-objective optimization. Thus, other attempts were made considering multi-objective optimization using GAs [6,27,28]. Abido and Elazouni [29] optimized the objectives of duration, financing cost, profit, and required credit for multiple projects using the SPEA technique. The NSGA-II technique was used to develop an optimization construction scheduling model for multiple projects with multi-mode activities and multi-resources considering both financial and resource features simultaneously [30,31]. The financial features were represented through the total cost, financing cost, required credit, and profit, while the resource features considered both the resource fluctuations and peak demand. Recently, Al-Shihabi and AlDurgam [32] developed a mixed-integer linear programming model for projects with multi-mode activities to optimize the project's profit, credit limit, and financing cost. Apart from the construction application of the finance-based scheduling, the multi-objective evolutionary optimization algorithms were also implemented in other scheduling applications [33-36].

It can be noticed that multi-objective optimization has been used extensively to model and solve problems in different areas of construction management. Particularly, the SPEA and NSGA-II techniques have been used in several research efforts to model and solve finance-based scheduling problems in the literature. These types of problems don't come up with a single schedule that optimizes the conflicting objectives. In fact, the results are in a form of several optimum schedules that are non-dominated. In other words, all resulted schedules are considered feasible and no one has better values than the other with respect to all objectives. Such sets of optimum schedules are known as Pareto-optimal front. Moreover, the solved finance-based scheduling problems are not considered as benchmark problems. That is to say, their respective true Pareto-optimal front is unknown. None of the previous studies were found to evaluate and assess the outputs and performance of the techniques used in this area. Accordingly, this study aims to compare the quality of solutions obtained using both NSGA-II and SPEA techniques in solving financed-based scheduling problems and assess their respective performance. This will be achieved after presenting the development and implementation of an adjusted finance-based scheduling multi-objective optimization model for multiple projects using the NSGA-II optimization technique. Benchmarking NSGA-II against SPEA based on the metrics of quality of solutions and performance helps the researchers in this field to identify the strength and weakness of these two techniques in view of the domain of finance-based scheduling.

3. Research Methodology

As shown in Figure 1, the research methodology commenced by studying the previous attempts made in different areas of construction scheduling optimization as well as the different techniques used in solving such problems. After that an optimization model formulation was developed for the finance-based scheduling problem to identify the decision variables, optimization objectives, and constraints. The decision variables are represented by the start times of the activities upon which the optimization objectives of duration, financing cost, required credit, and profit are determined under defined constraints. Such constraints incorporate the logical relationship between the activities and the credit limit. Consequently, a finance-based scheduling optimization model was developed using four major modules: (1) a scheduling module that allocates the start times of the activities among the multiple projects; (2) a cash flow module that is linked to the scheduling module to calculate the objectives values; (3) an NSGA-II based module that optimizes the objectives' values over successive iterations; and (4) a fuzzy-based module that selects the best compromise solution among the resulted non-dominated solutions. Accordingly, the developed model was implemented and tested using two case studies from literature that were implemented before using the SPEA technique. The implementation process considered first optimizing each objective separately. Then two objectives were optimized at a time to examine the effect of each objective over the other. Consequently, three-objective optimization took place to demonstrate the model's ability in producing good quality non-dominated schedules that optimize the objectives values. Finally, the results obtained by the NSGA-II technique were compared to that obtained using the SPEA technique in terms of solutions' quality and performance.

The model formulation and the finance-based scheduling optimization model presented in this study were adapted from the study carried by El-Abbasy et al. [31]. The optimization model developed by El-Abbasy et al. [31] using NSGA-II was adjusted to match the aspects and output variables of the optimization model developed by Abido and Elazouni [29] using SPEA for consistent comparisons between both algorithms. El-Abbasy et al. [31] model considered multi-mode activities with multi-resources, while in this study only single mode was considered for the activities. Moreover, El-Abbasy et al. [31] model considered the optimization of the objectives of duration, total cost, financing cost, required credit, profit, resource fluctuations, and peak demand. However, in this study, only the objectives of duration, financing cost, required credit, and profit were considered for optimization.

4. Model Formulation

The decision variables, optimization objectives, and constraints for the optimization model to be built are presented as follows:

4.1 Decision Variables

The optimization model to be developed aimed at producing different non-dominated schedules that result in optimized objectives values. Thus, the decision variables will be represented by the start times (*stm*) of all the activities in each project. The start time for a certain activity, based on its total float, can range between its corresponding early and late start times. This creates a huge number of possible combinations of alternatives. The more the number of activities and the more their corresponding total floats are, the more the alternatives will be resulting in large search space. For instance, assuming that we have a project of 20 activities of which 12 of them will have a total float of 3 days each. In this case, there can be 531,441 (i.e. 3^{12}) possible alternatives that represent the full search space. This search space can dramatically increase with the existence of more activities and more total floats. Accordingly, the optimization model serves here in finding the optimal or near optimal alternatives throughout such big search space. Optimal alternatives are the ones which are non-dominated with respect to all the multi-objective and simultaneously are not expected to be dominated. Practically speaking, finance-based scheduling problems are not considered as benchmark problems in which there optimal alternatives are pre-known. The only way to determine the optimal alternatives is to consider the full search space (i.e. all the possible alternatives) which requires a very high computational effort. Throughout optimization, an optimal alternative can be found without knowing since the full search space is not considered. For that reason, any optimization carried in this study will consider finding either an optimal or near optimal alternative. The former is not guaranteed while the latter is expected to be achieved throughout the evolutionary algorithms applied.

4.2 Optimization Objectives

The optimization model is supposed to minimize the conflicting objectives of duration, financing cost, and required credit and maximizes the profit objective. The duration objective is considered as the total time taken for finishing the construction of a set of multiple projects based on the projects' schedules. The projects' schedules are determined based on the allocated start time of each activity in each project. Considering the line of credit to be the contractor's external source of funding, the financing cost represents the periodical interest charged on any outstanding debt that is not timely paid to the funding source. Therefore, the total financing cost objective will be the summation of the interests charged in each period. For unit price contracts, in each period there will be a cumulative cash outflow that is determined by summing up the costs of work performed during that period. Furthermore, by the end of each period there will be a cash inflow which is the owner's payment to the contractor for the work performed in the previous period. Such owner's payment includes the costs of the work performed during a certain period in addition to the contractor's overheads and profit after deducting an agreed percentage which is known as the retained amount. Since the cash inflow is delayed by one or more period, there may be a negative cumulative balance in each period. Accordingly, the required credit objective is considered as the maximum negative cumulative balance throughout the different projects' periods. This defines the maximum cash amount that the contractor should secure from the external source of funding so that no any cash deficit occurs throughout construction. By the end of the projects and after all expenditures are charged and the contractor receives all his/her payments and any retained amount of money, the net cumulative balance represents the profit objective. The model is built to determine the effect of different combinations of activities' start times on these four objectives (*f*). The four objectives are expressed mathematically as follows:

$$f_1 = D \quad \text{(1)}$$

$$f_2 = \sum_{t=1}^{T}(I_t) \quad \text{(2)}$$

$$f_3 = -F, \quad F = min\{F'_t : t = 1,2, \ldots, T\} \quad \text{(3)}$$

$$f_4 = -N'_T \quad \text{(4)}$$

Where; D = total duration; I_t = financing cost at period t; F = maximum required credit, N'_T = cumulative net balance at period T, i.e. profit. Accordingly, the optimization model can be formulated as shown in Equations (5 – 9):

Decision Variables

$$s_p = \{stm_1, stm_2, \ldots, stm_v, \ldots, stm_V\}, \quad p = 1,2, \ldots, P \quad \text{(5)}$$

Where;

$$S = \{s_1, s_2, \ldots, s_p, \ldots, s_P\} \quad \text{(6)}$$

Minimize

$$f_i(S), \quad i = 1,2, \ldots OB \quad \text{(7)}$$

Subject to

$$stm_k - stm_v - d_v \geq 0, \quad \forall \quad k \in SCC_v \quad \text{(8)}$$

$$F \leq CLT \quad (9)$$

Where; P = total number of projects; f_i = ith objective; s_p = vector that represents a candidate activities' start times for the pth project; S = matrix that represents candidates' project schedule; stm_v = start time of activity v; d_v = duration of activity v; stm_k = start times of successors of activity v; V = number of project's activities; OB = number of objectives; SCC_v = set of successors of activity v; and CLT = credit limit.

As discussed above, one of the objectives to be optimized is the required credit which is to be unconditionally minimized or conditionally minimized below a defined top credit limit. When there is cash constraints and a top credit limit not to be exceeded, most probably the project duration should be extended beyond its planned duration. However, extending the project duration indefinitely will create a limitless search space. In other words, the number of possible schedules that fulfill the credit limit will be indefinite. Thus, an extension increment should be defined for each project in order to limit the search space. Such extension increment will adjust the total floats of the activities. Accordingly, instead of allocating the start times of the activities within their total floats, the allocation will be within their total floats plus the defined extension increment. The main goal here is to minimize such extension which is considered by minimizing the total duration in Equation 1.

5. Finance-Based Scheduling Optimization Model Development

As mentioned earlier, the finance-based scheduling optimization model development will comprise four major modules as briefly explained in the following sub-sections.

5.1 Scheduling Module

This module is indented to devise schedules for multiple construction projects and eventually determine their total durations. The module determines the project activities' start and finish times as shown in Equations (10) and (11), respectively. Thus, the total duration of each project is determined as shown in Equation (12).

$$(stm_v)_p \geq max{:}(ftm_r)_p \quad (10)$$

$$(ftm_v)_p = (stm_v)_p + (d_v)_p \quad (11)$$

$$D_p = max{:}(ftm_v)_p \quad (12)$$

Where; $(stm_v)_p$ = start time of activity v in project p; $(ftm_v)_p$ = finish time of activity v in project p; $(ftm_r)_p$ = finish time of activities preceding activity v in project p; $(d_v)_p$ = duration of activity v in project p; and D_p = total duration of project p. Since there can be several activities in a certain project without any successors, the D_p in Equation (12) is defined by the maximum finish time of all activities in a certain project. Using the same concept, the total overall multiple projects' duration D, which is the first objective to be optimized (see Equation 1), can be calculated as follows:

$$D = max{:}\ ftm_v \quad (13)$$

5.2 Cash Flow Module

This module calculates the periodical projects' cash flow parameters according to the schedules resulted from the previously discussed scheduling module. The module was adapted from Au and Hendrickson [37], however, it is adjusted to consider multiple projects in parallel. The main outputs of this module are the values of the optimization objectives of financing cost, required credit, and profit. The main cash flow parameters are illustrated in Figure 2. The cash outflow (E_t) includes the periodical contractor's direct and indirect costs, while the cash inflow (P_t) is the owner periodical payment to the contractor based on the periodical progress done. It is worth to mention that P_t should be adjusted to deduct from it the periodical retained amount and advance payment share (if any) and the late completion penalty (if any). Thus, for any periodical outstanding debt (F_t), an interest rate is applied to determine the periodical financing cost (I_t). The summation of the I_t results in the total financing cost which is the second objective to be optimized (see Equation 2). Adding both F_t and the accumulated financing costs (I'_t) will result in determining the periodical negative cumulative balance (F'_t). The minimum F'_t value represents the maximum required credit which is the third objective to be optimized (see Equation 3). The cumulative net balance value (N'_t) forms the negative cumulative balances after receiving the owner periodical payment. Finally, the cumulative net balance the end of the project (N'_T) represents the profit which is the fourth objective to be optimized (see Equation 4).

5.3 NSGA-II Optimization Module

This module aims to search for a set of optimum solutions (i.e. schedules) known as Pareto-optimal front that minimizes the objectives of duration, financing cost, and required credit and maximizes the profit objective. Such solutions are non-dominated and considered to be the best set found through several iterations using the NSGA-II. The well-known procedure of NSGA-II goes through three major phases: (1) initialization phase that generates an initial set of N possible schedules by randomly allocating different start times for the activities; (2) fitness evaluation phase that calculates for each generated schedule in phase 1 the projects' duration, financing costs, maximum required credit, and profit using the scheduling and cash flow modules; and (3) population generation phase that aims to improve the fitness of the schedules generated over successive generations. For more detailed computational procedure of these three phases, the reader is referred to El-Abbasy et al. [31].

5.4 Best Compromise Solution Selection Module

As mentioned earlier, the optimization model results in a set of optimum non-dominated solutions. If decision-maker is biased towards a specific objective, then simply the solution satisfying such preference is to be selected (e.g. a solution with the minimum required credit or a solution with the maximum profit). However, in case no preference exists, a best compromise solution can be selected using a fuzzy approach. Such best compromise solution is considered as the best balance between the conflicting objectives to be optimized among the obtained set of non-dominated optimum solutions. Simply, a normalized membership function μ^s defined by Dhillon et al. [38] for each solution is determined. The solution with the maximum μ^s is the best compromise solution. For more details about the best compromise solution determination, the reader is referred to El-Abbasy et al. [31].

6. Model Implementation And Testing

To implement and test the outputs of the developed optimization model using the NSGA-II technique, two case studies were extracted from literature that were solved using the SPEA technique [29]. This section will only discuss the results obtained using the NSGA-II technique, while the next section will compare the results obtained using both NSGA-II and SPEA techniques.

The implementation of the developed model involved coding and testing the integrated scheduling, cash flow, and NSGA-II optimization modules using the Java programming language.

As the number of objectives to be optimized simultaneously increases, the search space (i.e. the population size) should be increased to guarantee reaching near optimal good quality solutions. Moreover, increasing the number of objectives may result in reduced quality of desired objective values as the algorithm distributes its focus in attaining a tradeoff between the conflicting objectives rather than minimizing or maximizing each objective individually. In other words, most probably, better objectives' values could be obtained if each objective is optimized solely rather than optimizing more than one objective simultaneously. Thus, for each case study, the model was implemented through three optimization levels. The first optimization level considers only single-objective optimization (i.e. optimizing the duration, financing cost, required credit, and profit individually). The main purpose of this optimization level is to determine the extreme minimum values of the duration, financing cost, and required credit objectives and the extreme maximum value of the profit value that could be reached using the developed model. Such extreme objective values can be used as a benchmark later for comparison when optimizing more than one objective simultaneously. The second optimization level considers two-objective optimization at a time to examine the tradeoff between the duration, financing cost, and profit objectives against the required credit. Finally, the third optimization level, considers three-objective optimization by optimizing the objectives of duration, financing cost, and required credit simultaneously. It is worth to mention that in the third optimization level that either the profit or the financing cost objective is to be considered for optimization simultaneously with the duration and required credit. This is for a reason that both profit and financing cost are non-conflicting, i.e. as the financing cost is minimized the profit is maximized and vice versa. In the study carried by Abido and Elazouni [29], the financing cost objective was selected to be optimized with the duration and required credit simultaneously instead of the profit. Thus, the same was applied in this study in order to have a consistent comparison. The NSGA-II parameters used in each optimization level including the population size, number of generations, crossover and mutation rates are shown in Table 1.

6.1 Case Studies Overview

The first case study consists of two small construction parallel projects of 25 and 30 activities. On the other hand, to demonstrate the capability of the developed optimization model in solving larger scale projects, the second case study involves two larger construction parallel projects of 225 and 240 activities. As a reminder, both case studies were adopted from Abido and Elazouni [29] to ensure fair and consistent comparison as will be discussed in section 7. For all projects in both case studies, the time interval used for calculating the cash flow parameters was in weeks and each week comprises five working days. The full contractual terms, time, and financial data used for all projects are presented in Table 2. Moreover, Table 3 shows an example of the calculation of the activities' cost and price for the 25-activity project.

6.2 Discussion of Results

6.2.1 Single-Objective Optimization

The results of the single-objective optimization runs for both case studies are shown in Table 4. These results considers the optimization of each objective under study separately regardless of the values of the other non-considered objectives. For example, the minimum achieved required credit for case study 1 and 2, was $37,149.9 and $81,149.6, respectively. Achieving such credit values neglected optimizing their corresponding duration, financing cost, and profit values. The same applies for the other objectives. Moreover, the minimum achieved duration for case study 1 and 2, was 44 and 269 days, respectively. In fact, these values should be pre-known without performing any optimization for the duration objective solely since these values represents the original projects' schedules without applying the extension increment. Finally, as mentioned earlier, the values obtained in Table 4 will be considered as a benchmark to investigate how much the objectives' values of the extreme solutions obtained for the two-objective and three-objective runs are close to them.

6.2.2 Two-Objective Optimization

For case studies 1 and 2, Figures 3 and 4, respectively, shows the plot of three Pareto-optimal fronts representing the duration, financing cost, and profit versus the required credit relationships. Both figures shows the same trend of "duration VS required credit", "financing cost VS required credit", and "profit VS required credit".

Figures 3a and 4a illustrates that as the duration increases, the required credit decreases. This is for a reason that extending the duration will allow having gaps between the projects' activities beyond their respective initial total floats. Such gaps will reduce the activities' accumulation over a certain period and eventually reduce the cumulative contractor's periodical expenses. It is worth to mention that the same duration can result in several different required credits due to the different allocation of the activities' start times within the same time frame. For example, three solutions with a duration of 62 days in case study 1, can have a corresponding required credit of $38,000, $39,000, and $40,000, respectively. In such case, the optimization model selects the solution with the least required credit (i.e. $38,000) and considers the other two solutions to be dominated and carries them to the next front. Such dilemma can also result in having several solutions with very close required credit values but with big differences in their corresponding durations. For example, the first three solutions in Figure 4a have a duration with corresponding required credit of 354 days with $81,737, 333 days with $82,167, and 300 days with $83,478, respectively. With respect to optimization rules, these three solutions are deemed feasible and non-dominated. However, practically, the contractor may consider selecting the solution with the least duration as it will save him/her 54 days at just an additional required credit of $1,741.

Figures 3b and 4b shows that an increase in the required credit results in a decrease in the financing cost. This can be interpreted through the relationship between the duration and the required credit discussed previously. Minimizing the duration results in less overhead costs and any penalties for late completion. This means that higher required credit decreases the financing cost based on the decrease attained in the duration and the more likeliness in finding other schedules of same duration but with lower financing cost. Accordingly, higher required credit reduces the overhead costs, penalties, and financing costs, hence, higher profit values are achieved as demonstrated in Figures 3c and 4c.

Among the obtained solutions of each Pareto-optimal front in Figures 3 and 4, three remarkable solutions are of interest; two solutions with the extreme desired (optimum) values for the two objectives under study, and one solution representing the best compromise. Table 5 summarizes such remarkable solutions for each relationship for both case studies. Comparing the extreme desired objective values of the single-objective optimization in Table 4 with those achieved in

Table 5, shows that both results are very close to each other which is a good indication of the solutions' quality when optimizing two objectives simultaneously.

Referring to case study 1 in Table 5, when the only priority is to minimize the duration, then the "minimum duration" solution associated with the "duration VS required credit" Pareto-optimal front is to be selected to achieve a global minimum duration of 44 days with a minimized required credit of $61,425.6. On the other hand, when the only priority is to minimize the required credit, then the "minimum required credit" solution associated with the "duration VS required credit" Pareto-optimal front is to be selected to achieve a global minimum required credit of $38,788.7 with a minimized duration of 62 days. It is obvious that having a solution with both objectives globally minimized is impossible as both objectives are conflicting. It is usually more appealing for the contractor to execute the projects at their minimum possible duration to avoid additional overheads and liquidated damages. However, the contractor might still be governed by his/her credit limit which could be below the $61,425.6. Therefore, beside the discussed two extreme solutions, the optimization model provides additional solutions in between that are a trade-off between the duration and the required credit as shown in Figure 3a. In each solution, both the duration and required credit are minimized but not globally minimized. Such additional solutions are considered optimum since they are non-dominated. Finally, if the contractor have no certain priority towards either minimizing the duration or the required credit, the model provides the best compromise solution within the additional solutions. As shown in Table 5, the best compromise solution associated with the "duration VS required credit" Pareto-optimal front gives a schedule with a duration and required credit of 52 days and $42,736, respectively.

To illustrate the internal detailed calculations of the cash flow parameters, Figures 5 and 6 show the schedules of the 25- and 30-activity projects, respectively of the best compromise solution associated with the "duration VS required credit" Pareto-optimal front. The schedules in Figures 5 and 6 shows also the periodical (i.e. weekly) direct costs and earned values. The earned values are the direct costs multiplied by the bid price factor. Table 6 presents the weekly cash outflows (E_t) and inflows (P_t) for each project separately and for both projects together. Consequently, the E_t and P_t values of the two projects together are used to determine the other cash flow parameters as shown in Table 7.

6.2.3 Three-Objective Optimization

The obtained Pareto-optimal fronts for optimizing the duration, financing cost, and required credit objectives simultaneously, comprised 74 and 86 non-dominated solutions for case studies 1 and 2, respectively. Likewise the two-objective optimization results obtained in Table 5, Table 8 presents the remarkable solutions that are of interest for the three-objective optimization for both case studies. A solution for the minimum duration, minimum financing cost, minimum required credit, as well as the best compromise solution. Again, comparing the extreme desired objective values of the single-objective optimization in Table 4 with those achieved in Table 8, shows that both results are very close to each other. This shows the capability of the model to obtain a sound optimal or near optimal solutions when optimizing three objectives together.

7. Algorithms' Comparisons

7.1 Quality of Solutions

The quality of solutions in this study intends to compare the values of the optimized objectives resulted from each algorithm used (i.e. NSGA-II and SPEA). Starting with the single-objective optimization, Table 4 shows that with respect to the duration objective, both algorithms in both case studies provided identical results. In fact, when it comes to the duration objective, both algorithms will always provide identical results whether with two-objective or three-objective optimization. Referring again to Table 4, it is clear that NSGA-II provided better financing cost than SPEA in both case studies. As for the required credit, NSGA-II provided better minimized value than SPEA in case study 1 while a close - but still better - value in case study 2. Finally, NSGA-II provided a better profit value than SPEA in case study 2 while a close - but still better - value in case study 1.

Similarly, for the two-objective optimization, Table 5 shows the values of the objectives in each remarkable solution for both algorithms and it is clear that NSGA-II resulted in better values in all cases. Such better values were not only with respect to the objective under study, but also with the other associated objective. For example, considering the minimum required credit solution associated with the "financing cost VS required credit" Pareto-optimal front of case study 2 in Table 5, NSGA-II did not only provide a better required credit of $82,159.2 compared to $84,096.3 of SPEA. Actually, it also provided a better financing cost of $8,675.0 compared to $8,943.3 of SPEA within the same solution.

A better visualization for the quality of solutions obtained by both algorithms is depicted in Figures 3 and 4. Figures 3a and 3b shows how the Pareto-optimal front obtained using the NSGA-II is shifting away from that of SPEA towards a more feasible region. The shift was in the leftward direction (i.e. more minimized required credit) and simultaneously in the downward direction (i.e. more minimized financing cost and duration). The same shift trend towards a more feasible region can be seen in Figure 3c, however instead of shifting downward, the shift was upward (i.e. more maximized profit). Apart from the shifting, it can be seen also that all the solutions obtained using the NSGA-II dominates that of SPEA. Such improvement of objective values were also achieved in case study 2 as seen in Figure 4, however, the improvement was less than in case study 1. As shown in Figure 4, both Pareto-optimal fronts obtained by each algorithm were closer to each other than in case study 1. In other words, as the project size and eventually the number of activities increases, the ability of NSGA-II to surpass SPEA decreases somehow. This is due to the increased computational effort required by the algorithm to search for optimal or near optimal solutions which will eventually affect the solutions' quality.

For the three-objective optimization, Table 8 shows the values of the objectives in each remarkable solution for both algorithms and again the NSGA-II resulted in better values in all cases. Table 9 summarizes the improvement percentages of the NSGA-II over the SPEA for all types of optimization. The average improvement percentage was from 1.7% to 20.4%. Also, it is obvious from Table 9 that the average improvement percentage was always higher in case study 1 than in 2 due to the project size.

7.2 Performance

Comparing the values of the objectives obtained by each algorithm alone is not sufficient evaluation. In fact, a good algorithm should provide set of solutions in the obtained Pareto-optimal front having good convergence and diversity features. A good convergence means that the solutions set in the obtained Pareto-optimal front are closer to the true Pareto-optimal front. On the other hand diversity is represented by the uniformity and distribution of the solutions set in the obtained Pareto-optimal front. A good uniformity means that the solutions obtained are uniformly distributed along the obtained Pareto-optimal front, while a good distribution means that the solutions obtained cover the whole true Pareto-optimal front as much as possible [39]. Many performance metrics to measure the convergence and diversity are available in literature (e.g. generational distance, inverted generational distance, spacing, maximum spread, C-metric, hypervolume, overall non-dominated vector generation, etc.). Most of these metrics requires the pre-knowledge of the true Pareto-optimal front of the multi-objective optimization problem under study. Since the true Pareto-optimal front of our finance-based scheduling problem is unknown, the spacing SP [40], hypervolume HV [41], and maximum spread MS [42] performance metrics were selected in this study to assess and evaluate the performance of both algorithms used. Such performance metrics are briefly discussed as follows:

- Spacing (SP): this is a diversity that assesses the uniformity of the obtained set of solutions. The main idea is to measure the variance of the solutions found in the obtained Pareto-optimal front as shown in Equation 14. The smaller the value of this metric is the greater the number of solutions and the more equally spread solutions is along the obtained Pareto-optimal front [43].

$$SP = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\bar{d} - d_i)^2} \qquad (14)$$

Where n = number of obtained Pareto-optimal solutions, d_i = distance between solution i and the nearest solution with respect to each objective, and $\bar{d}$ = the average of all d_i.

- Maximum Spread (MS): another diversity metric to assess the distribution of the obtained set of solutions. Such assessment measures how well the true Pareto-optimal front is covered by the obtained Pareto-optimal front [44]. Its mathematical expression is shown in Equation 15. A value closer to one of this metric indicates better diversity in terms of distribution.

$$MS = \sqrt{\frac{1}{OB}\sum_{i=1}^{OB}\left\{\frac{\min(f_i^{max}, F_i^{max}) - \max(f_i^{min}, F_i^{min})}{F_i^{max} - F_i^{min}}\right\}^2} \qquad (15)$$

Where OB is the number of objectives, f_i^{max} and f_i^{min} are the maximum and minimum values of the ith objective in the obtained Pareto-optimal front, respectively, F_i^{max} and F_i^{min} are the maximum and minimum values of the ith objective in the true Pareto-optimal front, respectively. It is shown in the equation that this metric requires only the extreme solutions of the true Pareto-optimal front rather than the whole set. Although, the true Pareto-optimal front is unknown in our problem, the extreme objective values obtained during the single-objective optimization (see Table 4) will be considered as an approximation to the true Pareto-optimal front.

- Hypervolume (HV): this metric measures both convergence and diversity of the obtained Pareto-optimal front. The idea is to calculate the area or volume of the objective space that is dominated by the obtained Pareto-optimal front. Thus, it requires a reference point (solution) that is worse than the obtained non-dominated solutions (i.e. a dominated solution). Its mathematical expression is shown in Equation 16. A value closer to one of this metric indicates better convergence and diversity.

$$HV = volume(\bigcup_{i=1}^{n} v_i) \quad \text{.....................(16)}$$

Where v_i = volume enclosed between non-dominated solution i and the dominated reference solution.

All of the discussed performance metrics were measured for the two-objective and three-objective optimization runs. Apart from assessing the algorithms' performance with respect to convergence and diversity, the running (computational) times (RT) to reach the different obtained Pareto-optimal fronts is also considered. Table 10 summarizes the measured values of the four mentioned metrics (i.e SP, HV, MS, and RT) for both algorithms. The values show that both algorithms performed well with respect to convergence and diversity. However, a closer look will reveal that NSGA-II outperformed SPEA in terms of integrated convergence and diversity assessment with HV values more than SPEA in all runs. For uniformity assessment, NSGA-II also outperformed SPEA with SP values less than SPEA in all runs. However, SPEA outperformed NSGA-II in most of the runs in terms of distribution with higher MS values. In terms of running time, NSGA-II showed a significant faster running time to reach the obtained Pareto-optimal fronts in all runs.

Finally, Table 11 summarizes the improvement percentages of NSGA-II over SPEA in terms of the utilized performance metrics. Similar to the quality of solutions, the average improvement percentages decreases as the project size increases. The average improvement percentage of the metrics HV, SP, and MS ranged from 8.5% to 19.1%. On the other hand, the significant average improvement percentage of RT ranged from 87.0% to 98.2%.

8. Conclusions

This study presented an adjusted finance-based scheduling model using NSGA-II to optimize the conflicting objectives of duration, financing cost, required credit, and profit. Two case studies from literature were applied to demonstrate the ability of the model to obtain optimal or near optimal trade-off between different two-objective combinations as well as three conflicting objectives. The two-objective optimization showed that as the required credit increases, the duration and financing cost decreases, while the profit increases. Furthermore, by comparing the two-objective and three-objective optimization's extreme desired objective values with single-objective optimization, it was found that the results were very close indicating the capability of the model in providing good quality solutions when number of objectives to be simultaneously optimized increases. Consequently, NSGA-II was compared with SPEA in solving the same finance-based scheduling problems. Such comparison was in terms of quality of solutions obtained and performance of the algorithms. It was found that NSGA-II outperformed SPEA in most comparison aspects. With respect to the quality of obtained solutions, the average improvement percentage was from 1.7% to 20.4%. With respect to the performance metrics of SP, HV, and MS, the average improvement percentage ranged from 8.5% to 19.1%. However, SPEA outperformed NSGA-II mostly in terms of MS separately. Finally, NSGA-II showed a significant improvement over SPEA with respect to the running time to reach the obtained Pareto-optimal fronts with average improvement percentage from 87.0% to 98.2%. It was noticed that such improvements' efficiency decreases as the project size increases. Based on the experimental work conducted on the case studies in this study, it can be concluded that NSGA-II outperformed SPEA in solving finance-based scheduling problems. However, more case studies should be implemented to statistically support the better performance of NSGA-II over SPEA. Moreover, such conclusion is limited to multiple projects within the range of 200 activities each. Thus, further investigation should be carried to determine if NSGA-II will attain its superiority over SPEA with larger projects' size.

For practitioners, the developed optimization model is expected to help in efficiently prioritizing the allocation of the different activities' start times among multiple projects so that the contractor's credit limit is not exceeded. Moreover, satisfying the contractor's credit limit is accompanied with a simultaneous minimization and maximization of the financing cost and profit, respectively, with the least possible duration. On the other hand, for researchers, this study helped in identifying the weaknesses and strengths of implementing both NSGA-II and SPEA optimization techniques in solving the finance-based scheduling problem for multiple projects. However, as a future research, other metaheuristic techniques can be applied to solve finance-based scheduling problems to investigate their performance against NSGA-II as there will always be a room for improvement in solving such problems.

Regardless of the optimization techniques used, there are some limitations in the optimization model developed which can be addressed in future for more practicality. First, the model should cover all other types of precedence relationships (i.e. finish-to-finish, start-to-start, and start-to-finish) and should consider also the time lag between activities. Taking such relationships into consideration is essential to develop more practical schedules by overlapping the activities of a network to accelerate the execution of a project. Second, many factors exist during construction that may affect the cash flow including time delays, cost overruns, unconfirmed earned values, change orders, and changes of cost plan elements. Thus, for more practical schedules, activities' interruption or splitting caused by different parties can be considered. Moreover, uncertainties in the activities' duration and cost can be considered while scheduling the projects. Finally, the cash flow model of this study applies only for unit-price type contracts. Thus, the cash flow model structure can be adjusted to suit other different types of construction contracts. This can be done by studying and applying the different contracts' methods of payment as well as the timing of payment which can highly affect the project schedule.

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Table 1: NSGA-II Parameters

Parameter	Single-Objective Optimization		Two-Objective Optimization		Three-Objective Optimization	
	Cast Study 1	Case Study 2	Cast Study 1	Case Study 2	Cast Study 1	Case Study 2
Population Size	100	200	300	600	500	1000
Number of Generations	200	400	500	1000	1000	2000
Crossover Rate	0.9	0.9	0.9	0.9	0.9	0.9
Mutation Rate	0.1	0.1	0.1	0.1	0.1	0.1

Table 2: Case Studies' Data

Data Type	Item	Case Study 1*		Case Study 2**	
		25 Activity Project	30 Activity Project	225 Activity Project	240 Activity Project
TIME	Project Start Time (day #)	0	15	0	100
	Extension Increment (days)	25	25	95	95
	No. of Days per Week	5	5	5	5
	Original Duration (days)	27	29	243	169
	Original Duration (weeks)	6	6	49	34
FINANCIAL	Interest Rate % per Week	0.80	0.80	0.80	0.80
	Overheads %	17	15	15	15
	Mobilization Costs %	8	5	1	1
	Tax %	2	2	2	2
	Mark-Up %	12	20	10	10
	Bond Premium %	4	1	1	1
CONTRACT TERMS	Advance Payment % of Bid Price	9	10	2	2
	Weeks to Retrieve Advance Payment	[a]	[a]	[a]	[a]
	Retained % of Pay Requests	6	5	5	5
	Lag to Pay Retained Money After Last Payment (weeks)	0	0	0	0
	Weeks to Submit Pay Requests Regularly	1	1	1	1
	Lag to Pay Payment Requests (weeks)	1	1	1	1
	Late Completion Penalty per Day ($ / day)	1500	1000	1000	1000

[a]Number of weeks encompassing the total project duration
*Credit Limit = $82,500
**Credit Limit = $120,000

Table 3: Activities Cash Outflows and Inflows Rates (25-Activity Project)

Activity	Duration (days)	Daily Direct Cost ($/day)	Total Direct Cost ($)	Daily Price ($/day)	Activity	Duration (days)	Daily Direct Cost ($/day)	Total Direct Cost ($)	Daily Price ($/day)
A	2	1000	2000	1501.3	**N**	5	400	2000	600.5
B	3	1200	3600	1801.6	**O**	5	550	2750	825.7
C	2	1100	2200	1651.4	**P**	4	500	2000	750.6
D	3	900	2700	1351.2	**Q**	3	1350	4050	2026.7
E	3	1250	3750	1876.6	**R**	5	600	3000	900.8
F	3	1150	3450	1726.5	**S**	5	850	4250	1276.1
G	2	1050	2100	1576.4	**T**	6	700	4200	1050.9
H	3	950	2850	1426.2	**U**	4	1200	4800	1801.6
I	2	650	1300	975.8	**V**	3	1850	5550	2777.4
J	5	450	2250	675.6	**W**	5	650	3250	975.8
K	5	350	1750	525.5	**X**	5	600	3000	900.8
L	5	500	2500	750.6	**Y**	2	1000	2000	1501.3
M	1	1450	1450	2176.9					

Note: The prices in this table do not include the financing cost. Total Cash Outflow = 72,750; Overheads = 12,367.5; Mobilization = 6,809.4; Cash Outflow + Overheads + Mobilization = 91,926.9; Taxes = 1,838.5; Taxes + Cash Outflow + Overheads + Mobilization = 93,765.4; Profit = 11,251.9; Profit + Taxes + Cash Outflow + Overheads + Mobilization = 105,017.3; Bond Premium = 4,200.7; Total Bid Price = 109,218; and Bid Price Factor (109,218 / 72,750) = 1.501.

Table 4: Single-Objective Optimization Results

Case Study	Optimized Objective	Optimization Type	Optimized Objectives Results	
			NSGA-II	**SPEA**
(1) 25-30 Activity Projects	Duration	Minimization	44	44
	Financing Cost	Minimization	1,523.1	1,634.9
	Required Credit	Minimization	37,149.9	40,530.6
	Profit	Maximization	42,042.9	41,956.4
(2) 225-240 Activity Projects	Duration	Minimization	269	269
	Financing Cost	Minimization	6,012.1	6,277.4
	Required Credit	Minimization	81,149.6	81,174.3
	Profit	Maximization	195,446.1	190,630.2

Table 5: Two-Objective Optimization Results (Extreme and Best Compromise Solutions)

Case Study	Optimized Objectives	Solution Type	Optimized Objectives Results							
			NSGA-II				SPEA			
			Duration	Financing Cost	Required Credit	Profit	Duration	Financing Cost	Required Credit	Profit
(1) 25-30 Activity Projects	Duration VS Required Credit	Min. Duration	44	-	61,425.6	-	44	-	70,019.5	-
		Min. Required Credit	62	-	38,788.7	-	66	-	40,986.5	-
		Best Compromise	52	-	42,736.0	-	57	-	45,409.0	-
	Financing Cost VS Required Credit	Min. Financing Cost	-	1,538.8	65,403.6	-	-	1,654.2	82,854.3	-
		Min. Required Credit	-	1,877.1	38,480.2	-	-	2,933.9	44,477.1	-
		Best Compromise	-	1,684.7	45,255.6	-	-	1,805.2	50,878.3	-
	Profit VS Required Credit	Max. Profit	-	-	78,227.9	42,035.0	-	-	81,545.2	41,954.0
		Min. Required Credit	-	-	40,469.5	14,644.1	-	-	43,691.6	4,889.5
		Best Compromise	-	-	53,896.5	37,261.5	-	-	67,494.9	36,819.6
(2) 225-240 Activity Projects	Duration VS Required Credit	Min. Duration	269	-	102,032.7	-	269	-	108,500.0	-
		Min. Required Credit	354	-	81,736.8	-	360	-	82,500.0	-
		Best Compromise	281	-	86,090.6	-	285	-	90,000.0	-
	Financing Cost VS Required Credit	Min. Financing Cost	-	6,041.8	104,766.1	-	-	6,368.9	106,969.3	-
		Min. Required Credit	-	8,675.0	82,159.2	-	-	8,943.2	84,096.3	-
		Best Compromise	-	7,156.2	84,115.2	-	-	7,178.1	84,719.5	-
	Profit VS Required Credit	Max. Profit	-	-	105,607.1	192,009.5	-	-	109,590.0	185,870.0
		Min. Required Credit	-	-	81,350.1	29,750.1	-	-	82,031.0	27,174.0
		Best Compromise	-	-	91,826.4	168,427.0	-	-	93,274.0	151,306.0

Table 6: Best Compromise Solution Weekly Cash Outflow and Inflow Calculations

End of Week	Cash Outflow (E_t)				Cash Inflow (P_t)			
	Item	**25-Act. Project**	**30-Act. Project**	**Sum of Two Projects**	**Item**	**25-Act. Project**	**30-Act. Project**	**Sum of Two Projects**
0	**Mobilization & Bond**	11010.7[f]	-	11,010.7	**Advance Payment**	9829.7[g]	-	9829.7
1	**Direct Cost**	17250	-	-	**Earned Value**	-	-	-
	Overhead	2290.4[a]	-	-	**Deductions**	-	-	-
	Tax	340.5[b]	-	-	**Additions**	-	-	-
	Total	19880.8	-	19,880.8	**Net**	-	-	-
2	**Direct Cost**	10400	-	-	**Earned Value**	25897.4	-	-
	Overhead	2290.4[a]	-	-	**Deductions**	3192.1[c]	-	-
	Tax	340.5[b]	-	-	**Additions**	-	-	-
	Total	13030.8	-	13,030.8	**Net**	22705.3	-	22,705.3
3	**Direct Cost**	15550	-	-	**Earned Value**	15613.5	-	-
	Overhead	2290.4[a]	-	-	**Deductions**	2575.1[c]	-	-
	Tax	340.5[b]	-	-	**Additions**	-	-	-
	Total	18180.8	9577.3[f]	27,758.1	**Net**	13038.4	19779.1[g]	32,817.5
4	**Direct Cost**	15800	7900	-	**Earned Value**	23345.2	-	-
	Overhead	2290.4[a]	3426.7[a]	-	**Deductions**	3039.0[c]	-	-
	Tax	340.5[b]	551.7[b]	-	**Additions**	-	-	-
	Total	18430.8	11878.4	30,309.3	**Net**	20306.2	-	20,306.2
5	**Direct Cost**	10250	8600	-	**Earned Value**	23720.5	11792.8	-
	Overhead	2290.4[a]	3426.7[a]	-	**Deductions**	3061.5[c]	3062.0[c]	-
	Tax	340.5[b]	551.7[b]	-	**Additions**	-	-	-
	Total	12880.8	12578.4	25,459.3	**Net**	20659.0	8730.8	29,389.8
6	**Direct Cost**	3500	17300	-	**Earned Value**	15388.3	12837.8	-
	Overhead	916.1[a]	3426.7[a]	-	**Deductions**	2561.6[c]	3114.3[c]	-
	Tax	136.2[b]	551.7[b]	-	**Additions**	-	-	-
	Total	4552.3	21278.4	25,830.8	**Net**	12826.7	9723.5	22,550.2
7	**Direct Cost**	-	21700	-	**Earned Value**	5254.5	25824.8	-
	Overhead	-	3426.7[a]	-	**Deductions**	1953.6[c]	3763.6[c]	-
	Tax	-	551.7[b]	-	**Additions**	6553.2[e]	-	-
	Total	-	25678.4	25,678.4	**Net**	9854.1	22061.2	31,915.3
8	**Direct Cost**	-	25900	-	**Earned Value**	-	32392.9	-
	Overhead	-	3426.7[a]	-	**Deductions**	-	4092.0[c]	-
	Tax	-	551.7[b]	-	**Additions**	-	-	-
	Total	-	29878.4	29,878.4	**Net**	-	28300.9	28,300.9
9	**Direct Cost**	-	25300	-	**Earned Value**	-	38662.5	-
	Overhead	-	3426.7[a]	-	**Deductions**	-	4405.5[c]	-
	Tax	-	551.7[b]	-	**Additions**	-	-	-
	Total	-	29278.4	29,278.4	**Net**	-	34257.0	34,257.0
10	**Direct Cost**	-	21800	-	**Earned Value**	-	37766.9	-
	Overhead	-	3426.7[a]	-	**Deductions**	-	5360.7[d]	-
	Tax	-	551.7[b]	-	**Additions**	-	-	-
	Total	-	25778.4	25,778.4	**Net**	-	32406.2	32,406.2
11	**Direct Cost**	-	4000	-	**Earned Value**	-	32542.2	-
	Overhead	-	1370.7[a]	-	**Deductions**	-	9099.5[d]	-
	Tax	-	220.7[b]	-	**Additions**	-	-	-
	Total	-	5591.4	5,591.4	**Net**	-	23442.7	23,442.7
12	**Direct Cost**	-	-	-	**Earned Value**	-	5971.1	-
	Overhead	-	-	-	**Deductions**	-	4770.9[d]	-
	Tax	-	-	-	**Additions**	-	9889.6[e]	-
	Total	-	-	-	**Net**	-	11089.8	11,089.8

[a](Total Overheads / Original duration in days) x No. of days in corresponding week
[b](Total Tax / Original duration in days) x No. of days in corresponding week
[c]Retained Amount + Advance Payment Share
[d]Retained Amount + Advance Payment Share + Penalty
[e]Total Retained Money
[f]Mobilization + Bond
[g]Advance Payment % x Total Bid Price

Table 7: Best Compromise Solution Financial Parameters Calculations

Financial Parameter \ End of Week	0	1	2	3	4	5	6	7	8	9	10	11	12
Expenditures (E)	-11011	-19881	-13031	-27758	-30309	-25459	-25831	-25678	-29878	-29278	-25778	-5591	0
Income (P)	9830	0	22705	32817	20306	29390	22550	31915	28301	34257	32406	23443	11090
Cumulative Balance (F)	-11011	-21062	-34093	-39146	-36637	-41791	-38232	-41360	-39323	-40300	-31822	-5007	18436
Cumulative Net Balance (N)	-1181	-21062	-11387	-6328	-16331	-12401	-15681	-9445	-11022	-6043	584	18436	29525
Financing Cost (I)	0	-89	-221	-202	-172	-232	-203	-228	-195	-205	-151	-20	0
Compounded Financing Cost (I')	0	-89	-310	-515	-691	-929	-1139	-1376	-1582	-1800	-1966	-2002	-2018
Cumulative Balance Including Financing Cost (F')	-11011	-21151	-34403	-39661	-37328	-42719	-39370	-42736	-40905	-42101	-33788	-7009	16418
Cumulative Net Balance Including Financing Cost (N')	-1181	-21151	-11698	-6843	-17022	-13330	-16820	-10821	-12604	-7844	-1382	16434	27507

Table 8: Three-Objective Optimization Results (Extreme and Best Compromise Solutions)

Case Study	Solution Type	Optimized Objectives Results					
		NSGA-II			SPEA		
		Duration	Financing Cost	Required Credit	Duration	Financing Cost	Required Credit
(1) 25-30 Activity Projects	Min. Duration	44	1,805.4	70,238.1	44	1,870.4	76,419.2
	Min. Financing Cost	52	1,671.1	60,689.5	55	1,683.0	62,092.5
	Min. Required Credit	62	2,041.3	37,787.3	67	2,937.1	45,043.7
	Best Compromise	51	1,749.7	50,063.7	51	1,778.4	55,243.8
(2) 225-240 Activity Projects	Min. Duration	269	6,680.5	106,880.1	269	7,118.8	109,234.1
	Min. Financing Cost	316	6,035.5	104,732.4	323	6,368.9	106,969.3
	Min. Required Credit	323	8,611.7	82,193.3	344	8,692.8	84,117.1
	Best Compromise	275	6,351.0	98,281.9	283	6,808.9	103,739.4

Table 9: NSGA-II Improvement Percentages over SPEA with respect to Quality of Solutions

Optimization	Case Study	Optimized Objective(s)	Solution Type	Improvement Percentage in				Average Improvement Percentage
				Duration	Financing Cost	Required Credit	Profit	
SINGLE-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Duration	Min. Duration	0.0%	-	-	-	3.8%
		Financing Cost	Min. Financing Cost	-	6.8%	-	-	
		Required Credit	Min. Required Credit	-	-	8.3%	-	
		Profit	Max. Profit	-	-	-	0.2%	
	(2) 225-240 Activity Projects	Duration	Min. Duration	0.0%	-	-	-	1.7%
		Financing Cost	Min. Financing Cost	-	4.2%	-	-	
		Required Credit	Min. Required Credit	-	-	0.03%	-	
		Profit	Max. Profit	-	-	-	2.5%	
TWO-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Duration VS Required Credit	Min. Duration	0.0%	-	12.3%	-	20.4%
			Min. Required Credit	6.1%	-	5.4%	-	
			Best Compromise	8.8%	-	5.9%	-	
		Financing Cost VS Required Credit	Min. Financing Cost	-	7.0%	21.1%	-	
			Min. Required Credit	-	36.0%	13.5%	-	
			Best Compromise	-	6.7%	11.1%	-	
		Profit VS Required Credit	Max. Profit	-	-	4.1%	0.2%	
			Min. Required Credit	-	-	7.4%	199.5%	
			Best Compromise	-	-	20.1%	1.2%	
	(2) 225-240 Activity Projects	Duration VS Required Credit	Min. Duration	0.0%	-	6.0%	-	3.2%
			Min. Required Credit	1.7%	-	0.9%	-	
			Best Compromise	1.4%	-	4.3%	-	
		Financing Cost VS Required Credit	Min. Financing Cost	-	5.1%	2.1%	-	
			Min. Required Credit	-	3.0%	2.3%	-	
			Best Compromise	-	0.3%	0.7%	-	
		Profit VS Required Credit	Max. Profit	-	-	3.6%	3.3%	
			Min. Required Credit	-	-	0.8%	9.5%	
			Best Compromise	-	-	1.6%	11.3%	
THREE-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Duration VS Financing Cost VS Required Credit	Min. Duration	0.0%	3.5%	8.1%	-	7.1%
			Min. Financing Cost	5.5%	0.7%	2.3%	-	
			Min. Required Credit	7.5%	30.5%	16.1%	-	
			Best Compromise	0.0%	1.6%	9.4%	-	
	(2) 225-240 Activity Projects	Duration VS Financing Cost VS Required Credit	Min. Duration	0.0%	6.2%	2.2%	-	3.5%
			Min. Financing Cost	2.2%	5.2%	2.1%	-	
			Min. Required Credit	6.1%	0.9%	2.3%	-	
			Best Compromise	2.8%	6.7%	5.3%	-	

Table 10: Two- and Three-Objective Optimizations' Performance Metrics

Optimization	Case Study	Optimized Objectives	Performance Metrics							
			NSGA-II				SPEA			
			SP	HV	MS	RT (sec)	SP	HV	MS	RT (sec)
TWO-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Duration VS Required Credit	0.031	0.850	0.761	5.7	0.048	0.632	0.845	182.8
		Financing Cost VS Required Credit	0.034	0.948	0.599	4.9	0.041	0.810	0.805	723.4
		Profit VS Required Credit	0.047	0.854	0.824	4.3	0.060	0.578	0.928	18.2
	(2) 225-240 Activity Projects	Duration VS Required Credit	0.080	0.919	0.833	228.1	0.126	0.828	0.905	1444.6
		Financing Cost VS Required Credit	0.098	0.846	0.604	237.7	0.193	0.807	0.604	2470.7
		Profit VS Required Credit	0.066	0.714	0.794	219.6	0.088	0.636	0.835	1622.2
THREE-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Duration VS Financing Cost VS Required Credit	0.020	0.768	0.864	14.2	0.026	0.579	0.851	774.7
	(2) 225-240 Activity Projects	Duration VS Financing Cost VS Required Credit	0.037	0.617	0.755	1873.3	0.041	0.503	0.810	16724.8

Table 11: NSGA-II Improvement Percentages over SPEA with respect to Performance

Optimization	Case Study	Optimized Objectives	Improvement Percentage in SP	HV	MS	RT	SP, HV, and MS Average Improvement Percentage	RT Average Improvement Percentage
TWO-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Duration VS Required Credit	35.4%	34.5%	-9.9%	96.9%	14.1%	90.9%
TWO-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Financing Cost VS Required Credit	17.1%	17.0%	-25.6%	99.3%	14.1%	90.9%
TWO-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Profit VS Required Credit	21.7%	47.8%	-11.2%	76.4%	14.1%	90.9%
TWO-OBJECTIVE OPTIMIZATION	(2) 225-240 Activity Projects	Duration VS Required Credit	36.5%	11.0%	-8.0%	84.2%	14.0%	87.0%
TWO-OBJECTIVE OPTIMIZATION	(2) 225-240 Activity Projects	Financing Cost VS Required Credit	49.2%	4.8%	0.0%	90.4%	14.0%	87.0%
TWO-OBJECTIVE OPTIMIZATION	(2) 225-240 Activity Projects	Profit VS Required Credit	25.0%	12.3%	-4.9%	86.5%	14.0%	87.0%
THREE-OBJECTIVE OPTIMIZATION	(1) 25-30 Activity Projects	Duration VS Financing Cost VS Required Credit	23.1%	32.6%	1.5%	98.2%	19.1%	98.2%
THREE-OBJECTIVE OPTIMIZATION	(2) 225-240 Activity Projects	Duration VS Financing Cost VS Required Credit	9.8%	22.7%	-6.8%	88.8%	8.5%	88.8%

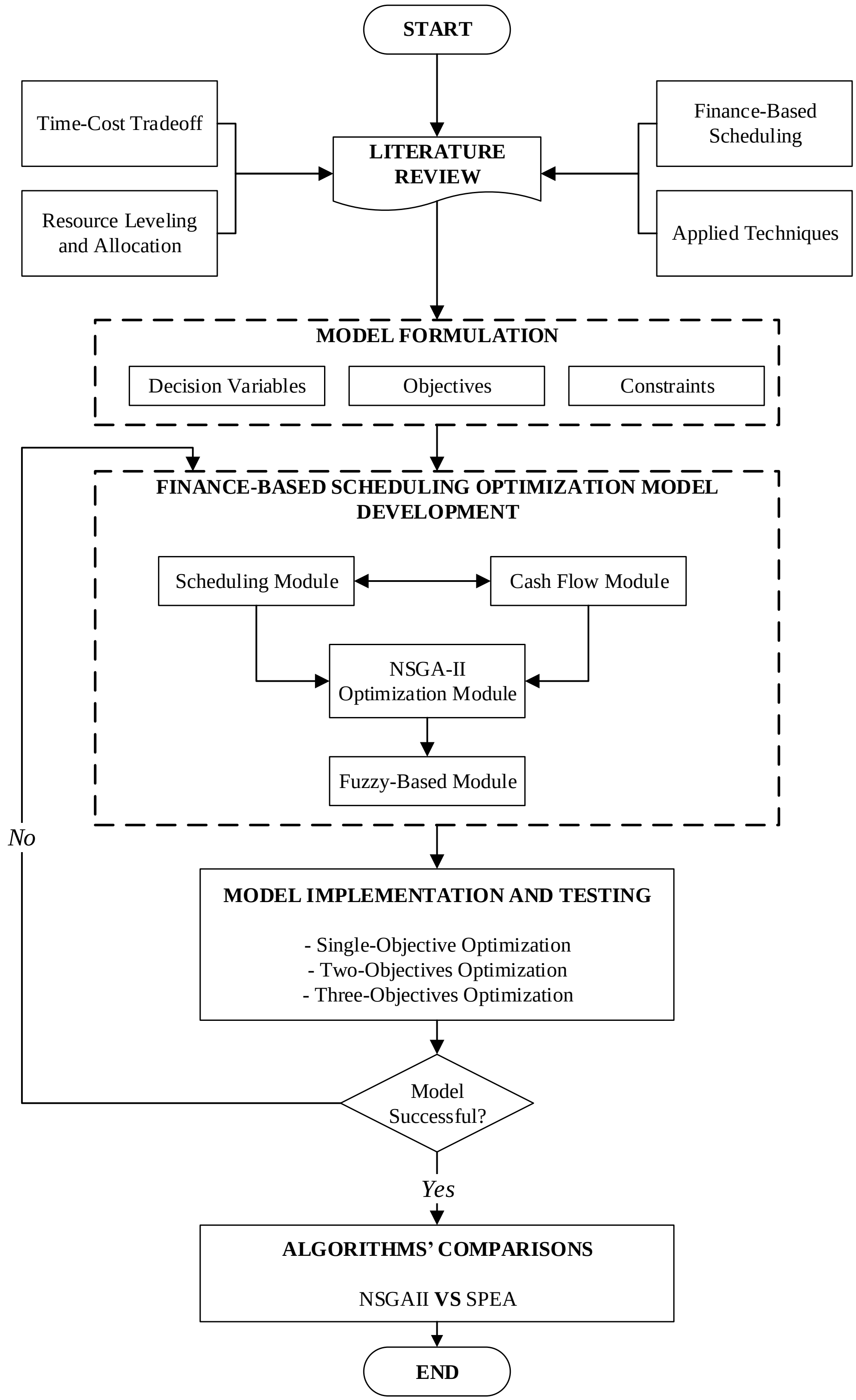

Figure 1: Research Methodology Framework

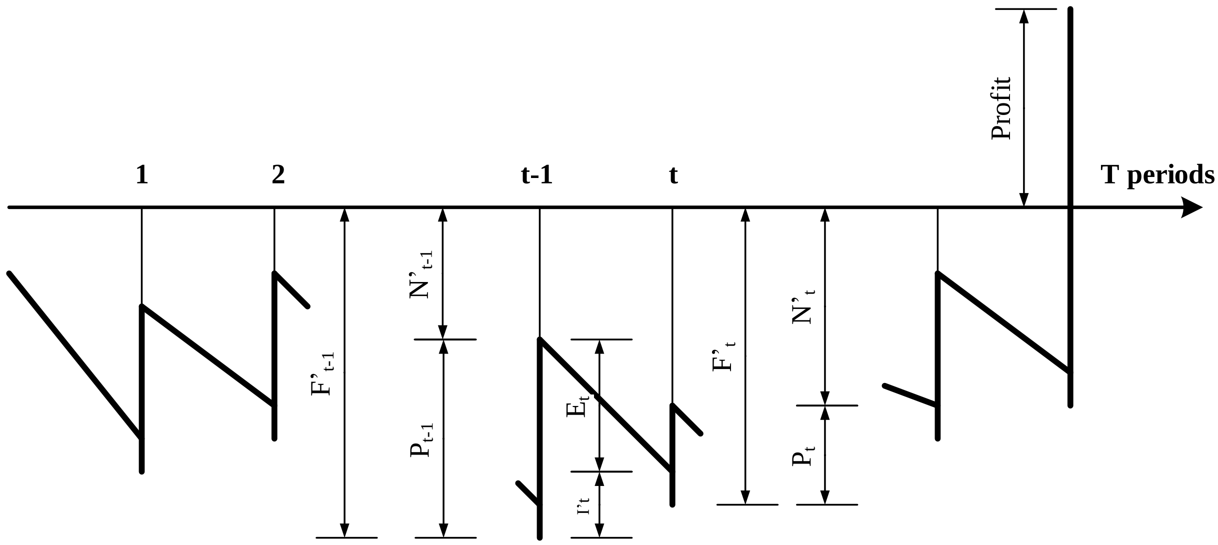

Figure 2: Cash Flow Profile [29]

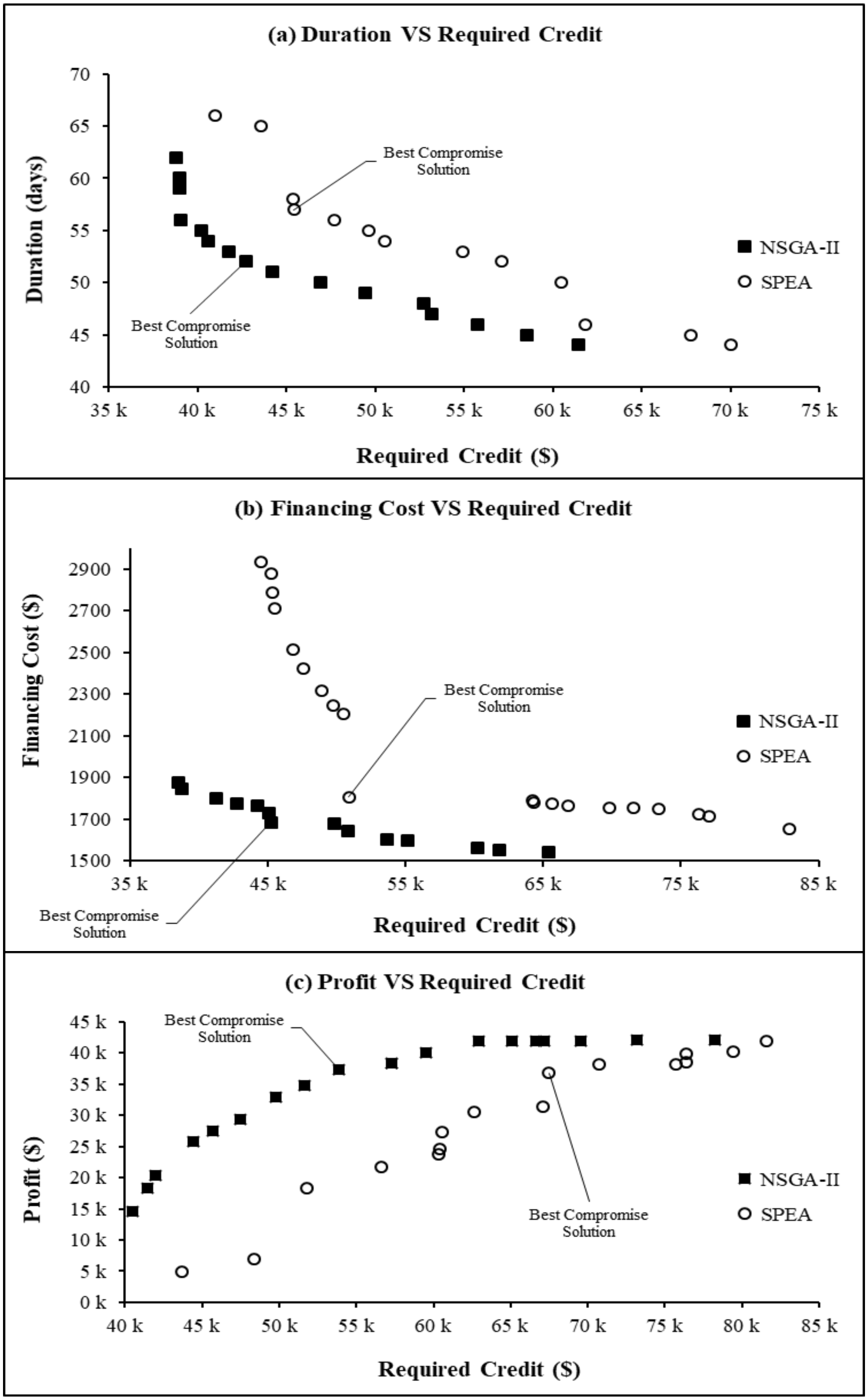

Figure 3: Two-Objective Optimization Pareto Fronts (Case Study 1)

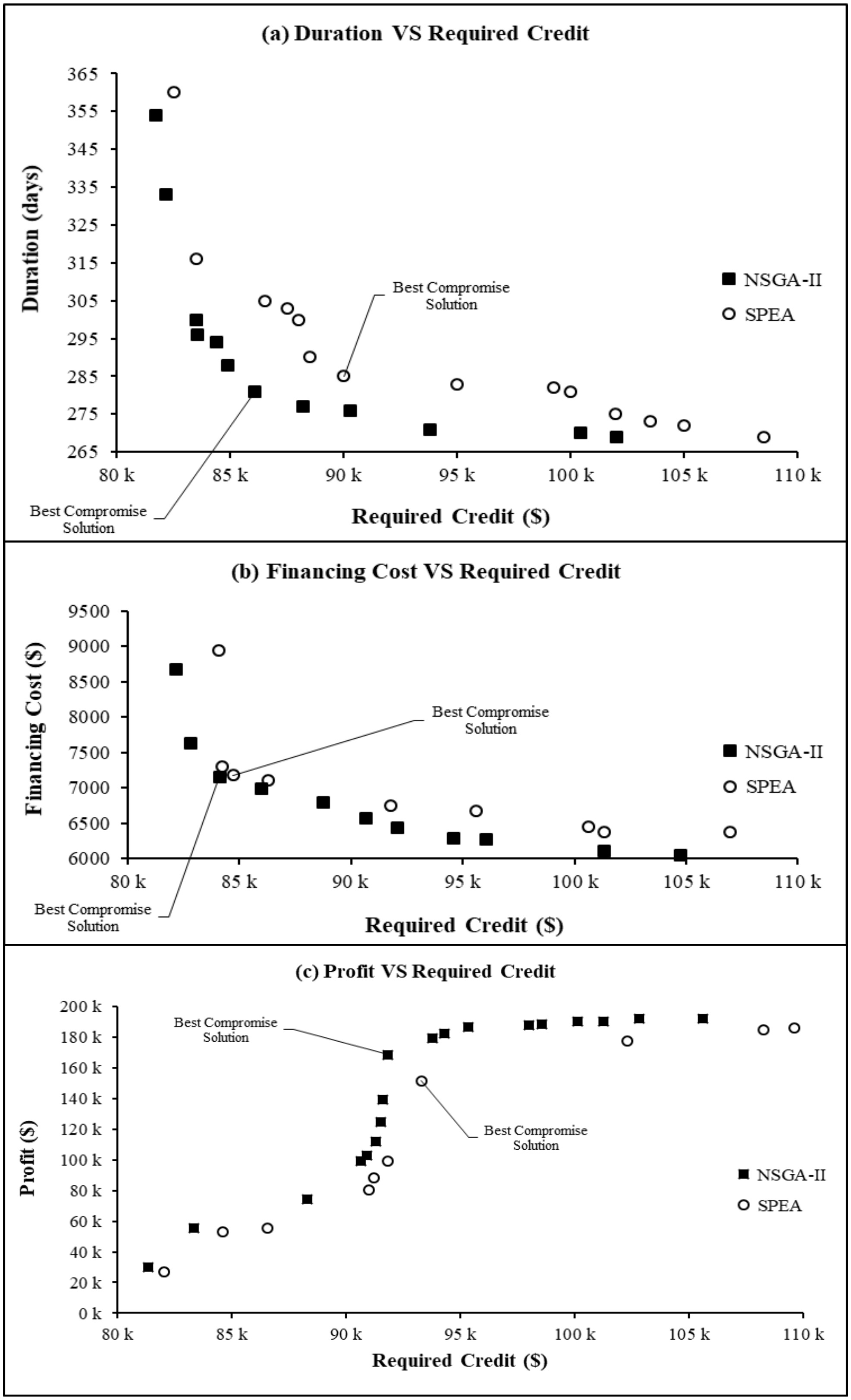

Figure 4: Two-Objective Optimization Pareto Fronts (Case Study 2)

Activity	Predecessor(s)	Daily Direct Cost	Daily Price	Week 1					Week 2					Week 3					Week 4					Week 5					Week 6				
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	-	1000	1501.3			■	■																										
B	-	1200	1801.5	■	■	■																											
C	-	1100	1651.4		■	■																											
D	-	900	1351.2	■	■	■																											
E	A	1250	1876.6					■	■	■																							
F	B,C	1150	1726.5				■	■	■																								
G	C	1050	1576.3										■	■																			
H	C,D	950	1426.2				■	■	■																								
I	D	650	975.8				■	■																									
J	E,F	450	675.6								■	■	■	■	■																		
K	F	350	525.4							■	■	■	■	■																			
L	H	500	750.6							■	■	■	■																				
M	H,I	1450	2176.9											■																			
N	J	400	600.5													■	■	■	■	■													
O	G,K,L	550	825.7												■	■	■	■	■														
P	L	500	750.6												■	■	■	■															
Q	L,M	1350	2026.7												■	■	■																
R	N	600	900.8																		■	■	■	■	■								
S	K,N	850	1276.1																		■	■	■	■	■								
T	O,P	700	1050.9																	■	■	■	■	■	■								
U	P	1200	1801.5																		■	■	■	■									
V	Q	1850	2777.4															■	■	■													
W	R	650	975.8																							■	■	■	■	■			
X	S,T,U	600	900.8																							■	■	■	■	■			
Y	U	1000	1501.3																									■	■				
Direct Cost per Week ($)				17250					10400					15550					15800					10250					3500				
Earned Value per Week ($)				25897.4					15613.5					23345.2					23720.5					15388.3					5254.5				

Figure 5: Best Compromise Solution Schedule of the 25-Activity Project

Activity	Predecessor(s)	Daily Direct Cost	Daily Price	Week 4					Week 5					Week 6					Week 7					Week 8					Week 9					Week 10					Week 11				
				16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
Aa	-	1700	2537.7	■																																							
Ab	Aa	1700	2537.7		■																																						
Ac	Ab	1700	2537.7									■																															
Ad	Ac	1700	2537.7													■																											
Ae	Ad	1700	2537.7																	■																							
Ba	Aa	1500	2239.1			■	■																																				
Bb	Ab,Ba	1500	2239.1					■	■																																		
Bc	Ac,Bb	1500	2239.1											■	■																												
Bd	Ad,Bc	1500	2239.1															■	■																								
Be	Ae,Bd	1500	2239.1																								■	■															
Ca	Aa	1800	2687.0							■	■	■																															
Cb	Ab,Ca	1800	2687.0													■	■	■																									
Cc	Ac,Cb	1800	2687.0																■	■	■																						
Cd	Ad,Cc	1800	2687.0																			■	■	■																			
Ce	Ae,Cd	1800	2687.0																								■	■	■														
Da	Ba,Ca	1900	2836.2													■	■	■	■																								
Db	Bb,Cb,Da	1900	2836.2																	■	■	■	■																				
Dc	Bc,Cc,Db	1900	2836.2																					■	■	■	■																
Dd	Bd,Cd,Dc	1900	2836.2																									■	■	■	■												
De	Be,Ce,Dd	1900	2836.2																													■	■	■	■								
Ea	Da	1600	2388.4																					■	■	■																	
Eb	Db,Ea	1600	2388.4																								■	■	■														
Ec	Dc,Eb	1600	2388.4																											■	■	■											
Ed	Dd,Ec	1600	2388.4																														■	■	■								
Ee	De,Ed	1600	2388.4																																	■	■	■					
Fa	Ea	2000	2985.5																												■	■											
Fb	Eb,Fa	2000	2985.5																														■	■									
Fc	Ec,Fb	2000	2985.5																																■	■							
Fd	Ed,Fc	2000	2985.5																																		■	■					
Fe	Ee,Fd	2000	2985.5																																				■	■			
Direct Cost per Week ($)				7900					8600					17300					21700					25900					25300					21800					4000				
Earned Value per Week ($)				11792.8					12837.8					25824.8					32392.9					38662.5					37766.9					32542.2					5971.0				

Figure 6: Best Compromise Solution Schedule of the 30-Activity Project