

This is the accepted manuscript of the following article: Li, Y., Ba, Y., Ng, C. T., Wu, J., Yuan, W., & Li, J. (2023). A principal-agent model for hazmat transportation in China with risk perception and regulatory policy. *International Journal of Shipping and Transport Logistics*, 17(1-2), 1-20, which has been published in final form at <https://dx.doi.org/10.1504/IJSTL.2023.132646>.

# A Principal-agent Model for Hazmat Transportation in China with Risk Perception and Regulatory Policy

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## Abstract

This paper develops one basic and two improved principal-agent models to analyze the hazmat transportation (HT) under a regulatory policy in China. Compared to the basic principal-agent model, the improved models focus on two important elements of HT: harmful risk and government regulatory policy. Under the contract, the principal is the shipper and the agent is the carrier. First, we study the implications of harmful properties of hazmat transported by the agent on the principal's contract. Second, we consider the regulatory policy with penalty from the government, which can drive the agent to meet the entry threshold of qualifications. Finally, an optimal contract is presented to promote the safety of HT. A good regulatory policy will reach a win-win solution both for the enterprises and the government. We conduct 12 numerical experiments for parameter analysis. The methodology and results provide a quantitative approach with significant insights for decision makers.

**Keywords:** hazmat transportation; principal-agent model; risk; government policy

## 1 Introduction

Focusing on China's economic development, industry is still a contributing factor to China's economic growth. In 2018, the added value of industrial GDP accounted for about 33.9% of the country's total GDP, reaching 3051.6 billion yuan. Moreover, industrial production, oil, natural gas, chemical materials, and other basic products are essential raw materials, which provide a market basis for the HT. Relevant data show that the transportation of hazmat in China's entire industry

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in 2017 had exceeded 1.6 billion tons, an annual increase of more than 10 percent. Among them, road transport accounted for more than 60% of the total transport of hazmat, and more than 30% of the total annual road transport. By 2018, there were more than 210000 vehicles used in hazmat transport and the gross tonnage exceeded 2.2 million tons. With the increase of market demand, the volume of HT is increasing.

However, in recent years, safety accidents of hazmat transport have occurred from time to time, resulting in life and property losses and other serious consequences. Examples are: the rear-end accident of hazmat transport vehicle occurred in Jinji high-speed railway of Shanxi in March 2014, explosion accident of sodium chlorate transport occurred in Zhangshi high-speed railway of Hebei in May 2017, and explosion accident of tank car occurred in Wenling, Zhejiang, in June 2020. The above three accidents caused 75 deaths and 206 injuries, with direct economic losses of more than 150 million yuan. The accident investigation reports show that the above vehicles are all vehicles entrusted to transport hazmat, and all the personnel involved have speculation of different degrees. The government has formulated laws and regulations, such as "some opinions on strengthening the supervision and management of HT safety". However, from the investigation and analysis of major safety accidents in recent years, there are still hidden dangers in the principal-agent relationship of HT. The agent does not pay enough attention to the serious harm of HT, and the government regulators should strengthen the influence on the design of principal-agent contract for HT.

Motivated by this, in this paper, we present an economic method to study the HT. We build one basic and two improved principal-agent models for shippers and carriers of HT and optimize the principal-agent contracts through incentives and penalties. Besides, we consider the risk perception factor and entry threshold factor, numerically analyze the key factors in the participants' decisions, and provide management enlightenment and policy suggestions.

The remainder of this paper is organized as follows. Section 2 is the literature review, which presents three research streams most related to our paper. Section 3 develops the models, which include the basic model and two improved models. Numerical results are provided in Section 4. Section 5 gives our summary.

## 2 Literature Review

Our work is related to three streams of research in the literature. We will give a brief review for each of them.

### 2.1 Hazmat transportation related literature

Risk assessment is an important research field of HT. Different methodologies and techniques were used to evaluate the risks. For example, Zhong et al. (2018) utilized the accident rate and the whole accident consequences to measure the transportation risk. Torretta et al. (2017) used the leak rate of accident and the accident consequences to calculate the risk. Some researchers evaluate the risk from the perspective of a specific loss, such as population and environment. Erkut et al. (1997) dealt with the definition of expected risk of a road by the accident rate of transportation and the exposed populations in the vicinity of the road. Pirmin et al. (2020) presented the risk assessment of multi-modal transportation based on population. Erkut et al. (2000) proposed a model composed of accident rate, exposed population, and risk perception coefficient to measure the risk under a

certain transportation type. Saat et al. (2014) considered the risk of the soil, underground water, and exposed population to evaluate the accident consequences of the hazmat railway transportation. Other examples can be found in Abkowitz et al. (1992), Wan et al. (2019), Alghanmi et al. (2020), Lun et al. (2020), Yang et al. (2021). In the above, exposed population is a commonly used risk index in risk assessment of HT. Combined with risk perception coefficient, exposed population can be used to build the primary risk index as risk perception indicates the risk preference of the HT carrier. This paper will provide a new HT principal-agent model by embedding risk perception.

## 2.2 Principal-agent theory and applications

Principal-agent theory is applied to two (or more) parties where one, designated as the agent, who acts for, on behalf of, or as a representative for the others; and one, designated as the principal, in a particular domain of decision problems (Ross, 1973). Since its birth, the principal-agent theory has attracted much attention and has been widely used in various fields (Sun et al., 2018; Tsionas et al., 2020; Wang et al., 2020). Laffont and Martimort (2002) gave an excellent summary of the principal-agent theory in industrial organizations, labor economics, and behavioral economics. In addition, the principal-agent theory is also extensively applied in management science. Iyer et al. (2005) developed a principal-agent model for product specification and production of an automobile manufacturer. Cai and Singham (2017) studied the implications of uncertainty in agents demands on the principals contracts in Carbon capture and storage, where the demands are heterogeneously distributed. Schosser (2019) established a new risk allocation scheme that both the principal and agent are risk-averse to obtain consistent incentives for the firm and the manager. Chu et al. (2020) established an optimal incentive contract with multiple express companies and a common agency to ensure that the principals (express companies) have no motivation to take hidden actions to influence the agency's workload division. Yang et al. (2021) considered the unique characteristics of the ocean freight transportation and developed optimal contract under two scenarios. In summary, principal-agent theory has been used in both economics and management science. However, the research objects of the current transportation management are most general cargo, rarely hazmat. This paper focuses on the hazmat which has serious risk, and uses the principal-agent models to develop the contract and analyze their behaviors.

## 2.3 Policy effective and game theory

Game theory, such as Nash equilibrium and Stackelberg game, has been applied to HT and regulation, safety investment decisions of chemical enterprises, cargo transportation, and other fields. Reilly et al. (2011) developed a three-player game involving a government agency, a carrier, and a terrorist to determine when and where specific facilities should be restricted. Mohri et al. (2020) proposed a bi-objective Stackelberg game to minimize the overall risk in hazmat routing and scheduling. Li et al. (2016) noticed the problem of sporadic hazmat express and studied a three-party game model. Besides, Wu et al. (2020) addressed the investment issue of chemical cluster for domino accidents using an N-player game. Talarico et al. (2015) presented a decision model by game theory to allocate security resources for preventing intentional acts on transportation infrastructure within a chemical supply chain. Song et al. (2019) applied Nash game to analyze how two competing ocean shipping companies integrate downstream transportation services in multi-modal transportation. Moreover, some researchers studied the principal-agent issues between the government and enterprises. For

example, Liang et al. (2019) provided an agent-based model for policy making on energy-efficiency retrofit policies to improve the effectiveness of the incentive policies. Based on the above research, the government regulation is an important method to control the risk; and methods such as spot check, award, or punishment are the general regulatory strategies. But, the regulatory mechanism combined with transportation qualification of hazmat transport enterprises has not been studied in the previous models. This paper designs and introduces the government regulatory mechanism based on an entry threshold, constructs the principal-agent model of HT under government regulation, and discusses the influence of main parameters in the regulation mechanism of the system.

From the above, we find that both technology and policy factors in HT are hot and attractive. The gaps are listed as follows. (1) The classical principal-agent model used in the transportation field does not consider the harmful property of the shipping goods (e.g., hazmat). However, the risk cost of the agent is closely related to the hazmat. (2) How do the conflict-involved players react under the regulatory policy, such as the entry threshold? (3) Most research on policy study is qualitative analysis, not from the economic perspective with quantitative analysis. In summary, the risk cost will be reconsidered in the principal-agent model, and the regulatory policy, such as the entry threshold, will be added to the model and influence the results. All are significant for HT management. In addition, in 2018, we investigated more than 40% of chemical enterprises in Beijing, China, and confirmed that there is such a problem in reality.

Motivated by these, in this paper, we present an economic method to study the HT. We formulate the model by the principal-agent theory, and present the optimal contract under the hazmat risk and regulatory policy, respectively.

### 3 Models

In this section, we apply the principal-agent theory on HT problem and focus on the relationship between players of the hazmat carrier and the hazmat shipper. We study it from an economic standpoint and analyze it by different mathematical models.

#### 3.1 Basic setting

##### (1) The agent

The agent is the hazmat carrier (e.g., HT enterprises) who will sign transportation contracts with the principal and needs to have the qualification of HT aiming to ensure the safety and punctuality of the transportation process.

##### (2) The principal

The principal is the hazmat shipper (e.g., hazmat manufacturers) who delegates the agent to transport hazardous materials and pays for the transportation.

##### (3) Assumptions

In order to clarify the key problems in the principal-agent of HT, the following assumptions are put forward. The players in the contract are completely rational and the goal of their decision-making is to maximize the profit. The principal is risk neutral, but the penalty from the regulatory

policy of the government must be accepted. The agent is risk averse and considers the risk cost. The government's regulatory measures must be applied in the game as exogenous variables.

(4) Notations

$\lambda$ : output coefficient of the agent's effort, where  $\lambda > 0$ .

$\xi$ : exogenous random variable, which follows a normal distribution, i.e.,  $\xi \sim N(0, \sigma^2)$ , where  $\sigma$  is a constant.

$\theta$ : fixed income of the agent in the contract.

$b_1$ : effort cost coefficient of the agent. The bigger  $b_1$ , the lower the level of transportation technology of the agent, where  $b_1 > 0$ .

$\omega$ : the minimum expected income of the agent, which is a constant.

$\rho$ : the risk aversion of the agent,  $\rho > 0$ .

$p_t$ : the accident rate of HT,  $0 < p_t < 1$ .

$pop$ : the exposed population near the HT accidents segment,  $pop > 0$ .

$\alpha$ : the preference factor for risk of the agent,  $\alpha > 0$ .

$\pi$ : the output function of the agent.

$C(e_1)$ : the cost of the agent's effort.

$C_{con}$ : the contract price.

$C'$ : the risk cost of the agent.

$C_r$ : the risk cost of the agent under the improved model.

$U_0$ : the principal's expected profit.

$U_1$ : the agent's expected profit.

Decision variables

$\beta$ : incentive coefficient, where  $\beta > 0$ , decided by the principal.

$e_1$ : the effort of the agent for HT, where  $e_1 > 0$ , decided by the agent.

### 3.2 Model I: basic principal-agent model

Similar to the basic principal-agent model (Ross, 1973; Laffont and Martimort, 2002; Makris, 2003), the output of the agent is related to its effort and the output coefficient, so  $\pi$  can be formulated as  $\pi = \lambda e_1 + \xi$ . Based on the contract, the principal cooperates with the agent, where the contract price is modeled as  $C_{con} = \theta + \beta\pi$ . The cost of agent's effort is  $C(e_1) = \frac{1}{2}b_1e_1^2$ . According to the Arrow-Pratt theory (Hakansson and Arrow, 1972), the risk cost of the agent can be denoted as  $C' = \frac{1}{2}\rho Var(C_{con})$ . The utility function of the principal equals the output of the agent minus the contract price, so the principal's expected profit  $U_0 = E(\pi - C_{con})$ , which can also be expressed as  $U_0 = \lambda(1 - \beta)e_1 - \theta$ . The utility function of the agent equals the contract price minus effort cost and risk cost, and the agent's expected profit  $U_1 = E(C_{con} - C(e_1) - C')$ , which can be reorganized as  $U_1 = \theta + \beta\lambda e_1 - \frac{1}{2}b_1e_1^2 - \frac{1}{2}\rho\beta^2\sigma^2$ . Whether the agent will accept the contract depends on its minimum expected income  $\omega$ . In other words, if  $\theta + \beta\lambda e_1 - \frac{1}{2}b_1e_1^2 - \frac{1}{2}\rho\beta^2\sigma^2 \geq \omega$ , the agent will accept the contract; otherwise, it will not accept the contract.

**Theorem 3.1** *The optimal contract design of the basic principal-agent model is  $(e_1^*, \beta^*)$ , where  $e_1^* = \frac{\lambda^3}{\lambda^2 b_1 + b_1^2 \rho \sigma^2}$  and  $\beta^* = \frac{\lambda^2}{\lambda^2 + b_1 \rho \sigma^2}$ .*

**Proof.** The objective function of the principal is given in (1) and the participation constraint and incentive constraint are given in (2) and (3), respectively.

$$\max_{\beta} \lambda(1 - \beta)e_1 - \theta \quad (1)$$

$$\text{s.t. } \theta + \beta\lambda e_1 - \frac{1}{2}b_1e_1^2 - \frac{1}{2}\rho\beta^2\sigma^2 \geq \omega \quad (2)$$

$$e_1 \in \arg \max_{e_1} \theta + \beta\lambda e_1 - \frac{1}{2}b_1e_1^2 - \frac{1}{2}\rho\beta^2\sigma^2 \quad (3)$$

where (2) is the participation constraint and (3) is the incentive constraint.

By solving the optimization model (1)-(3), we can get the solution as follows:

$$\beta^* = \frac{\lambda^2}{\lambda^2 + b_1\rho\sigma^2} \quad (4)$$

$$e_1^* = \frac{\lambda^3}{\lambda^2b_1 + b_1^2\rho\sigma^2} \quad (5)$$

### 3.3 Model II: improved basic model for hazmat transportation (HT principal-agent model)

Considering the harmful properties of hazmat, the agent should think about the accident rate and the special consequences of HT when calculating the risk cost. As with the risk perception in Erkut et al. (2007), let  $R_t = p_t \text{pop}^\alpha$  to help measure the risk cost. Hence, this paper improves the risk cost of the basic model by including the special risks of transporting goods. The mathematical formula of the new risk cost is shown as follows:

$$C_r = \frac{1}{2}R_t \text{Var}(C_{con}) = \frac{1}{2}p_t \text{pop}^\alpha \text{Var}(C_{con}) \quad (6)$$

So the expected profit of the agent  $U_1 = E(C_{con} - C(e_1) - C_r)$  can be rewritten as  $U_1 = \theta + \beta\lambda e_1 - \frac{1}{2}b_1e_1^2 - \frac{1}{2}p_t \text{pop}^\alpha \beta^2 \sigma^2$ .

**Theorem 3.2** *The optimal contract design of the HT principal-agent model is  $(e_1^{**}, \beta^{**})$ , where  $e_1^{**} = \frac{\lambda^3}{\lambda^2b_1 + b_1^2p_t \text{pop}^\alpha \sigma^2}$  and  $\beta^{**} = \frac{\lambda^2}{\lambda^2 + b_1p_t \text{pop}^\alpha \sigma^2}$ .*

**Proof.** The objective function, the participation constraint, and incentive constraint of the HT principal-agent model are as follows:

$$\max_{\beta} \lambda(1 - \beta)e_1 - \theta \quad (7)$$

$$\text{s.t. } \theta + \beta\lambda e_1 - \frac{1}{2}b_1e_1^2 - \frac{1}{2}p_t \text{pop}^\alpha \beta^2 \sigma^2 \geq \omega \quad (8)$$

$$e_1 \in \arg \max_{e_1} \theta + \beta\lambda e_1 - \frac{1}{2}b_1e_1^2 - \frac{1}{2}p_t \text{pop}^\alpha \beta^2 \sigma^2 \quad (9)$$

Similar to the proof of Theorem 3.1, by solving the optimization model (7)-(9), we can get the optimal solution as follows.

$$e_1^{**} = \frac{\lambda^3}{\lambda^2 b_1 + b_1^2 p_t \text{pop}^\alpha \sigma^2} \quad (10)$$

$$\beta^{**} = \frac{\lambda^2}{\lambda^2 + b_1 p_t \text{pop}^\alpha \sigma^2} \quad (11)$$

By Theorem 3.2, we can obtain Corollaries 1 and 2, with proofs given in the Appendix.

**Corollary 1** *With the other conditions unchanged, in the HT principal-agent model, the optimal effort of agent  $e_1^{**}$  (i) increases in output coefficient  $\lambda$ , (ii) decreases in effort cost coefficient  $b_1$ , accident rate  $p_t$ , exposed population  $\text{pop}$ , and extrinsic factor  $\xi$ , and (iii) increases at first then decreases in risk preference  $\alpha$ .*

**Corollary 2** *With the other conditions unchanged, in the HT principal-agent model, the optimal incentive coefficient  $\beta^{**}$  (i) increases in output coefficient  $\lambda$ , (ii) decreases in effort cost coefficient  $b_1$ , accident rate  $p_t$ , exposed population  $\text{pop}$ , and extrinsic factor  $\xi$ , and (iii) increases at first then decreases in risk preference  $\alpha$ .*

The results of the basic principal-agent model and the HT principal-agent model are compared on risk cost, effort, and incentive coefficient in Table 1. It is evident that, in the basic model, the above three variables are only related to the output coefficient, effort cost coefficient, risk aversion, and extrinsic factor, which does not display the relationship between hazmat and risk cost. While in the HT principal-agent model, the risk cost is the function of accident rate and exposed population, which means the two factors are most related to risk cost, and they will influence the effort and incentive coefficient.

Hence, the HT principal-agent model can well reveal the different between HT and general cargo transportation, so that the principal and agent will negotiate the contract considering accident rate and risk consequences, which is more closed to the actual activity and good for society safety.

Table 1 The results of the basic model and the HT principal-agent model

	Basic model	HT principal-agent model
Risk cost	$C' = \frac{\rho \sigma^2 \lambda^4}{2(\lambda^2 + b_1 \rho \sigma^2)^2}$	$C_r = \frac{p_t \text{pop}^\alpha \sigma^2 \lambda^4}{2(\lambda^2 + b_1 p_t \text{pop}^\alpha \sigma^2)^2}$
Effort	$e_1^* = \frac{\lambda^3}{\lambda^2 b_1 + b_1^2 \rho \sigma^2}$	$e_1^{**} = \frac{\lambda^3}{\lambda^2 b_1 + b_1^2 p_t \text{pop}^\alpha \sigma^2}$
Incentive coefficient	$\beta^* = \frac{\lambda^2}{\lambda^2 + b_1 \rho \sigma^2}$	$\beta^{**} = \frac{\lambda^2}{\lambda^2 + b_1 p_t \text{pop}^\alpha \sigma^2}$

### 3.4 Model III: HT principal-agent model under regulatory policy

For the regulatory policy, let  $e_b$  denote the entry threshold stipulated by the government to HT enterprises, where the entry threshold refers to transportation qualification and comprehensive requirements such as the facilities, equipment, and personal operation.  $p$  refers to the probability

of spot check by the government. Let  $k$  be the penalty coefficient.  $C_r'$  is defined as the risk cost of the agent under regulatory policy.  $C_{pun}$  refers to the penalty function, when the spot check result from the government to the agent is unqualified. In fact, if the agent's effort is greater than or equal to the entry threshold, the spot check result is qualified and there is no need for the penalty. It is the same as the HT principal-agent model proposed in Section 3.3, which will not be discussed below. However, when the agent's effort is lower than the entry threshold, the spot check result is unqualified and both the principal and agent should be penalized. The penalty is related to the penalty coefficient, entry threshold and agents effort, and the formula is as follows.

$$C_{pun} = k(e_b - e_1)^2, \quad e_1 < e_b \quad (12)$$

The utility function of the principal becomes the output of the agent minus the contract price and the penalty, so the principal's expected profit changes to  $U_0 = E(\pi - C_{con} - pC_{pun})$ , which can also be expressed as  $U_0 = \lambda(1 - \beta)e_1 - \theta - pk(e_b - e_1)^2$ . The utility function of the agent becomes the contract price minus effort cost, risk cost and penalty, and agents expected profit changes to  $U_1 = E(C_{con} - C(e_1) - C_r' - pC_{pun})$ , which can be rewritten as  $U_1 = \theta + \beta\lambda e_1 - \frac{1}{2}b_1 e_1^2 - \frac{1}{2}p_t pop^\alpha \beta^2 \sigma^2 - pk(e_b - e_1)^2$ .

**Theorem 3.3** *The optimal contract design of the HT principal-agent under regulatory policy is  $(e_1^{***}, \beta^{***})$ , where  $e_1^{***} = \frac{(b_1 + 2pk)\lambda^3 + 2pk e_b b_1 \lambda^2}{(p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k \lambda^2 p)(b_1 + 2pk)} + \frac{2pk e_b}{b_1 + 2pk}$  and  $\beta^{***} = \frac{(b_1 + 2pk)\lambda^2 + 2pk \lambda e_b b_1}{p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k \lambda^2 p}$ .*

**Proof.** Considering the regulatory policy based on the entry threshold of the government, the objective function, the participation constraint, and incentive constraint of HT principal-agent are as follows:

$$\max_{\beta} \lambda(1 - \beta)e_1 - \theta - pk(e_b - e_1)^2 \quad (13)$$

$$\text{s.t.} \quad \theta + \beta\lambda e_1 - \frac{1}{2}b_1 e_1^2 - \frac{1}{2}p_t pop^\alpha \beta^2 \sigma^2 - pk(e_b - e_1)^2 \geq \omega \quad (14)$$

$$e_1 \in \arg \max_{e_1} \theta + \beta\lambda e_1 - \frac{1}{2}b_1 e_1^2 - \frac{1}{2}p_t pop^\alpha \beta^2 \sigma^2 - pk(e_b - e_1)^2 \quad (15)$$

Similar to the proof of Theorem 3.1, by solving the optimization model (13)-(15), we can get the solution as follows:

$$e_1^{***} = \frac{\beta\lambda + 2pk e_b}{b_1 + 2pk} \quad (16)$$

$$\beta^{***} = \frac{(b_1 + 2pk)\lambda^2 + 2pk \lambda e_b b_1}{p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k \lambda^2 p} \quad (17)$$

$$e_1^{***} = \frac{(b_1 + 2pk)\lambda^3 + 2pk e_b b_1 \lambda^2}{(p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k \lambda^2 p)(b_1 + 2pk)} + \frac{2pk e_b}{b_1 + 2pk} \quad (18)$$

By Theorem 3.3, we can obtain Corollaries 3 and 4, with proofs given in the Appendix.



**Corollary 3** *In the HT principal-agent model under regulatory policy, when  $e_b b_1 > \beta \lambda$ , the agent's effort  $e_1$  increases in the probability of spot check  $p$  and penalty coefficient  $k$ ; when  $e_b b_1 < \beta \lambda$ , the agent's effort  $e_1$  decreases in the probability of spot check  $p$  and penalty coefficient  $k$ ; and when  $e_b b_1 = \beta \lambda$ , the agent gets to the optimal effort  $e_1^{***}$ .*

**Corollary 4** *With the other conditions unchanged, in the HT principal-agent model under regulatory policy, the optimal effort  $e_1^{***}$  and the optimal incentive coefficient  $\beta^{***}$  increase in the entry threshold set by the government  $e_b$ .*

In terms of the HT principal-agent model under regulatory policy, by considering the government penalty, the agent's effort depends on the incentive coefficient, entry threshold, probability of spot check, and penalty coefficient. Here, the agent's effort increases with the increase of entry threshold and incentive level of the agent. The incentive coefficient is related to the output coefficient, level of transportation technology, risk perception, and entry threshold. When the accident rate and the exposed population increase, the risk perception becomes higher too, even though the incentive level increases. However, the agent would not like to accept the contract.

The results of the HT principal-agent model without regulatory policy and the model under regulatory policy are compared on risk cost, effort, and incentive coefficient in Table 2. It is evident that adding the regulatory policy, such as government penalty, makes the risk cost, effort, and incentive coefficient being affected by the entry threshold, probability of spot check, and penalty coefficient. In other words, under these conditions, the principal and agent should think about the regulatory policy carefully so that they can be away from the penalty when designing the contract.

Table 2 The results of the HT principal-agent model and that under regulatory policy

	HT principal-agent model without regulatory policy	HT principal-agent model under regulatory policy
Risk cost	$C_r = \frac{p_t pop^\alpha \sigma^2 \lambda^4}{2(\lambda^2 + b_1 p_t pop^\alpha \sigma^2)^2}$	$C_r' = \frac{p_t pop^\alpha \sigma^2 ((b_1 + 2pk)\lambda^2 + 2pk\lambda e_b b_1)^2}{2(p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k\lambda^2 p)^2}$
Effort	$e_1^{**} = \frac{\lambda^3}{\lambda^2 b_1 + b_1^2 p_t pop^\alpha \sigma^2}$	$e_1^{***} = \frac{(b_1 + 2pk)\lambda^3 + 2pk e_b b_1 \lambda^2}{(p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k\lambda^2 p)(b_1 + 2pk)} + \frac{2pk e_b}{b_1 + 2pk}$
Incentive coefficient	$\beta^{**} = \frac{\lambda^2}{\lambda^2 + b_1 p_t pop^\alpha \sigma^2}$	$\beta^{***} = \frac{(b_1 + 2pk)\lambda^2 + 2pk\lambda e_b b_1}{p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k\lambda^2 p}$

## 4 Sensitivity analysis

Because the equilibrium solution of the optimal contract of HT principal-agent model under regulatory policy is very complicated, it is difficult to analyze the characteristics of the analysis formulas. Therefore, this section conducts 12 numerical experiments to display the impact of the three factors in government regulatory policy, i.e., entry threshold, probability of spot check, and penalty coefficient, on the agent's effort, incentive coefficient, and expected profits of players. As with the reference Erkut et al.(2007), let  $p_t = 1 \times 10^{-5}$ ,  $\alpha = 2$ . We use the population unit square kilometer near the HT accident segment as the exposed population. According to the average population density of the first-tier cities, let  $pop = 1000$ , and the value of other parameters is given in the tables below.

#### 4.1 The influence of entry threshold under different probabilities of spot check

With different probabilities of spot check, we explore the impact of entry threshold  $e_b$  set by the government on the agent's effort  $e_1$ , incentive coefficient  $\beta$ , expected profit of agent  $U_1$ , and the expected profit of principal  $U_0$ . Let the high, medium, and low probabilities of spot check be 0.8, 0.5, and 0.2, respectively. The other related parameters are given in Table 3. Through the numerical experiments, the results of the influence of entry threshold on the agent's effort and incentive coefficient are shown in Fig. 1, and that on the expected profit of agent and expected profit of principal are shown in Fig. 2.

Table 3 Other parameters for sensitivity analysis of entry threshold

Notations	$\lambda$	$b_1$	$k$	$\sigma$	$Q$	$\omega$
Value	1000	3	5	10	1000	2000

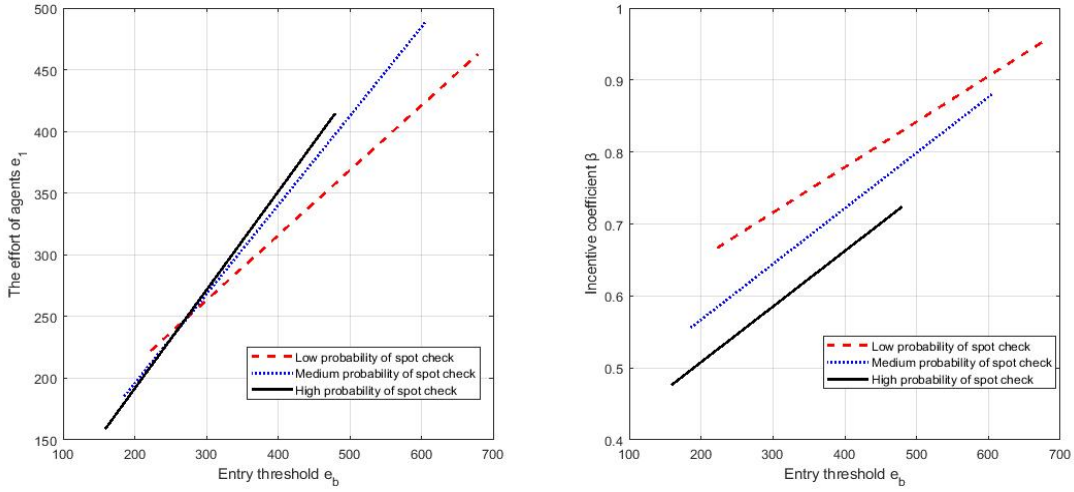


Figure 1: The influence of entry threshold on the agent's effort and incentive coefficient

Under the current conditions, Fig. 1 (left) shows that no matter what the level of probability of spot check is, the agent's effort increases in the entry threshold. When  $e_b \in (272, 274)$ , the agent's effort hardly changes with the probability of spot check. When  $e_b > 274$ , the agent's effort increases in the probability of spot check; while  $e_b < 272$ , it decreases in the probability of spot check. As for Fig. 1 (right), overall, the incentive coefficient grows with the entry threshold. However, when the entry threshold is fixed, the incentive coefficient under the higher probability of spot check is lower than that under the lower probability of spot check. It is because when the agent's effort cannot meet the entry threshold of the government but the probability of spot check is at a high level, the principal is likely to be punished. In addition, to maintain its profit, it will reduce the contract price and decrease the incentive coefficient either.

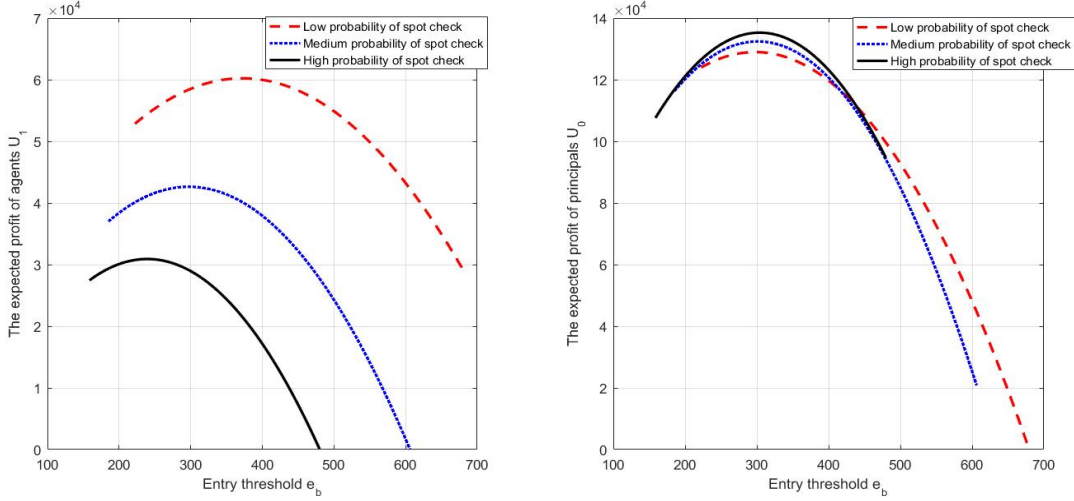


Figure 2: The influence of entry threshold on the expected profits of agent and principal

As for Fig. 2 (left), under the same conditions, the expected profit of the agent under the higher probability of spot check is smaller than that under the lower probability of spot check. With different probabilities of spot check, the expected profit of agent increases at first then decreases with the growing of entry threshold. In Fig. 2 (right), the results show that the influence of the probability of spot check on the principal's expected profit is less than that on the agent's expected profit. For any probability of spot check, the principal's expected profit increases at first then decreases. Therefore, when the parameters are fixed, improving the entry threshold appropriately, the incentive coefficient will rise, which will drive the agent to improve the effort, so that the expected profits of the two players will both grow. But, an excessive entry threshold will reduce the expected profit of the two players, and affect the economic activity of the HT industry.

#### 4.2 The influence of penalty coefficient under different entry thresholds

With different entry thresholds set by the government, we explore the impact of penalty coefficient  $k$  on the agent's effort  $e_1$ , incentive coefficient  $\beta$ , expected profit of agent  $U_1$ , and the expected profit of principal  $U_0$ . Let the high, medium, and low entry thresholds be 350, 250, and 150, respectively. The other related parameters are given in Table 4. After the numerical experiments, the results of the influence of penalty coefficient on the agent's effort and incentive coefficient are shown in Fig. 3, and that on the expected profit of the agent and the principal are shown in Fig. 4.

Table 4 Other parameters for sensitivity analysis of penalty coefficient

Notations	$\lambda$	$b_1$	$p$	$\sigma$	$Q$	$\omega$
Value	1000	3	0.5	10	1000	2000

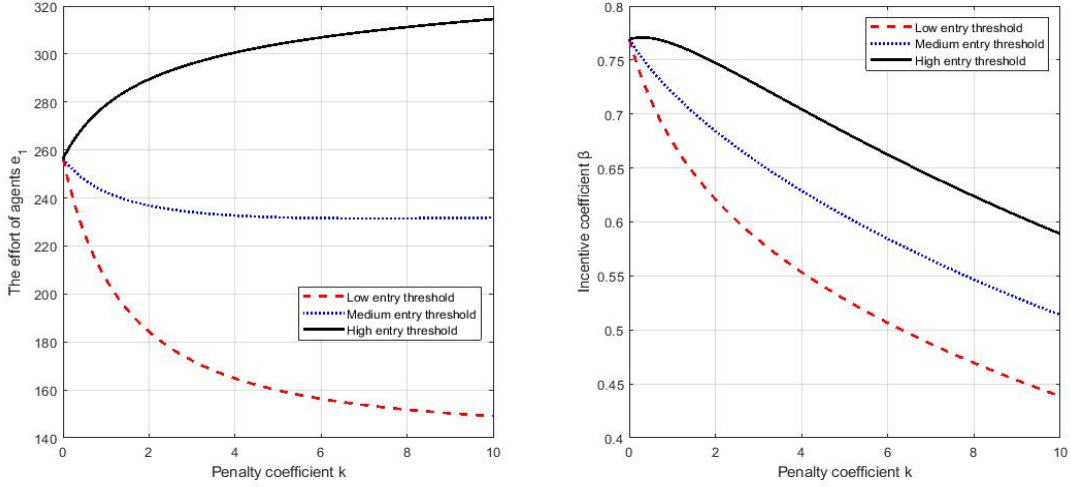


Figure 3: The influence of penalty coefficient on the agent's effort and incentive coefficient

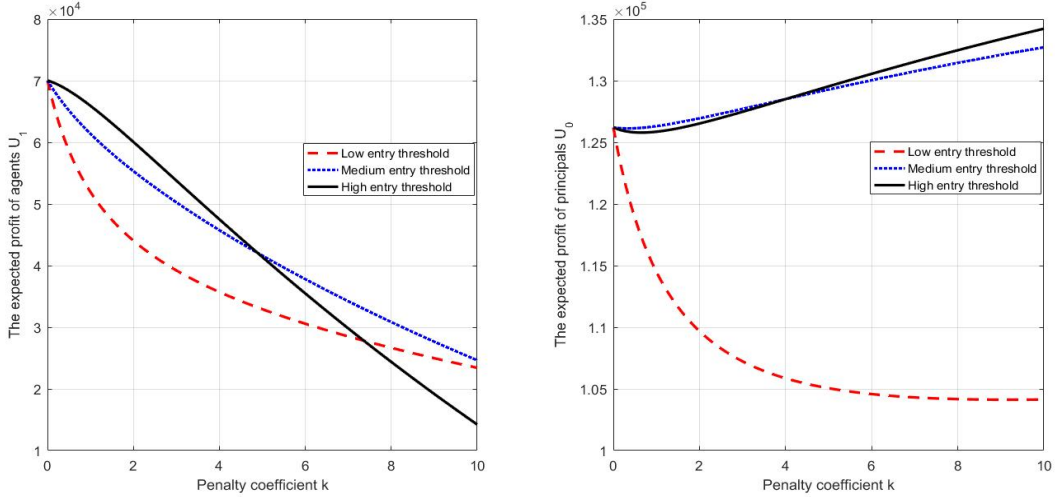


Figure 4: The influence of penalty coefficient on the expected profit of agents and principals

We can estimate from Fig. 3 (left) that, without other parameters changing, when the government sets medium and low entry thresholds, the agent's effort decreases in the penalty coefficient. But the agent's effort rises by the increase of the penalty coefficient under high entry threshold. When the penalty coefficient is given, the agent's effort under the higher entry threshold is greater than that under the lower entry threshold. The reason is that the penalty from the government to the unqualified agent is proportional to the square of the difference between the agent's effort and entry threshold. In Fig. 3 (right), under different entry thresholds, it is evident that the incentive coefficient decreases in the penalty coefficient. Because when the agent's effort is less than the entry threshold, the greater the penalty coefficient is, the more the penalty to the two players will be, and in order to maintain its expected profit, the principal will reduce the contract price, so the incentive coefficient

will decrease too. When the penalty coefficient is unchanged, raising the entry threshold will improve the incentive coefficient, which is an effective method to reduce the penalty.

It can be seen from Fig. 4 (left) that with the increase of penalty coefficient, the expected profit of the agent decreases no matter how high or low the entry threshold is, in which the speed of decrease under the high entry threshold is the fastest. As the growing of penalty coefficient in Fig. 4 (right), the expected profit of the principal increases gradually under the high and medium entry thresholds, while decreases significantly under the low entry threshold. Therefore, when the threshold is low, reducing the penalty coefficient will improve the agent's effort, the incentive coefficient and the expected profit of both players.

### 4.3 The influence of the probability of spot check under different levels of transportation technology

With different levels of transportation technology, we explore the impact of the probability of spot check  $p$  on the agent's effort  $e_1$ , incentive coefficient  $\beta$ , expected profit of agent  $U_1$ , and expected profit of principal  $U_0$ . Let the high, medium, and low levels of transportation technology be 0.8, 2, and 4, respectively. The other related parameters are given in Table 5. Through the numerical experiments, the results of the influence of the probability of spot check on the agent's effort and incentive coefficient are shown in Fig. 5, and that on the expected profit of agent and expected profit of principal are shown in Fig. 6.

Table 5 Other parameters for sensitivity analysis of the probability of spot check

Notations	$\lambda$	$k$	$e_b$	$\sigma$	$Q$	$\omega$
Value	1000	5	280	10	1000	2000

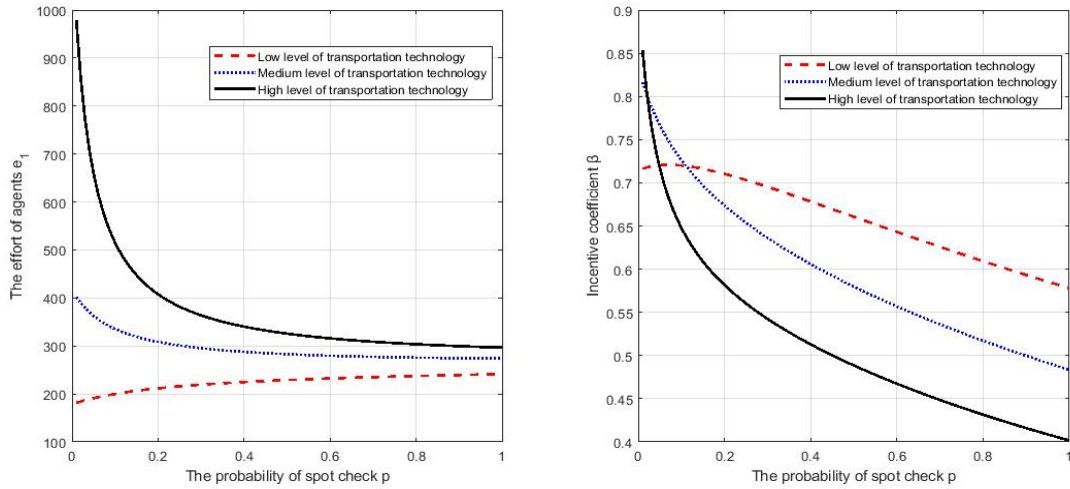


Figure 5: The influence of the probability of spot check on the agent's effort and incentive coefficient

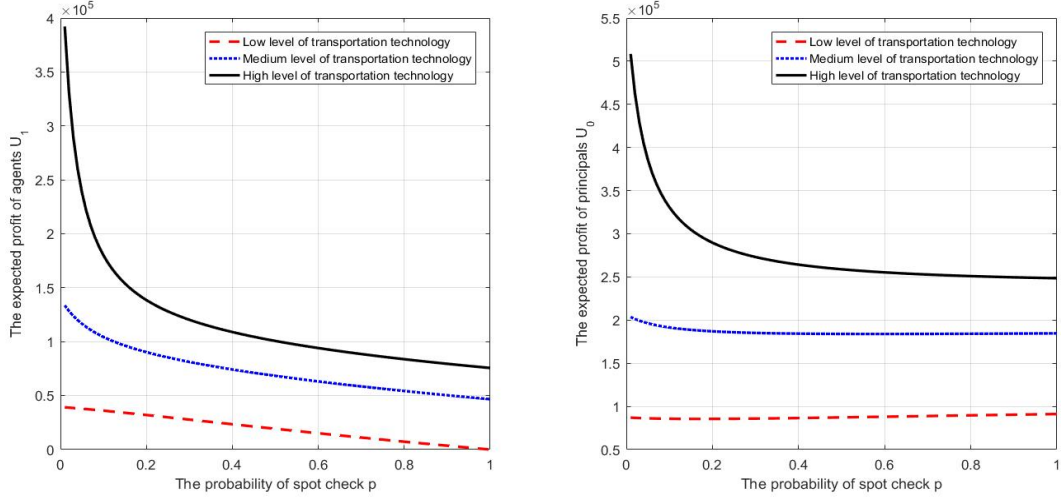


Figure 6: The influence of the probability of spot check on the expected profits of agent and principal

The results in Fig. 5 (left) show that, under the given parameters, either the level of transportation technology of the agent is medium or high, the agent's effort decreases in the probability of spot check. Given a fixed probability, the higher the level of transportation technology of the agent is, the greater the agent's effort will be. It can be seen from Fig. 5 (right), when the probability of spot check  $p < 0.02$ , the agent with high level of transportation technology will attain a higher incentive coefficient. By comparison, as the probability increases, the incentive coefficient from principal to the agent who has medium or low level of transportation will decrease.

In Fig. 6 (left), under the same probability of spot check, the expected profit of the agent under the higher level of agent's transportation technology is greater than that under the lower level of agent's transportation technology. With the increase of the probability of spot check, the expected profit of agent decreases no matter what the level of transportation technology is, in which the expected profits of high-tech agent is significantly affected. Fig. 6 (right) shows that the expected profit of the principal under the high level of the agent's transportation technology is bigger than that under the low level of the agent's transportation technology. As the probability increases, under the high level of the agent's transportation technology, the expected profit of the principal decreases rapidly and tends to be stable. Meanwhile, the effect of the principal who cooperates with the agent with medium and low technical level is less. Therefore, it can improve the probability of spot check of medium and low-tech agent, which has little impact on the expected profits of the principal and agent but strengthens the safety of HT.

## 5 Conclusions

This paper focuses on the principal-agent theory on hazmat transportation. First, combined with the characteristics of hazmat risk, we propose the HT principal-agent model by embedding risk perception. Through the comparison of the results, we notice the influence of harmful properties of hazmat on primary equilibrium solution. Then, we provide the HT principal-agent model under regulatory policy which is based on the entry threshold set by government. Finally, we conduct 12

numerical experiments to analyze the sensitivity analysis of the important parameters and discuss the effect of the main factors constituting the government's supervision mechanism on the agent's effort, incentive coefficient and expected profits of the principal and agent in detail.

The main conclusions are as follows: First, under the given conditions, the increase of the entry threshold will increase the agent's effort, but the change of the probability of spot check under the same conditions has no obvious effect on the agent's effort; the incentive coefficient will grow with the increase of the entry threshold, while the increase of the probability of spot check under the same parameters will cut down the incentive coefficient. Either the expected profit of the agent or principal first increases and then decreases in the entry threshold. Second, with the given conditions, the rise of penalty coefficient has different effects on the agent's effort with different government entry thresholds. When the entry threshold is of high level, the agent's effort grows with the increase of penalty coefficient. Under the medium and high entry thresholds, the principal's expected profit increases in the penalty coefficient; while under the low entry threshold, it decreases. Third, under given conditions, if government improves the probability of spot check, it will make the effort of low-tech agent increases slowly, but the effort of high-tech agent decreases rapidly; meanwhile, the expected profit of high-tech agent decreases significantly, and the expected profit of the corresponding principal decreases. Therefore, the government should make regulatory policy reasonably to promote the player with suitable effort to improve the safety of hazmat transportation.

## Acknowledgments

This research is supported by National Natural Science Foundation of China (Grant No. 71571010), Beijing Social Science Foundation (No. 17GLB014), Funds for First-Class Discipline Construction (XK1802-5), and BUCT(G-JD202002).

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## Appendix

### Proof for Corollary 1

Based on Eq. (12), the first derivative of the optimal agent's effort to parameters  $\lambda$ ,  $b_1$ ,  $p_t$ ,  $pop$ ,  $\xi$ ,  $\alpha$  are as follows:

$$\frac{\partial e_1^{**}}{\partial \lambda} = \frac{\lambda^4 b_1 + 3\lambda^2 b_1^2 p_t pop^\alpha \sigma^2}{(\lambda^2 b_1 + b_1^2 p_t pop^\alpha \sigma^2)^2} > 0, \quad \frac{\partial e_1^{**}}{\partial b_1} = -\frac{\lambda^3 (\lambda^2 + 2b_1 p_t pop^\alpha \sigma^2)}{(\lambda^2 b_1 + b_1^2 p_t pop^\alpha \sigma^2)^2} < 0, \quad \frac{\partial e_1^{**}}{\partial p_t} = -\frac{\lambda^3 b_1^2 pop^\alpha \sigma^2}{(\lambda^2 b_1 + b_1^2 p_t pop^\alpha \sigma^2)^2} < 0,$$

$$\frac{\partial e_1^{**}}{\partial pop} = -\frac{\lambda^3 b_1^2 p_t \alpha pop^{\alpha-1} \sigma^2}{(\lambda^2 b_1 + b_1^2 p_t pop^\alpha \sigma^2)^2} < 0, \quad \frac{\partial e_1^{**}}{\partial \sigma^2} = -\frac{\lambda^3 b_1^2 p_t pop^\alpha}{(\lambda^2 b_1 + b_1^2 p_t pop^\alpha \sigma^2)^2} < 0; \quad \frac{\partial e_1^{**}}{\partial \alpha} = -\frac{\lambda^3 b_1^2 p_t pop^\alpha \sigma^2 \ln \alpha}{(\lambda^2 b_1 + b_1^2 p_t pop^\alpha \sigma^2)^2}.$$

when  $0 < \alpha < 1$ ,  $\frac{\partial e_1^{**}}{\partial \alpha} > 0$ ; and when  $\alpha > 1$ ,  $\frac{\partial e_1^{**}}{\partial \alpha} < 0$ .

### Proof for Corollary 2

Based on Eq.(13), similar to the proof of Corollary 1, the first derivative of the optimal incentive coefficient to parameters  $\lambda$ ,  $b_1$ ,  $p_t$ ,  $pop$ ,  $\xi$ ,  $\alpha$  are as follows:

$$\frac{\partial \beta^{**}}{\partial \lambda} = \frac{2\lambda b_1 p_t pop^\alpha \sigma^2}{(\lambda^2 + b_1 p_t pop^\alpha \sigma^2)^2} > 0, \quad \frac{\partial \beta^{**}}{\partial b_1} = -\frac{\lambda^2 p_t pop^\alpha \sigma^2}{(\lambda^2 + b_1 p_t pop^\alpha \sigma^2)^2} < 0, \quad \frac{\partial \beta^{**}}{\partial p_t} = -\frac{\lambda^2 b_1 pop^\alpha \sigma^2}{(\lambda^2 + b_1 p_t pop^\alpha \sigma^2)^2} < 0,$$

$$\frac{\partial \beta^{**}}{\partial pop} = -\frac{\lambda^2 b_1 p_t \alpha pop^{\alpha-1} \sigma^2}{(\lambda^2 + b_1 p_t pop^\alpha \sigma^2)^2} < 0, \quad \frac{\partial \beta^{**}}{\partial \sigma^2} = -\frac{\lambda^2 b_1 p_t pop^\alpha}{(\lambda^2 + b_1 p_t pop^\alpha \sigma^2)^2} < 0; \quad \frac{\partial \beta^{**}}{\partial \alpha} = -\frac{\lambda^2 b_1 p_t pop^\alpha \sigma^2 \ln \alpha}{(\lambda^2 + b_1 p_t pop^\alpha \sigma^2)^2},$$

when  $0 < \alpha < 1$ ,  $\frac{\partial \beta^{**}}{\partial \alpha} > 0$ ; and when  $\alpha > 1$ ,  $\frac{\partial \beta^{**}}{\partial \alpha} < 0$ .

### Proof for Corollary 3

We have the equilibrium solution  $e_1^{***} = \frac{\beta\lambda + 2pk e_b}{b_1 + 2pk}$  as with the formula (16). For the convenience of observation and calculation, let  $pk = t$ . The derivation of  $t$  leads to  $\frac{\partial e_1^{***}}{\partial t} = \frac{2(e_b b_1 - \beta\lambda)}{(b_1 + 2t)^2}$ . It is evident that when  $e_b b_1 - \beta\lambda > 0$ ,  $\frac{\partial e_1^{***}}{\partial t} > 0$ , which means the agent's effort  $e_1$  increases by the increase of the product of the probability of spot check and penalty coefficient. Similarly, when  $e_b b_1 - \beta\lambda < 0$ ,  $\frac{\partial e_1^{***}}{\partial t} < 0$ , which means the agent's effort  $e_1$  decreases by the increase of the product of the probability of spot check and penalty coefficient. Then when  $e_b b_1 - \beta\lambda = 0$ , we have  $\frac{\partial e_1^{***}}{\partial t} = 0$ , where the agent gets the optimal effort.

#### **Proof for Corollary 4**

The first derivation of formula (18) to the entry threshold  $e_b$  leads to  $\frac{\partial e_1^{***}}{\partial e_b} = \frac{2pk b_1 \lambda^2}{(p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2)(b_1 + 2pk)} + \frac{2pk}{b_1 + 2pk} > 0$ . Obviously, it is greater than zero. Hence, the optimal agent's effort  $e_1^{***}$  increases with the increase of entry threshold  $e_b$ . Similarly, the first derivation of formula (19) to the entry threshold  $e_b$  leads to  $\frac{\partial \beta^{***}}{\partial e_b} = \frac{2pk \lambda b_1}{p_t pop^\alpha \sigma^2 (b_1 + 2pk)^2 + b_1 \lambda^2 + 4k \lambda^2 p} > 0$  which is greater than zero. It is evident that the optimal incentive coefficient  $\beta^{***}$  increases with the increase of entry threshold  $e_b$ .