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The Fu (2009) Positive Relation between Idiosyncratic Volatility and Expected Returns Is Due to Look-Ahead Bias*

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Abstract

Expected idiosyncratic volatility and its positive relation to expected returns of Fu (2009) can be closely replicated, but only when we include information up to time t to estimate the idiosyncratic volatility at time t. Since this involves look-ahead bias, we re-estimate expected idiosyncratic volatility using information only up to time $t-1$. We find no significant relation between idiosyncratic volatility and returns, and our results are robust to the sample periods extended to before and after that of Fu (2009). Our findings are consistent with the fact that idiosyncratic risk is not priced.

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1. Introduction

Risk and return is a basic concept in finance. Risk is related to the volatility of an asset, which can be decomposed into systematic volatility and idiosyncratic volatility (IVOL). When IVOL can be completely eliminated by forming portfolios in the limit, under the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965a), IVOL should not be priced. In contrast, under the incomplete information CAPM of Merton (1987), IVOL and expected stock returns are positively related, especially for less recognized stocks. Lintner (1965b) and Lehmann (1990) find that IVOL is positively related to expected returns. Using firm size as a proxy for investor recognition, Carroll and Wei (1988) find that for the 1931 to 1985 sample period, the relation between IVOL and expected returns is positive for small portfolios but negative for big ones. Ang et al. (2006) note that previous studies on the relation between IVOL and expected returns do not measure IVOL at the firm level, or that they sort on portfolios with other measures like firm size and/or market beta. After correcting these, however, they find that historical IVOL and future stock returns are negatively related.

Fu (2009) argues that IVOL does not follow a random walk and that expected IVOL (EIVOL) should be estimated using a dynamic econometric model. He proposes using the exponential, generalized, autoregressive, conditional heteroskedasticity (EGARCH) model to estimate EIVOL. Fu (2009, p. 29) further notes that "My EGARCH(p, q) model involves $p + q + 3$ parameters. Using the full period data to estimate these parameters, $\ldots$, incurs a look-ahead bias. To avoid this concern, I estimate EGARCH parameters by using an expanding window of data .... In other words, the EGARCH parameters used to forecast conditional idiosyncratic volatility at month t are estimated on the basis of the data up through month $t - 1$." Fu (2009) finds that his EIVOL measure and future stock returns are positively related, which supports the prediction of the incomplete information CAPM of Merton (1987).

In this paper, we comprehensively replicate Fu's (2009) study and intend to resolve the debate in the literature about the relation between expected IVOL and expected stock returns. Specifically, we replicate the cross-sectional regression analysis of Fu (2009) to test the relation between EIVOL under different EGARCH specifications estimated using an expanding window of data and stock returns during his original sample period, as done in Fink et al. (2012) and Guo et al. (2014). We also replicate Fu's (2009) other analyses, including the sorting portfolio analysis and an additional analysis that explains the puzzling finding of the strongly negative relation between historical IVOL and future stock returns documented by Ang et al. (2006). Furthermore, we extend the work of Fu (2009) to include periods both before and after the original sample period, and to examine the effects of varying the data window and/or data frequency when estimating IVOL on the statistical inference about the relation between IVOL and future stock returns.

We find that we can closely replicate all results as reported by Fu (2009) only when we use the in-sample data to estimate EIVOL. Our findings not only hold for the original sample period in his study but also remain robust to pre- and post-sample periods. We conjecture that Fu (2009) estimates the EIVOL of time t using data up to time t to obtain results. This could lead to look-ahead bias in the estimation. We reestimate expected IVOL using EGARCH, just as in Fu (2009) but, to eliminate the bias, we use data only up to $t-1$. When doing so and running the Fama–MacBeth (1973) regressions, we find that there is no significant relation between our EIVOL measure and expected returns just as was reported by Fink et al. (2012) and Guo et al. (2014). Our findings are consistent with the portfolio theory and the CAPM that IVOL is not priced. In addition, when we control for our EIVOL measure, we do not find a positive relation between firm size and stock returns, as documented in Fu (2009).

We next sort all stocks by EIVOL and perform a portfolio analysis. While Fu (2009) finds a monotonic pattern of value-weighted returns increasing as his measure of EIVOL increases, we do not find such a pattern when we sort on our measure of EIVOL. Instead, we find a hump-shaped pattern with the highest value-weighted return in our middle portfolio

(6^{th} of 10).

Unlike Fink et al. (2012) and Guo et al. (2014), who consider approximately the same sample window as that in Fu (2009), which encompasses the sample range used in Ang et al. (2006), we extend the analyses by using data up to 2017 and back to 1926. The pre- and post-sample analysis is motivated by the recent studies of Linnainmaa and Roberts (2018) and Wahal (2019). Data mining is an important concern in research that touches on asset pricing anomalies. Linnainmaa and Roberts (2018) investigate a number of accounting-based anomalies in three sample periods (in-, pre-, and post-sample), and find that most of the anomalies exist only in the in-sample periods of the original studies. In particular, they find no economically or statistically significant premium on the profitability and investment factors in their early sample period (i.e., 1926 to 1969). Wahal (2019) tests whether the investment and profitability factors are priced in an early sample period, from 1946 to 1963 and finds that there is no reliable relation between investment and future stock returns. In contrast, we find that our results are robust during the recent post-sample period (January 2007 to December 2017), the full-sample period (July 1963 to December 2017), and the pre-sample period (July 1926 to June 1963).

We further examine the relation between IVOL or EIVOL and stock returns by choosing different IVOL or EIVOL estimation methods. Bali and Cakici (2008) find that thereis no robust relation between IVOL and returns in the cross-section as documented in Ang et al. (2006). To estimate IVOL, like Ang et al. (2006), we vary the time window and choose daily or monthly return data for estimation. Consistent with their findings, we discover that the cross-sectional relation between IVOL and returns is sensitive to estimation methodology, suggesting that the evidence in Ang et al. (2006) can be replicated, but this is not a robust finding. We also estimate EIVOL using a rolling window of data and find that the results are qualitatively similar to EIVOL estimated using an expanding window with all available information up to the estimation month. The results suggest that the evidence in Fu (2009) with a look-ahead bias and our evidence without such a bias are replicable and robust.

In sum, our study and those of Fink et al. (2012) and Guo et al. (2014), which have attempted to replicate the results as originally documented by Fu (2009) with the absence of look-ahead bias, have found strong, unequivocal evidence that his study is subject to this look-ahead bias. We conclude that his core finding, overturning the result in Ang et al. (2006), is itself overturned, with the evidence in Ang et al. (2006) left standing.

Our replication evidence points to the fact that future studies should not consider Fu's result as given. Many papers frequently cite Fu (2009) as evidence that IVOL is positively priced in the cross section of stocks, which is consistent with the prediction of under-diversification or the incomplete information CAPM (Diavatopoulos et al., 2008; Chua et al., 2010; Miffre et al., 2013; Umutlu, 2015; Lee and Mauck, 2016). Several studies estimate expected IVOL using a process similar to that of Fu (2009) (Peterson and Smedema, 2011; Chichernea et al., 2015).

2. Empirical Analysis

In this section, we first replicate the results reported by Fu (2009) for his original sample period of July 1963 to December 2006. We then extend the analysis to a post-sample period from January 2007 to December 2017, a full-sample period from July 1963 to December 2017, and a pre-sample period from July 1926 to June 1963. For ease of comparison, in the first panel of each table, we present the results as reported in Fu (2009). For all empirical analyses, the results for the pre-sample period are reported in the final panel of each table.

2.1. Data and Sample Description

We include stocks traded on the NYSE, Amex, or Nasdaq during our four respective sample periods. Daily and monthly data are obtained from the Center for Research in Security Prices (CRSP), and annual accounting data are retrieved from the CRSP/Compustat Merged Database. We retrieve daily and monthly factor data and the book value of the common equity of individual stocks in the pre-sample period from Kenneth French's website.

2.2. Idiosyncratic Volatility and Its Properties

The IVOL at time t is unobservable at time $t-1$ and needs to be estimated. We first estimate a stock's IVOL and examine its time-series properties. Following Ang et al. (2006) and Fu (2009), we define IVOL as the volatility of idiosyncratic return relative to the Fama and French (1993) three-factor model. Specifically, for a stock i in month t, we regress the stock's daily excess return on the three daily Fama and French (1993) factors:

$$R_{i,d} - r_{f,d} = \alpha_{i,t} + \beta_{i,t}(R_{m,d} - r_{f,d}) + s_{i,t}\text{SMB}_d + h_{i,t}\text{HML}_d + \varepsilon_{i,d}, \tag{1}$$

where d denotes the day in month t, $R_{i,d}$ is stock i's daily return, $r_{f,d}$ is the daily risk-free rate, and $R_{m,d}$ is the daily return on the market portfolio. SMB_d is the daily size factor, and HML_d is the daily value factor. Stock i's monthly IVOL in month t, $\text{IVOL}_{i,t}$, is equal to the product of a standard deviation of the daily idiosyncratic return $\varepsilon_{i,d}$ and the square root of the number of trading days in that month. We calculate IVOL for stocks with at least 15 trading days in a month.[1]

[Insert Table 1 here]

Table 1 reports the time-series properties of IVOL. We first calculate the IVOL statistics for each stock and then report the average of the statistics across stocks. As reported in Panel B of Table 1, for the in-sample period from July 1963 to December 2006, there are 26,187 stocks with an average IVOL of 16.04%; the mean standard deviation of IVOL is 10.01%. Our results are very close to those reported by Fu (2009). As reported in Panel

[1] In his study, Fu (2009) also requires that stocks have nonzero monthly trading volumes. However, when we exclude observations with zero monthly trading volumes, the sample size (24,342 stocks) becomes smaller than that reported by Fu (26,189 stocks). We examine the time-series properties of IVOL for the full sample and the sample with nonzero monthly trading volumes, and we find that the results for the two samples are very similar. To match the sample size that Fu reports, we report results only for the full sample. Results of the sample containing nonzero monthly trading volumes are available upon request.

A of Table 1, his sample includes 26,189 stocks with an average IVOL of 16.89%, and the average standard deviation of IVOL is 9.94%. There are 10,710 stocks traded in the recent post-sample period from January 2007 to December 2017 (Table 1, Panel C), and the average IVOL is 11.54%, which is lower than that in the original sample period; the mean standard deviation of IVOL is 7.96%. During the pre-sample period (July 1926–June 1963, as reported in Panel E of Table 1), there are 2,648 stocks with an average IVOL of 13.38%.

The final columns in Table 1 report the autocorrelations of IVOL at various lags. During the original sample period (Panel B of Table 1), we find that the average autocorrelation at the first lag is 0.33 and it decreases to 0.11 at the 13$^{\text{th}}$ lag, which is close to Fu's (2009) results, as reported in Panel A. As shown in Panel C of Table 1, the average autocorrelations of IVOL seem to be larger during the recent post-sample period. In the pre-sample period (Panel E of Table 1), the average autocorrelation of IVOL at each lag is smaller than that in the later sample periods, while the rate of decay of the autocorrelation of IVOL seems to be slower. The autocorrelation coefficient of IVOL at the first lag is 0.27 and remains as high as 0.23 at the 13$^{\text{th}}$ lag. Overall, the results suggest that IVOL may not follow a random walk process, as claimed by Fu (2009).

We conduct a Dickey–Fuller test to see whether IVOL follows a random walk process. We run the following two time-series regressions for stocks with at least 30 observations of monthly IVOL:[2]

$$\text{Model 1}: \text{IVOL}_{i,t+1} - \text{IVOL}_{i,t} = \gamma_{0,i} + \gamma_{1,i}\text{IVOL}_{i,t} + \eta_i, \tag{2}$$

$$\text{Model 2}: \ln \text{IVOL}_{i,t+1} - \ln \text{IVOL}_{i,t} = \gamma_{0,i} + \gamma_{1,i}\ln \text{IVOL}_{i,t} + \eta_i. \tag{3}$$

Our parameter of interest is $\gamma_{1,i}$. The T-statistic of the estimated $\gamma_{1,i}$ is compared to the Dickey–Fuller critical values. The null hypothesis that the IVOL of a stock follows a random walk is rejected when the T-statistic of $\gamma_{1,i}$ is smaller than the corresponding critical value.

[2]We follow the same criterion as in Fu (2009).

[Insert Table 2 here]

Table 2 tabulates the distributions of the estimated parameter γ_1 of the two regression models across individual stocks. Panel A reports Fu's (2009) results; our replication results are reported in Panel B during the original sample period. For the first regression model, there are 20,762 stocks in Panel B (Fu (2009) reports 20,979 stocks as shown in Panel A). Our replication shows that the mean and median of γ_1 are -0.61 and -0.61 (the corresponding values are -0.61 and -0.60, as in Fu (2009)), and the mean and median of the T-statistic of γ_1 are -6.82 and -6.40, respectively (the corresponding T-statistics are -6.81 and -6.40, as reported in Fu (2009)). In the final column, we show the percentage of stocks for which the null hypothesis of a random walk is rejected. We reject the null hypothesis for 89.70% of the stocks, while Fu (2009) rejects the null hypothesis for 89.97% of stocks in his sample. The results for the second regression model for lnIVOL are similar. For the post-sample period (Panel C of Table 2) and the pre-sample period (Panel E of Table 2), the null hypothesis that IVOL follows a random walk is also largely rejected.

2.3. The Relation between EGARCH EIVOL and Stock Return

The results of the unit-root test of IVOL indicate that the IVOL process is not a random walk, suggesting that one-month lagged IVOL may not be a good proxy for investors' EIVOL in the next month. To find a better measure of EIVOL, Fu (2009) proposes using the EGARCH(p, q) model of Nelson (1991), where $1 \leqslant p \leqslant 3$ and $1 \leqslant q \leqslant 3$, to represent the conditional variance of idiosyncratic return relative to the Fama and French (1993) three-factor model:

$$
\begin{aligned}
R_{i,t} - r_{f,t} &= \alpha_i + \beta_i (R_{m,t} - r_{f,t}) + s_i \mathrm{SMB}_t + h_i \mathrm{HML}_t + \varepsilon_{i,t}, \qquad (4) \\
\varepsilon_{i,t} &\sim N(0, \sigma^2_{i,t}), \\
\ln(\sigma^2_{i,t}) &= \kappa_i + \sum_{l=1}^{p} \gamma_{i,l} \ln(\sigma^2_{i,t-l}) + \sum_{k=1}^{q} \theta_{i,k} \left[\frac{|\varepsilon_{i,t-k}|}{\sigma_{i,t-k}} - E\left\{ \frac{|\varepsilon_{i,t-k}|}{\sigma_{i,t-k}} \right\} \right] + \sum_{k=1}^{q} \xi_{i,k} \left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right),
\end{aligned}
$$

where subscript i denotes the stock and t denotes the month. The idiosyncratic return $\varepsilon_{i,t}$ is assumed to be normally distributed with mean zero and with conditional variance $\sigma_{i,t}^2$, which is a function of past p-periods of variance of residuals and past q-periods of residual returns. The true conditional variance of EIVOL is unobservable, so we need to estimate it from the data.

For a stock i in month t, we estimate its EIVOL conditional on information available up to month T, $E_T(\text{IVOL}_{i,t})$, as follows. First, we regress the stock's monthly excess returns on the Fama and French (1993) three factors over the period from month 1 to month T:

$$R_{i,\tau} - r_{f,\tau} = \alpha_{i,T} + b_{i,T}(R_{m,\tau} - r_{f,\tau}) + s_{i,T}\text{SMB}_\tau + h_{i,T}\text{HML}_\tau + \varepsilon_{i,\tau}, 1 \leqslant \tau \leqslant T, \quad (5)$$

and obtain a time series of residual returns $\{\varepsilon_{i,1}, \varepsilon_{i,2}, ..., \varepsilon_{i,T}\}$. We then estimate the parameters of the EGARCH model. The estimated parameters maximize the sum of the log likelihood:

$$\sum_{\tau=1}^{T} L(R_{i,\tau}) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{\tau=1}^{T}\log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{T}\frac{\varepsilon_{i,\tau}^2}{\sigma_{i,\tau}^2}. \quad (6)$$

Finally, the EIVOL for month t is calculated recursively, using the estimated parameters:

$$\ln(\sigma_{i,t}^2) = \kappa_{i,T} + \sum_{l=1}^{p}\gamma_{i,T,l}\ln(\sigma_{i,t-l}^2) + \sum_{k=1}^{q}\theta_{i,T,k}\left[\frac{|\varepsilon_{i,t-k}|}{\sigma_{i,t-k}} - E\left\{\frac{|\varepsilon_{i,t-k}|}{\sigma_{i,t-k}}\right\}\right] + \sum_{k=1}^{q}\xi_{i,T,k}\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right), \quad (7)$$

and taking the square root of the expected conditional idiosyncratic variance for month t gives us the expected conditional IVOL for month t, $E_T(\text{IVOL}_{i,t})$. As in Fu (2009), we estimate nine EGARCH specifications with different orders of GARCH terms and/or ARCH terms. For a stock i in month t, there are up to nine converged EIVOL; for our analyses, we choose the one with the lowest Akaike information criterion (AIC). To ensure precision of estimation, we choose stocks that each have at least 30 monthly returns.[3] Note that we denote the parameters of the EGARCH model and EIVOL with T to emphasize that the

[3] Fu (2009) sets the same requirement.

parameters and EIVOL are estimated using information available up to time T.

To preclude look-ahead bias, when estimating the EIVOL for month t, we should use only the information available up to month $t-1$ (i.e., $T = t-1$) to derive a strictly out-of-sample EIVOL. Otherwise, the inclusion of data from month t when estimating the EIVOL for month t would lead to look-ahead bias and a spurious relation between the EIVOL for month t and the stock return in month t. To determine how look-ahead bias affects the relation between EIVOL and stock returns, we also calculate the in-sample EIVOL by including the month t stock return in the information set to estimate the EGARCH model's parameters (i.e., we set T to equal t). Fu (2009) claims to use an information set up to month $t-1$ to estimate EIVOL for month t. In the remainder of this paper, we denote EIVOL as reported by Fu (2009) as $E^F_{t-1}(\text{IVOL}_t)$. For our replicated results and those from the extended sample period, the out-of-sample EIVOL (which we estimate using an information set up to month $t-1$) is denoted as $E_{t-1}(\text{IVOL}_t)$. Meanwhile, our own estimated in-sample EIVOL that uses an information set up to month t is denoted as $E_t(\text{IVOL}_t)$.

2.3.1. Summary Statistics of Variables

We then examine the relation between various measures of EIVOL and stock returns. Monthly stock return (RET) is the monthly holding period return. The monthly excess return (XRET) is the monthly stock return in excess of the one-month Treasury-bill rate.

In addition to IVOL and EIVOL, we also calculate and report other variables that have been found to predict future stock returns—namely, market beta, size, book-to-market ratio, past stock returns, and stock liquidity. The full-sample portfolio market BETA, market value of equity (ME), and book-to-market ratio (BE/ME) are calculated as per Fama and French (1992). The market value of equity (ME) is defined as the product of the monthly closing price and the month-end number of shares outstanding in June of the current calendar year. The book-to-market ratio is equal to the book value of equity for the fiscal year ending in the previous calendar year divided by market capitalization at the end of December of the previous calendar year. RET(−7,−2) is the cumulative return from month $t-7$ to month $t-2$. The average turnover, TURN, and the coefficient of variation of turnovers, CVTURN, are calculated as per Chordia et al. (2001) using the data from the past 36 months.

[Insert Table 3 here]

Table 3 reports the summary statistics of variables for the pooled sample for the four sample periods. As in Fu (2009), to reduce the impact of extreme values, in each month, we winsorize each of ME, BE/ME, IVOL, E(IVOL), RET(−7,−2), TURN, and CVTURN at the 0.5% level. We also exclude observations with monthly returns exceeding 300%. We report ME, BE/ME, TURN, and CVTURN as their natural logarithm. Overall, our replication results (as shown in Panel B of Table 3) are similar to those of the original study (as shown in Panel A of Table 3). During the original sample period of Fu (2009), stocks have an average RET of 1.18% and an average XRET of 0.71%, which are the same as those reported by Fu (2009). The mean and standard deviation of monthly IVOL are 14.06% and 13.78%, respectively, which are close to the corresponding values of 14.17% and 13.91% as reported in Fu (2009). On average, the EIVOL estimated by Fu (2009) is 12.67%. During the same sample period, our in-sample EIVOL and out-of-sample EIVOL have a mean of 12.48% and 11.78%, respectively.

Compared to the original sample period, in the recent post-sample period (Panel C of Table 3), the average stock return, IVOL, and EIVOL are much lower, and firm size is on average larger. Panel E of Table 3 tabulates the statistics of the variables of the pooled sample in the pre-sample period. In this early sample, stocks earn an average monthly return of 1.29%, which is higher than that in the later sample periods. Consistent with the lower historical IVOL in the pre-sample period, both the out-of-sample and in-sample EIVOL are smaller than in other periods. For other firm characteristics, compared to stocks traded on and after July 1963, stocks traded before July 1963 feature a smaller firm size, higher book-to-market, and lower share turnover.

2.3.2. Cross-Sectional Pairwise Correlation Matrix

For each month, we calculate the cross-sectional Pearson correlation between each pair of variables and report the time-series average of the correlations; Table 4 tabulates these results. Correlation coefficients marked by an asterisk (*) are significant at the 1% level. As Panels A and B show, during the sample period of Fu (2009), we can closely replicate the time-series means of cross-sectional correlations between pairs of variables as in the original study, except for the average correlation between the out-of-sample EIVOL and stock return. The average correlation between simultaneous IVOL and stock return is 0.12 for our replication and is 0.14 in Fu (2009). We find that $E_t(\text{IVOL}_t)$ (the in-sample EIVOL) is significantly and positively correlated (correlation coefficient = 0.10) with stock returns at the 1% level, which is very close to the average correlation between $E^F_{t-1}(\text{IVOL}_t)$ and RET_t of 0.09 as reported in Fu (2009). In constrast, we find that EIVOL with an absence of look-ahead bias, $E_{t-1}(\text{IVOL}_t)$, is negatively correlated (although insignificantly) with stock returns. Fu (2009) finds that $E^F_{t-1}(\text{IVOL}_t)$ has a significant correlation of 0.46 with IVOL_t. We find that both $E_t(\text{IVOL}_t)$ and $E_{t-1}(\text{IVOL}_t)$ are significantly and positively correlated with IVOL_t, but the correlation of $E_t(\text{IVOL}_t)$ is larger and closer to that reported by Fu (2009).

For both our replication result and the result reported in Fu (2009), the correlation between market beta and stock return is not significant; this finding is consistent with that of Fama and French (1992). Firm size and stock returns are significantly and negatively correlated, and BE/ME and stock returns are significantly and positively correlated. These findings are consistent with the size effect and the value effect shown in Fama and French (1992). The significantly positive correlation between RET (−7,−2) and RET confirms the momentum effect found by Jegadeesh and Titman (1993). Consistent with Fu (2009), we find that IVOL is negatively correlated with firm size and book-to-market equity, and positively with TURN and CVTURN. Therefore, on average, small stocks, growth stocks, and liquid stocks have higher IVOL values than do large stocks, value stocks, and illiquid stocks.

[Insert Table 4 here]

The average correlations between variables during the recent post-sample period (Panel C of Table 4) are largely similar to those in the original sample period, but there are also several important differences. The average correlation between firm size and stock returns becomes significantly positive. The momentum effect is also weak in the recent post-sample period. Another difference is that value stocks tend to have a higher IVOL than do growth stocks. Panel E of Table 4 shows that during the pre-sample period before July 1963, firm size is positively correlated with stock returns, but the average correlation is insignificant, suggesting that there is no strong size effect in the early sample period. Firm size and share turnover are significantly and negatively correlated, but book-to-market equity and share turnover are significantly and positively correlated. These findings suggest that small firms and value firms tend to have a higher turnover than large firms and growth firms. These relations reverse on and after July 1963.

Additionally, the relation between EIVOL and book-to-market equity is different before and after July 1963. Before July 1963, both in-sample EIVOL and out-of-sample EIVOL are significantly and positively correlated with book-to-market equity, suggesting that value stocks are expected to have higher IVOL than growth stocks. After June 1963, however, the average correlations between in-sample EIVOL or out-of-sample EIVOL and the book-to-market equity become significantly negative.

2.3.3. Month-to-Month Fama–MacBeth (1973) Regressions

In this section, we investigate the predictability of various variables on future stock return by running month-to-month Fama–MacBeth (1973) regressions. Specifically, in each month t, we regress monthly stock returns on a number of explanatory variables across stocks, as follows.

$$R_{i,t} = \gamma_{0,t} + \sum_{k=1}^{K} \gamma_{k,t} X_{k,i,t} + \varepsilon_{i,t}, \tag{8}$$

where i denotes stocks, t denotes months, and k denotes variables. $R_{i,t}$ is stock i's monthly stock return in month t, and $X_{k,i,t}$ are the set of explanatory variables. The regressions are run in each month, and the time series of regression coefficients are derived. For each regression coefficient, its time-series average, variance, and T-statistic are obtained by:

$$\begin{aligned} \bar{\gamma}_k &= \frac{1}{T_m}\sum_{t=1}^{T_m}\gamma_{k,t}, \\ \mathrm{Var}(\gamma_k) &= \frac{\sum_{t=1}^{T_m}(\gamma_{k,t}-\bar{\gamma}_k)^2}{T_m-1}, \\ t(\bar{\gamma}_k) &= \frac{\bar{\gamma}_k}{\sqrt{\frac{\mathrm{Var}(\gamma_k)}{T_m}}}, \end{aligned} \tag{9}$$

where T_m is the number of months during a sample period (i.e., 522 months in the original sample period, 132 months in the post-sample period, 654 months in the full-sample period, and 444 months in the pre-sample period).

[Insert Table 5 here]

Table 5 reports the regression results. For each regression model, we report the time-series average of each regression coefficient, the T-statistic of each coefficient, and the time-series average adjusted R^2. The first regression model includes three independent variables, namely, market beta, firm size, and book-to-market equity. The second regression model includes three more explanatory variables: RET(−7,−2), ln(TURN), and ln(CVTURN). Panels A and B of Table 5 show the results in the original sample period. Our regression results (Panel B) are similar to those reported in Fu (2009), as shown in Panel A. In both models 1 and 2, the average coefficient on market beta is not significantly different from zero. Consistent with the finding in Fama and French (1992), firm size has a significant and negative average coefficient, and the book-to-market equity has a significant and positive average coefficient. These results suggest that during the original sample period, small stocks and value stocks earn higher average returns than do large and growth stocks. The significantly positive average coefficient on RET(−7,−2) indicates that stocks that performed well in the previous six months continue to perform well in the following month, which confirms the momentum effect documented by Jegadeesh and Titman (1993). The average slopes on the two liquidity variables are significantly negative, which is consistent with the finding of Chordia et al. (2001) that both the level and variability of turnover are negatively related to future stock returns.

Regression model 3 examines the relation between the out-of-sample EIVOL, $E^F_{t-1}(\mathrm{IVOL}_t)$, and stock returns in month t. Fu (2009) claims to use information available up to last month, $t-1$, to estimate EIVOL in month t, and finds a significantly positive average coefficient of 0.11 (T-stat = 9.05) on $E^F_{t-1}(\mathrm{IVOL}_t)$, as reported in Panel A of Table 5. Contrary to his finding, as shown in Panel B of Table 5, we find that $E_{t-1}(\mathrm{IVOL}_t)$ has a small average coefficient of 0.01 that is insignificant (T-stat = 0.62). One possible reason for the inconsistent results between our and his findings is that Fu (2009) might have included stock return in month t in his information set when estimating EIVOL for month t. The inclusion of stock return in month t would create look-ahead bias, which would result in a spurious relation between EIVOL and stock returns. To test this conjecture, in regression model 3′, we regress stock return in month t on $E_t(\mathrm{IVOL}_t)$, which is the in-sample EIVOL estimated using information available up to month t. The average slope on $E_t(\mathrm{IVOL}_t)$ is 0.12 with a T-statistic of 8.74, which is very similar to what is reported by Fu (2009).

Regression models 4 and 5 in Panel A of Table 5 examine the predictability of $E^F_{t-1}(\mathrm{IVOL}_t)$ while controlling for other stock return predictors. Fu (2009) finds that the significantly positive relation between $E^F_{t-1}(\mathrm{IVOL}_t)$ and RET_t is robust after controlling for market beta, firm size, book-to-market equity, past stock return, and stock liquidity. He also reports that the negative relation between firm size and stock return reverses and becomes significantly positive. However, after controlling for other variables, as in Fu (2009), we are unable to find a significantly positive relation between $E_{t-1}(\mathrm{IVOL}_t)$ and RET_t. As model 5 shows (Table 5, Panel B), after controlling for $E_{t-1}(\mathrm{IVOL}_t)$, firm size has a significantly negative average slope, indicating that the small firm size anomaly still exists. In regression models 4′ and

5′ (Table 5, Panel B), we replace the out-of-sample EIVOL with the in-sample EIVOL. The average coefficient of $E_t(\text{IVOL}_t)$ is significantly positive, which is the same as in regression model 3′. That is, controlling for other variables would not attenuate the strongly positive relation between $E_t(\text{IVOL}_t)$ and stock return. Moreover, as in Fu (2009), we also find that after including $E_t(\text{IVOL}_t)$, the average slope of firm size is significantly positive. Thus, our results show that, in order to replicate the primary results of Fu (2009) (that EIVOL is positively related to stock returns) or the secondary results (that firm size is positively related to stock returns when EIVOL is controlled), EIVOL with look-ahead bias must be used. When we use EIVOL with absence of look-ahead bias, we cannot replicate his results.

Regression models 6 and 7 examine the relation between lagged and contemporaneous IVOL and stock returns. As Panel B of Table 5 shows, the average slope on the one-month-lagged IVOL, IVOL_{t-1}, is -0.03 with a T-statistic of 4.23, which is very close to the findings reported by Fu (2009) (Table 5, Panel A). The significantly negative slope is consistent with the finding of Ang et al. (2006) that high-IVOL stocks in the previous month earn lower returns in the current month. In contrast, the average coefficient on contemporaneous IVOL, IVOL_t, is 0.26 (T-stat = 18.82) (Table 5, Panel B), which is also similar to that reported in Fu's original study (Table 5, Panel A).

Panel C of Table 5 presents the regression results in the recent post-sample period. The results are similar to those in the original sample period, except for the coefficients on firm size and RET(-7,-2), which become insignificant. This suggests that the small firm size effect and the momentum effect are weaker in the recent post-sample period. There is no significant relation between $E_{t-1}(\text{IVOL}_t)$ and stock return, while the relation between $E_t(\text{IVOL}_t)$ and stock returns is significantly positive. Moreover, the IVOL puzzle documented by Ang et al. (2006) remains intact. Panel E of Table 5 reports the results in the pre-sample period. We show that the results are in general similar to those in later sample periods. In the first regression model, which includes only three independent variables (market beta, firm size, and book-to-market equity), the regression coefficient on firm size is marginally significantly negative, and that on book-to-market equity is significantly positive. These findings suggest that in the early pre-sample period, the value premium exists and the size effect is not strong. The significantly negative regression coefficient on IVOL (after controlling for other variables) indicates that the IVOL puzzle also exists in the pre-sample period, that is, before July 1963. For the relation between EIVOL and future stock returns, we again find a significantly positive relation only between in-sample EIVOL and future stock returns. In contrast, we fail to find any reliable relation between the out-of-sample EIVOL and subsequent stock returns. When we include the in-sample EIVOL in the regression, the coefficient on firm size becomes significantly positive.

In short, we fail to find a significant association between $E_{t-1}(\mathrm{IVOL}_t)$ and stock returns, which is divergent from the finding reported in Fu (2009). However, when we include month t's stock return in the information set to estimate EIVOL, our results are similar to those of Fu (2009). Our findings suggest that when estimating EIVOL, Fu (2009) might have used an information set available up to month t, and that the strongly positive relation between EIVOL and stock return could therefore be attributed to look-ahead bias.

2.3.4. Sorting Portfolio Analysis

We next conduct a portfolio analysis to examine the relation between EIVOL and stock returns. At the end of month $t-1$, we sort all stocks into a decile portfolio in two different ways: by (i) $E_{t-1}(\mathrm{IVOL}_t)$ and (ii) $E_t(\mathrm{IVOL}_t)$. Stocks in Portfolio 1 (10) are expected to have the lowest (highest) IVOL in month t. For each portfolio, we calculate the value-weighted and equal-weighted monthly portfolio return in month t. Portfolios are rebalanced each month. We regress monthly value-weighted portfolio excess returns on the monthly Fama and French (1993) three factors. The intercepts from this time-series regression are portfolio risk-adjusted returns (alphas).

[Insert Table 6 here]

Table 6 reports the time-series average of value-weighted and equal-weighted monthly returns and the pooled means or medians of the firm characteristics of stocks of decile portfolios sorted by $E_{t-1}(\text{IVOL}_t)$ or $E_t(\text{IVOL}_t)$. As Panel B of Table 6 shows, during the original sample period from July 1963 to December 2006, for decile portfolios sorted by $E_{t-1}(\text{IVOL}_t)$, the patterns of variables other than portfolio returns are similar to those reported in Fu (2009) (Panel A of Table 6). Stocks with a high $E_{t-1}(\text{IVOL}_t)$ tend to be small and growth stocks. BETA and ex-post IVOL increase across decile portfolios sorted by $E_{t-1}(\text{IVOL}_t)$. However, the patterns of average returns on portfolios sorted by $E_{t-1}(\text{IVOL}_t)$ (Table 6, Panel B) differ from the patterns documented by Fu (2009) (Table 6, Panel A). Fu (2009) finds that as the average $E^F_{t-1}(\text{IVOL}_t)$ increases from 3.19% to 36.35%, the time-series average of the value-weighted (equal-weighted) portfolio monthly return increases monotonically, from 0.90% (0.54%) to 2.65% (5.33%), across portfolios sorted by $E^F_{t-1}(\text{IVOL}_t)$. Portfolio 10 also has the highest monthly risk-adjusted return (1.45%). However, as Panel B of Table 6 shows, we find no such monotonic pattern of returns across the decile portfolios sorted by $E_{t-1}(\text{IVOL}_t)$. As average $E_{t-1}(\text{IVOL}_t)$ increases from 3.00% to 34.44% across the decile portfolios, the pattern of the value-weighted stock returns becomes humpe-shaped: the average return first increases from Portfolio 1 to Portfolio 6 and then decreases from Portfolio 6 to Portfolio 10. Portfolio 10 earns the lowest average value-weighted return and lowest risk-adjusted return. The equal-weighted portfolio returns tend to be higher for portfolios with high $E_{t-1}(\text{IVOL}_t)$, but the pattern is nonmonotonic, and the return spread between Portfolio 10 and Portfolio 1 is 0.22% (= 1.32% − 1.10%), much smaller than that in the original study (4.79% = 5.33% − 0.54%), as shown in Panel A of Table 6. The weak association between $E_{t-1}(\text{IVOL}_t)$ and stock returns is consistent with the insignificant coefficient on $E_{t-1}(\text{IVOL}_t)$ in the Fama-MacBeth cross-sectional regressions.

To investigate whether look-ahead bias gives rise to a spuriously positive relation between in-sample EIVOL and stock returns, we sort stocks by $E_t(\text{IVOL}_t)$. Panel B of Table 6 shows that both the value-weighted and equal-weighted portfolio returns increase monotonically across the decile portfolios. Portfolio 1 (10) earns a value-weighted monthly return of 0.89% (2.58%) and an equal-weighted monthly return of 0.58% (5.89%); these results are similar to the portfolio returns reported in Fu (2009) (Table 6, Panel A). The finding that stocks with higher $E_t(\text{IVOL}_t)$ earn higher average returns confirms the significantly positive slope of $E_t(\text{IVOL}_t)$ in the cross-sectional regressions. The patterns of other variables for decile portfolios sorted by in-sample EIVOL are also similar to those reported in Fu (2009).

The results from the portfolio analysis in the recent post-sample period (Panel C of Table 6) are similar to those in the original sample period, except that the pooled medians of the firm size of the portfolios are much higher than the pooled medians in the early sample periods, because in recent years exchange-listed firms have larger market capitalization. For the pre-sample period before July 1963 (Panel E of Table 6), the results are similar to those in the periods on and after July 1963, except that the average firm size is smaller than that in the later sample periods and that book-to-market equity increases as EIVOL increases (for both in-sample and out-of-sample EIVOL). These latter findings are consistent with the significantly positive correlations between in-sample or out-of-sample EIVOL and book-to-market equity.

Similar to the cross-sectional regressions, the results from the portfolio analysis show that the strongly positive relation between EIVOL and the stock returns in month t exists only when EIVOL is estimated using an information set available up to month t.

2.4. The Relation between Lagged IVOL and Stock Return

Ang et al. (2006) find that stocks with higher IVOL (relative to the three-factor model of Fama and French (1993)) in month $t-1$ will, on average, earn lower returns in month t. We first replicate their results by investigating the properties of portfolios sorted by one-month-lagged IVOL. Specifically, at the end of month $t-1$, stocks are sorted into quintile portfolios by IVOL_{t-1}, which is the standard deviation (muliplied by the square root of number of trading days in a month) of residuals in the regression of daily excess individual stock returns on the daily three factors of Fama and French (1993) in month $t-1$. Portfolio

1 (5) includes stocks with the lowest (highest) IVOL in month $t-1$. For each of the quintile portfolios, we calculate value-weighted returns and equal-weighted excess and raw returns in month $t-1$ and month t. We then regress the time series of value-weighted monthly excess returns on the contemporaneous monthly three factors of Fama and French (1993). The intercepts of the time-series regressions are the portfolios' risk-adjusted returns (alphas).

[Insert Table 7 here]

Table 7 reports the average portfolio returns and firm characteristics of quintile portfolios sorted by one-month-lagged IVOL. Panel B of Table 7 shows that during the sample period of Fu (2009), although stocks with the highest IVOL in the last month earn the lowest value-weighted excess return, the relation between the one-month-lagged IVOL and the average ex-post portfolio return is nonmonotonic. Specifically, both value-weighted excess returns and equal-weighted raw returns in month t are highest for Portfolio 3 rather than for Portfolio 1. For the risk-adjusted returns, Portfolios 4 and 5 (the portfolios with high lagged IVOL values) have significantly negative alphas, while Portfolios 1 and 2 (portfolios with low lagged IVOL values) earn insignificant but positive abnormal returns. The results suggest that the negative relation between lagged IVOL and stock returns is driven by the low ex-post returns of stocks with high lagged IVOL, which tend to be small firms with low market shares. Fu (2009) also sorts stocks by one-month-lagged IVOL, and our findings are qualitatively similar to his results (as shown in Panel A of Table 7). Fu (2009) argues that stocks with very high IVOL_{t-1} also have very high lagged stock returns, and that it reverses in month t (Huang et al., 2010). Therefore, the IVOL puzzle is derived from the short-term return reversal of stocks with high IVOL values. To test this hypothesis, we also calculate the raw, excess, and abnormal portfolio returns in month $t-1$ of the quintile portfolios sorted by IVOL_{t-1}. Raw, excess, and risk-adjusted returns increase monotonically from Portfolio 1 to Portfolio 5. The positive association between IVOL and portfolio returns in the same month is consistent with the significantly positive average coefficient on IVOL_t in the cross-sectional regression with RET_t as the dependent variable. The results also confirm Fu's (2009) conjecture that stocks with high $IVOL_{t-1}$ earn high average returns in month $t-1$. The results continue to hold for the recent post-sample period (Panel C of Table 7) and the pre-sample period (Panel E of Table 7).

[Insert Table 8 here]

To further examine the short-term return reversal of stocks with high lagged IVOL, as per Fu (2009), we focus on the 40% of stocks with the highest $IVOL_{t-1}$. At the end of each month $t-1$, we sort the 40% of stocks with high $IVOL_{t-1}$ into quintile portfolios by RET_{t-1}, and calculate portfolio returns in month t. Table 8 reports the average raw, excess, and abnormal portfolio monthly returns. Panel B shows that as RET_{t-1} increases from −20.11% for Portfolio 1 to 30.88% for Portfolio 5 by construction, all measures of future returns decrease monotonically across portfolios. This suggests a strong short-term return reversal effect for stocks with high IVOL values. The results are consistent with those of Fu (2009), as reported in Panel A. The short-term return reversal effect remains strong in the recent post-sample period (Panel C of Table 8) and in the pre-sample period (Panel E of Table 8). The results of the portfolio-based analysis confirm Fu's (2009) conjecture that IVOL can be explained by the return reversal of stocks that have very high IVOL values.

2.5. Robustness Test

2.5.1. IVOL Estimation Method and the IVOL Puzzle

Bali and Cakici (2008) find that the negative relation between lagged IVOL and stock return is sensitive to various factors, including the data frequency used to estimate IVOL, the weighting scheme used to calculate average returns of portfolios sorted by lagged IVOL, break points used to sort stocks, and the screens used to filter stocks. Specifically, they find that the strong negative relation only exists when the full sample of all stocks traded on NYSE, Amex, and Nasdaq is sorted into quintile portfolios using CRSP break points of IVOL estimated using daily data in the last month, and portfolio returns are weighted by stocks' market capitalizations. The significantly negative relation between IVOL and stock returns disappears when IVOL is estimated using monthly returns in the previous 24 to 60 months (as available); stocks are sorted by NYSE break points of IVOL; the sample only includes NYSE stocks or excludes extremely illiquid, smallest, and lowest priced stocks; or stock returns are equal-weighted or weighted by the inverse of IVOL.

We undertake a robustness test to examine how the relation between lagged or contemporaneous IVOL and stock returns is affected by the manner in which IVOL is estimated. We estimate (monthly) IVOL using data of daily or monthly frequency during different time windows. When daily data are used (Methods 1, 2, and 3 in Table 9), IVOL is estimated using daily data in the 3 previous months (from month $t-2$ to month t), the 6 previous months (from $t-5$ to t), or the 12 previous months (from month $t-11$ to month t). For each stock i, the excess daily returns of each individual stock are regressed on the daily three factors of Fama and French (1993) during a time window. The (monthly) IVOL of the stock is the product of the standard deviation of the regression residuals and the square root of the average number of trading days per month in a time window. When monthly data are used (Methods 4, 5, and 6 in Table 9), IVOL is estimated using monthly data in the previous year (from month $t-11$ to month t), the three previous years (from month $t-35$ to month t), the five previous years (from month $t-59$ to month t), and the whole sample period up to month t (from month 1 to month t). For each stock i, the excess monthly returns of each individual stock are regressed on the monthly three factors of Fama and French (1993) during a time window. The (monthly) IVOL of the stock is the standard deviation of the regression residuals. Then the cross-sectional regressions analysis of Fama–MacBeth (1973) is used to examine the relation between lagged or contemporaneous IVOL and stock returns. In each month t, RET_t is regressed on one-month-lagged IVOL, IVOL_{t-1}, or contemporaneous IVOL, IVOL_t, across stocks and on other stock return predictors as control variables. The estimated regression coefficients are the means of the time series of estimated slopes in each month.

[Insert Table 9 here]

Table 9 reports the regression results. Panel A of the table shows that during the original sample period of Fu (2009), the average coefficient on IVOL_t is always significantly positive, regardless of the data frequency and the time window of the data used to estimate IVOL. Moreover, the strongly positive relation is robust after controlling for other variables. In contrast, the coefficient on IVOL_{t-1} seems to be sensitive to how IVOL is estimated. For the univariate regressions in which returns are regressed on IVOL_{t-1} only, the average coefficient on IVOL_{t-1} is marginally significantly negative when daily data in the three previous months are used to estimate IVOL, but insignificant when IVOL is estimated in other ways. After controlling for other variables, the average slopes on IVOL_{t-1} become more negative but are only significant when short-time-window data are used to estimate IVOL (i.e., three months of daily data or one year of monthly data).

As Panel B of Table 9 shows, the regression results are similar for the recent post-sample period. The average slopes on IVOL_t are significantly positive. The average slopes on IVOL_{t-1} appear to be more negative than those in the early in-sample period. Again, the strength of the negative relation depends on the methods used to calculate IVOL, and it tends to decrease when a longer time window of data is used to estimate IVOL. Using the early pre-sample period before July 1963 (Panel D of Table 9), we again find that the negative relation between historical IVOL and future stock returns is sensitive to how IVOL is calculated. The coefficient on IVOL is significantly negative (after including controlling variables) only when IVOL is estimated using daily stock returns in the three previous months, while the coefficient is insignificant when IVOL is calculated in other ways.

In summary, consistent with Bali and Cakici (2008), we find that the negative relation between lagged IVOL and stock returns is sensitive to the way in which IVOL is estimated.

2.5.2. Estimation of EIVOL in a Rolling Window

Our second robustness check is about the way in which we estimate EGARCH EIVOL. The EGARCH model captures the time-varying conditional volatility of stock return, and in that model, expected volatility is a function of historical volatility and historical error terms. In our main empirical analysis, following Fu (2009), we use all available information (stock return from month 1 to month $t-1$ for the out-of-sample EIVOL and from month 1 to month t for the in-sample EIVOL) to estimate EIVOL for month t. Using long-history data to estimate EIVOL can increase estimation reliability. However, for stocks that have been traded on exchanges over a very long time, using all available information could render EIVOL dependent on information far into the history while weakening the role of recent information in EIVOL estimation. Therefore, one possible reason for our failure to find a positive relation between out-of-sample EIVOL and future stock return is that when estimating EIVOL using all available data, puts less weight on recent information, compared to the cases where we use shorter and more recent sample periods. To take this into consideration, we estimate a rolling-window out-of-sample EIVOL for month t ($E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$) using information from the previous 24 to 60 months (as available), with the period ending in the last month $t-1$. This is the same time window used to estimate market beta. Again, to examine the effect of look-ahead bias on the results, we also estimate a rolling-window in-sample EIVOL for month t ($E_t^{\text{Roll}}(\text{IVOL}_t)$), using information on the previous 24–60 months (as available), with the period ending in month t. We reconduct the main empirical analysis for EIVOL, estimated using the rolling window of data for three sample periods, namely, the original sample period (July 1963 to December 2006), the post-sample period (January 2007 to December 2017), and the pre-sample period (July 1926 to June 1963). Table 10 reports the results with respect to the two rolling-window EIVOLs.

[Insert Table 10 here]

Panel A of Table 10 reports the summary statistics of in-sample and out-of-sample

EIVOLs estimated using a rolling window of data. In general, EIVOL estimated using the rolling window data has a distribution similar to that of EIVOL estimated using all available information, except that the average value of the rolling-window EIVOL seems to be slightly smaller. Panel B of Table 10 tabulates correlations between EIVOL and other variables. Like the EIVOL results estimated using all available information, only $E_t^{\text{Roll}}(\text{IVOL}_t)$ (in-sample EIVOL) is strongly and positively correlated with future stock returns. In contrast, the correlation coefficient between $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$ (out-of-sample EIVOL) and future stock returns or log future stock returns is, in some cases, either insignificant or even significantly negative. The results from cross-sectional regressions (Panel C of Table 10) are consistent with the pairwise correlations. The regression coefficient on out-of-sample $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$ is not significant, suggesting that there is no reliable relation between EIVOL and stock returns. The coefficient on in-sample $E_t^{\text{Roll}}(\text{IVOL}_t)$ is significantly positive, and the strong relation is robust after controlling for other variables.

We then conduct portfolio analysis by sorting stocks into portfolios either by in-sample $E_t^{\text{Roll}}(\text{IVOL}_t)$ or out-of-sample $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$ and then compare subsequent portfolio returns. As Panel D of Table 10 shows, there is a monotonically increasing pattern of portfolio returns only when stocks are sorted by in-sample $E_t^{\text{Roll}}(\text{IVOL}_t)$; the portfolio with the highest EIVOL earns a very high average return. When stocks are sorted by out-of-sample $E_{t-1}^{Roll}(\text{IVOL}_t)$, we again find a humped pattern of portfolio returns. Specifically, the portfolio return first increases as we move from portfolios with low EIVOL to portfolios with mid-level EIVOL, but it then tends to decrease as we move to portfolios with high EIVOL.

Compared to the results with in-sample EIVOL estimated using all information, the in-sample EIVOL estimated using a rolling window is correlated more strongly with future stock returns. Moreover, the coefficient on in-sample EIVOL in the regression models (with future stock returns as the dependent variable) is both larger and more statistically significant. The findings are consistent with the look-ahead bias explanation. That is, when EIVOL is estimated using a shorter time window of data, the stock return in month t could have a greater impact on the estimation of model parameters and EIVOL. In this way, the look-ahead bias is more pronounced, which leads to a more spurious relation between in-sample EIVOL and stock returns in month t. Our finding of a stronger relation between in-sample EIVOL estimated using a shorter time window of data and stock returns is consistent with the implication of simulations analysis as reported in Guo et al. (2014) that look-ahead bias is monotonically decreasing in the length of the return series used in estimation. The results are similar for the original sample period, the recent post-sample period, and the early pre-sample period.

In sum, using a rolling window of data up to the last month of the period to estimate EGARCH EIVOL and using information strictly out of sample, we still fail to find a reliable relation between EIVOL and subsequent stock returns.

3. Conclusion

Fu (2009) points out the fact that idiosyncratic volatility (IVOL) does not follow a random walk. He introduces a dynamic econometric model, EGARCH, to estimate expected IVOL (EIVOL). He finds that his measure of EIVOL is significantly and positively related to expected stock returns, which overturns the finding of a negative association between lagged realized IVOL and expected returns by Ang et al. (2006). In this paper, we comprehensively replicate the study of Fu (2009) and intend to resolve the debate on the association between expected IVOL and expected stock returns. Unlike previous replication studies based on the original sample period of Fu (2009) by Fink et al. (2012) and Guo et al. (2014), we extend the sample period to include the pre- and post-sample periods. Moreover, we also examine the effects of varying data window and/or data frequency when estimating lagged IVOL and EIVOL on the relation between expected returns and lagged IVOL or EIVOL

Fu (2009, p. 29) emphasizes that “the EGARCH parameters used to forecast conditional idiosyncratic volatility at month t are estimated on the basis of the data up through month

t–1" to avoid look-ahead bias. However, in fact we find that he uses data available up to month t when estimating EIVOL at month t. After correcting this look-ahead bias, we find that the relation between EIVOL and expected returns is no longer significant, which is consistent with the evidence in Fink et al. (2012) and Guo et al. (2014). That is, his main result that overturns the evidence of Ang et al. (2006) is itself overturned.

We also find that our replication results on the association between EIVOL and expected returns with or without look-ahead bias are robust to varying data window in estimating EIVOL. In contrast, the relation between lagged IVOL and expected returns is sensitive to varying data frequency in estimating lagged IVOL, consistent with the evidence in Bali and Cakici (2008).

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Table 1 Time-series properties of idiosyncratic volatility

Description: This table summarizes the time-series statistics of individual stock IVOLs. The IVOL is estimated as follows. For each stock i in every month t, the excess daily returns of each individual stock are regressed on the daily Fama and French (1993) three factors:

$$R_{i,d} - r_{f,d} = \alpha_{i,t} + b_{i,t}(R_{m,d} - r_{f,d}) + s_{i,t}\text{SMB}_d + h_{i,t}\text{HML}_d + \varepsilon_{i,d}.$$

The (monthly) IVOL of the stock is the product of the standard deviation of the regression residuals and the square root of the number of observations in the month. We first compute the time-series statistics of IVOL for each stock and then the mean statistics across all stocks. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: Summary statistics and autocorrelations of IVOL of the replication and extensions are very similar to those reported by Fu (2009). Results indicate that IVOL does not follow a random walk process.

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 27

	N	Mean	S.D.	C.V.	Skew	Autocorrelation at lags								
						1	2	3	4	5	6	11	12	13
IVOL	26,189	16.87	9.94	0.55	1.65	0.33	0.27	0.24	0.20	0.19	0.18	0.12	0.14	0.11
$\ln\left(\frac{\text{IVOL}_t}{\text{IVOL}_{t-1}}\right)$	26,068	−0.004	0.54	366.24	−0.03	−0.42	−0.04	0.01	−0.02	−0.01	0.01	−0.02	0.03	−0.02

Panel B Sample Period July 1963 to December 2006, replication results

	N	Mean	S.D.	C.V.	Skew	Autocorrelation at lags								
						1	2	3	4	5	6	11	12	13
IVOL	26,187	16.04	10.01	0.54	1.64	0.33	0.27	0.24	0.20	0.19	0.18	0.13	0.14	0.11
$\ln\left(\frac{\text{IVOL}_t}{\text{IVOL}_{t-1}}\right)$	26,001	−0.005	0.54	377.91	−0.03	−0.43	−0.04	0.01	−0.02	−0.01	0.01	−0.02	0.03	−0.02

Panel C Sample Period January 2007 to December 2017, post-sample results

	N	Mean	S.D.	C.V.	Skew	Autocorrelation at lags								
						1	2	3	4	5	6	11	12	13
IVOL	10,710	11.54	7.96	0.63	1.92	0.39	0.34	0.33	0.26	0.25	0.25	0.16	0.18	0.14
$\ln\left(\frac{\text{IVOL}_t}{\text{IVOL}_{t-1}}\right)$	10,675	0.001	0.57	386.68	−0.04	−0.43	−0.04	0.05	−0.05	−0.01	0.03	−0.04	0.07	-0.05

Panel D Sample Period July 1963 to December 2017, full-sample results

	N	Mean	S.D.	C.V.	Skew	Autocorrelation at lags								
						1	2	3	4	5	6	11	12	13
IVOL	30,034	15.49	10.93	0.72	1.86	0.34	0.28	0.26	0.21	0.20	0.19	0.13	0.14	0.10
$\ln\left(\frac{\text{IVOL}_t}{\text{IVOL}_{t-1}}\right)$	29,860	−0.003	0.56	482.31	−0.05	−0.43	−0.04	0.02	−0.03	-0.01	0.02	−0.02	0.04	−0.03

Panel E Sample Period July 1926 to June 1963, pre-sample results

	N	Mean	S.D.	C.V.	Skew	Autocorrelation at lags								
						1	2	3	4	5	6	11	12	13
IVOL	2,648	13.38	7.76	0.51	1.56	0.27	0.21	0.19	0.17	0.20	0.20	0.25	0.25	0.23
$\ln\left(\frac{\text{IVOL}_t}{\text{IVOL}_{t-1}}\right)$	2,636	−0.010	1.47	554.35	0.00	−0.43	−0.05	0.00	−0.03	0.03	0.00	0.00	0.02	−0.01

Table 2 Do monthly idiosyncratic volatilities follow a random walk process?

Description: This table presents statistics of the estimations from the time-series regressions in which the changes in (natural logarithm of) IVOL of an individual stock i are regressed on the level of (natural logarithm of) IVOL in the past month. The two regression models are

$$\begin{aligned}\text{Model 1} : \text{IVOL}_{i,t+1} - \text{IVOL}_{i,t} &= \gamma_{0,i} + \gamma_{1,i}\text{IVOL}_{i,t} + \eta_i, \\ \text{Model 2} : \ln \text{IVOL}_{i,t+1} - \ln \text{IVOL}_{i,t} &= \gamma_{0,i} + \gamma_{1,i}\ln \text{IVOL}_{i,t} + \eta_i.\end{aligned}$$

The reported statistics are the cross-sectional mean and median, the lower and the upper quartiles of the coefficient estimate γ_1, and its associated T-statistic. The T-statistics are compared to the Dickey–Fuller critical values to examine whether the null hypothesis of a random walk is rejected. The last column reports the percentage of firms for which the random walk hypothesis is rejected at the 1% level. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: Distributions of coefficients and T-statistics of the Dickey–Fuller test of the replication are consistent in signs and magnitudes with those reported by Fu (2009). The null hypothesis that IVOL follows a random walk is largely rejected.

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 28

Model	Variables	N	Mean	Median	Q1	Q3	RW rejected
1	γ_1	20,979	−0.61	−0.60	−0.76	−0.45	89.97
	$t(\gamma_1)$	20,979	−6.81	−6.40	−8.43	−4.85	
2	γ_1	20,979	−0.56	−0.55	−0.70	−0.41	87.81
	$t(\gamma_1)$	20,979	−6.38	−5.99	−7.86	−4.51	

Panel B Sample Period July 1963 to December 2006, replication results

Model	Variables	N	Mean	Median	Q1	Q3	RW rejected
1	γ_1	20,762	−0.61	−0.61	−0.76	−0.45	89.70
	$t(\gamma_1)$	20,762	−6.82	−6.40	−8.47	−4.81	
2	γ_1	20,762	−0.58	−0.56	−0.72	−0.42	87.85
	$t(\gamma_1)$	20,762	−6.41	−5.97	−7.92	−4.50	

Panel C Sample Period January 2007 to December 2017, post-sample results

Model	Variables	N	Mean	Median	Q1	Q3	RW rejected
1	γ_1	9,064	−0.56	−0.53	−0.70	−0.40	92.93
	$t(\gamma_1)$	9,064	−7.87	−7.43	−9.59	−5.62	
2	γ_1	9,064	−0.52	−0.49	−0.64	−0.37	93.21
	$t(\gamma_1)$	9,064	−7.34	−6.98	−8.83	−5.30	

Panel D Sample Period July 1963 to December 2017, full-sample results

Model	Variables	N	Mean	Median	Q1	Q3	RW rejected
1	γ_1	24,135	−0.60	−0.59	−0.77	−0.43	89.02
	$t(\gamma_1)$	24,135	−7.07	−6.70	−8.78	−5.01	
2	γ_1	24,135	−0.58	−0.55	−0.72	−0.41	88.11
	$t(\gamma_1)$	24,135	−6.47	−6.12	−8.08	−4.48	

Panel E Sample Period July 1926 to June 1963, pre-sample results

Model	Variables	N	Mean	Median	Q1	Q3	RW rejected
1	γ_1	1,528	−0.55	−0.55	−0.73	−0.36	95.99
	$t(\gamma_1)$	1,528	−8.74	−8.80	−10.68	−6.88	
2	γ_1	1,528	−0.52	−0.50	−0.70	−0.32	94.89
	$t(\gamma_1)$	1,528	−8.28	−8.39	−10.23	−6.44	

Table 3 Variable descriptive statistics for the pooled sample

Description: This table reports the pooled descriptive statistics of stocks. RET is the monthly raw return reported in percentage. XRET is the raw return net of the one-month T-bill rate. BETA is the portfolio beta estimated from the full period using 100 size and preranking beta portfolios. The market value of equity (ME) is the product of monthly closing price and the number of outstanding shares in June. Book-to-market equity (BE/ME) is the fiscal year-end book value of common equity divided by the calendar year-end market value of equity. IVOL is the monthly idiosyncratic volatility. $E_t(\text{IVOL}_t)$ is the in-sample EIVOL estimated using information set up to month t. $E^F_{t-1}(\text{IVOL}_t)$ is the EIVOL reported by Fu (2009). $E_{t-1}(\text{IVOL}_t)$ is the out-of-sample EIVOL estimated using information set up to month $t-1$. RET(−7,−2) is the cumulative return from month $t-7$ to $t-2$. TURN is the average turnover and CVTURN is the coefficient of variation of turnovers in the previous 36 months. Variables with skewness greater than 3 are reported as the natural logarithm. The smallest and largest 0.5% of the observations in each month for ME, BE/ME, IVOL, E(IVOL), RET(−7, −2), TURN, and CVTURN are set equal to the next smallest or largest values. Observations with monthly returns greater than 300% have been excluded. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: Both the replicated in-sample EIVOL and the replicated out-of-sample EIVOL have distributions similar to the distributions of the EIVOL in Fu (2009).

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 29

Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
RET (%)	1.18	16.86	0.15	−6.52	6.78	2.35	2,947,826
XRET(%)	0.71	16.87	−0.27	−6.99	6.34	2.35	2,947,826
$\ln(1+\text{RET})$ (%)	−0.12	16.09	0.15	−6.74	6.56	−0.54	2,947,826
IVOL	14.17	13.91	10.41	6.33	17.41	6.94	2,946,521
$E^F_{t-1}(\text{IVOL}_t)$	12.67	10.91	10.29	6.46	15.18	2.38	2,867,821
BETA	1.22	0.36	1.17	0.94	1.46	0.31	1,721,356
ln (ME)	4.29	2.03	4.16	2.82	5.64	0.34	2,804,878
ln (BE/ME)	−0.39	1.09	−0.35	−0.97	0.20	0.27	2,145,253
RET(−7, −2)	1.07	0.39	1.03	0.85	1.22	2.90	2,758,743
ln (TURN)	1.39	1.09	1.39	0.67	2.13	−0.08	2,041,658
ln (CVTURN)	4.15	0.44	4.14	3.92	4.48	0.14	2,038,647

Panel B Sample Period July 1963 to December 2006, replication results

Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
RET (%)	1.18	16.86	0.15	−6.45	6.76	2.35	2,947,518
XRET(%)	0.71	16.87	−0.22	−6.94	6.31	2.35	2,947,518
$\ln(1+\text{RET})$ (%)	−0.12	16.09	0.15	−6.67	6.54	−0.53	2,947,518
IVOL	14.06	13.78	10.10	6.32	17.38	7.05	2,909,370
$E_{t-1}(\text{IVOL}_t)$	11.78	10.86	9.85	6.01	14.93	2.78	2,866,374
$E_t(\text{IVOL}_t)$	12.48	10.04	9.78	6.01	14.67	2.54	2,860,509
BETA	1.22	0.31	1.21	1.00	1.43	0.23	1,724,536
ln (ME)	4.35	1.97	4.17	2.87	5.65	0.38	2,779,887
ln (BE/ME)	−0.40	1.12	−0.37	−1.01	0.19	0.28	2,139,250
RET(−7, −2)	1.07	0.41	1.03	0.85	1.21	2.90	2,758,060
ln (TURN)	1.39	1.11	1.40	0.68	2.12	−0.06	2,055,982
ln (CVTURN)	4.19	0.49	4.17	3.86	4.48	0.14	2,043,104

Table 3 Variable descriptive statistics for the pooled sample, Cont.

Panel C Sample Period January 2007 to December 2017, post-sample results

Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
RET (%)	0.59	14.18	0.42	−4.85	5.32	2.36	898,204
XRET(%)	0.54	14.19	0.36	−4.91	5.26	2.36	898,204
$\ln(1 + \text{RET})$ (%)	−0.37	14.09	0.42	−4.97	5.18	−1.32	898,204
IVOL	10.47	11.93	7.37	4.63	12.74	10.41	772,197
$E_{t-1}(\text{IVOL}_t)$	10.42	8.41	8.37	5.42	14.46	4.12	757,670
$E_t(\text{IVOL}_t)$	10.38	8.86	8.36	5.44	14.33	5.20	750,858
BETA	1.25	0.33	1.24	1.01	1.50	0.18	772,124
ln (ME)	5.91	2.07	5.83	4.47	7.30	0.16	863,213
ln (BE/ME)	−0.45	1.19	−0.47	−1.11	0.09	0.60	604,351
RET(−7, −2)	1.04	0.37	1.03	0.86	1.17	2.85	742,912
ln (TURN)	2.38	1.06	2.49	1.72	3.09	−0.39	627,957
ln (CVTURN)	3.93	0.53	3.85	3.54	4.25	0.60	646,662

Panel D Sample Period July 1963 to December 2017, full-sample results

Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
RET (%)	1.04	16.28	0.15	−6.11	6.34	2.37	3,845,722
XRET(%)	0.67	16.29	−0.08	−6.49	5.99	2.37	3,845,722
$\ln(1 + \text{RET})$(%)	−0.17	15.64	0.15	−6.31	6.15	−0.67	3,845,722
IVOL	13.31	13.23	9.47	5.88	16.40	8.27	3,681,567
$E_{t-1}(\text{IVOL}_t)$	11.65	10.80	9.75	5.89	14.84	2.76	3,624,044
$E_t(\text{IVOL}_t)$	12.04	10.78	9.69	5.89	14.60	2.52	3,617,906
BETA	1.25	0.33	1.24	1.01	1.50	0.18	3,059,189
ln (ME)	4.72	2.10	4.55	3.15	6.12	0.27	3,643,100
ln (BE/ME)	−0.41	1.14	−0.40	−1.03	0.17	0.36	2,743,601
RET(−7, −2)	1.06	0.40	1.03	0.85	1.20	2.91	3,500,972
ln (TURN)	1.63	1.18	1.64	0.85	2.45	−0.08	2,683,939
ln (CVTURN)	4.12	0.51	4.11	3.76	4.44	0.19	2,689,766

Panel E Sample Period July 1926 to June 1963, pre-sample results

Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
RET (%)	1.29	13.64	0.71	−4.57	5.88	3.21	398,744
XRET(%)	1.17	13.65	0.18	−4.71	5.77	3.21	398,744
$\ln(1 + \text{RET})$ (%)	0.47	12.59	0.71	−4.68	5.72	0.10	398,744
IVOL	11.36	12.07	7.52	5.45	11.90	5.27	398,185
$E_{t-1}(\text{IVOL}_t)$	8.03	9.02	7.02	4.81	10.11	4.70	369,390
$E_t(\text{IVOL}_t)$	7.93	10.26	6.99	4.83	9.99	4.29	368,660
BETA	1.38	0.37	1.40	1.10	1.67	−0.05	296,778
ln (ME)	3.23	1.56	3.08	2.10	4.27	0.41	370,386
ln (BE/ME)	0.11	0.96	0.05	−0.43	0.60	−0.13	305,843
RET(−7, −2)	1.08	0.35	1.05	0.90	1.20	4.31	379,767
ln (TURN)	0.57	0.99	0.55	−0.12	1.22	0.23	313,896
ln (CVTURN)	4.19	0.45	4.18	3.88	4.48	0.13	313,896

Table 4 Cross-sectional simple correlations

Description: This table presents the time-series means of the cross-sectional Pearson correlations. Variables are defined in Table 3. $E^F_{t-1}(\text{IVOL}_t)$ is the EIVOL reported by Fu (2009). $E_t(\text{IVOL}_t)$ is the in-sample EIVOL estimated using information set up to month t. $E_{t-1}(\text{IVOL}_t)$ is the out-of-sample EIVOL estimated using information set up to month $t-1$. The correlation coefficients denoted by * are significant at the 1% level based on their time-series standard error. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: The results indicate that the strongly positive association between EIVOL and future stock returns exists only when EIVOL is estimated using in-sample data.

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 30

	$\ln(1+\text{RET}_t)$	IVOL_t	$E^F_{t-1}(\text{IVOL}_t)$	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)
RET_t	0.98*	0.14*	0.09*	−0.01	−0.01*	0.03*	0.02*	−0.02*	0.00
$\ln(1+\text{RET}_t)$		0.05*	0.03*	−0.03*	0.02*	0.04*	0.04*	−0.03*	−0.02*
IVOL_t			0.46*	0.34*	−0.39*	−0.05*	−0.12*	0.16*	0.31*
$E^F_{t-1}(\text{IVOL}_t)$				0.35*	−0.34*	−0.11*	−0.04*	0.20*	0.30*
BETA					−0.34*	−0.04*	−0.03*	0.41*	0.23*
ln (ME)						−0.21*	−0.03*	0.04*	−0.57*
ln (BE/ME)							0.06*	−0.12*	0.16*
RET(−7, −2)								0.00	0.06*
ln (TURN)									0.02*
ln (CVTURN)									

Panel B Sample Period July 1963 to December 2006, replication results

	$\ln(1+\text{RET}_t)$	IVOL_t	$E_t(\text{IVOL}_t)$	$E_{t-1}(\text{IVOL}_t)$	BETA	ln(ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)
RET_t	0.98*	0.12*	0.10*	−0.01	−0.01	−0.01*	0.02*	0.02*	−0.02*	0.00
$\ln(1+\text{RET}_t)$		0.05*	0.03*	−0.02*	−0.03*	0.02*	0.03*	0.03*	−0.03*	−0.01*
IVOL_t			0.42*	0.27*	0.30*	−0.35*	−0.03*	−0.10*	0.12*	0.25*
$E_t(\text{IVOL}_t)$					0.32*	−0.30*	−0.10*	−0.03*	0.20*	0.30*
$E_{t-1}(\text{IVOL}_t)$					0.30*	−0.25*	−0.08*	−0.03*	0.17*	0.25*
BETA						−0.25*	−0.05*	−0.02*	0.39*	0.18*
ln (ME)							−0.26*	−0.01*	0.04*	−0.52*
ln (BE/ME)								0.04*	−0.10*	0.14*
RET(−7, −2)									0.01	0.07*
ln (TURN)										0.01*
ln (CVTURN)										

Table 4 Cross-sectional simple correlations, Cont.

Panel C Sample Period January 2007 to December 2017, post-sample results

	$\ln(1+\text{RET}_t)$	IVOL_t	$E_t(\text{IVOL}_t)$	$E_{t-1}(\text{IVOL}_t)$	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)
RET_t	0.97*	0.12*	0.07*	−0.01	0.00	0.01*	0.02*	0.01	−0.01	−0.01*
$\ln(1+\text{RET}_t)$		0.02	0.02	−0.04*	−0.02*	0.03*	0.01	0.02*	−0.02*	−0.04*
IVOL_t			0.50*	0.34*	0.28*	−0.33*	0.04*	−0.13*	0.00*	0.34*
$E_t(\text{IVOL}_t)$					0.31*	−0.26*	−0.03*	-0.09*	0.13*	0.35*
$E_{t-1}(\text{IVOL}_t)$					0.31*	−0.22*	−0.02*	−0.02	0.12*	0.30*
BETA						−0.18*	0.06*	−0.04*	0.24*	0.20*
ln (ME)							−0.26*	0.03*	0.29*	−0.58*
ln (BE/ME)								−0.01	−0.10*	0.13*
RET(−7, −2)									0.00	0.01
ln (TURN)										0.04*
ln (CVTURN)										

Panel D Sample Period July 1963 to December 2017, full-sample results

	$\ln(1+\text{RET}_t)$	IVOL_t	$E_t(\text{IVOL}_t)$	$E_{t-1}(\text{IVOL}_t)$	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)
RET_t	0.97*	0.12*	0.08*	−0.01	−0.01	−0.01	0.02*	0.02*	−0.02*	0.00
$\ln(1+\text{RET}_t)$		0.05*	0.02*	−0.03*	−0.03*	0.02*	0.02*	0.03*	−0.03*	−0.02*
IVOL_t			0.43*	0.28*	0.28*	−0.35*	−0.01*	−0.11*	0.09*	0.27*
$E_t(\text{IVOL}_t)$					0.31*	−0.29*	−0.08*	−0.04*	0.18*	0.31*
$E_{t-1}(\text{IVOL}_t)$					0.29*	−0.25*	−0.07*	0.02*	0.16*	0.26*
BETA						−0.19*	−0.03*	−0.02*	0.37*	0.16*
ln (ME)							−0.26*	0.00	0.09*	−0.53*
ln (BE/ME)								0.03*	−0.10*	0.14*
RET(−7, −2)									0.01	0.05*
ln (TURN)										0.03*
ln (CVTURN)										

Panel E Sample Period July 1926 to June 1963, pre-sample results

	$\ln(1+\text{RET}_t)$	IVOL_t	$E_t(\text{IVOL}_t)$	$E_{t-1}(\text{IVOL}_t)$	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)
RET_t	0.99*	0.09*	0.07*	−0.01	−0.01	0.00	0.14*	0.03*	−0.02*	−0.01
$\ln(1+\text{RET}_t)$		0.06*	0.03*	−0.02*	−0.02*	0.01*	0.14*	0.03*	−0.02*	−0.02
IVOL_t			0.45*	0.38*	0.42*	−0.48*	0.34*	−0.08*	0.10*	0.27*
$E_t(\text{IVOL}_t)$					0.43*	−0.40*	0.25*	0.02	0.24*	0.31*
$E_{t-1}(\text{IVOL}_t)$					0.41*	−0.37*	0.22*	0.03*	0.23*	0.29*
BETA						−0.65*	0.42*	0.00	0.47*	0.37*
ln (ME)							−0.43*	−0.01	−0.29*	−0.47*
ln (BE/ME)								0.09*	0.14*	0.24*
RET(−7, −2)									0.02	0.04*
ln (TURN)										0.21*
ln (CVTURN)										

Table 5 Fama–MacBeth regressions

Description: The table presents the time-series averages of the slopes in cross-sectional regressions using the standard Fama and MacBeth (1973) methodology. The T-statistic is the average slope divided by its time-series standard error and is reported in brackets. The dependent variable (RET_t) is the percentage monthly return. In regression models 3, 4, and 5 of Panel A, $E^F_{t-1}(\mathrm{IVOL}_t)$ is the EIVOL reported by Fu (2009). In regression models 3, 4, and 5 of Panels B, C, D, and E, we use $E_{t-1}(\mathrm{IVOL}_t)$, which is the out-of-sample EIVOL estimated using information set up to month t. In regression models 3′, 4′, and 5′ of Panels B, C, D, and E, we use $E_t(\mathrm{IVOL}_t)$, which is the in-sample EIVOL estimated using information set up to month $t-1$. IVOL_{t-1} and IVOL_t are the IVOL of the previous month $t-1$ and the current month t. BETA, ME, and BE/ME are estimated as per Fama and French (1992). TURN is the average turnover and CVTURN is the coefficient of variation of turnovers in the previous 36 months. RET(−7, −2) is the cumulative return from month $t-7$ to $t-2$. The smallest and largest 0.5% of the explanatory variables (except BETA) are set to be equal to the next smallest and largest values. The last column reports the average R-squares of the cross-sectional regressions. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: The results indicate that there is a strong association between the in-sample EIVOL and future stock returns, while the out-of-sample EIVOL is not correlated with future stock returns. The coefficient on firm size becomes significantly positive after controlling for the in-sample EIVOL, while firm size still has a significantly negative coefficient after controlling for the out-of-sample EIVOL.

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 31

Model	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	$E^F_{t-1}(\mathrm{IVOL}_t)$	IVOL_{t-1}	IVOL_t	$\overline{R^2}$(%)
1	0.02	−0.12	0.23							3.82
	(0.08)	(−3.11)	(4.97)							
2	0.14	−0.17	0.19	0.64	−0.12	−0.44				5.73
	(0.93)	(−4.52)	(4.38)	(3.09)	(−2.05)	(−6.79)				
3							0.11			3.02
							(9.05)			
4		0.25	0.60				0.13			4.98
		(7.28)	(12.58)				(11.41)			
5		0.19	0.48	0.93	−0.48	−0.73	0.15			6.89
		(5.01)	(10.70)	(4.74)	(−7.34)	(−11.82)	(13.65)			
6		−0.21	0.18	0.67	−0.09	−0.39		−0.02		5.56
		(−5.76)	(4.04)	(3.36)	(−1.24)	(−6.48)		(−3.73)		
7		0.41	0.44	1.61	−0.55	−0.83			0.31	10.42
		(14.53)	(10.57)	(8.55)	(−8.54)	(−13.59)			(20.56)	

Table 5 Fama–MacBeth regressions, Cont.

Panel B Sample Period July 1963 to December 2006, replication results

Model	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	E_{t-1}(IVOL$_t$)	E_t(IVOL$_t$)	IVOL$_{t-1}$	IVOL$_t$	$\overline{R^2}$(%)
1	0.05	−0.13	0.20								3.67
	(0.21)	(−2.93)	(3.83)								
2	0.22	−0.17	0.16	0.84	−0.18	−0.39					5.58
	(1.37)	(−4.12)	(3.62)	(3.94)	(−3.07)	(−6.16)					
3							0.01				1.11
							(0.62)				
3′								0.12			2.85
								(8.74)			
4		−0.12	0.21				0.00				3.06
		(−3.01)	(4.10)				(0.29)				
4′		0.24	0.45					0.14			4.75
		(6.84)	(9.61)					(10.00)			
5		−0.16	0.18	0.86	−0.16	−0.36	0.01				5.36
		(−4.01)	(3.99)	(3.85)	(−2.46)	(−6.21)	(1.00)				
5′		0.24	0.49	1.10	−0.46	−0.66		0.16			7.08
		(6.80)	(11.06)	(5.30)	(−7.63)	(−11.97)		(12.01)			
6		−0.19	0.17	0.88	−0.12	−0.31			−0.03		5.64
		(−5.47)	(3.77)	(4.29)	(−1.74)	(−5.64)			(−4.23)		
7		0.34	0.30	1.76	−0.47	−0.66				0.26	9.35
		(11.61)	(6.86)	(8.87)	(−7.40)	(−11.69)				(18.82)	

Panel C Sample Period January 2007 to December 2017, post-sample results

Model	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	E_{t-1}(IVOL$_t$)	E_t(IVOL$_t$)	IVOL$_{t-1}$	IVOL$_t$	$\overline{R^2}$(%)
1	0.26	0.04	0.15								2.27
	(0.54)	(0.70)	(2.04)								
2	0.17	−0.04	0.13	0.14	−0.20	−0.51					3.61
	(0.49)	(−0.79)	(1.88)	(0.29)	(−2.48)	(−3.87)					
3							−0.02				1.11
							(−1.02)				
3′								0.07			2.31
								(2.99)			
4		0.01	0.17				−0.02				1.87
		(0.25)	(1.89)				(−1.55)				
4′		0.23	0.19					0.09			3.31
		(4.34)	(2.37)					(3.11)			
5		−0.05	0.13	0.04	−0.15	−0.49	−0.01				3.23
		(−1.10)	(1.90)	(0.09)	(−1.67)	(−4.22)	(−1.19)				
5′		0.18	0.20	0.61	−0.45	−0.96		0.12			4.72
		(4.11)	(2.78)	(1.40)	(−5.62)	(−8.95)		(4.97)			
6		−0.11	0.13	−0.10	−0.13	−0.38			−0.05		3.44
		(−2.31)	(1.82)	(−0.20)	(−1.29)	(−3.37)			(−3.93)		
7		0.44	0.17	1.28	−0.39	−1.28				0.27	7.98
		(9.77)	(2.61)	(2.79)	(−3.90)	(−11.11)				(11.84)	

Table 5 Fama–MacBeth regressions, Cont.

Panel D Sample Period July 1963 to December 2017, full-sample results

Model	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	$E_{t-1}(\text{IVOL}_t)$	$E_t(\text{IVOL}_t)$	IVOL_{t-1}	IVOL_t	$\overline{R^2}(\%)$
1	0.09	−0.09	0.19								3.39
	(0.43)	(−2.57)	(4.30)								
2	0.21	−0.14	0.16	0.70	−0.18	−0.41					5.18
	(1.43)	(−4.16)	(4.06)	(3.54)	(−3.71)	(−7.25)					
3							0.00				1.11
							(0.16)				
3′								0.11			2.75
								(9.20)			
4		−0.10	0.20				−0.00				2.82
		(−2.79)	(4.51)				(−0.36)				
4′		0.24	0.40					0.13			4.46
		(7.92)	(9.71)					(10.33)			
5		−0.14	0.17	0.70	−0.16	−0.39	0.00				4.93
		(−4.15)	(4.40)	(3.36)	(−2.87)	(−7.54)	(0.47)				
5′		0.23	0.43	1.00	−0.46	−0.72		0.15			6.60
		(7.72)	(11.22)	(5.34)	(−9.02)	(−14.62)		(12.97)			
6		−0.18	0.16	0.68	−0.12	−0.32			−0.03		5.19
		(−5.91)	(4.17)	(3.49)	(−2.08)	(−6.55)			(−5.49)		
7		0.36	0.28	1.66	−0.46	−0.79				0.26	9.07
		(14.32)	(7.33)	(9.06)	(−8.32)	(−15.20)				(21.94)	

Panel E Sample Period July 1926 to June 1963, pre-sample results

Model	BETA	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	$E_{t-1}(\text{IVOL}_t)$	$E_t(\text{IVOL}_t)$	IVOL_{t-1}	IVOL_t	$\overline{R^2}(\%)$
1	0.24	−0.12	0.26								6.70
	(0.68)	(−1.76)	(2.32)								
2	0.38	−0.17	0.21	0.64	−0.18	−0.31					9.70
	(1.41)	(−2.96)	(2.09)	(1.57)	(−2.68)	(−2.70)					
3							0.03				1.72
							(1.66)				
3′								0.19			3.56
								(8.61)			
4		−0.12	0.27				−0.00				4.98
		(−2.17)	(2.44)				(−0.20)				
4′		0.18	0.11					0.20			6.58
		(3.47)	(1.08)					(10.26)			
5		−0.19	0.24	0.71	−0.15	−0.34	0.00				8.99
		(−3.66)	(2.39)	(1.61)	(−2.02)	(−3.19)	(0.24)				
5′		0.03	0.10	0.75	−0.24	−0.73		0.22			10.49
		(0.72)	(1.12)	(1.77)	(−3.34)	(−6.66)		(12.42)			
6		−0.24	0.23	0.57	−0.14	−0.27			−0.03		9.57
		(−4.79)	(2.40)	(1.34)	(−1.91)	(−2.53)			(−3.49)		
7		0.24	0.00	1.34	−0.12	−0.64				0.24	12.91
		(4.83)	(0.05)	(3.39)	(−1.57)	(−6.38)				(14.88)	

Table 6 Summary statistics for portfolios formed on conditional idiosyncratic volatility

Description: In Panel A, each month 10 portfolios are formed on $E^F_{t-1}(\text{IVOL}_t)$, which is the EIVOL reported by Fu (2009). In Panel B, C, D, and E, each month 10 portfolios are formed on $E_{t-1}(\text{IVOL}_t)$, the out-of-sample EIVOL estimated using information set up to month $t-1$, or $E_t(\text{IVOL}_t)$, the in-sample EIVOL estimated using information set up to month t. The first portfolio (Low) consists of the 10% of stocks with the lowest EIVOL; the last portfolio (High) consists of the 10% of stocks with the highest EIVOL. Portfolios are updated monthly. The first and second rows present the time-series means of the value-weighted and the equal-weighted portfolio returns of month t, respectively. The other rows show the pooled means of IVOL of month t (IVOL_t) and market beta (BETA); medians of market capitalization (ME, reported in million $) and book to market ratio (BE/ME)are within the particular portfolio. The last row reports the alphas (intercepts) from the time-series regressions of the value-weighted portfolio excess returns on the Fama–French three factors. IVOL, returns, and alphas are reported as percentages. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: Value- and equal-weighted returns of portfolios of stocks sorted by the in-sample EIVOL increase monotonically, which is consistent with Fu (2009). Portfolios of stocks sorted by the out-of-sample EIVOL have a hump-shaped return pattern.

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 33

Variables	Portfolios formed on $E^F_{t-1}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.90	0.96	0.97	0.98	1.00	1.02	1.17	1.18	1.28	2.65
Port.EWRET$_t$	0.54	0.77	0.79	0.80	0.78	0.82	0.85	0.91	1.41	5.33
$E^F_{t-1}(\text{IVOL}_t)$	3.19	5.17	6.52	7.80	9.19	10.78	12.73	15.34	19.58	36.35
IVOL$_t$	6.74	7.80	8.98	10.29	11.80	13.50	15.46	17.72	20.81	27.29
BETA	0.90	1.00	1.08	1.16	1.23	1.29	1.36	1.40	1.44	1.46
ME($mil, med)	113.03	177.16	161.38	119.04	85.80	63.04	45.68	33.83	23.72	14.19
BE/ME(med)	0.90	0.78	0.75	0.74	0.73	0.71	0.68	0.64	0.59	0.52
FF-3F α	0.03	0.01	−0.02	−0.02	−0.05	−0.06	0.04	0.01	0.13	1.45

Table 6 Summary statistics for portfolios formed on conditional idiosyncratic volatility, Cont.

Panel B Sample Period July 1963 to December 2006, replication results

Variables	Portfolios formed on $E_t(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.89	0.95	0.98	0.99	1.00	1.03	1.18	1.18	1.27	2.58
Port.EWRET$_t$	0.58	0.74	0.79	0.80	0.77	0.85	0.87	0.88	1.47	5.89
$E_t(\text{IVOL}_t)$	3.06	5.29	6.59	7.74	9.94	11.32	12.97	15.11	18.37	31.17
IVOL$_t$	6.85	7.84	8.55	10.53	11.71	14.19	15.97	18.04	20.77	26.97
BETA	0.94	1.03	1.10	1.17	1.22	1.28	1.33	1.37	1.41	1.44
ME($mil, med)	110.71	171.18	147.71	111.75	82.27	66.55	48.27	34.67	24.35	14.82
BE/ME(med)	0.84	0.76	0.71	0.70	0.71	0.70	0.68	0.65	0.60	0.54
FF-3F α	0.01	0.05	0.01	−0.02	−0.02	−0.07	0.06	0.09	0.16	1.42

Variables	Portfolios formed on $E_{t-1}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.90	0.92	0.98	0.98	1.04	1.08	0.97	0.98	0.96	0.60
Port.EWRET$_t$	1.10	1.15	1.21	1.23	1.30	1.29	1.36	1.35	1.29	1.32
$E_{t-1}(\text{IVOL}_t)$	3.00	5.26	6.58	7.75	9.97	11.38	13.07	15.29	18.76	34.44
IVOL$_t$	7.29	8.41	9.01	10.98	12.15	14.58	16.30	18.23	20.56	24.28
BETA	0.95	1.03	1.10	1.17	1.23	1.28	1.33	1.37	1.41	1.44
ME($mil, med)	107.53	168.43	146.80	111.10	81.69	65.84	47.73	34.32	24.20	15.74
BE/ME(med)	0.84	0.76	0.70	0.70	0.71	0.70	0.69	0.66	0.60	0.52
FF-3F α	-0.01	0.03	0.02	−0.03	0.02	0.01	−0.08	−0.08	−0.15	−0.46

Panel C Sample Period January 2007 to December 2017, post-sample results

Variables	Portfolios formed on $E_t(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.84	0.82	0.81	0.82	0.86	0.86	0.79	0.85	0.95	2.03
Port.EWRET$_t$	0.43	0.50	0.59	0.62	0.59	0.69	0.67	0.65	0.75	3.39
$E_t(\text{IVOL}_t)$	2.78	4.80	6.17	7.39	9.69	11.17	13.00	15.38	18.98	32.28
IVOL$_t$	4.20	5.34	6.21	8.12	9.23	11.48	13.00	14.89	17.64	24.15
BETA	1.00	1.02	1.09	1.16	1.23	1.28	1.34	1.38	1.42	1.47
ME ($mil, med)	308.64	717.92	859.04	930.37	738.40	620.83	425.40	283.78	174.35	85.41
BE/ME (med)	0.78	0.70	0.63	0.61	0.60	0.61	0.62	0.62	0.61	0.59
FF-3F α	0.13	0.18	0.02	0.01	−0.03	−0.04	−0.19	−0.29	−0.20	0.88

Variables	Portfolios formed on $E_{t-1}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.73	0.68	0.80	0.74	0.80	0.77	0.66	0.83	0.69	0.90
Port.EWRET$_t$	0.53	0.66	0.79	0.82	0.80	0.90	0.89	0.83	0.68	0.66
$E_{t-1}(\text{IVOL}_t)$	2.73	4.76	6.14	7.37	9.68	11.19	13.04	15.49	19.23	34.43
IVOL$_t$	4.40	5.68	6.55	8.42	9.51	11.78	13.25	15.08	17.62	22.05
BETA	1.01	1.03	1.09	1.16	1.23	1.28	1.34	1.38	1.42	1.47
ME ($mil, med)	307.15	709.76	843.76	926.30	742.40	619.19	417.07	282.50	173.66	89.32
BE/ME (med)	0.78	0.70	0.63	0.61	0.60	0.61	0.62	0.62	0.61	0.58
FF-3F α	-0.03	0.03	0.11	−0.08	−0.12	−0.16	−0.36	−0.26	−0.44	−0.26

Table 6 Summary statistics for portfolios formed on conditional idiosyncratic volatility, Cont.

Panel D Sample Period July 1963 to December 2017, full-sample results

Variables	Portfolios formed on $E_t(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.89	0.92	0.93	0.93	0.98	1.01	1.08	1.10	1.20	2.46
Port.EWRET$_t$	0.55	0.71	0.77	0.76	0.75	0.80	0.82	0.85	1.31	5.28
$E_t(\text{IVOL}_t)$	3.00	5.19	6.50	7.67	9.89	11.29	12.98	15.16	8.49	31.39
IVOL$_t$	6.31	7.33	8.08	10.04	11.20	13.63	15.36	17.40	20.13	26.39
BETA	0.95	1.03	1.11	1.17	1.23	1.29	1.34	1.38	1.41	1.45
ME ($mil, med)	140.78	228.66	216.34	168.96	125.24	100.43	72.35	51.17	35.60	21.32
BE/ME (med)	0.83	0.75	0.69	0.68	0.68	0.68	0.67	0.65	0.60	0.55
FF-3F α	0.06	0.08	0.01	−0.02	−0.01	−0.04	0.01	0.01	0.08	1.23

Variables	Portfolios formed on $E_{t-1}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.87	0.87	0.94	0.94	0.99	1.02	0.91	0.95	0.91	0.66
Port.EWRET$_t$	1.00	1.05	1.12	1.15	1.20	1.21	1.27	1.25	1.15	1.16
$E_{t-1}(\text{IVOL}_t)$	2.95	5.16	6.49	7.67	9.91	11.34	13.07	15.33	18.85	34.44
IVOL$_t$IVOL	6.71	7.86	8.51	10.46	11.62	14.01	15.69	17.59	19.97	23.83
BETA	0.95	1.03	1.11	1.17	1.24	1.29	1.34	1.38	1.41	1.45
ME ($mil, med)	136.79	225.11	215.04	167.73	123.62	98.93	71.44	50.61	35.35	22.56
BE/ME (med)	0.83	0.75	0.69	0.68	0.68	0.68	0.67	0.65	0.60	0.54
FF-3F α	0.02	0.04	0.04	−0.02	0.01	0.00	−0.14	−0.13	−0.21	−0.47

Panel E Sample Period July 1926 to June 1963, pre-sample results

Variables	Portfolios formed on $E_t(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.79	0.96	0.92	1.00	0.98	1.06	1.15	1.18	1.42	2.96
Port.EWRET$_t$	0.58	0.79	0.83	0.93	0.95	1.04	1.04	1.21	1.74	4.48
$E_{t-1}(\text{IVOL}_t)$	2.55	4.07	4.96	5.76	6.58	7.50	8.59	10.02	12.21	19.61
IVOL$_t$	7.14	7.58	8.33	9.18	10.06	11.11	12.45	14.05	16.61	23.37
BETA	1.07	1.14	1.21	1.27	1.34	1.39	1.45	1.52	1.59	1.68
ME ($mil, med)	51.38	47.80	38.83	31.43	26.50	21.78	17.97	14.40	10.35	5.71
BE/ME (med)	0.76	0.86	0.92	0.98	1.03	1.10	1.18	1.25	1.33	1.48
FF-3F α	0.02	0.05	−0.07	−0.06	−0.12	−0.15	−0.12	−0.11	0.08	1.53

Variables	Portfolios formed on $E_{t-1}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.87	1.03	0.94	1.25	1.06	1.05	1.00	0.92	1.06	1.02
Port.EWRET$_t$	1.06	1.15	1.25	1.32	1.68	1.39	1.37	1.40	1.32	1.32
$E_{t-1}(\text{IVOL}_t)$	2.48	4.02	4.91	5.72	6.57	7.51	8.64	10.12	12.44	21.15
IVOL$_t$	7.98	8.01	8.74	9.52	10.31	11.35	12.51	14.11	16.37	21.78
BETA	1.08	1.15	1.21	1.28	1.34	1.39	1.45	1.52	1.59	1.67
ME ($mil, med)	48.95	47.74	38.17	31.05	26.23	21.90	17.96	14.28	10.47	6.19
BE/ME (med)	0.78	0.86	0.93	0.99	1.02	1.11	1.18	1.24	1.33	1.44
FF-3F α	0.09	0.10	−0.03	0.21	−0.03	−0.12	−0.20	−0.36	−0.23	−0.24

Table 7 Return dispersion of portfolios sorted by idiosyncratic volatility

Description: This table reports differences in the monthly percentage returns of portfolios sorted by IVOLs. In each month t, stocks are sorted into quintiles on the basis of one-month-lagged IVOL (IVOL_{t-1}). Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) IVOL in the last month. N is the number of firm-month observations for the pooled sample. The numbers presented in other columns are means with T-statistics in brackets, if any. ME is market capitalization (reported in millions of $). MKT share is the percentage of market capitalization of a portfolio out of total market capitalization of stocks in the sample. RET is the equal-weighted average raw return of a portfolio. VWXRET is the value-weighted excess returns for a portfolio. The last column reports the alphas (intercepts) from the time-series regressions of the value-weighted portfolio excess returns on the Fama–French three factors. IVOL, returns, alphas, and percentage of market shares are reported as percentages. T-statistics are reported in brackets. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: For portfolios of stocks sorted by one-month-lagged IVOL of the replication, there is a negative yet not strictly monotonic relation between IVOL and future portfolio returns, but one-month-lagged portfolio returns increase monotonically as IVOL increases. The replicated results are consistent with those reported by Fu (2009).

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 35

IVOL Portfolio	N	IVOL $t-1$	ME ($mil)	MKT share (%)	RET t	VWXRET t	FF-3F α t	RET $t-1$	VWXRET $t-1$	FF-3F α $t-1$
1 (Low)	574,915	4.30	1885.04	43.00	1.10	0.52	0.07	0.44	0.39	−0.04
							(1.75)			(−0.94)
2	574,293	7.58	1451.81	33.08	1.34	0.57	0.03	0.55	0.63	0.05
							(0.76)			(1.07)
3	574,694	11.06	653.93	14.91	1.37	0.64	0.0	0.61	0.76	−0.02
							(0.83)			(−0.13)
4	574,707	16.17	294.70	6.72	1.19	0.29	−0.35	0.77	0.79	0.26
							(−3.60)			(2.65)
5 (High)	574,915	32.32	100.44	2.29	1.08	−0.40	−1.15	4.11	1.66	0.85
							(−7.00)			(3.14)
5 − 1							−1.22			0.89
							(−6.45)			(3.26)

Panel B Sample Period July 1963 to December 2006, replication results

IVOL Portfolio	N	IVOL $t-1$	ME ($mil)	MKT share (%)	RET t	VWXRET t	FF-3F α t	RET $t-1$	VWXRET $t-1$	FF-3F α $t-1$
1 (Low)	577,453	3.17	1874.60	43.42	1.07	0.47	0.05	0.41	0.36	−0.04
							(0.75)			(−0.67)
2	577,453	6.88	1536.91	32.83	1.33	0.53	0.04	0.59	0.61	0.09
							(0.89)			(2.11)
3	577,453	10.25	722.93	15.55	1.44	0.62	0.02	0.68	0.77	0.14
							(0.26)			(2.24)
4	577,453	15.23	293.02	5.96	1.25	0.30	−0.41	0.83	0.81	0.03
							(−4.09)			(0.29)
5 (High)	578,489	32.01	106.13	2.24	1.11	−0.38	−1.28	3.88	1.76	0.73
							(−7.53)			(2.64)
5 − 1							−1.32			0.77
							(−6.96)			(2.61)

Table 7 Return dispersion of portfolios sorted by idiosyncratic volatility, Cont.

Panel C Sample Period January 2007 to December 2017, post-sample results

IVOL Portfolio	N	IVOL $t-1$	ME ($mil)	MKT share (%)	RET t	VWXRET t	FF-3F α t	RET $t-1$	VWXRET $t-1$	FF-3F α $t-1$
1 (Low)	153,906	2.97	7472.94	40.78	0.77	0.76	0.15	0.41	0.52	−0.07
							(2.27)			(−0.98)
2	153,906	5.25	5829.23	31.12	0.87	0.67	−0.09	0.63	0.76	−0.02
							(−1.36)			(−0.29)
3	153,906	7.67	3191.51	16.98	0.86	0.65	−0.26	0.69	0.74	−0.17
							(−1.89)			(−1.26)
4	153,906	11.46	1538.58	8.13	0.80	0.66	−0.38	0.76	1.00	−0.10
							(−1.58)			(−0.38)
5 (High)	154,169	24.38	568.49	2.98	0.47	0.37	−0.70	1.79	2.15	0.73
							(−2.03)			(1.22)
5 − 1							−0.85			0.80
							(−2.31)			(1.28)

Panel D Sample Period July 1963 to December 2017, full-sample results

IVOL Portfolio	N	IVOL $t-1$	ME ($mil)	MKT share (%)	RET t	VWXRET t	FF-3F α t	RET $t-1$	VWXRET $t-1$	FF-3F α $t-1$
1 (Low)	731,359	2.96	2967.33	48.28	1.01	0.53	0.09	0.41	0.41	−0.00
							(1.72)			(−0.04)
2	731,359	6.42	2173.50	30.48	1.24	0.56	0.01	0.64	0.64	0.08
							(0.34)			(2.21)
3	731,359	9.58	1073.54	13.26	1.33	0.63	−0.04	0.69	0.75	0.07
							(−0.75)			(1.18)
4	731,359	14.24	509.15	5.85	1.16	0.37	−0.40	0.81	0.89	0.03
							(−4.32)			(0.27)
5 (High)	732,658	29.47	184.52	2.13	0.98	−0.23	−1.16	3.35	1.86	0.72
							(−7.58)			(2.92)
5 − 1							−1.25			0.72
							(−7.25)			(2.72)

Panel E Sample Period July 1926 to June 1963, pre-sample results

IVOL Portfolio	N	IVOL $t-1$	ME ($mil)	MKT share (%)	RET t	VWXRET t	FF-3F α t	RET $t-1$	VWXRET $t-1$	FF-3F α $t-1$
1 (Low)	79,154	4.27	277.54	60.08	1.16	0.87	0.13	0.72	0.54	−0.20
							(4.73)			(−5.44)
2	79,154	6.81	99.97	19.53	1.28	0.92	−0.06	0.53	0.87	−0.17
							(−1.10)			(−2.87)
3	79,154	9.24	60.25	11.00	1.34	0.96	−0.20	0.87	1.20	−0.01
							(−2.79)			(−0.06)
4	79,154	13.02	35.64	6.35	1.38	0.83	−0.40	1.51	2.03	0.67
							(−4.21)			(5.28)
5 (High)	80,056	27.46	16.58	3.03	1.67	0.89	−0.26	3.61	3.30	1.93
							(−1.46)			(7.68)
5 − 1							−0.39			2.13
							(−2.11)			(7.99)

Table 8 Return dispersion of high-IVOL stocks sorted by the one-month-lagged return

Description: This table examines the impact of return reversal on the high-IVOL stocks. At month $t-1$, we identify 40% of the stocks that have the highest IVOLs and divide them into quintiles on the basis of their contemporaneous returns. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) RET_{t-1}. N is the number of firm-month observations for the pooled sample. RET is the raw monthly return. VWXRET (EWXRET) are the time-series mean value-weighted (equal-weighted) excess returns for the portfolio, which are used to calculate the FF-3F alphas in the time-series regressions. ME is market capitalization (reported in millions of $). IVOL is the idiosyncratic volatility. IVOL, returns, and alphas are reported as percentages. T-statistics are reported in brackets. Panel A shows the results reported in Fu (2009) for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006. Panel B reports our replicated results for the same sample period as in Fu (2009). Panel C reports the post-sample results for stocks traded in the three stock exchanges from January 2007 to December 2017. Panel D reports the full-sample results for stocks traded in the three stock exchanges from July 1963 to December 2017. Panel E reports the pre-sample results for stocks traded in the three stock exchanges from July 1926 to June 1963.

Interpretation: The results indicate that there is a strong short-term return reversal effect among stocks with very high IVOL.

Panel A Sample Period July 1963 to December 2006, as reported by Fu (2009) on p. 36

Portfolio sorted by RET_{t-1}	N	RET $t-1$	RET t	EWXRET t	VWXRET t	IVOL t	ME $t-1$	FF-3F α EWXRET_t	FF-3F α VWXRET_t
1 (Low)	232,405	−22.67	3.35	2.84	0.56	28.10	155.23	1.77	−0.33
								(6.55)	(−1.52)
2	223,492	−8.39	1.17	0.93	0.35	21.63	193.63	−0.07	−0.41
								(−0.46)	(−2.67)
3	228,808	0.00	0.90	0.67	0.09	20.32	193.69	−0.48	−0.57
								(−3.62)	(−4.54)
4	233,511	9.15	0.45	0.02	−0.06	19.35	288.15	−0.83	−0.70
								(−7.23)	(−6.02)
5 (High)	231,406	33.78	−0.21	−0.62	−0.10	21.15	216.79	−1.40	−0.69
								(−9.49)	(−5.66)

Panel B Sample Period July 1963 to December 2006, replication results

Portfolio sorted by RET_{t-1}	N	RET $t-1$	RET t	EWXRET t	VWXRET t	IVOL t	ME $t-1$	FF-3F α EWXRET_t	FF-3F α VWXRET_t
1 (Low)	230,215	−20.11	3.09	2.62	0.62	25.37	173.12	1.50	−0.17
								(5.59)	(−0.87)
2	230,215	−7.07	1.43	0.96	0.23	19.87	187.29	−0.06	−0.57
								(−0.37)	(−4.07)
3	230,215	0.30	0.97	0.50	0.19	18.62	190.42	−0.48	−0.52
								(−3.64)	(−3.81)
4	230,215	8.61	0.54	0.07	0.02	17.79	225.73	−0.82	−0.71
								(−7.16)	(−6.19)
5 (High)	231,255	30.88	−0.16	−0.63	−0.08	19.51	231.69	−1.46	−0.71
								(−9.93)	(−4.36)

Table 8 Return dispersion of high-idiosyncratic volatility stocks sorted by the one-month-lagged return, Cont.

Panel C Sample Period January 2007 to December 2017, post-sample results

Portfolio sorted by RET_{t-1}	N	RET $t-1$	RET t	EWXRET t	VWXRET t	IVOL t	ME $t-1$	FF-3F α $EWXRET_t$	FF-3F α $VWXRET_t$
1 (Low)	61,427	−22.04	1.36	1.31	0.78	20.75	942.37	0.24	−0.42
								(0.50)	(−0.91)
2	61,427	−8.11	0.54	0.49	0.60	15.84	1125.35	−0.47	−0.46
								(−1.58)	(−1.61)
3	61,427	−0.74	0.50	0.45	0.42	14.72	1053.74	−0.43	−0.59
								(−1.73)	(−2.22)
4	61,427	7.23	0.52	0.46	0.26	14.24	1152.84	−0.39	−0.72
								(−1.61)	(−2.96)
5 (High)	61,690	28.22	0.25	0.20	0.15	16.01	1009.31	−0.70	−0.75
								(−2.25)	(−2.39)

Panel D Sample Period July 1963 to December 2017, full-sample results

Portfolio sorted by RET_{t-1}	N	RET $t-1$	RET t	EWXRET t	VWXRET t	IVOL t	ME $t-1$	FF-3F α $EWXRET_t$	FF-3F α $VWXRET_t$
1 (Low)	291,642	−20.50	2.74	2.36	0.66	24.44	336.87	1.26	−0.27
								(5.37)	(−1.49)
2	291,642	−7.28	1.25	0.87	0.57	19.05	371.39	−0.10	−0.29
								(−0.69)	(−2.36)
3	291,642	0.09	0.88	0.49	0.20	17.84	342.22	−0.42	−0.57
								(−3.58)	(−4.75)
4	291,642	8.33	0.54	0.15	0.04	17.07	370.27	−0.70	−0.72
								(−6.76)	(−6.85)
5 (High)	292,245	30.34	-0.08	−0.47	−0.03	18.80	321.00	−1.29	−0.73
								(−9.67)	(−5.14)

Panel E Sample Period July 1926 to June 1963, pre-sample results

Portfolio sorted by RET_{t-1}	N	RET $t-1$	RET t	EWXRET t	VWXRET t	IVOL t	ME $t-1$	FF-3F α $EWXRET_t$	FF-3F α $VWXRET_t$
1 (Low)	31,466	−13.04	4.06	3.95	2.59	22.32	23.31	2.24	1.26
								(9.24)	(5.00)
2	31,466	−4.19	1.92	1.81	1.37	17.44	25.08	0.29	0.04
								(2.04)	(0.24)
3	31,466	1.03	1.37	1.26	1.06	16.48	27.02	−0.14	−0.12
								(−1.25)	(−0.82)
4	31,466	6.95	0.77	0.66	0.59	16.27	29.33	-0.73	-0.57
								(−6.28)	(−3.80)
5 (High)	32,327	21.50	−0.41	−0.52	−0.38	18.47	26.59	−1.84	−1.55
								(−10.93)	(−7.67)

Table 9 Robustness check 1: Idiosyncratic volatility calculated under different methods

Description: The table presents the time-series averages of the slopes in Fama–MacBeth cross-sectional regressions where the dependent variable is the realized monthly return (RET_t) and one-month-lagged (IVOL_{t-1}) or contemporaneous IVOL (IVOL_t) calculated using data of different frequency during different time windows. Under Methods 1, 2, and 3, IVOL is estimated using daily data in the previous three months (from month $t-2$ to month t), previous six months (from month $t-5$ to month t), or previous 12 months (from month $t-11$ to month t). For each stock i, the excess daily returns of each individual stock are regressed on the daily Fama–French three factors during a time window:

$$R_{i,d} - r_{f,d} = \alpha_{i,t} + b_{i,t}(R_{m,d} - r_{f,d}) + s_{i,t}\text{SMB}_d + h_{i,t}\text{HML}_d + \varepsilon_{i,d}.$$

The (monthly) IVOL of the stock is the product of the standard deviation of the regression residuals and the square root of the average number of trading days per month in a time window. Under Methods 4, 5, 6, and 7, IVOL is estimated using monthly data in the previous year (from month $t-11$ to month t), previous three years (from month $t-35$ to month t), previous five years (from month $t-59$ to month t), and the whole sample period up to month t (from month 1 to month t). For each stock i, the excess monthly returns of each individual stock are regressed on the monthly Fama–French three factors during a time window. The (monthly) IVOL of the stock is the standard deviation of the regression residuals. Other control variables are defined in Table 5. The T-statistic is the average slope divided by its time-series standard error and is reported in brackets. Panels A.1 and A.2 report results for stocks traded in the NYSE, Amex, or Nasdaq from July 1963 to December 2006 for IVOL calculated using daily data and monthly data, respectively. Panels B.1 and B.2 report the results for stocks traded in the three stock exchanges from January 2007 to December 2017 for IVOL calculated using daily data and monthly data, respectively. Panels C.1 and C.2 report the results for stocks traded in the three stock exchanges from July 1963 to December 2017 for IVOL calculated using daily data and monthly data, respectively. Panels D.1 and D.2 report the results for stocks traded in the three stock exchanges from July 1926 to June 1963 for IVOL calculated using daily data and monthly data, respectively.

Interpretation: The regression coefficient of stock returns on contemporaneous IVOL is significantly positive regardless of the data frequency and the time window of the data used to estimate IVOL. However, the negative relation between lagged IVOL and stock returns is sensitive to the way that IVOL is estimated, which is consistent with Bali and Cakici (2008).

Table 9 Robustness check 1: Idiosyncratic volatility calculated under different methods, Cont.

Panel A.1 Sample Period July 1963 to December 2006, IVOL estimated using daily data

Method	Model	ln(ME)	Ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
1	1						−0.02		2.38
							(−1.31)		
	2							0.10	3.61
								(6.51)	
	3	−0.17	0.16				−0.03		3.73
		(−4.97)	(3.18)				(−2.71)		
	4	0.19	0.31					0.14	4.93
		(6.25)	(6.44)					(8.90)	
	5	−0.21	0.15	0.95	−0.12	−0.34	−0.03		5.68
		(−6.25)	(3.19)	(4.59)	(−1.93)	(−6.14)	(−3.21)		
	6	0.15	0.25	1.36	−0.38	−0.66		0.17	6.82
		(4.98)	(5.56)	(6.92)	(−6.38)	(−11.36)		(11.01)	
2	1						−0.01		2.56
							(−0.67)		
	2							0.06	3.17
								(3.85)	
	3	−0.16	0.17				−0.02		3.83
		(−4.71)	(3.44)				(−1.79)		
	4	0.08	0.27					0.09	4.42
		(2.50)	(5.63)					(5.41)	
	5	−0.20	0.15	0.97	−0.13	−0.35	−0.02		5.76
		(−6.21)	(3.33)	(4.69)	(−2.13)	(−6.32)	(−2.03)		
	6	0.04	0.22	1.10	−0.30	−0.56		0.11	6.35
		(1.32)	(5.02)	(5.44)	(−5.06)	(−9.56)		(6.97)	
3	1						0.00		2.69
							(0.09)		
	2							0.04	3.03
								(2.61)	
	3	−0.13	0.18				−0.01		3.86
		(−3.97)	(3.65)				(−0.75)		
	4	0.02	0.25					0.06	4.20
		(0.49)	(5.18)					(3.38)	
	5	−0.17	0.16	0.96	−0.16	−0.40	−0.00		5.80
		(−5.44)	(3.55)	(4.53)	(−2.65)	(−7.10)	(−0.21)		
	6	−0.02	0.21	1.04	−0.27	−0.53		0.08	6.15
		(−0.55)	(4.73)	(4.99)	(−4.55)	(−9.22)		(5.04)	

Table 9 Robustness check 1, Cont.

Panel A.2 Sample Period July 1963 to December 2006, IVOL estimated using monthly data

Method	Model	ln (ME)	Ln(BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
4	1						−0.01		1.79
							(−0.86)		
	2							0.18	3.17
								(9.94)	
	3	−0.17	0.16				−0.03		3.28
		(−4.18	(3.11)				(−2.38)		
	4	0.24	0.45					0.23	4.65
		(6.53)	(9.24)					(13.21)	
	5	−0.21	0.12	0.97	−0.13	−0.33	−0.03		5.38
		(−5.51)	(2.65)	(4.16)	(−2.09)	(−5.80)	(−3.62)		
	6	0.16	0.35	0.65	−0.55	−0.89		0.27	6.78
		(4.31)	(7.59)	(2.88)	(−9.68)	(−14.27)		(16.15)	
5	1						−0.00		2.03
							(−0.20)		
	2							0.08	2.45
								(4.43)	
	3	−0.16	0.16				−0.02		3.38
		(−4.06)	(3.42)				(−1.07)		
	4	0.04	0.34					0.09	3.75
		(0.99)	(7.30)					(5.36)	
	5	−0.18	0.15	0.91	−0.17	−0.39	0.00		5.37
		(−5.06)	(3.24)	(3.97)	(−2.85)	(−6.79)	(0.04)		
	6	−0.01	0.31	0.96	−0.37	−0.59		0.12	5.74
		(−0.17)	(6.79)	(4.33)	(−6.66)	(−9.95)		(7.65)	
6	1						−0.00		2.03
							(−0.19)		
	2							0.08	2.45
								(4.44)	
	3	−0.16	0.16				−0.02		3.38
		(−4.06)	(3.43)				(−1.06)		
	4	0.04	0.34					0.09	3.75
		(1.00)	(7.31)					(5.37)	
	5	−0.18	0.15	0.91	−0.17	−0.39	0.00		5.37
		(−5.06)	(3.25)	(3.97)	(−2.86)	(−6.78)	(0.05)		
	6	−0.01	0.31	0.96	−0.37	−0.59		0.12	5.74
		(−0.17)	(6.79)	(4.32)	(−6.66)	(−9.94)		(7.65)	
7	1						−0.00		2.02
							(−0.20)		
	2							0.08	2.45
								(4.45)	
	3	−0.16	0.16				−0.02		3.38
		(−4.06)	(3.43)				(−1.06)		
	4	0.04	0.34					0.09	3.75
		(1.00)	(7.32)					(5.38)	
	5	−0.18	0.15	0.90	−0.17	−0.39	0.00		5.37
		(−5.05)	(3.24)	(3.97)	(−2.86)	(−6.77)	(0.05)		
	6	−0.01	0.31	0.96	−0.37	−0.59		0.12	5.74
		(−0.17)	(6.78)	(4.32)	(−6.66)	(−9.95)		(7.65)	

Table 9 Robustness check 1, Cont.

Panel B.1 Sample Period January 2007 to December 2017, IVOL estimated using daily data

Method	Model	ln(ME)	Ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
1	1						−0.05		1.66
							(−2.61)		
	2							0.05	2.44
								(2.18)	
	3	−0.07	0.14				−0.06		2.29
		(−1.25)	(1.79)				(−3.42)		
	4	0.28	0.17					0.09	3.19
		(4.71)	(2.31)					(3.73)	
	5	−0.12	0.12	−0.06	−0.11	−0.33	−0.05		3.59
		(−2.64)	(1.68)	(−0.11)	(−1.13)	(−3.08)	(−3.60)		
	6	0.19	0.15	0.55	−0.30	−0.98		0.12	4.57
		(4.46)	(2.28)	(1.20)	(−3.22)	(−8.46)		(5.35)	
2	1						−0.04		1.78
							(−2.35)		
	2							0.02	2.14
								(0.79)	
	3	−0.07	0.13				−0.06		2.40
		(−1.20)	(1.66)				(−3.09)		
	4	0.17	0.16					0.04	2.79
		(2.96)	(2.07)					(1.82)	
	5	−0.12	0.11	0.01	−0.10	−0.31	−0.06		3.67
		(−2.65)	(1.55)	(0.02)	(−1.03)	(−2.92)	(−3.25)		
	6	0.10	0.14	0.31	−0.26	−0.84		0.07	4.11
		(2.41)	(2.05)	(0.65)	(−2.81)	(−7.35)		(3.28)	
3	1						−0.04		1.80
							(−1.91)		
	2							0.00	2.00
								(0.08)	
	3	−0.07	0.12				−0.06		2.44
		(−1.17)	(1.50)				(−2.69)		
	4	0.10	0.14					0.01	2.63
		(1.61)	(1.83)					(0.47)	
	5	−0.11	0.10	−0.00	−0.09	−0.3	−0.05		3.69
		(−2.52)	(1.42)	(−0.01)	(−1.01)	(−2.80)	(−2.78)		
	6	0.04	0.12	0.18	−0.22	−0.71		0.04	3.92
		(0.99)	(1.82)	(0.37)	(−2.45)	(−6.18)		(1.67)	

Table 9 Robustness check 1, Cont.

Panel B.2 Sample Period January 2007 to December 2017, IVOL estimated using monthly data

Method	Model	ln(ME)	Ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
4	1						−0.04		1.56
							(−1.91)		
	2							0.11	2.66
								(3.63)	
	3	−0.03	0.14				−0.06		2.20
		(−0.56)	(1.59)				(−2.66)		
	4	0.28	0.19					0.16	3.37
		(5.02)	(2.55)					(4.90)	
	5	−0.09	0.10	0.07	−0.08	−0.37	−0.05		3.51
		(−2.08)	(1.50)	(0.12)	(−1.00)	(−3.23)	(−2.99)		
	6	0.23	0.19	0.22	−0.56	−1.16		0.21	4.77
		(5.25)	(2.80)	(0.50)	(−7.34)	(−10.00)		(7.27)	
5	1						−0.03		1.61
							(−1.13)		
	2							0.06	2.12
								(1.96)	
	3	−0.02	0.13				−0.04		2.30
		(−0.34)	(1.45)				(−1.58)		
	4	0.18	0.20					0.08	2.82
		(2.95)	(2.29)					(2.55)	
	5	−0.07	0.11	0.08	−0.11	−0.41	−0.03		3.49
		(−1.56)	(1.49)	(0.15)	(−1.46)	(−3.57)	(−1.29)		
	6	0.18	0.22	0.29	−0.53	−1.10		0.15	4.10
		(3.73)	(2.99)	(0.61)	(−7.30)	(−8.79)		(4.78)	
6	1						−0.02		1.54
							(−1.00)		
	2							0.04	1.90
								(1.54)	
	3	−0.01	0.13				−0.03		2.25
		(−0.22)	(1.46)				(−1.37)		
	4	0.15	0.20					0.06	2.61
		(2.42)	(2.22)					(2.01)	
	5	−0.06	0.11	0.08	−0.13	−0.44	−0.02		3.44
		(−1.38)	(1.56)	(0.16)	(−1.65)	(−4.00)	(−0.97)		
	6	0.15	0.22	0.26	−0.46	−0.95		0.12	3.85
		(3.06)	(2.94)	(0.54)	(−6.31)	(−8.05)		(4.13)	
7	1						−0.01		1.24
							(−0.71)		
	2							0.03	1.42
								(1.20)	
	3	0.01	0.15				−0.02		1.92
		(0.22)	(1.66)				(−1.09)		
	4	0.11	0.21					0.04	2.09
		(1.98)	(2.27)					(1.61)	
	5	−0.05	0.13	0.07	−0.16	−0.50	−0.01		3.28
		(−0.98)	(1.85)	(0.13)	(−1.80)	(−4.40)	(−0.46)		
	6	0.08	0.21	0.15	−0.33	−0.68		0.07	3.44
		(1.73)	(2.88)	(0.30)	(−3.94)	(−6.15)		(3.57)	

Table 9 Robustness check 1, Cont.

Panel C.1 Sample Period July 1963 to December 2017, IVOL estimated using daily data

Method	Model	ln(ME)	Ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
1	1						−0.02		2.24
							(−2.10)		
	2							0.09	3.38
								(6.85)	
	3	−0.15	0.16				−0.04		3.44
		(−5.08)	(3.62)				(−3.75)		
	4	0.21	0.28					0.13	4.58
		(7.66)	(6.83)					(9.63)	
	5	−0.19	0.14	0.75	−0.12	−0.34	−0.04		5.26
		(−6.76)	(3.58)	(3.82)	(−2.20)	(−6.87)	(−4.26)		
	6	0.16	0.23	1.19	−0.36	−0.72		0.16	6.36
		(6.19)	(6.00)	(6.56)	(−7.12)	(−13.89)		(12.17)	
2	1						−0.02		2.41
							(−1.38)		
	2							0.05	2.96
								(3.90)	
	3	−0.14	0.16				−0.03		3.54
		(−4.81)	(3.81)				(−2.77)		
	4	0.10	0.25					0.08	4.09
		(3.51)	(5.99)					(5.69)	
	5	−0.18	0.14	0.78	−0.12	−0.34	−0.03		5.34
		(−6.72)	(3.67)	(4.00)	(−2.36)	(−6.96)	(−3.03)		
	6	0.05	0.21	0.94	−0.29	−0.62		0.10	5.90
		(2.05)	(5.42)	(5.01)	(−5.73)	(−11.77)		(7.66)	
3	1						−0.01		2.51
							(−0.51)		
	2							0.03	2.82
								(2.50)	
	3	−0.12	0.17				−0.02		3.57
		(−4.10)	(3.95)				(−1.66)		
	4	0.03	0.23					0.05	3.88
		(1.14)	(5.48)					(3.32)	
	5	−0.16	0.15	0.77	−0.15	−0.38	−0.01		5.37
		(−5.97)	(3.82)	(3.88)	(−2.83)	(−7.60)	(−1.14)		
	6	−0.00	0.19	0.86	−0.26	−0.57		0.07	5.70
		(−0.17)	(5.06)	(4.49)	(−5.13)	(−11.00)		(5.30)	

Table 9 Robustness check 1, Cont.

Panel C.2 Sample Period July 1963 to December 2017, IVOL estimated using monthly data

Method	Model	ln(ME)	Ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
4	1						−0.02		1.74
							(−1.50)		
	2							0.17	3.07
								(10.54)	
	3	−0.14	0.16				−0.03		3.07
		(−4.13)	(3.48)				(−3.28)		
	4	0.25	0.40					0.22	4.39
		(7.89)	(9.49)					(14.00)	
	5	−0.19	0.12	0.79	−0.12	−0.34	−0.04		5.00
		(−5.86)	(3.01)	(3.65)	(−2.30)	(−6.64)	(−4.53)		
	6	0.17	0.31	0.57	−0.55	−0.94		0.26	6.37
		(5.68)	(8.07)	(2.80)	(−11.52)	(−17.12)		(17.68)	
5	1						−0.01		1.95
							(−0.56)		
	2							0.08	2.39
								(4.83)	
	3	−0.13	0.16				−0.02		3.10
		(−3.92)	(3.71)				(−1.40)		
	4	0.07	0.31					0.08	3.43
		(2.05)	(7.58)					(5.66)	
	5	−0.16	0.14	0.74	−0.16	−0.39	−0.00		4.95
		(−5.28)	(3.56)	(3.50)	(−3.17)	(−7.67)	(−0.46)		
	6	0.03	0.29	0.82	−0.40	−0.70		0.12	5.41
		(1.07)	(7.41)	(4.10)	(−8.59)	(−12.77)		(8.98)	
6	1						−0.01		1.93
							(−0.51)		
	2							0.08	2.34
								(4.70)	
	3	−0.13	0.16				−0.02		3.15
		(−3.87)	(3.72)				(−1.40)		
	4	0.06	0.32					0.08	3.52
		(1.86)	(7.54)					(5.67)	
	5	−0.16	0.14	0.74	−0.16	−0.40	−0.00		4.98
		(−5.23)	(3.59)	(3.50)	(−3.24)	(−7.85)	(−0.33)		
	6	0.03	0.29	0.82	−0.39	−0.66		0.12	5.36
		(0.86)	(7.39)	(4.03)	5 (−8.30)	(−12.43)		(8.68)	
7	1						−0.01		1.87
							(−0.39)		
	2							0.07	2.24
								(4.60)	
	3	−0.12	0.16				−0.02		3.09
		(−3.74)	(3.81)				(−1.36)		
	4	0.05	0.32					0.08	3.41
		(1.64)	(7.55)					(5.60)	
	5	−0.16	0.15	0.74	−0.17	−0.41	−0.00		4.95
		(−5.09)	(3.69)	(3.47)	(−3.32)	(−8.02)	(−0.09)		
	6	0.01	0.29	0.80	−0.36	−0.61		0.11	5.28
		(0.39)	(7.36)	(3.87)	(−7.63)	(−11.61)		(8.34)	

Table 9 Robustness check 1, Cont.

Panel D.1 Sample Period July 1926 to June 1963, IVOL estimated using daily data

Method	Model	ln(ME)	Ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
1	1						0.00		3.51
							(0.08)		
	2							0.10	4.46
								(5.19)	
	3	−0.16	0.23				−0.03		5.92
		(−3.01)	(2.32)				(−2.21)		
	4	0.15	0.07					0.13	6.90
		(3.02)	(0.80)					(6.71)	
	5	−0.23	0.21	0.67	−0.13	−0.28	−0.03		9.76
		(−4.64)	(2.33)	(1.59)	(−1.74)	(−2.64)	(−2.47)		
	6	0.05	0.07	0.88	−0.14	−0.56		0.15	10.73
		(1.10)	(0.84)	(2.26)	(−1.88)	(−5.40)		(8.05)	
2	1						0.01		3.71
							(0.62)		
	2							0.06	4.21
								(3.47)	
	3	−0.14	0.22				−0.02		6.04
		(−2.73)	(2.18)				(−1.50)		
	4	0.05	0.12					0.07	6.48
		(0.98)	(1.30)					(3.91)	
	5	−0.22	0.19	0.67	−0.12	−0.30	−0.03		9.88
		(−4.41)	(2.20)	(1.57)	(−1.69)	(−2.75)	(−1.64)		
	6	−0.04	0.11	0.69	−0.13	−0.47		0.09	10.29
		(−0.82)	(1.33)	(1.68)	(−1.83)	(−4.44)		(4.92)	
3	1						0.02		3.79
							(0.88)		
	2							0.05	4.05
								(2.47)	
	3	−0.11	0.18				−0.01		6.15
		(−2.11)	(1.84)				(−0.78)		
	4	−0.00	0.14					0.04	6.37
		(−0.06)	(1.41)					(2.02)	
	5	−0.18	0.15	0.59	−0.11	−0.34	−0.01		9.98
		(−3.67)	(1.76)	(1.37)	(−1.46)	(−3.14)	(−0.47)		
	6	−0.09	0.11	0.63	−0.12	−0.44		0.05	10.19
		(−1.76)	(1.38)	(1.50)	(−1.65)	(−4.14)		(2.94)	

Table 9 Robustness check 1, Cont.

Panel D.2 Sample Period July 1926 to June 1963, IVOL estimated using monthly data

Method	Model	ln(ME)	Ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$(%)
4	1						0.03		2.28
							(1.27)		
	2							0.20	3.31
								(7.46)	
	3	−0.14	0.26				−0.02		5.31
		(−2.52)	(2.33)				(−1.06)		
	4	0.15	0.11					0.22	6.32
		(2.71)	(1.03)					(10.05)	
	5	−0.22	0.24	0.76	−0.12	−0.29	−0.02		9.52
		(−4.08)	(2.37)	(1.64)	(−1.69)	(−2.55)	(−1.48)		
	6	0.01	0.10	0.26	−0.19	−0.66		0.24	10.41
		(0.26)	(1.02)	(0.58)	(−2.67)	(−5.93)		(11.31)	
5	1						0.04		3.05
							(1.34)		
	2							0.13	3.56
								(4.13)	
	3	−0.13	0.24				−0.01		5.78
		(−2.49)	(2.17)				(−0.56)		
	4	0.08	0.12					0.13	6.32
		(1.45)	(1.18)					(4.80)	
	5	−0.20	0.21	0.72	−0.13	−0.30	−0.01		9.68
		(−3.78)	(2.17)	(1.59)	(−1.76)	(−2.65)	(−0.48)		
	6	−0.03	0.12	0.61	−0.22	−0.60		0.16	10.07
		(−0.65)	(1.27)	(1.39)	(−3.18)	(−5.41)		(6.37)	
6	1						0.04		3.09
							(1.46)		
	2							0.11	3.49
								(3.66)	
	3	−0.13	0.25				−0.01		5.82
		(−2.55)	(2.21)				(−0.53)		
	4	0.04	0.15					0.10	6.27
		(0.82)	(1.43)					(3.79)	
	5	−0.20	0.22	0.73	−0.13	−0.32	−0.01		9.63
		(−3.84)	(2.21)	(1.62)	(−1.86)	(−2.79)	(−0.30)		
	6	−0.05	0.14	0.67	−0.22	−0.54		0.13	9.92
		(−1.04)	(1.50)	(1.53)	(−3.23)	(−4.86)		(5.36)	
7	1						0.05		2.43
							(2.07)		
	2							0.08	2.68
								(3.47)	
	3	−0.11	0.28				0.01		5.26
		(−2.18)	(2.47)				(0.37)		
	4	0.01	0.20					0.07	5.59
		(0.28)	(1.83)					(3.56)	
	5	−0.19	0.25	0.77	−0.16	−0.36	0.01		9.32
		(−3.73)	(2.44)	(1.72)	(−2.15)	(−3.22)	(0.80)		
	6	−0.07	0.17	0.72	−0.20	−0.46		0.08	9.48
		(−1.55)	(1.74)	(1.64)	(−2.74)	(−4.32)		(4.41)	

Table 10 Robustness check 2: expected idiosyncratic volatility estimated in a rolling window

Description: The table presents the results of EIVOL estimated in a rolling window of 24–60 months of data. Results are reported for three sample periods: the original sample period, as in Fu (2009), from July 1963 to December 2006; the post-sample period, from January 2007 to December 2017; and the pre-sample period, from July 1926 to June 1963. $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$, which is the out-of-sample rolling-window EIVOL, is estimated using information in the previous 24–60 months (as available) ending in the last month $t-1$. $E_{t}^{\text{Roll}}(\text{IVOL}_t)$, which is the in-sample rolling-window EIVOL, is estimated using information in the previous 24–60 months (as available) ending in month t. Other variables are defined as in Table 3. Panel A reports the summary statistics of the pooled sample. The smallest and largest 0.5% of EIVOL in each month are set equal to the next smallest or largest values. Panel B tabulates the time-series average of the cross-sectional Pearson correlations between $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$ or $E_{t}^{\text{Roll}}(\text{IVOL}_t)$ and other variables. Panel C presents the time-series averages of the slopes in cross-sectional regressions with stock return as the dependent variable and $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$ or $E_{t}^{\text{Roll}}(\text{IVOL}_t)$ as the independent variable (along with other controlling variables). The T-statistic is the average slope divided by its time-series standard error and is reported in brackets. Panel D reports the time-series means of the value-weighted and the equal-weighted raw and risk-adjusted portfolio returns in month t, the pooled means of IVOL of month t (IVOL_t) and market beta (BETA), the medians of market capitalization (ME, reported in millions of \$), and the book-to-market ratio (BE/ME) of 10 portfolios formed on either the out-of-sample $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$ or the in-sample $E_{t}^{\text{Roll}}(\text{IVOL}_t)$.

Interpretation: The results are consistent with Guo et al. (2014) that look-ahead bias is monotonically decreasing in the length of the return series used in estimation. The out-of-sample EIVOL estimated in a rolling window ($E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$) is not positively correlated with future stock returns.

Table 10 Robustness check 2: expected idiosyncratic volatility estimated in a rolling window, Cont.

Panel A Summary statistics of EIVOL estimated in a rolling window

Sample Period July 1963 - December 2006							
Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
$E_t^{\text{Roll}}(\text{IVOL}_t)$	10.46	9.65	9.13	5.27	14.23	3.15	2,827,747
$E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$	10.81	11.65	9.17	5.22	14.52	3.61	2,826,769
Sample Period January 2007 - December 2017							
Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
$E_t^{\text{Roll}}(\text{IVOL}_t)$	9.24	8.75	8.00	4.40	12.75	3.46	746,041
$E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$	9.46	10.58	8.00	4.34	12.91	3.80	752,723
Sample Period July 1926 - June 1963							
Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
$E_t^{\text{Roll}}(\text{IVOL}_t)$	6.94	6.14	5.88	3.78	8.86	4.18	367,187
$E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$	7.11	7.10	5.89	3.73	9.04	4.65	365,473

Panel B Correlation between EIVOL estimated in a rolling window and other variables

Sample Period July 1963 - December 2006

	RET_t	$\ln(1+\text{RET}_t)$	IVOL_t	BETA	ln(ME)	ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)
$E_t^{\text{Roll}}(\text{IVOL}_t)$	0.14*	0.07*	0.43*	0.30*	−0.33*	−0.07*	0.01	0.19*	0.30*
$E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$	−0.01*	−0.03*	0.26*	0.28*	−0.29*	−0.08*	0.04*	0.16*	0.25*

Sample Period January 2007 - December 2017

	RET_t	$\ln(1+\text{RET}_t)$	IVOL_t	BETA	ln(ME)	ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)
$E_t^{\text{Roll}}(\text{IVOL}_t)$	0.10*	0.02	0.51*	0.34*	-0.26*	0.00	−0.03*	0.13*	0.34*
$E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$	−0.01	−0.04*	0.31*	0.30*	−0.23*	−0.01	0.01	0.12*	0.29*

Sample Period July 1926 - June 1963

	RET_t	$\ln(1+\text{RET}_t)$	IVOL_t	BETA	ln(ME)	ln(BE/ME)	RET(−7, −2)	ln(TURN)	ln(CVTURN)
$E_t^{\text{Roll}}(\text{IVOL}_t)$	0.11*	0.06*	0.46*	0.25*	−0.37*	0.22*	0.04*	0.21*	0.29*
$E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$	−0.01	−0.02*	0.35*	0.23*	−0.33*	0.19*	0.05*	0.19*	0.26*

Table 10 Robustness check 2: expected idiosyncratic volatility estimated in a rolling window, Cont.

Panel C Month-to-month cross -sectional regressions

Sample Period July 1963 - December 2006

Model	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	$E^{\text{Roll}}_{t-1}(\text{IVOL}_t)$	$E^{\text{Roll}}_{t}(\text{IVOL}_t)$	$\overline{R^2}$(%)
3						0.00		1.07
						(0.36)		
3′							0.13	3.11
							(8.89)	
4	−0.14	0.30				−0.01		2.94
	(−3.50)	(5.11)				(−0.96)		
4′	0.26	0.51					0.17	4.96
	(7.73)	(9.59)					(11.52)	
5	−0.19	0.22	1.01	−0.16	−0.41	−0.00		5.15
	(−4.95)	(4.49)	(4.81)	(−2.43)	(−7.23)	(−0.68)		
5′	0.25	0.53	1.34	−0.52	−0.80		0.19	7.21
	(7.57)	(11.18)	(7.02)	(−8.63)	(−15.07)		(14.54)	

Sample Period January 2007- December 2017

Model	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	$E^{\text{Roll}}_{t-1}(\text{IVOL}_t)$	$E^{\text{Roll}}_{t}(\text{IVOL}_t)$	$\overline{R^2}$(%)
3						-0.02		0.97
						(-1.31)		
3′							0.10	3.06
							(3.64)	
4	0.02	0.12				−0.02		1.76
	(0.31)	(1.31)				(−1.86)		
4′	0.31	0.17					0.16	4.14
	(6.07)	(2.10)					(5.29)	
5	−0.06	0.06	0.08	−0.14	−0.49	−0.01		3.18
	(−1.23)	(0.82)	(0.16)	(−1.52)	(−4.09)	(−1.51)		
5′	0.28	0.19	0.65	−0.58	−1.21		0.20	5.64
	(6.32)	(2.64)	(1.61)	(−7.06)	(−11.14)		(7.42)	

Sample Period July 1926 - June 1963

Model	ln (ME)	ln (BE/ME)	RET(−7, −2)	ln (TURN)	ln (CVTURN)	$E^{\text{Roll}}_{t-1}(\text{IVOL}_t)$	$E^{\text{Roll}}_{t}(\text{IVOL}_t)$	$\overline{R^2}$(%)
3						0.02		1.49
						(1.21)		
3′							0.26	4.60
							(12.16)	
4	−0.12	0.27				−0.01		4.98
	(−2.18)	(2.33)				(−0.97)		
4′	0.27	0.09					0.30	7.47
	(5.13)	(0.95)					(14.94)	
5	−0.20	0.22	0.69	−0.14	−0.32	−0.01		9.25
	(−3.63)	(2.17)	(1.54)	(−1.89)	(−2.85)	(−0.71)		
5′	0.08	0.09	0.52	−0.28	−0.82		0.32	11.63
	(1.67)	(1.01)	(1.20)	(−3.86)	(−7.68)		(17.29)	

Table 10 Robustness check 2: expected idiosyncratic volatility estimated in a rolling window, Cont.

Panel D Summary statistics for portfolios formed on EIVOL estimated in a rolling window

Sample Period July 1963 - December 2006

Variables	Portfolios formed on $E_t^{\text{Roll}}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.83	0.85	0.86	0.91	0.93	0.97	0.99	1.11	1.19	2.63
Port.EWRET$_t$	0.66	0.69	0.72	0.73	0.75	0.86	0.85	0.92	1.36	6.73
$E_{t-1}(\text{IVOL}_t)$	2.75	4.64	5.86	7.03	9.27	10.70	12.42	14.68	18.13	31.44
IVOL$_t$	6.59	7.68	8.61	10.67	11.93	14.32	16.05	18.06	20.72	26.92
BETA	0.98	1.06	1.12	1.18	1.24	1.29	1.34	1.38	1.41	1.44
ME ($mil, med)	173.67	194.98	127.01	95.16	74.00	62.65	47.40	35.03	24.93	15.39
BE/ME (med)	0.80	0.75	0.72	0.71	0.70	0.68	0.65	0.61	0.55	0.48
FF-3F α	−0.06	−0.10	−0.16	−0.11	−0.03	−0.11	−0.09	−0.01	0.02	1.46

Variables	Portfolios formed on $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.88	1.01	0.97	1.06	1.00	0.94	1.02	1.05	0.88	0.56
Port.EWRET$_t$	1.02	1.17	1.23	1.24	1.28	1.31	1.40	1.39	1.11	1.03
$E_{t-1}(\text{IVOL}_t)$	2.68	4.57	5.80	6.99	9.26	10.72	12.50	14.87	18.60	35.32
IVOL$_t$	7.04	8.30	9.22	11.23	12.45	14.83	16.44	18.26	20.43	23.90
BETA	0.95	1.06	1.12	1.18	1.24	1.29	1.34	1.38	1.40	1.44
ME ($mil, med)	166.34	190.82	125.23	93.58	72.31	60.94	46.78	34.67	24.74	16.16
BE/ME (med)	0.81	0.75	0.72	0.71	0.70	0.69	0.65	0.61	0.55	0.47
FF-3F α	−0.01	0.08	0.02	0.08	0.01	−0.10	−0.03	−0.03	−0.21	−0.50

Sample Period January 2007 - December 2017

Variables	Portfolios formed on $E_t^{\text{Roll}}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.78	0.77	0.75	0.75	0.80	0.83	0.86	0.89	0.97	1.93
Port.EWRET$_t$	0.38	0.58	0.59	0.56	0.51	0.56	1.05	1.03	1.20	4.14
$E_{t-1}(\text{IVOL}_t)$	2.46	4.05	5.16	6.26	8.46	9.86	11.58	13.87	17.41	31.21
IVOL$_t$	4.16	5.31	6.28	8.21	9.28	11.50	12.94	14.85	17.47	24.41
BETA	1.02	1.04	1.10	1.16	1.22	1.27	1.33	1.38	1.43	1.48
ME ($mil, med)	400.59	687.65	753.47	720.50	629.86	544.27	411.24	290.86	192.41	100.50
BE/ME (med)	0.67	0.64	0.62	0.61	0.60	0.61	0.60	0.59	0.57	0.54
FF-3F α	0.11	0.10	−0.09	−0.04	−0.13	−0.17	−0.29	−0.14	−0.23	0.66

Variables	Portfolios formed on $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.76	0.63	0.77	0.76	0.77	0.92	0.80	0.77	0.82	0.47
Port.EWRET$_t$	0.58	0.67	0.77	0.80	0.86	0.87	0.86	0.82	0.64	0.67
$E_{t-1}(\text{IVOL}_t)$	2.40	3.98	5.09	6.21	8.43	9.86	11.63	14.00	17.74	34.18
IVOL$_t$	4.43	5.79	6.74	8.68	9.80	11.90	13.35	15.06	17.41	21.35
BETA	1.01	1.04	1.10	1.16	1.22	1.28	1.33	1.38	1.42	1.48
ME ($mil, med)	390.84	675.86	730.37	703.31	619.86	545.96	405.71	290.84	192.97	103.95
BE/ME (med)	0.68	0.64	0.62	0.61	0.61	0.61	0.60	0.59	0.56	0.53
FF-3F α	0.11	−0.05	0.04	−0.06	−0.11	0.00	−0.17	−0.29	−0.28	−0.72

Sample Period July 1926 -June 1963

Variables	Portfolios formed on $E_t^{\text{Roll}}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.85	0.84	0.82	0.94	0.90	0.97	1.22	1.19	1.51	3.67
Port.EWRET$_t$	0.65	0.76	0.71	0.74	0.72	0.87	1.00	1.14	1.76	5.13
$E_{t-1}(\text{IVOL}_t)$	1.98	3.27	4.10	4.86	5.66	6.55	7.60	8.97	11.09	18.33
IVOL$_t$	7.40	7.70	8.51	9.27	10.23	11.11	12.31	13.89	16.33	23.32
BETA	1.06	1.13	1.18	1.22	1.27	1.30	1.34	1.37	1.39	1.49
ME ($mil, med)	79.77	55.96	33.28	25.84	21.36	19.85	16.43	13.23	10.13	6.08
BE/ME (med)	0.84	0.90	0.94	0.98	1.03	1.07	1.15	1.20	1.25	1.39
FF-3F α	0.06	−0.04	−0.21	−0.07	−0.23	−0.20	−0.05	−0.10	0.18	2.13

Variables	Portfolios formed on $E_{t-1}^{\text{Roll}}(\text{IVOL}_t)$									
	Low	2	3	4	5	6	7	8	9	High
Port.VWRET$_t$	0.87	1.05	1.12	1.04	1.11	1.18	1.13	0.86	0.97	0.99
Port.EWRET$_t$	1.13	1.19	1.26	1.25	1.38	1.48	1.30	1.40	0.99	1.16
$E_{t-1}(\text{IVOL}_t)$	1.91	3.19	4.03	4.81	5.63	6.54	7.64	9.08	11.36	20.39
IVOL$_t$	8.02	8.32	8.89	9.71	10.35	11.44	12.55	14.03	16.27	21.03
BETA	1.07	1.12	1.18	1.23	1.27	1.30	1.34	1.37	1.39	1.42
ME ($mil, med)	75.02	55.21	33.28	25.17	21.12	19.59	16.63	13.14	10.32	6.67
BE/ME (med)	0.85	0.90	0.94	0.99	1.03	1.09	1.14	1.20	1.24	1.34
FF-3F α	0.08	0.14	0.11	0.03	0.01	0.04	−0.10	−0.43	−0.27	−0.28