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Impacts of uncertain information delays on distributed real-time optimal controls for building HVAC systems deployed on IoT-enabled field control networks

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Abstract

Distributed optimal control deployed on field control networks is receiving increasing attention with the rapid development of the Internet of Things (IoT), including its applications in HVAC systems. Information delays refer to the time delays in information exchange between devices integrated in communication networks. They could affect the performance of distributed optimal control, but are rarely concerned in HVAC field. This study investigates and quantifies the impacts of information delays on the performance of distributed optimal control strategies for HVAC systems through theoretical analysis and case studies, including a typical central cooling plant and a typical multi-zone air-conditioning system. The uncertain information delays are modeled by a Markov chain according to the characteristics of networks. Their impacts are quantified by comparing the performance of the distributed optimal control strategies involving the information delays with ideal performance. Results show that information delays significantly affected the convergence rate and control accuracy of the distributed optimal control strategies. These delays can result in a difference in optimized cooling tower outlet water temperature of up to 0.6 K and a number of iterations of up to180 (about nine times than in ideal conditions). Test results indicate the necessity of considering the impacts of information delays when developing distributed optimal control strategies for HVAC systems. This necessity exists for both future IoT-enabled and current LAN-based field control networks.

Keywords: distributed optimal control, networked control, IoT-enabled network, air-conditioning system, information delays, uncertainty

[1] The short version of the paper was presented at ICAE2020, Dec 1-10, 2020. This paper is a substantial extension of the short version of the conference paper.

Nomenclature

Symbols		*Subscripts*	
C	*specific heat, kJ/ (kg K)*	*a*	*air*
CO2	*CO_2 concentration (ppm)*	*amb*	*ambient air*
E	*energy*	*chi*	*chiller*
f,g,h	*function*	*co*	*coordinator agent*
INp	*indoor pollution index*	*com*	*communication process*
Ld	*Length of information delay*	*Con*	*condenser*
M	*flow rate, kg/s*	*cpt*	*Computation process*
Md	*maximum possible delay length*	*ct*	*cooling tower*
N	*number*	*cwo*	*cooling tower outlet water*
Obj	*objective value*	*fan*	*fan of PAU*
P	*power, kW*	*G*	*generation*
Q	*ventilation air volume, L/s*	*in*	*inlet*
SI	*Sampling interval*	*Limit*	*upper limit*
T	*temperature, ℃*	*opt*	*optimal*
t	*time cost, s*	*out*	*outlet*
X	*value range*	*tot*	*total*
x	*control variable*		
Greek symbols			
γ	*weighting factor*		
λ	*Lagrange multiplier*		

1 Introduction

The IoT (Internet of Things) is widely recognized as one of the most important directions for future technology development and has attracted vast attention from a wide range of application fields. Distributed intelligence is an important trend of development in many application fields since centralized decision-making causes heavy computational load on central stations, severe network traffic and reduced reliability [1]. A number of solutions have been proposed, such as fog computing, edge computing and mist computing [2]. In recent years, smart IoT sensors and control devices have found increasing application in the building industry and are expected to play a major role in the field control networks of the building automation systems (BASs) of the next generation [3]. It is therefore important to develop effective means to facilitate online optimal control and make use of the huge capacity of the great number of smart IoT sensors/devices distributed in field control networks. This is an urgent issue to be addressed for the further development and wider applications of BASs and smart buildings, since online optimal control is an essential means of enhancing building energy efficiency in actual operation. Many researchers and engineers have made serious efforts to develop optimal control methods and strategies for HVAC (heating, ventilation and air conditioning) systems [4], such as model-based optimal control strategies. However, these optimal control strategies are generally centralized and have a number of major limitations. Firstly, the model-based performance prediction and optimization of these strategies must typically be performed on BAS central computer stations, due to the high computational load and programming complexity, particularly for complex HVAC systems. Secondly, these control strategies often lack generality as they need to be designed for specific systems. This makes it inconvenient and costly to adjust or customize these strategies for the application on different HVAC systems. Furthermore, the requirement of a central station greatly limits application: it is impractical to implement a large number of optimal control strategies on a small number of computer stations as there are many local sub-systems to be optimized. Distributed optimal control is an effective means to overcome these drawbacks and to be deployed on IoT-enabled field control networks.

Recently, many distributed control strategies have been developed for building services systems adopting the concept of multi-agent system because of its scalability and modularity [5]. Individual agents that can each perceive their environment and act according to their goals and tendencies [6] form a system achieving a common goal through coordination [7]. Most of these studies focus on adjusting the set-points of building services systems, such as lighting and HVAC systems, according to information collected by agents (e.g. temperature, humidity and occupant behavior) to facilitate intelligent energy management while maintaining a comfortable indoor environment (i.e. building thermal management) [8,9]. Klein et al. [10], Yang and Wang [11] and Michailidis et al. [12] proposed multi-agent systems which collect information about occupant preferences and behavior as a basis for the control of HVAC and lighting systems. Yang and Wang [13] proposed a multi-agent control system using particle swarm optimization to improve the energy intelligence of the multi-zone buildings. Lymperopoulos and Ioannou [14] proposed a distributed adaptive scheme for the temperature regulation of multi-zone HVAC systems. However, in these studies, only the "software distribution" is addressed - the algorithm is either distributed in different computational packages, typically operating on one computer station, or the implementation issue is ignored. Few researchers addressed the practical implementation of distributed optimal control, the implementation and distribution of decomposed optimization tasks on physical devices. Wang et al. [15] and Cai et al. [16] presented a generic structure of an agent-based optimal control strategy for HVAC systems. Both chose to decompose the optimization problems from the system level to the components level. Local optimization of each component is conducted by a corresponding agent deployed on an individual device; global optimization is achieved by coordinating these agents. These studies proposed a basic means of constructing agent-based optimal control strategies for HVAC systems. The structure of multi-agent systems and an optimization technique for distributed optimization were also provided. However, these studies did not consider the requirements and constraints in practical applications when the distributed control strategies are implemented on physical platforms, such as the convergence rate, computational load distribution and information delays. When developing the distributed optimal control strategy, the authors of this paper [17] addressed the convergence rate and computational load distribution for the deployment in future IoT-enabled field control networks as well as current LAN-based field control networks. However, research on the impacts of information delays are still missing. This study will address this problem.

The information delays refer to the time delays in the information exchange between different devices over communication networks. In the control field, the impacts of information delays on the performance of networked feedback control loops have received attention from researchers and engineers since the 1980s, concerning the application scenarios that involve long-distance data transmission, via local area networks, between controllers connecting sensors and control actuation devices [18]. This issue has received increasing attention in recent years as with the rapid emergence of IoT technologies and field networks, more and more feedback control loops are established across standalone smart sensors and actuators as well as controllers integrated through communication networks [19]. Li et. al. [20] proposed a method for stability analysis of the feedback control of a multi-input, multi-output plant with varying network delays. Long et. al. [21] analyzed the impacts of time-varying delays on the performance and stability of networked load frequency control systems. Nilsson et. al. [22] evaluated the performance of different control schemes with varying delays, assumed to be less than the sampling interval, and proposed a new scheme for handling these delays using timestamps. Their results show the necessity of accounting for information delays while designing the control scheme of feedback control systems.

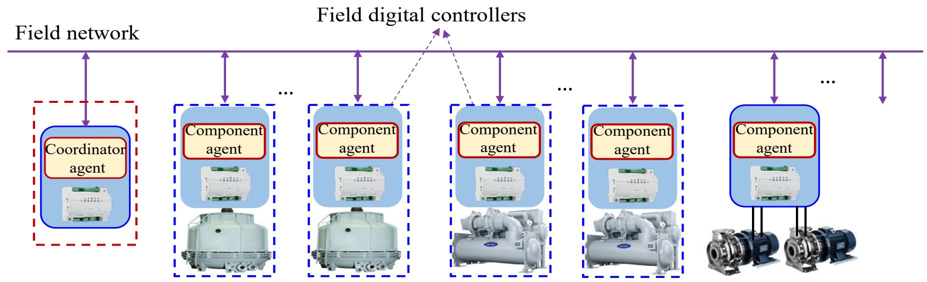
Previous studies didn't consider the impacts of information delays on distributed optimization as they focused on "software distribution" [23]. However, information exchange between different local control devices integrated in a field control network is essential during distributed optimal control. Global optimization can be affected by information delays. In the optimization field, a few researchers have investigated the impacts of delays on distributed optimization algorithms and developed robust optimization algorithms capable of coping with communication delays. It has been found that information delays can result in these algorithms either converging to incorrect values or failing to converge at all [24]. Tsianos and Rabbat [25] proposed two ways to model constant delays and uncertain delays. They analyzed the impacts of delays by involving the delay models in the distributed optimization algorithm and found that delays slow down optimization. Nedi´c et al. [26] investigated the convergence rate of a consensus problem using a common consensus algorithm under bounded delay conditions. By introducing additional delays to the subgradient projection algorithm, Lin et al. [27] solved the distributed optimization problems with arbitrary bounded communication delays. The above algorithms use a diminishing step-size which results in a low convergence rate. For algorithms assuming a fixed step-size, Yang et al. [28] demonstrated negative impacts on the convergence rate and optimization results due to information delays. Zhao et al. [29] pointed out that there always exists a sufficiently small step-size which can guarantee convergence of an algorithm under finite constant time delays. Charalambous et al. [30] proposed an algorithm which achieves convergence in a small finite number of steps whether delays exist or not.

However, existing studies have not considered the impacts of information delays on hierarchical distributed optimization algorithms such as the dual decomposition method and ADMM (alternating direction method of multipliers). Moreover, only theoretical analysis has been conducted on the impacts of information delays on other kinds of distributed optimization algorithms. No study can be found on the delays in distributed online optimal control of HVAC systems deployed on local control devices integrated in BAS field control networks. Investigating the impacts of information delays here is essential as the information delays could affect the performance of the distributed optimal control strategies. It is also essential for the development and applications of IoT technologies in the building automation field, which requires reliable and energy efficient distributed real-time optimal control strategies. To fulfill these research gaps and facilitate the application of distributed optimal control in building filed control networks, this study investigates the impacts of the information delays on two typical hierarchical distributed optimization algorithms, one based on dual decomposition and the subgradient method and the other using ADMM. It quantifies the impacts on the performance of distributed optimal control for HVAC systems implemented on both current and future BAS platforms at the field control network level. At first, the potential impacts of the information delays are investigated qualitatively by analyzing the mathematical schemes of distributed optimal control strategies with information delays involved. These impacts are further quantified through evaluating the performance of the distributed optimal control strategies for two typical HVAC systems which account for information delays, a central cooling plant and an air-side air conditioning system. The robustness of the two commonly used distributed optimization algorithms with uncertain information delays are evaluated and compared.

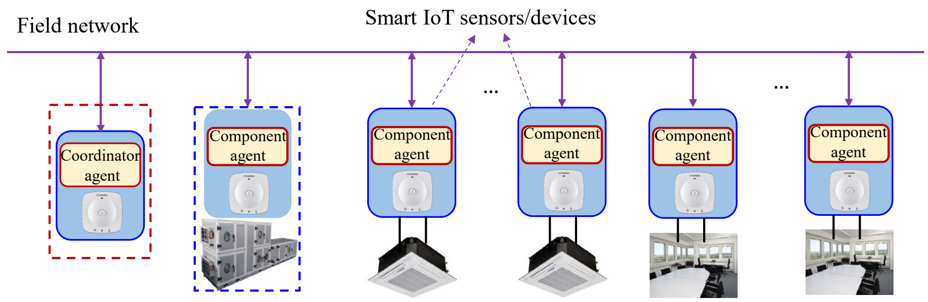
2 Distributed optimal controls and information delays when deployed on BAS field control networks

2.1 Deployment framework of distributed optimal control strategies for HVAC systems

Fig. 1 shows the deployment frameworks of the proposed distributed optimal control strategy for HVAC systems on the field control networks of current and future BASs respectively. For the optimal control of typical HVAC systems and subsystems, distributed optimal control strategies can be deployed on integrated local BAS field controllers or devices, such as central cooling plants of different configurations, all air systems of constant air volume (CAV) and variable air volume (VAV) and dedicated outdoor air systems (DOAS). A distributed optimal control strategy consists of a number of agents, each responsible for solving a local optimization problem and implemented on their corresponding local control devices (i.e. digital controllers or smart sensors). The optimal set-points are achieved by coordinating these agents through information exchange. Using local measurements and received information, the component agents will conduct local optimizations for the corresponding components and send the local optimization results to the coordinator agent. Then, the coordinator agent will check the convergence status according to the received results. If it has not converged, the global parameter will be updated and sent to the component agents to be used at the next iteration. If it has converged, a signal will be sent to the component agents to inform them to execute control actions. Information exchange between the component agents and the coordinator agent through field networks is the key issue to achieving global optimization for the distributed optimal control. Therefore, the impact of information delays on the performance of distributed optimal control strategies should be investigated quantitatively and in detail.



(a) Distributed optimal control deployed on DDC field control networks



(b) Distributed optimal control deployed on smart IoT sensor/device networks

Fig.1 Illustration of deployment frameworks of distributed optimal control of HVAC systems on current and future BASs

2.2 Description and modelling of information delays in distributed optimization

Fig. 2 illustrates information exchange between control devices (or agents) in a synchronized network and how information delays of different lengths affect an optimization process. In ideal conditions, the actual information delays are less than the sampling interval (referred to as "regular delays" hereafter). Unless a delay larger than the sampling interval occurs, the intermediate local optimization results of the current sampling step can be used by other agents at the next sampling step. It is also possible that data is lost during communication as packet loss. In general, information delays are caused by the time cost of computation (t_{cpt}) and communication (t_{com}). Since the field controllers or devices generally sample the data from their buffers and process them periodically with a predefined sampling interval, the effective delay of the information exchange between controllers is discrete in terms of the number of sampling intervals. According to the time spent for these processes and the sampling interval (SI), the possible maximum information delay (Md) can be obtained using Eq. (1). It is worth noting that the field controllers or devices can store some received data in their buffers and will always use the latest available data. Therefore, packet loss will not result in a situation where no information is available for the use of an agent.

$$Md = \begin{cases} \left\lfloor \frac{t_{com}+t_{cpt}}{SI} \right\rfloor & (bounded\ delay) \\ +\infty & (packet\ loss) \end{cases} \quad (1)$$

Synchronizing the operation of multiple individual local controllers or devices is not easy in engineering practice [19]. Normally, in building automation systems, the controllers or devices normally operate at the same sampling interval, but their operation is not synchronized. In such conditions, even under regular delays, some devices can experience effective information delays of one sampling interval when receiving actual data from other devices.

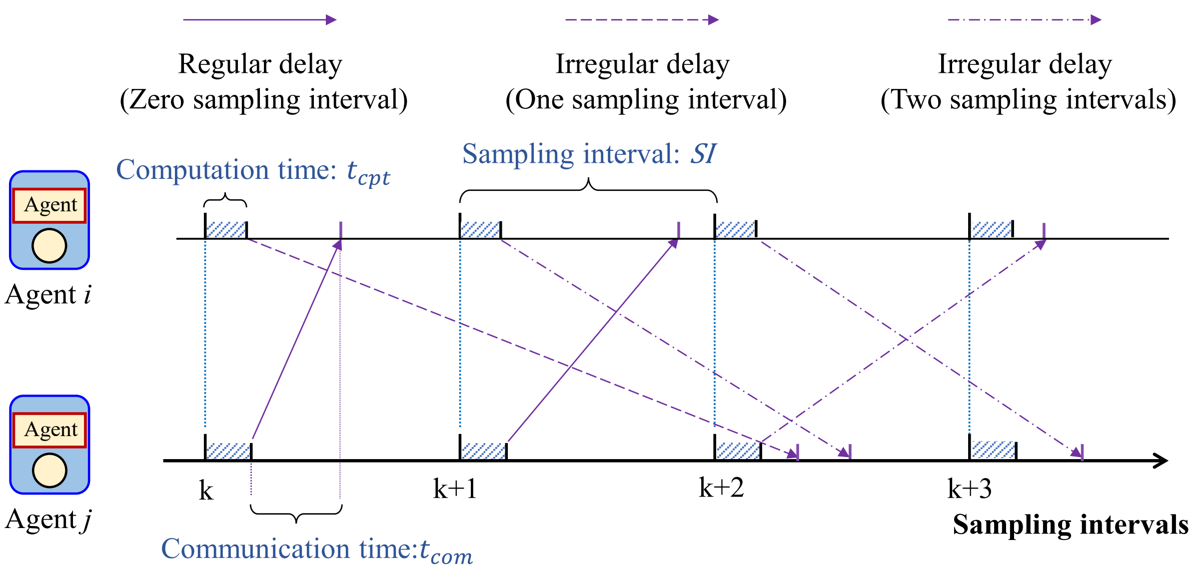


Fig. 2. Information delays in the distributed optimization deployed among devices in a synchronized network

Markov chains are commonly used to model the uncertain network delays since the current time delays are usually related to previous time delays [31]. The Markov chain can be expressed by a state space and transition probability matrix. The state space includes all possible values of the delays and the transition matrix gives the probabilities of the delays changing from one value to another value. In this study, the state space is set to [0, 1, 2] representing the possible lengths of effective information delays in terms of the number of sampling steps. A delay of zero steps represents a regular delay. The transition matrix is given in Eq. (2).

$$Tran = \begin{bmatrix} 0.4 & 0.6 & 0.0 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} \quad (2)$$

Packet loss is also included in this model since local devices always use the latest available data. This is also the reason that the probability of transition from a delay of zero step to a delay of two steps is zero. This model will be used in tests to quantify the impacts of information delays on the performance of distributed optimal control strategies in Section 3 and 4.

3 Impacts on distributed optimal control for a central cooling plant

The system involved in this test case is shown in Fig. 3. It consists of three centrifugal chillers, three constant speed cooling water pumps and six cooling towers. Each chiller has a cooling capacity of 7235kW with a COP of 5.6 at the rated working conditions. The rated water flow rate and power load of the condenser cooling water pumps are 410.6L/s and 185kW, respectively. The nominal heat rejection capacity and power consumption of the six cooling towers are 5234kW and 155kW respectively. For these six cooling towers, the first cooling tower is assumed to be a new cooling tower with rated efficiency, and different degrees of performance degradation are assumed for the other five cooling towers. The heat transfer efficiencies of these five cooling towers are 95.3%, 89.1%, 84.5% 77.6% and 74.6% of the rated efficiency respectively. The optimization objective of the central cooling plant is to find the individual optimal cooling tower outlet water temperature (T_{cwoi}) which minimizes system energy consumption (P_{tot}) as shown in Eq (3). $T_{con,in}$ is the condenser inlet water temperature. N_{ct} and N_{chi} are the number of cooling towers and chillers in operation respectively.

$$\underset{\{Tcwoi\}}{Min}\, P_{tot} = \sum_{j=1}^{N_{chi}} P_{chi,j} + \sum_{i=1}^{N_{ct}} P_{ct,i} \tag{3}$$

$$\text{Subject to : } T_{con,in} = T_{cwo} = \frac{1}{N_{ct}} \sum_{i=1}^{N_{ct}} T_{cwoi}$$

The simplified chiller model proposed by Stoecker [32] is used to predict the power consumption of chillers in this study. A simplified model proposed by Wang and Ma [33] is used to calculate the heat transfer from the cooling water to the inlet air.

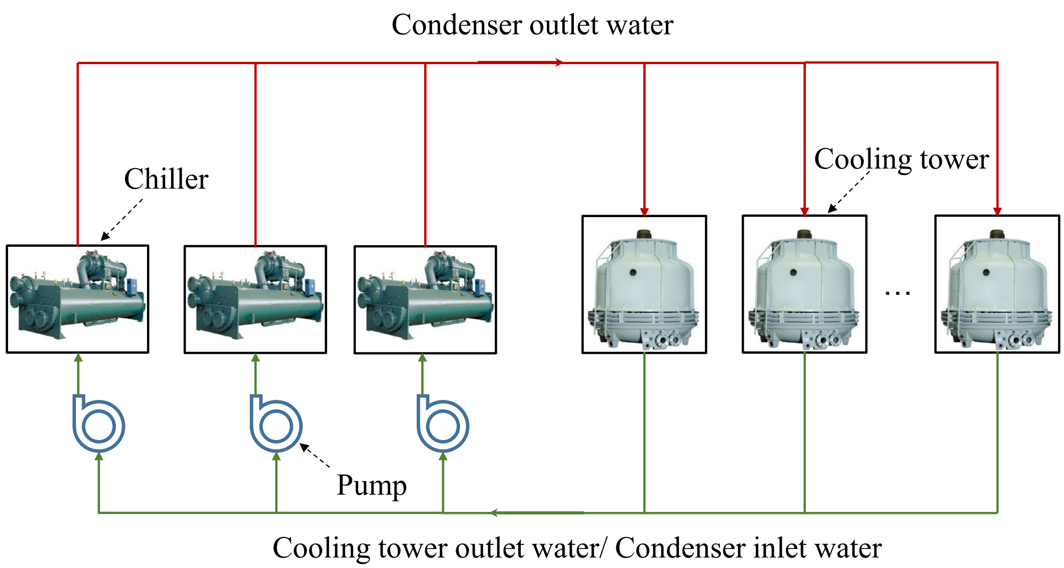


Fig .3. Schematic of the condenser cooling system of a central cooling plant concerned

3.1 Description of the distributed optimal control strategy

The distributed optimal control strategy is formulated based on the dual decomposition and subgradient method [34]. The original optimization problem can be decomposed into a number of sub-problems and a master problem. The sub-problems are expressed by Eqs. (4) and (5) and the master problem is described by Eq. (6). M_{cw} is the mass flow rate of the cooling water, C is the specific heat, and λ is the Lagrange multiplier.

$$\underset{(T_{con,in})}{Min} \sum_{j=1}^{N_{chi}} (P_{chi,j} - \lambda \cdot CM_{cwj}(T_{con,out} - T_{con,in})) \tag{4}$$

$$\underset{\{T_{cwoi}\}}{Min} \sum_{i=1}^{N_{ct}} (P_{ct,i} + \lambda \cdot CM_{cwi}(T_{con,out} - T_{cwoi})) \tag{5}$$

$$\lambda \cdot (T_{cwo} - T_{con,in}) = 0 \tag{6}$$

The cooling tower agents and chiller agents are designed to solve the corresponding sub-problems, while the coordinator agent solves the master problem and determines the convergence status. The optimization process of the control strategy is shown in Fig. 4. With the measurements and initial value of λ, the chiller agents and cooling tower agents conduct local optimizations and send the achieved local optimization results to the coordinator agent. The coordinator agent determines the convergence status and updates the value of λ if convergence is not achieved. More detailed information about the control strategy can be found in [17].

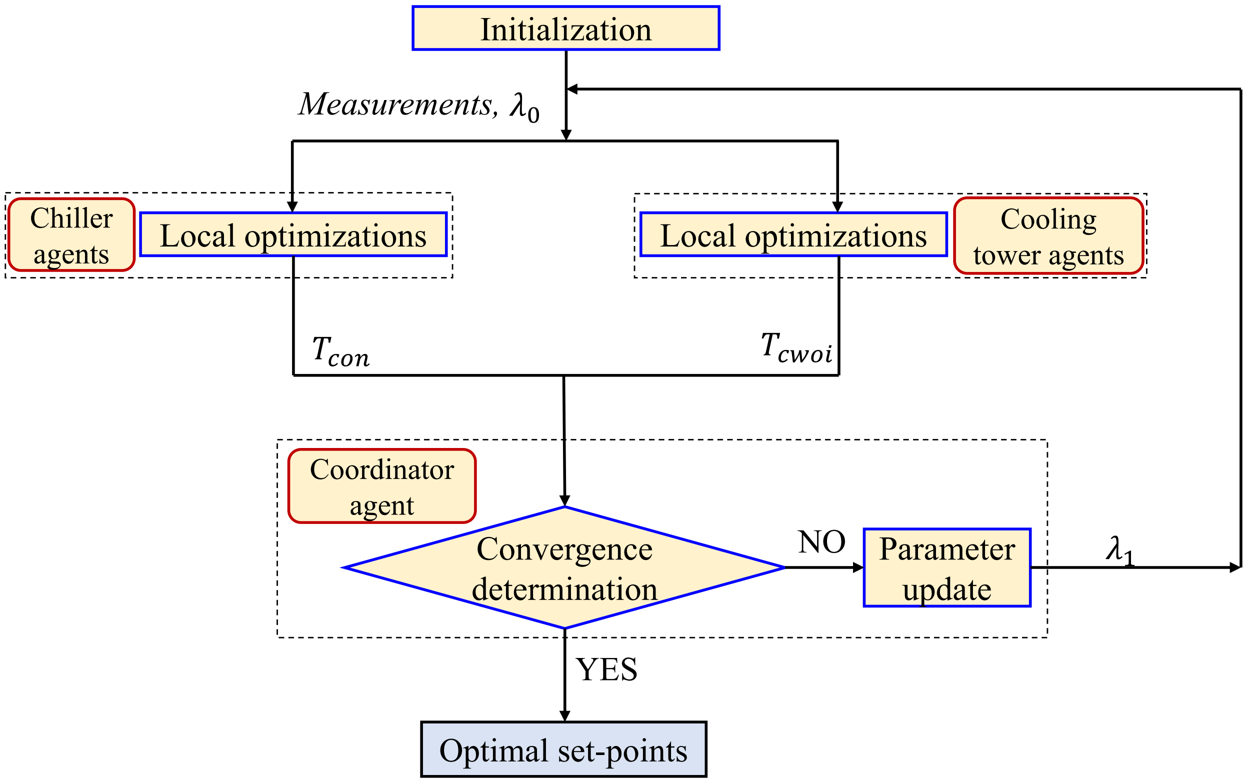


Fig .4. Optimization process of the distributed optimal control strategy

3.2 Qualitative analysis on the impacts of information delays on distributed optimization

Uncertain information delays can affect both the control accuracy and the convergence rate of distributed optimal control strategies. The impacts on control accuracy mean that the control strategy obtains the biased optimal results, which will result in the reduced energy efficiency of the target energy system. The impacts on convergence rate mean that more iteration steps are needed to achieve convergence of the optimization, which may result in convergence failure within the preset optimization interval. In this case, a simple near-optimal strategy will need to be adopted to generate set-points, which will also result in the reduced energy efficiency.

To analyze the impacts of information delays on distributed optimization, formulas for the distributed optimization with information delays involved are formulated. It is worth noting that, since the optimization interval concerned in the real-time optimal control is short, usually in minutes, the system is assumed to be static during the optimization interval. The length of information delay is denoted as *Ld*. In this case, the objective is to find the optimal cooling tower outlet water temperature. At step *k*, the chiller agents and cooling tower agents update their local optimization results with the latest information available, as shown in Eqs. (7) and (8). The coordinator agent determines the convergence state and updates the multiplier according to Eqs. (9) and (10).

$$T_{con,k} = f_{chi}\left(\lambda_{k-Ld_{chico}}\right) \tag{7}$$

$$T_{cwo,k} = f_{ct}\left(\lambda_{k-Ld_{ctco}}\right) \tag{8}$$

$$Conv_k = g\left(T_{con,k-Ld_{cochi}}, T_{cwo,k-Ld_{coct}}\right) \tag{9}$$

$$\lambda_k = h\left(T_{con,k-Ld_{cochi}}, T_{cwo,k-Ld_{coct}}\right) \tag{10}$$

$$Conv_k = g\left[f_{chi}(\lambda_{k-Ld_{chico}-Ld_{cochi}}), f_{ct}(\lambda_{k-Ld_{ctco}-Ld_{coct}})\right] \tag{11}$$

$$\lambda_k = h\left[f_{chi}\left(\lambda_{k-Ld_{chico}-Ld_{cochi}}\right), f_{ct}\left(\lambda_{k-Ld_{ctco}-Ld_{coct}}\right)\right] \tag{12}$$

Using Eqs. (7) and (8), Eqs. (9) and (10) can be transformed into Eqs. (11) and (12), which show how the information delays affect the convergence determination and multiplier update respectively. The impacts of information delays on the multiplier update are illustrated in Fig. 5. Compared with ideal conditions, the scenarios with irregular information delays lead to faster Lagrange multiplier updates in the first several steps, finally converging to biased optimization results at a lower convergence speed. Similar phenomena in the distributed optimization of economic dispatch problems have been reported in previous studies [35,36].

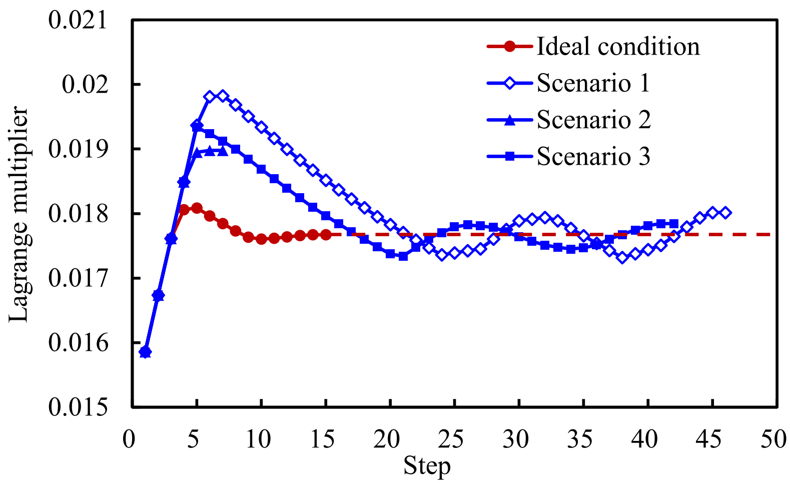


Fig .5. Evolution of the Lagrange multiplier

The impacts on the convergence determination are determined by the magnitude of delays during information exchange between chiller agents and the coordinator agent (i.e. Ld_{chi}=Ld_{chico} +

Ld_{cochi}), and that between the cooling tower agents and the coordinator agent (Ld_{ct}=Ld_{ctco} + Ld_{coct}). According to Ld_{chi} and Ld_{ct}, information delays can be divided into two categories, 'uniform delay' ($Ld_{chi} = Ld_{ct}$) and 'nonuniform delay' ($Ld_{chi} \neq Ld_{ct}$). In scenarios with nonuniform delays, the local optimal results used for convergence determination by the coordinator agent are from the same iteration step. Therefore, if the coordinator agent sends the actual optimal result to the cooling tower agents when convergence is achieved, uniform delay will not affect the optimization results. In scenarios with nonuniform delays, the coordinator agent determines convergence state with the local optimization results of different iteration steps. As shown in Fig. 6, due to the nonuniform delay, $T_{cwo,k-Ld_{ctco}}$ will be generated as the optimal value, which results in biased optimization results as shown in Eq. (13). Such impacts have not been reported in previous studies, since using the coordinator agent to determine convergence status is the mechanism only used in hierarchical optimization algorithms.

$$bias = T_{cwo,opt} - T_{cwo,k-Ld_{coct}} \tag{13}$$

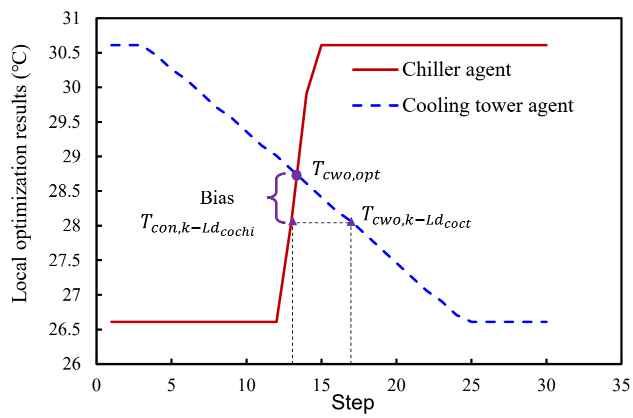
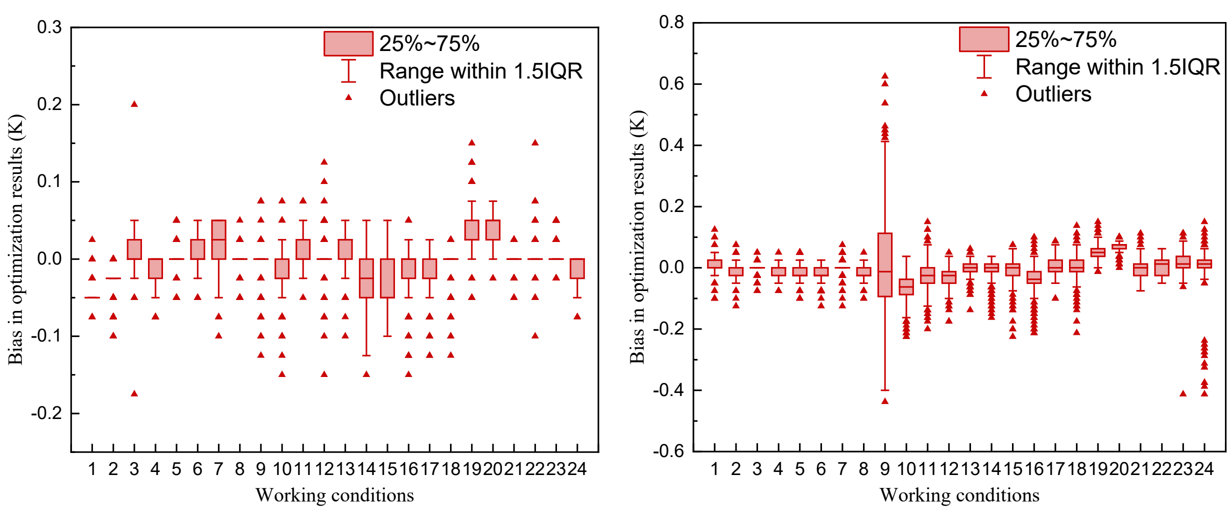


Fig .6. Impacts of nonuniform information delays on optimization results

3.3 Experimental and quantitative assessment on the impacts of information delays on distributed optimization

A distributed optimal control strategy for the central cooling plant was developed incorporating information delays to quantify their impacts. The delays were modelled using the Markov chain constructed in Section 2.2. In the tests, the optimization results, energy performance and the convergence rates of the distributed optimal control strategy were obtained and compared with those in ideal conditions. Twenty-four working conditions in a spring day and twenty-four working conditions in a summer day were chosen as the test conditions. Since the information delays are uncertain and information exchange is required at each iteration, during one global optimization interval, there can be hundreds of instances of information exchange, with delays of different lengths. In order to present an overview of the impacts of information delays, 1000 simulations were conducted for each working condition in the tests.

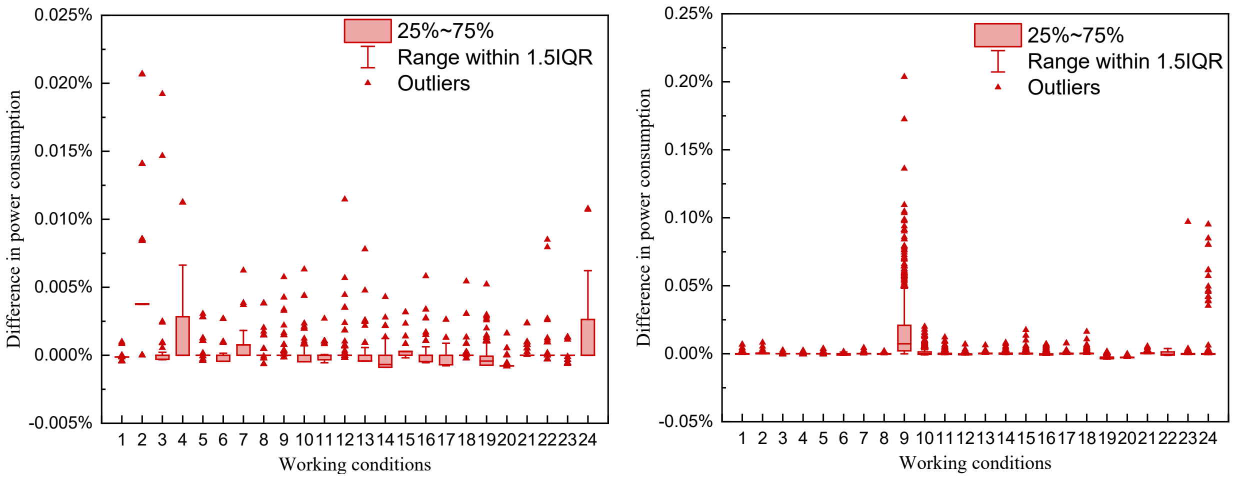
Fig. 7 shows the distribution of the biases in the optimization results in the spring and summer test conditions. The biases in the optimization results were very small in most optimizations, and the biases in the summer test conditions were larger than those in the spring test conditions. The maximum biases were 0.2 K and 0.6 K in the spring and summer test conditions respectively. As a result, the impacts on power consumption could be neglected in most optimizations, and the impacts at the summer test conditions were more obvious as shown in Fig. 8. The maximum increase was 0.02% and 0.2% in the spring and summer test conditions respectively. The impact on the convergence rate was obvious, and remained similar in the spring and summer test conditions as shown in Fig. 9. In ideal conditions, all the optimizations were completed within 40 steps, satisfying the one-minute optimization interval. However, due to information delays the convergence rate was slowed in most optimizations. In the spring test conditions, convergence could not be achieved within one minute in 1,533 optimizations, accounting for 6.1% of the total. In the worst case, convergence was achieved after 216 steps, about seven times as many as in ideal conditions. In the summer test conditions, convergence could not be achieved within one minute in 1,652 optimizations, accounting for 6.6% of the total, and in the worst case, convergence was achieved after 182 steps, about nine times as many as in ideal conditions.



(a) Spring test day

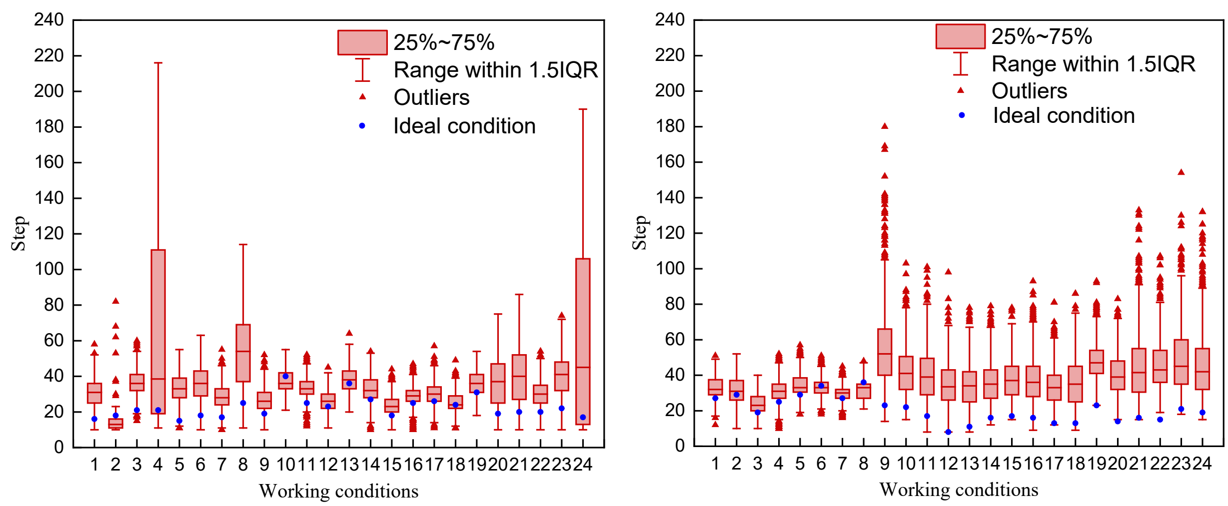
(b) Summer test day

Fig. 7. Statistical distribution of the bias in optimization results under information delays



(a) Spring test day (b) Summer test day

Fig. 8. Statistical distribution of the difference in system power consumption under information delays



(a) Spring working conditions (b) Summer working conditions

Fig. 9. Statistical distribution of the iteration steps under information delays

3.4 Sensitivity analysis of the impacts of information delays

Sensitivity analysis was also conducted to analyze the impacts of information delays. In this study, the length of information delays and the step-size for the update of the Lagrange multiplier are the main factors concerned, as shown in Eqs. (11) and (12). The control strategy was tested in scenarios with different types/lengths of information delays and step-sizes in various working conditions of a typical year. Three indicators were chosen to evaluate the impacts of information delays, bias in the optimization results, increase in energy consumption and increase in iteration step numbers compared with in ideal conditions.

3.4.1 Effects of information delay length

To investigate the effects of the information delay length, the control strategy was tested in scenarios with constant information delays of different lengths. The tested uniform and nonuniform delay scenarios and the corresponding results are shown in Table 1 and 2 respectively. In these tests, the step-size for the update of the Lagrange multiplier was set to 5×10^{-8}, reflecting ideal conditions. According to the convergence rate in ideal conditions, the optimal control interval was set to one minute. If the convergence cannot be achieved within the preset control interval, a near-optimal approach will be adopted, i.e. the cooling tower outlet water temperature will be set to 5 K higher than the current wet-bulb temperature.

According to the results of the tested uniform delay scenarios, the impacts of the uniform delay increased dramatically with increases in delay length. As delay length increased from 1 sampling interval to 4 sampling intervals, the average bias in optimization results increased from 0.05 K to 0.44 K and the total energy consumption increase rose from 7,853.3 kWh to 37,397.2 kWh (i.e. from 0.06% to 0.28%). As aforementioned, the uniform delay will not affect the optimization results if convergence can be achieved. The increase of these impacts was caused by the increasingly slowed convergence rate. The average increase of iteration steps rose from 9 to 33 and the number of non-convergence optimizations increased from 306 to 2,986 (i.e. from 4.50% to 43.92%) as the delay length grew.

Table 1 Statistical performance data of optimal control strategy under different uniform delays

Uniform delays	Ld_{ct}=1 Ld_{chi}=1	Ld_{ct}=2 Ld_{chi}=2	Ld_{ct}=3 Ld_{chi}=3	Ld_{ct}=4 Ld_{chi}=4
Average bias in optimization results (K)	0.05	0.23	0.30	0.44
Maximum bias in optimization results (K)	2.20	2.48	2.93	3.03
Total energy consumption increase (kWh)	7,853.3	18,589.9	27,518.2	37,397.2
Increase ratio of total energy consumption	0.06%	0.14%	0.21%	0.28%
Average increase of iteration step numbers	9	20	28	33
Number of non-convergence optimizations	306	1163	2145	2986

According to the results of the tested nonuniform delay scenarios, the value of Ld_{chi} had the largest effects on the impacts of the information delays. This is because the impacts of the information delays on the local optimization of chiller agents are more obvious than those on the local optimization of cooling tower agents. The tested scenarios include two groups: in one group Ld_{chi} was larger than Ld_{ct} and in the other group it was smaller than Ld_{ct}. Under the same lengths of the information delays, the impacts in the $Ld_{chi} > Ld_{ct}$ scenarios were larger than those in the $Ld_{chi} < Ld_{ct}$ scenarios. In scenarios where $Ld_{chi} > Ld_{ct}$, the impacts of information delays increased with the increase of difference between Ld_{chi} and Ld_{ct} as well as the sum of these two delays. In the scenario where [Ld_{ct}=0, Ld_{chi}=1], the impacts on the optimization results and convergence rate were the lowest, and in the scenario where [Ld_{ct}=3, Ld_{chi}=4], the impacts were the largest. In these two scenarios, the average biases in optimization results were 0.04 K and 0.45 K, the total energy consumption difference was 3868.1 kWh and 36581.0 kWh (0.06% and 0.28%), and the number of non-convergence optimizations was 184 and 2959 (2.71% and 43.53%) respectively. In the scenarios where $Ld_{chi} < Ld_{ct}$, the impacts of the information delays did not always increase with the increase of the difference or the sum of the delays. In the scenarios where Ld_{chi}>0, the impacts of the information delays almost decreased with the increase of the delays among the cooling tower agents and the coordinator agent. This is because the increase in the difference of delays reduces the number of non-convergence optimizations. In the scenarios where all optimizations convergenced, i.e. the scenarios where Ld_{chi}=0, the increase in the delays among the cooling tower agents and the coordinator agent led to the increase in the impacts on optimization results.

Table 2 Statistical performance data of optimal control strategy under different nonuniform delays

Ld_{ct}<Ld_{chi}	Dif = 1				Dif = 2			Dif = 3		Dif=4
	Ld_{ct}=0 Ld_{chi}=1	Ld_{ct}=1 Ld_{chi}=2	Ld_{ct} =2 Ld_{chi}=3	Ld_{ct}=3 Ld_{chi}=4	Ld_{ct}=0 Ld_{chi}=2	Ld_{ct}=1 Ld_{chi}=3	Ld_{ct}=2 Ld_{chi}=4	Ld_{ct}=0 Ld_{chi}=3	Ld_{ct}=1 Ld_{chi}=4	Ld_{ct}=0 Ld_{chi}=4
Average bias in optimization results (K)	0.04	0.19	0.32	0.45	0.17	0.32	0.44	0.30	0.45	0.44
Maximum bias in optimization results (K)	2.20	2.48	2.75	2.98	2.48	2.68	2.97	2.68	2.80	2.80
Total energy consumption increase (kWh)	3,868.1	16,338.4	26,877.1	36581.0	13,923.0	26,184.6	34,782.5	23,065.7	35,108.3	33539.3
Increase ratio of total energy consumption	0.03%	0.12%	0.20%	0.27%	0.10%	0.20%	0.26%	0.17%	0.26%	0.25%
Average increase in iteration step numbers	8	20	28	33	19	27	32	26	32	31
Number of non-convergence optimizations	184	1091	2033	2959	901	1891	2794	1642	2710	2582
Ld_{ct}>Ld_{chi}	Ld_{ct}=1 Ld_{chi}=0	Ld_{ct}=2 Ld_{chi}=1	Ld_{ct}=3 Ld_{chi}=2	Ld_{ct}=4 Ld_{chi}=3	Ld_{ct}=2 Ld_{chi}=0	Ld_{ct}=3 Ld_{chi}=1	Ld_{ct}=4 Ld_{chi}=2	Ld_{ct}=3 Ld_{chi}=0	Ld_{ct}=4 Ld_{chi}=1	Ld_{ct}=4 Ld_{chi}=0
Average bias in optimization results (K)	0.01	0.02	0.14	0.29	0.015	0.04	0.13	0.03	0.05	0.04
Maximum bias in optimization results (K)	0.14	1.67	2.48	2.68	0.33	1.44	2.27	0.36	1.46	0.36
Total energy consumption increase (kWh)	23.4	1,549.1	13,725.6	26,317.9	107.6	1,605.2	11,930.0	225.8	1,296.4	413.3
Increase ratio of total energy consumption	0.00%	0.01%	0.10%	0.20%	0.00%	0.01%	0.09%	0.00%	0.01%	0.00%
Average increase in iteration step numbers	1	8	19	28	1	8	18	-1	8	-1
Number of non-convergence optimizations	0	63	818	1916	0	58	707	0	38	0

3.4.2 Effects of step-size

To investigate the effects of the step-size for the update of the Lagrange multiplier on the impacts of information delays, the control strategy was tested with different step-sizes. Tests were conducted in two scenarios with typical information delays and the corresponding statistical results are shown in Table 3 and Table 4 respectively. The first scenario involves uniform delays with the largest sum of delays where [Ld_{ct}=4, Ld_{chi}=4], and the other scenario involves nonuniform delays with the largest difference of delays where [Ld_{ct}=0, Ld_{chi}=4]. The step-size varies from 1×10^{-8} to 5×10^{-8} with an increment of 1×10^{-8}.

It can be seen that the best choice of the step-size in ideal conditions, i.e. 5×10^{-8}, was not the best choice when information delays exist. Using an appropriate step-size can reduce the impacts of information delays on the optimization results and the convergence rate. The best choice of step-size was 2×10^{-8} under both test scenarios, as it had the least impacts on the convergence rate and a negligible impact on optimization results. In the two test scenarios, the average increase of iteration step numbers was 18 and 17, and the number of non-convergence optimizations was 397 and 350 (5.84% and 5.15%) respectively.

Table 3 Statistical performance data of optimal control strategy using different step-sizes under uniform delays

Step-size	1×10^{-8}	2×10^{-8}	3×10^{-8}	4×10^{-8}	5×10^{-8}
Average bias in optimization results (K)	0.47	0.08	0.27	0.36	0.44
Maximum bias in optimization results (K)	2.93	2.20	2.80	2.82	3.03
Total energy consumption increase (kWh)	23,449.7	6,924.6	22,719.9	28,743.7	37,397.2
Increase ratio of total energy consumption	0.18%	0.05%	0.17%	0.21%	0.28%
Average increase of iteration step numbers	30	18	25	29	33
Number of non-convergence optimizations	3,198	397	1,514	2,262	2,986

Table 4 Statistical performance data of optimal control strategy using different step-sizes under nonuniform delays

Step-size	1×10^{-8}	2×10^{-8}	3×10^{-8}	4×10^{-8}	5×10^{-8}
Average bias in optimization results (K)	0.48	0.08	0.26	0.39	0.44
Maximum bias in optimization results (K)	2.97	2.48	2.70	2.80	2.80
Total energy consumption increase (kWh)	23,941.6	6,409.5	21,115.4	30,507.4	33,539.3
Increase ratio of total energy consumption	0.18%	0.05%	0.16%	0.23%	0.25%
Average increase of iteration step numbers	30	17	25	28	31

Number of non-convergence optimizations	3,223	350	1,329	2,095	2,582

4 Impacts on distributed optimal control of a multi-zone air-conditioning system

The second test case is the impacts of information delays on the distributed multi-objective optimal control of a multi-zone air-conditioning system with a dedicated outdoor air system (DOAS) using fan-coil units (FCU), as shown in Fig. 10. The optimization objective is to minimize the energy consumption of the primary air handling unit (PAU) while maintaining indoor air quality through optimizing the ventilation air volume of individual rooms (Q_i) and the total PAU ventilation air volume (Q_{tot}). The objective function is shown in Eq. (14). *INp* is the indoor pollution index, *E* is the energy consumption, and γ is the weighting factor.

$$\min_{Q_i, Q_{Tot}} Obj = \sum_{i=1}^{n} INp_i + \gamma \cdot E \qquad (14)$$
$$\textit{subject to: } \sum_{i=1}^{n} Q_i = Q_{\mathrm{tot}}$$

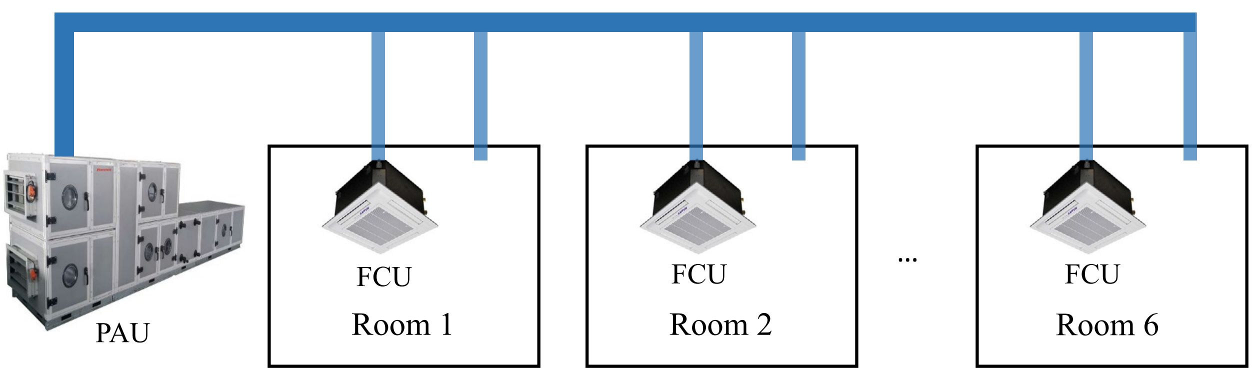


Fig. 10. Schematic of the air conditioning system concerned

In this test case, CO_2 is the pollutant concerned. The pollution index is defined as Eq. (15), which is related to the difference between the steady-state CO_2 concentration ($CO2_i$) and the recommended limit ($CO2_{limit}$) of 800 ppm [37].

$$INp_i = \max\{0, CO2_i - CO2_{limit}\}^2 \qquad (15)$$

The steady-state CO_2 concentration can be obtained according to the CO_2 concentration of ambient air ($CO2_{amb}$) and the CO_2 generation of the occupants ($MCO2_G$) as shown in Eq. (16).

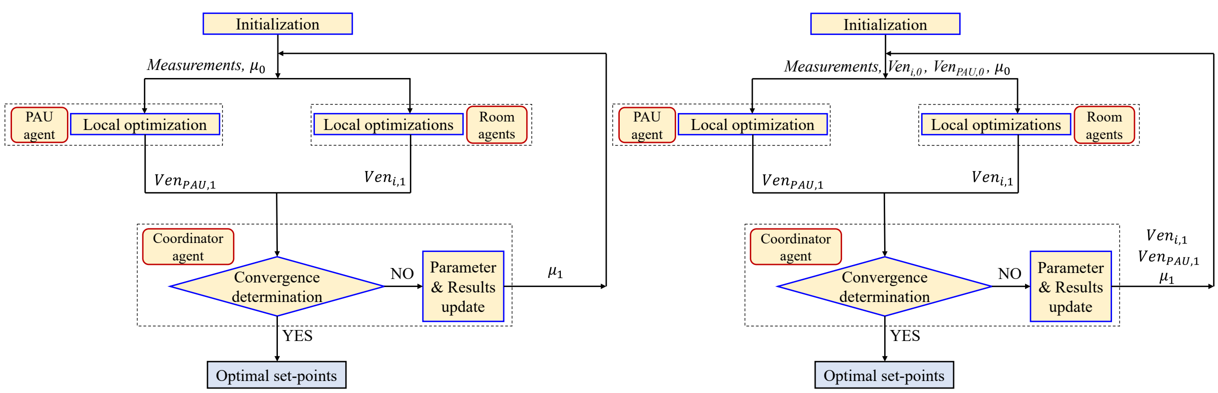
$$CO2_i = CO2_{amb} + \frac{MCO2_G \cdot N_i}{Q_i} \qquad (16)$$

The energy consumption of PAU (E) can be calculated through Eq. (17), it is the sum of the needed chiller energy consumption to generate the required cooling (E_{chi}) [38] and the energy consumption of the fan (E_{fan}).

$$E = E_{chi} + E_{fan} \tag{17}$$

4.1 Description of the distributed optimal control strategy

In this test case, two distributed optimization algorithms were adopted to formulate the distributed optimal control strategy, a subgradient method based on dual decomposition and the alternating direction method of multipliers (ADMM). The first one is simpler with lower computational load and communication load, while the second one requires weaker assumptions to guarantee convergence. The impacts of the information delays on the performance of the distributed optimal control strategies using the two algorithms were tested and compared. Fig. 11 illustrates the optimization processes of the distributed optimal control strategies using the two methods. It can be found that the main difference is that, using ADMM, more information exchange between component agents and the coordinator agent is required. More detailed information regarding the control strategy can be found in [39].



(a) Subgradient method (b) ADMM

Fig. 11. Optimization process of the distributed optimal control strategies using different methods

4.2 Comparison on impacts of information delays using different optimization methods

In order to compare the impacts of the information delays on the performance of the two optimal control strategies, they were tested using the same uncertain information delays as well as the same critical condition with constant information delays. The uncertain information delays are modelled using the Markov chain constructed in Section 2.2, and the constant information delays are set to the possible maximum length, 2 sampling intervals. Eleven working conditions (working hours) in a spring day and eleven working conditions in a summer day were selected as the test working conditions. The occupancy profiles of the six rooms are shown in Fig. 12.

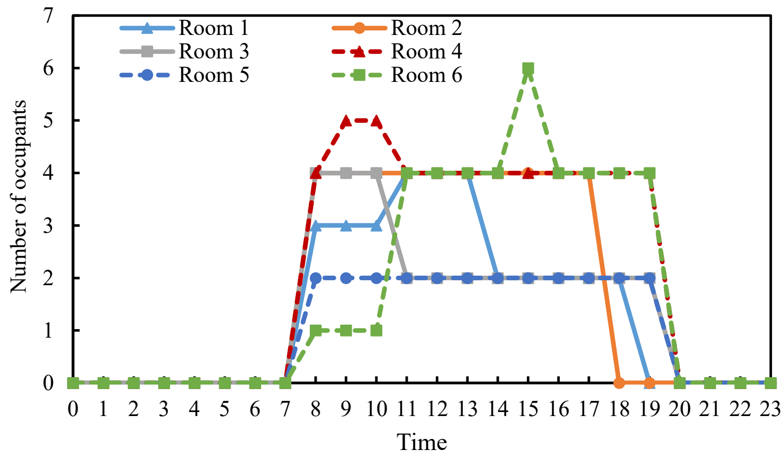
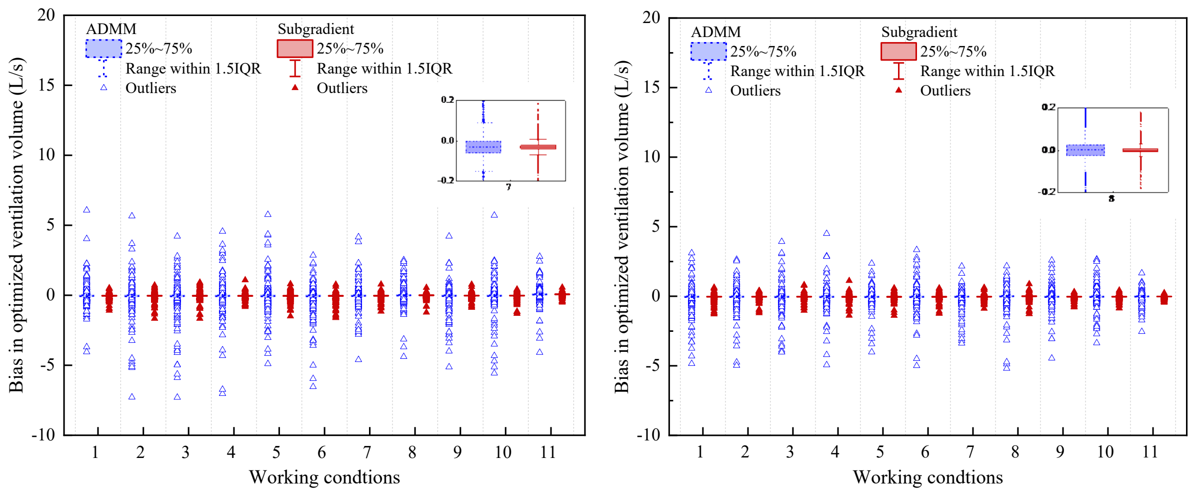


Fig. 12. Occupancy profiles of the six rooms concerned

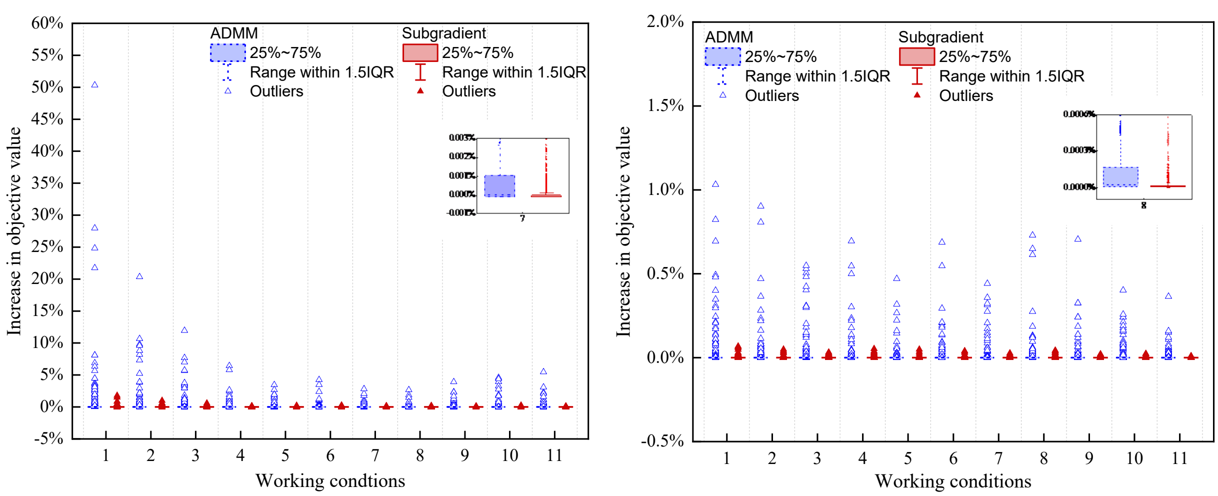
4.2.1 Impacts of uncertain information delays

The test results show that information delays had a larger impact on the control strategy using ADMM than when using the subgradient method. This is due to the fact that the amount of information exchange required by ADMM is larger than that required by the subgradient method. Fig. 13 shows the impacts of the information delays on the optimization results of the two control strategies in spring and summer working conditions respectively. It can be found that the biases in the optimized ventilation air volume using ADMM was larger than that using the subgradient method. As a result, the increase in the achieved objective value using ADMM was larger than that using the subgradient method in the tests as shown in Fig. 14. Using the subgradient method, in the spring and summer working conditions, the maximum biases in optimized ventilation air volume were 1.08 L/s and 1.11 L/s, and the maximum increase in the achieved objective values were 1.79% and 0.07% respectively. While using the ADMM, the maximum bias in optimized ventilation air volume was 6.07 L/s and 4.49 L/s, and the maximum increase in the achieved objective values was 50.33% and 1.03% respectively. The reason that the maximum increase in the achieved objective value in the spring working conditions was much larger is that the objective value achieved in ideal condition was much smaller. Fig. 15 shows the impacts of information delays on the convergence rate in the spring and summer working conditions respectively. The information delays resulted in the increase in the number of iteration steps in most of the test scenarios using the two methods, and the impacts on the strategy using ADMM was larger than that on the strategy using subgradient method. In ideal conditions, the numbers of iteration steps using the ADMM and subgradient method were similar, around 20 steps in testing. With the information delays involved, in the spring and summer working conditions, the median of the iteration steps using ADMM was around 62 and the maximum number of iteration steps was 147 and 148 respectively. When using subgradient method, the median of the iteration steps was around 40 and the maximum number of iteration steps was 51 and 59 respectively.



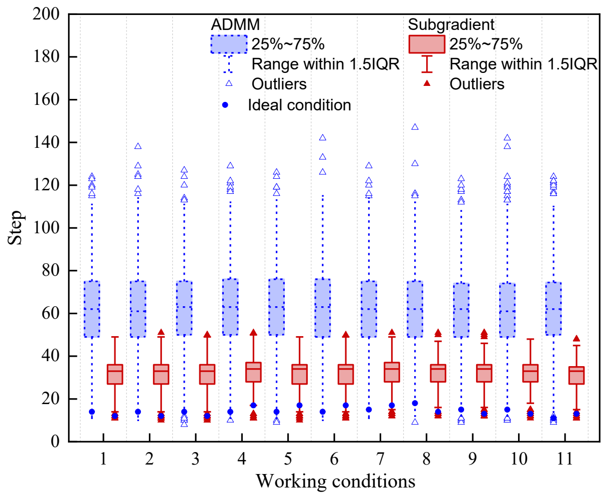
(a) Spring working conditions (b) Summer working conditions

Figure. 13 Statistical distribution of the bias in optimized ventilation volume under information delays

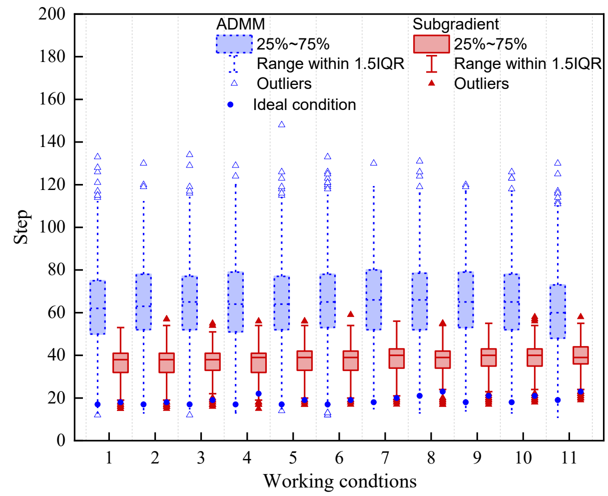


(a) Spring working conditions (b) Summer working conditions

Figure. 14 Statistical distribution of the difference in the objective value under information delays



(a) Spring working conditions



(b) Summer working conditions

Figure. 15 Statistical distribution of the number of iteration steps under information delays

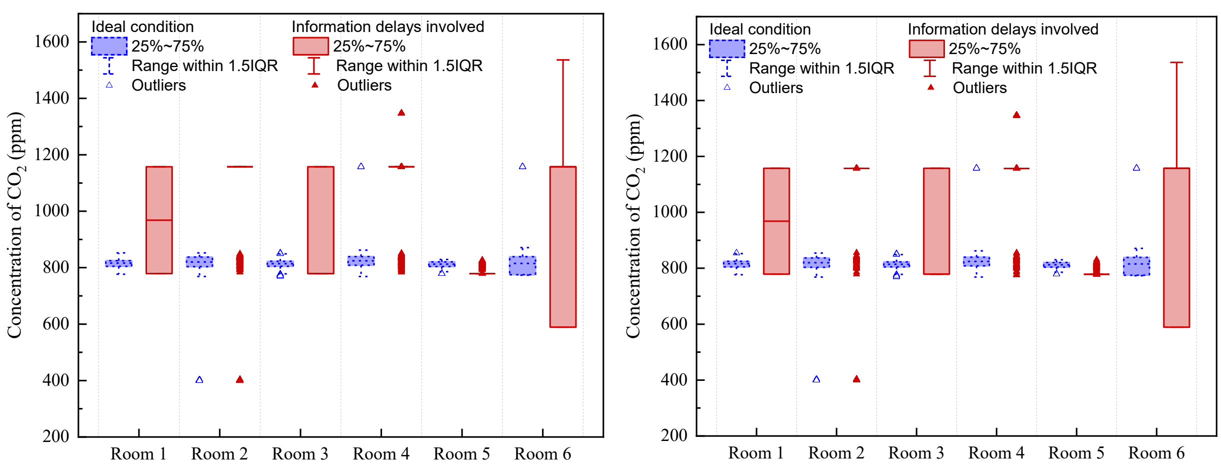
4.2.2 Impacts of information delays under critical conditions

Both control strategies were tested on a typical year with constant information delays of two sampling intervals. According to the requirements of optimal control in practical applications and the convergence rate in ideal conditions, the optimal control interval was set to one minute in the tests. If convergence is not achieved within one minute, a simple ventilation strategy is adopted: the ventilation air volume is set to 22L/s for each room according to the design occupancy and room size. Table 9 shows the statistical results of the tests. The constant information delays obviously affected ventilation volumes and convergence rate of both control strategies, with the impacts on the control strategy using ADMM being larger. Using the subgradient method, the average biases in total ventilation volume and individual ventilation volume were 70.22 L/s and 1.23 L/s respectively. Using ADMM, the average biases in total ventilation volume and individual ventilation volume were 87.40 L/s and 2.16 L/s respectively. The average increases in the number of iteration steps of the two control strategies were similar (i.e. 41 and 43 respectively) while the number of non-convergence optimizations using ADMM was 3,927 (i.e. 97.81%), which is 148 larger than that using subgradient method.

Table 9 Statistical performance data of two control strategies under constant information delays of two sampling intervals

Optimization algorithm	Subgradient method	ADMM
Average bias in total ventilation volume (L/s)	70.22	87.40
Maximum bias in total ventilation volume (L/s)	144.15	180.72
Average bias in individual ventilation volume (L/s)	1.23	2.16
Maximum bias in individual ventilation volume (L/s)	44.44	44.44
Average increase in iteration step numbers	41	43
Number of non-convergence optimizations	3,779	3,927

Fig. 16 shows the distribution of the CO_2 concentration in the six rooms in the test working conditions using the set-points given by the control strategies using the subgradient method and ADMM respectively. It can be found that in ideal conditions, both control strategies successfully maintained CO_2 concentration around 800 ppm, the recommended upper limit, except for three non-convergence optimizations. Due to information delays, only the CO_2 concentration in Room 5 was maintained around 800 ppm, and the CO_2 concentration in other rooms were much higher than the recommended upper limit in most of the test working conditions. This indicates that the control strategies failed to achieve a proper compromise between maintaining the indoor air quality and energy consumption due to the information delays, although the energy consumption when using the strategies under the information delays was reduced as shown in Fig. 17.



(a) Subgradient method (b) ADMM

Figure. 16. Statistical distribution of CO_2 concentration in six rooms using two control strategies under information delays

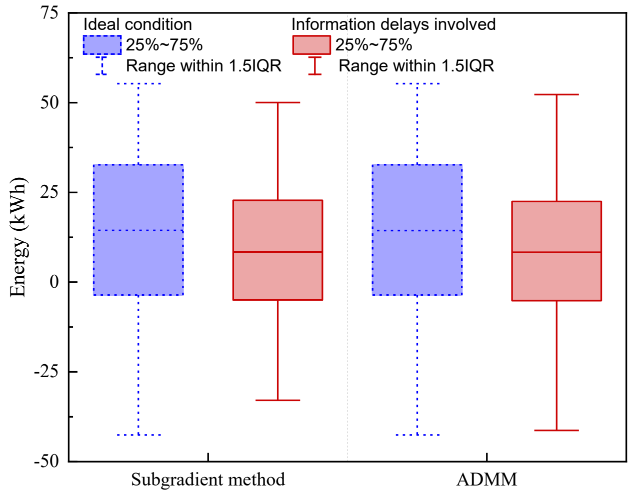


Figure. 17. Statistical distribution of energy consumption using two control strategies under information delays

5 Conclusions

The impacts of information delays on the performance of distributed optimal control for HVAC systems deployed on field control networks are investigated through qualitative analysis and quantitative assessment using two case studies. The information delays are caused by the time cost of local optimization and the information exchange between devices as well as the asynchronous operation of different devices. Due to the information delays, the optimal control strategies resulted in biased optimization results or failed to achieve the optimization results within the preset optimal control interval. These impacts can result in reduced energy performance for the distributed optimal control strategies. The lengths of information delays and the step-size for the update of the Lagrange multiplier are two critical factors which could significantly affect the impacts of the information delays. Considering these two factors when developing distributed optimal control strategies could improve the reliability and energy performance of the distributed optimal control strategies. Moreover, the distributed optimal control strategy using the subgradient method shows greater robustness than when using ADMM under the same information delays. According to the test results and analysis, detailed conclusions can be drawn as follows:

- Information delays could affect the performance of the distributed optimal control strategy significantly. Under uncertain information delays, the maximum bias of optimized cooling tower outlet water temperature was up to 0.6 K, the largest number of iteration steps increased to 180 (about nine times of that in ideal conditions), and the power consumption of the cooling plant was increased by 0.2%.

- The impacts of information delays increased dramatically with an increase in information delay length. With an increase in delay length from 1 sampling interval to 4 sampling intervals, the average bias in optimization results increased from 0.05 K to 0.44 K, the increase of annual energy consumption rose from 7,853.3 kWh to 37,397.2 kWh (0.06% to 0.28%) and the number of non-convergence optimizations increased from 306 to 2,986 (4.50% to 43.92%).
- The best step-size for the update of the Lagrange multiplier determined in ideal conditions is not the best choice when information delays exist. Proper selection of the appropriate step-size effectively reduced the impacts of information delays and therefore improved the energy performance. The annual energy consumption increase was reduced from 37,397.2 kWh to 6924.6 kWh (from 0.28% to 0.05%).
- The impacts of information delays on the performance of distributed optimal control strategies using ADMM is much larger than that using the subgradient method since ADMM requires more information exchange than the subgradient method. Under the same uncertain information delays, using ADMM and the subgradient method, the maximum biases in optimized ventilation air volume were 6.07 L/s and 1.08 L/s, and the maximum numbers of iteration steps increased from 18 and 17 to 147 and 51 respectively.

It is worth noticing that although the energy savings achieved by using distributed optimal control strategy is not significant compared with the near-optimal control strategy, which is 70967.61 kWh (i.e. 0.53%) in ideal conditions, it will not lead to any additional cost after the control strategy is implemented. Having confirmed the impacts and identified the critical factors, further research efforts will be devoted to developing delay-tolerant methods for the distributed optimal control strategies of building HVAC systems to eliminate or reduce the impacts of information delays to allow for practical implementation.

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References

[1] Byers CC, Wetterwald P. Fog Computing Distributing Data and Intelligence for Resiliency and Scale Necessary for IoT: The Internet of Things (Ubiquity Symposium). Ubiquity 2015;2015:4:1–4:12.

[2] Yousefpour A, Fung C, Nguyen T, Kadiyala K, Jalali F, Niakanlahiji A, et al. All one needs to know about fog computing and related edge computing paradigms: A complete survey. Journal of Systems Architecture 2019;98:289–330.

[3] Khan R, Khan SU, Zaheer R, Khan S. Future Internet: The Internet of Things Architecture, Possible Applications and Key Challenges. 2012 10th International Conference on Frontiers of Information Technology, 2012, p. 257–60.

[4] Wang SW, Ma Z. Supervisory and Optimal Control of Building HVAC Systems: A Review. HVAC&R Research 2008;14:3–32.

[5] Windham A, Treado S. A review of multi-agent systems concepts and research related to building HVAC control. Science and Technology for the Built Environment 2015;22:50–66.

[6] Wooldridge M, Jennings NR. Intelligent agents: theory and practice. The Knowledge Engineering Review 1995;10:115–152.

[7] Schumacher M. Objective Coordination in Multi-agent System Engineering: Design and Implementation. Berlin, Heidelberg: Springer-Verlag; 2001.

[8] Yang Cao, Tao Wang, Xinyi Song, An energy-aware, agent-based maintenance-scheduling framework to improve occupant satisfaction, Automation in Construction, Volume 60, 2015, Pages 49-57, ISSN 0926-5805.

[9] Hagras H, Callaghan V, Colley M, Clarke G. A hierarchical fuzzy–genetic multi-agent architecture for intelligent buildings online learning, adaptation and control. Information Sciences 2003;150:33–57.

[10] Klein L, Kwak J, Kavulya G, Jazizadeh F, Becerik-Gerber B, Varakantham P, et al. Coordinating occupant behavior for building energy and comfort management using multi-agent systems. Automation in Construction 2012;22:525–36.

[11] Yang R, Wang L. Development of multi-agent system for building energy and comfort management based on occupant behaviors. Energy and Buildings 2013;56:1–7.

[12] Michailidis IT, Schild T, Sangi R, Michailidis P, Korkas C, Fütterer J, et al. Energy-efficient HVAC management using cooperative, self-trained, control agents: A real-life German building case study. Applied Energy 2018;211:113–25.

[13] Lymperopoulos G, Ioannou P. Building temperature regulation in a multi-zone HVAC system using distributed adaptive control. Energy and Buildings 2020;215:109825.

[14] Yang R, Wang L. Multi-zone building energy management using intelligent control and optimization. Sustainable Cities and Society 2013;6:16–21.

[15] Wang SW, Xing J, Jiang Z, Dai Y. A decentralized, model-free, global optimization method for energy saving in heating, ventilation and air conditioning systems. Building Services Engineering Research and Technology 2019:0143624419862707.

[16] Cai J, Kim D, Jaramillo R, Braun JE, Hu J. A general multi-agent control approach for building energy system optimization. Energy and Buildings 2016;127:337–51.

[17] Su B, Wang SW. An agent-based distributed real-time optimal control strategy for building HVAC systems for applications in the context of future IoT-based smart sensor networks. Applied Energy 2020;274:115322.

[18] Doyle J, Stein G. Robustness with observers. IEEE Transactions on Automatic Control 1979;24:607–11.
[19] Ploplys NJ, Kawka PA, Alleyne AG. Closed-loop control over wireless networks. IEEE Control Systems Magazine 2004;24:58–71.
[20] Feng-Li Lian, Moyne J, Tilbury D. Analysis and modeling of networked control systems: MIMO case with multiple time delays. Proceedings of the 2001 American Control Conference. (Cat. No.01CH37148), vol. 6, 2001, p. 4306–12 vol.6.
[21] Long F, Zhang C, He Y, Wu M. Stability analysis of load frequency control systems with two delays in each area. IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society, 2017, p. 5656–60.
[22] Nilsson J, Bernhardsson B, Wittenmark B. Stochastic analysis and control of real-time systems with random time delays. Automatica 1998;34:57–64.
[23] Nedic A, Ozdaglar A. Distributed Subgradient Methods for Multi-Agent Optimization. IEEE Transactions on Automatic Control 2009;54:48–61.
[24] Lobel I, Ozdaglar A. Distributed Subgradient Methods for Convex Optimization Over Random Networks. IEEE Transactions on Automatic Control 2011;56:1291–306.
[25] Tsianos KI, Rabbat MG. Distributed consensus and optimization under communication delays. 2011 49th Annual Allerton Conference on Communication, Control, and Computing (Allerton), 2011, p. 974–82.
[26] Nedić A, Ozdaglar A. Convergence rate for consensus with delays. Journal of Global Optimization 2010;47:437–56.
[27] Lin P, Ren W, Song Y. Distributed multi-agent optimization subject to nonidentical constraints and communication delays. Automatica 2016;65:120–31.
[28] Yang S, Liu Q, Wang J. Distributed Optimization Based on a Multiagent System in the Presence of Communication Delays. IEEE Transactions on Systems, Man, and Cybernetics: Systems 2017;47:717–28.
[29] Zhao C, Duan X, Shi Y. Analysis of Consensus-Based Economic Dispatch Algorithm Under Time Delays. IEEE Transactions on Systems, Man, and Cybernetics: Systems 2020;50:2978–88.
[30] Charalambous T, Yuan Y, Yang T, Pan W, Hadjicostis CN, Johansson M. Distributed Finite-Time Average Consensus in Digraphs in the Presence of Time Delays. IEEE Transactions on Control of Network Systems 2015;2:370–81.
[31] Lin Xiao, Hassibi A, How JP. Control with random communication delays via a discrete-time jump system approach. Proceedings of the 2000 American Control Conference. ACC (IEEE Cat. No.00CH36334), vol. 3, 2000, p. 2199–204 vol.3.
[32] Stoecker WF (ed). Procedures for simulating the performance of components and systems for energy calculations. Third edition 1975.
[33] Ma Z, Wang SW. An optimal control strategy for complex building central chilled water systems for practical and real-time applications. Building and Environment 2009;44:1188–98.
[34] Palomar DP, Mung Chiang. A tutorial on decomposition methods for network utility maximization. IEEE Journal on Selected Areas in Communications 2006;24:1439–51.
[35] Tao Yang, Di Wu, Yannan Sun, Jianming Lian. Impacts of time delays on distributed algorithms for economic dispatch. 2015 IEEE Power Energy Society General Meeting, 2015, p. 1–5.

[36] Zhao C, Duan X, Shi Y. Analysis of Consensus-Based Economic Dispatch Algorithm Under Time Delays. IEEE Transactions on Systems, Man, and Cybernetics: Systems 2020;50:2978–88.
[37] Indoor Air Quality Management Group of the Government of the Hong Kong Special Administrative Region. Guidance Notes for the Management of Indoor Air Quality in Offices and Public Places. 2003.
[38] Cheng Y, Zhang S, Huan C, Oladokun MO, Lin Z. Optimization on fresh outdoor air ratio of air conditioning system with stratum ventilation for both targeted indoor air quality and maximal energy saving. Building and Environment 2019;147:11–22.
[39] Li WZ, Wang SW. A multi-agent based distributed approach for optimal control of multi-zone ventilation systems considering indoor air quality and energy use. Applied Energy 2020;275:115371.