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Measuring Route Diversity for Urban Rail Transit Networks: A Case Study of the Beijing Metro Network

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***Abstract*—Most stations and tracks in metro networks are irreplaceable due to daily operations. If any of them were disrupted, it would impact not only the individual metro line but also the whole metro network. Therefore, metro managers need to have a good understanding of alternative routes between each pair of stations in the metro network. In the event of incidents, metro managers can make use of this information to reroute passengers to minimize the impact of disruptions. This paper aims to develop a route diversity index to address two questions: "*how many reasonable routes are there for passengers between any two stations in normal operations or in the event of a disruption?*" and "*which stations are most vulnerable (i.e., the largest impact to the overall metro network when they are disrupted)?*" To implement this measure in practice, definitions of routes and route diversity and a solution algorithm based on characteristics of metro networks are described to calculate the route diversity index. To show proof of the concept, a simple network example and a real-world network based on the Beijing Metro network in China are presented to demonstrate the feasibility of the route diversity index and its application to a real-world metro network.**

Keywords: Metro systems; reasonable routes; route diversity; vulnerability

I. Introduction

Metro networks have played a key role for mitigating congestion and pollution in many metropolitan cities around the world [1]. In mainland China, metro networks have received rapid development in past decades, expanding from three cities (i.e., Beijing, Shanghai, and Guangzhou) with five metro lines in 2000 to twenty two cities with eighty-eight metro lines spanning a total distance of more than 3000 kilometers by the end of 2014 [2, 3].

Although metro networks operate with large transport capacity and efficient energy consumption [4, 5], they are susceptible to incidents and are also vulnerable to disruptions [6]. If one or more stations were disrupted by incidents, there would be a great impact not only on the individual metro line but also on the whole metro network. History has shown that metro networks are not incidents free; therefore, it is difficult, if not impossible, to maintain normal operations at all the times. The definition of an incident is an event that can directly or indirectly result in considerable reductions or interruptions in the serviceability of a link, route, or network [7]. Common incidents in metro networks include collapses, leaks, fires, and terrorist attacks (see Fig. 1 for an example). A set of metro incidents around the world in the past decade is provided in Table I.

When any incident occurs, metro operation managers need to be able to provide alternate route information to reroute passengers [18]. Therefore, it is important to know how many reasonable routes are there between any two stations in normal operation or in the event of a disruption. In addition, metro managers should know which stations are vulnerable (i.e., have the largest impact to the overall metro network when they are disrupted) in order to make use of the limited human, material, and financial resources to protect these stations. The main contribution of this paper is to develop a route diversity index to tackle two critical questions:

- How many reasonable routes are there for passengers between any two stations in the metro network?
- Which stations are most vulnerable?

The reasonable route adopted in this paper refers to the definition in Dial [19], which is "*a route consists of the links that take network users further away from the origin and closer to the destination.*" In other words, reasonable routes are comprised of a sequence of links and nodes such that the shortest route cost from the origin to the head of the link is strictly greater than that to the tail of the link, and the shortest route cost from the head of the link to the destination is strictly less than that from the tail of the link to the destination.

The remainder of this paper is organized as follows. In Section 2, the vulnerability of metro networks and the methods for finding routes are reviewed. In Section 3, we first describe the problem and provide some definitions. Then, we introduce the principle of the metro network simplification. Finally, we present the solution algorithm for calculating the number of reasonable routes and the route diversity index. In Section 4, we present two numerical examples to demonstrate the feasibility of the measurement and its application on the real-world metro networks. Conclusions are given in Section 5.

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(a) Collapse (b) Leak (c) Fire (d) Terrorist attack

Fig. 1. Examples of some metro incidents [8–11].

TABLE I
SOME METRO INCIDENTS AROUND THE WORLD IN THE PAST DECADE

Location	Time	Incident type	Source
London Underground (UK)	July 2005	Terrorist attack	British Red Cross [12]
Hangzhou Metro (China)	Nov. 2008	Collapse	Xinhua Online [8]
Beijing Metro (China)	Feb. 2010	Leak	Sohu [9]
Moscow Metro (Russia)	Mar. 2010	Terrorist attack	CNN World [13]
Delhi Metro (India)	Nov. 2012	Fire	NDTV [14]
Moscow Metro (Russia)	Jun. 2013	Fire	Trend News Agency [10]
Nanjing Metro (China)	Nov. 2013	Collapse	People's Daily Online [15]
Cairo Metro (Egypt)	June 2014	Terrorist attack	The Wall Street Journal [11]
Mumbai Metro (India)	July 2014	Leak	Dnaindia [16]
Zhengzhou Metro (China)	Sept. 2014	Leak	China Daily [17]

II. LITERATURE REVIEW

The incidents in metro networks could result in significant productivity loss and widespread confusion. Therefore, the safety, reliability, robustness and vulnerability of metro networks have been studied in past decades [20–25]. Zhang *et al.* [20] presented a probabilistic decision approach for safety risk analysis for metro construction in complex project environments. A fuzzy importance measure was deployed for the sensitivity analysis of basic event to reveal the critical basic events for reducing the risk limit. Derrible and Kennedy [25] analyzed the complexity and robustness of metro networks using two particular concepts: scale-free patterns and small-worlds. The Tokyo Metro network was studied as a real-world example for large networks analysis. Recently, reducing the vulnerability of metro networks has been studied as a direct method to minimize the impact of disruptions. Vulnerability is a general concept, and there are many different definitions. For example, Holmgren [26] defines vulnerability as a collection of properties of an infrastructure system that may weaken or limit its ability to maintain its intended function when exposed to threats and hazards. In transportation networks, vulnerability is commonly seen as the complement of reliability [7, 27]. Deng *et al.* [28] use the Space L model to examine the topological vulnerability of the Nanjing Metro network in China. They studied the network topological characteristics and functional properties of metro networks. Li and Kim [29] developed a survivability index to measure both reliability and vulnerability of the Beijing Metro network. The survivability index measures the topological connectivity under various simulated failures of transfer stations and variations in passenger flow in response to disruptive factors. In contrast, this paper develops the route diversity index to measure to address two critical questions: "*how many reasonable routes are there for passengers between any two stations in the metro network?*" and "*which stations are most vulnerable?*".

The procedure for counting the set of reasonable routes in this paper is based on three different algorithms: Dijkstra's shortest route algorithm [30], Dial's STOCH loading algorithm [19], and a combinatorial algorithm by Meng *et al.* [31]. As early as 1950s, Dijkstra [30] proposed the Dijkstra's algorithm to find the shortest routes between nodes in a graph, which was widely applied to transportation networks. In 1968, Hart *et al.* [32] proposed the A^* algorithm to enhance the performance of the shortest route algorithm for finding the minimum cost routes. However, some users in the real-world transportation network may not select the shortest route in many cases due to various reasons (e.g., perception errors of network conditions, personal preferences, etc.). Considering the users' multiple route choices in practice, Dial [19] proposed the concept of reasonable routes to describe a route of an origin-destination (O-D) pair that could be used by network users with error-prone perception on the route cost. The key idea of Dial's algorithm is to find the reasonable routes subject to the links that take network users further away from the origin and closer to the destination. Furthermore, Meng *et al.* [31] designed a combinatorial algorithm with polynomial-time complexity to compute the number of different reasonable routes of an O-D pair for a transportation network. This algorithm consists of two parts: (1) constructing a sub-network G_r for each origin r by deleting the links and nodes that cannot be reachable from the origin r; and (2) counting the number of efficient routes from origin r to all nodes in the subnetwork G_r. The whole calculation involves three nested layers with a time complexity of $O(|N|^3)$ where $|N|$ is the number of nodes.

III. Methodology

Metro networks in many metropolitan cities around the world are generally large scale. For example, the Beijing Metro network and the New York Metro network respectively have 233 stations and 421 stations (stations connected by transfers are counted as a single station). In these large-scale networks, there may be thousands of different routes between stations with long distance. However, most of these routes take too much time and money; they are unreasonable or unacceptable to passengers. In this section, we provide some definitions related to the concept of reasonable routes, introduce the unique structure of metro networks, and present the procedure for calculating the number of reasonable routes and the route diversity index for a metro network.

A. Problem Statement

Consider a directed metro network $G = (N, A)$, where N is a finite set of nodes (stations) and A is a finite set of links (tracks between adjacent stations). We define that $|N|$ and $|A|$ denote the numbers of nodes and links, respectively. Let a be a link in A. If link $a \in A$ connects from node a_t to node a_h, node a_t and node a_h are considered to as the tail and head of the link, respectively. It is known that any node in N can be considered as an origin or a destination. Let m and n be two distinct nodes in N. The (m, n) is defined as an O-D pair from node m to node n. For clarity, some other definitions are presented as follows.

Definition 1. A route between the O-D pair (m, n) is defined as an alternating sequence of nodes and links beginning with node m and ending with node n.

Definition 2. A simple route between the O-D pair (m, n) is defined as the following: neither a node nor a link appears more than once in the sequence of a route between the O-D pair (m, n). Note that two simple routes are considered as different (or unique) if and only if there is at least one distinct node or link between these two simple routes.

Definition 3. A shortest route between the O-D pair (m, n) is defined as a simple route whose nodes and links sum to the smallest total travel time cost.

Definition 4. A reasonable route between the O-D pair (m, n) is defined such that all links in a simple route k satisfies the following constraint:

$$f_{\min}(m, a_h) > f_{\min}(m, a_t), \ \forall a \in A_k, \tag{1}$$

where $f_{\min}(m, a_h)$ and $f_{\min}(m, a_t)$ denote the shortest route costs from origin m to the head and tail of link a, respectively; and A_k denotes the set of links in route k. This means that the network users on a reasonable route will get further away from the origin and closer to the destination.

According to the characteristics of metro networks, all sections are two-way links and all stations are interlinked nodes in normal operation conditions, so the number of O-D pairs of the network G is $|N|(|N| - 1)$.

Definition 5. Route diversity of the network G is defined as the average number of reasonable routes between all O-D pairs. Mathematically, a route diversity index $\mathfrak{R}$ can be expressed as:

$$\mathfrak{R} = \frac{\sum_{m=1}^{|N|} \left(\sum_{n=1}^{m-1} x(m, n) + \sum_{n=m+1}^{|N|} x(m, n) \right)}{|N|(|N| - 1)}, \tag{2}$$

where $x(m, n)$ denotes the number of reasonable routes between O-D pair (m, n).

B. Metro Network Simplification

In contrast to the road network, the connected metro lines are only conjoined by one or several fixed transfer stations. The successive stations in the same line divided by transfer stations (such as stations 1 and 2, stations 4 and 5, etc. in Fig. 2a) have similar location and character in the network, so we respectively consider them as a single zone. Note that there is only one route between any two stations in the same zone, and there are the same number of routes between any two stations in different zones or between any two stations in the same zone and transfer stations. For example, the network shown in Fig. 2a are divided into 5 zones and 2 transfer stations, and there are the same number of routes between stations 1 and 9, 1 and 10, 2 and 9, and 2 and 10. Furthermore, stations in the same zone can be considered as a single node, so that the network can be simplified to facilitate the calculations of the number of reasonable routes. For example, the original network shown in Fig. 2a has 12 nodes and 26 links (two-way links are considered as two single links), while the simplified network shown in Fig. 2b only has 7 nodes and 16 links.

Remark 1. For the simplified network in Fig. 2b, we can calculate the number of reasonable routes between O-D pair (1, 6), which also represents the number of reasonable routes between O-D pairs (1, 9), (1, 10), (2, 9) and (2, 10) of the original network in Fig. 2a. Therefore, we evaluate the simplified network to represent the original network.

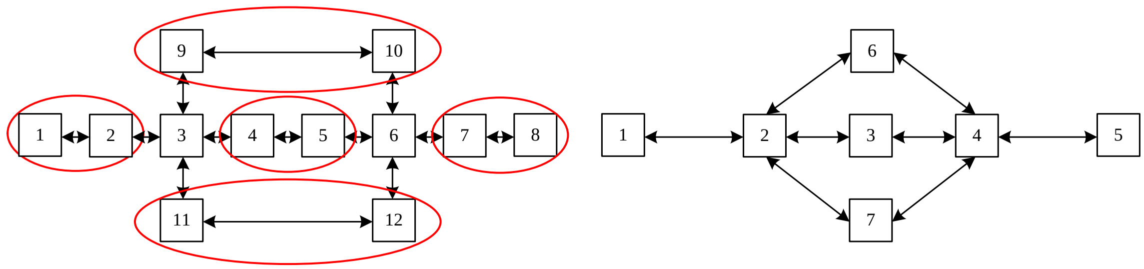
With the given original metro network $G_o = (N_o, A_o)$, we can obtain the simplified network $G = (N, A)$. Based on the above explanations, we calculate the number of reasonable routes between each O-D pair and the route diversity index of the simplified network to represent the passenger route choice and network vulnerability. In addition, the combinatorial time complexity can be effectively reduced.

C. Solution Algorithm

In what follows, we present the detailed procedure of the solution algorithm for calculating the number of reasonable routes between each O-D pair and the route diversity index for a metro network.

With the given network $G = (N, A)$, we define $\boldsymbol{u} = \{u(m, n), m = 1, 2, \cdots, |N|, n = 1, 2, \cdots, |N|\}$ as the node adjacent matrix. For each $m, n \in N$, it has the following expression:

$$u(m, n) = \begin{cases} 1, & \text{if there is a link from node } n \text{ to} \\ & \text{node } m \text{, and } m \neq n \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$



(a) Original network. (b) Simplified network.

Fig. 2. Principle of a metro network simplification.

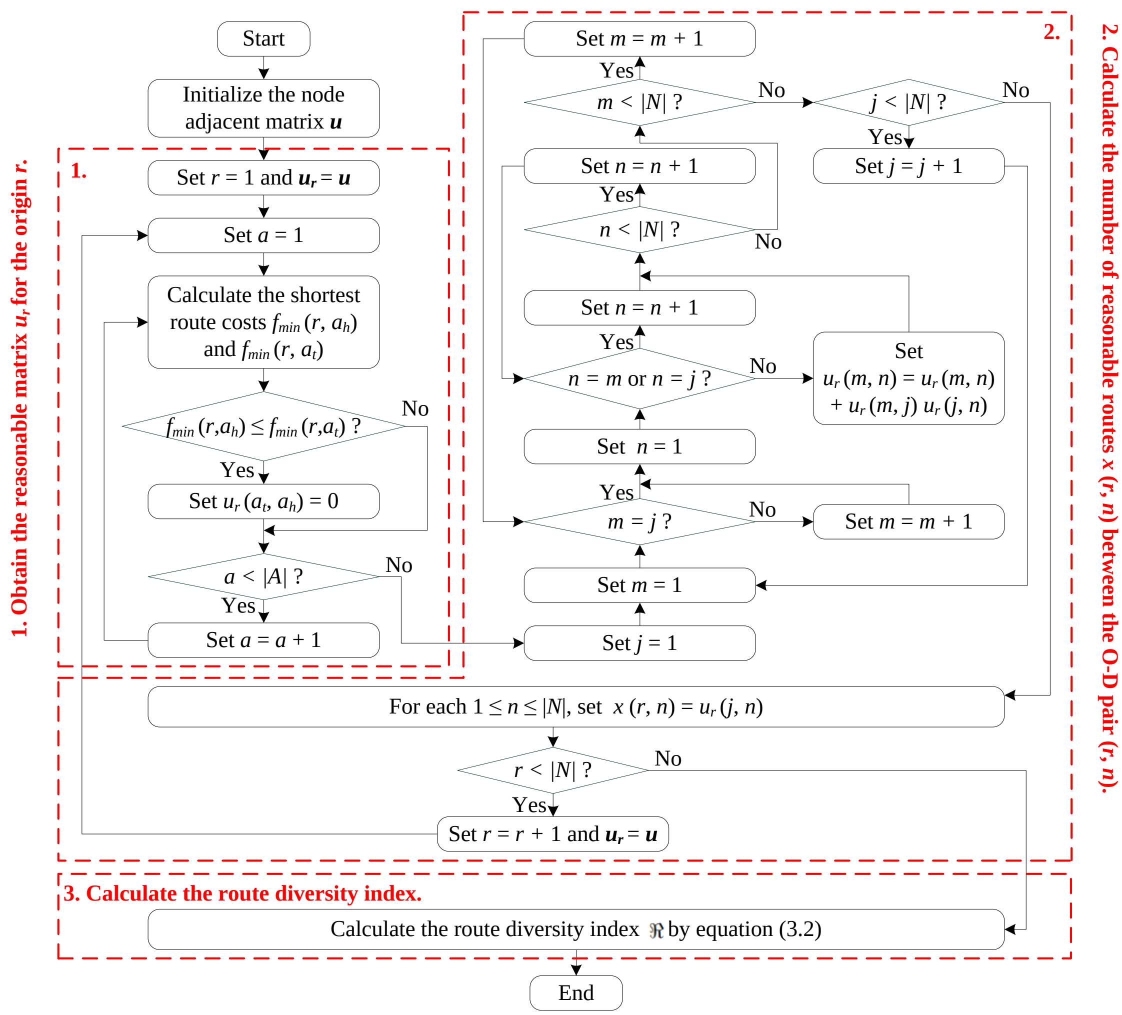


Fig. 3. Flow chart of the solution algorithm.

The procedure for calculating the number of reasonable routes between each O-D pair and the route diversity index is depicted in Fig. 3, and summarized in Algorithm 1.

Remark 2. The whole procedure consists of three parts: (1) obtaining the reasonable adjacent matrix for the origin r; (2) calculating the number of reasonable routes between O-D pair (r, n) for each $1 \leq r \leq N$ and $1 \leq n \leq N$; and (3) evaluating the route diversity index of the network.

Remark 3. The combinatorial time complexity of the whole procedure focuses on the second part (see Fig. 3) that involves four nested layers (i.e., the complexity is $O(|N|^4)$). Therefore, the simplification principle (see Fig. 2) can reduce the complexity by $O(|N_o|^4) - O(|N|^4)$.

IV. Numerical Examples

This section presents two numerical examples, a simple network and the Beijing Metro network, to demonstrate the feasibility of the route diversity index and its application to a real-world metro network.

Algorithm 1 Calculation of the route diversity index

Initialization: Input all sorted nodes $\{r \in N | r = 1, 2, \cdots, |N|\}$, all sorted links $\{a \in A | a = 1, 2, \cdots, |A|\}$, and the node adjacent matrix $\boldsymbol{u}$.

Procedure:

for $1 \leq r < |N|$ **do**
  set $\boldsymbol{u_r} = \boldsymbol{u}$
  for $1 \leq a < |A|$ **do**
    calculate the shortest route costs from origin r to the head a_h and tail a_t of link a using the Dijkstra's algorithm [30], and denote the costs from origin r to node a_h and node a_t by $f_{\min}(r, a_h)$ and $f_{\min}(r, a_t)$
    if $f_{\min}(r, a_h) \leq f_{\min}(r, a_t)$ **then**
      $u_r(a_t, a_h) = 0$
    else
      keep $u_r(a_t, a_h)$
    end if
  end for
  obtain the updated reasonable node adjacent matrix $\boldsymbol{u_r}$ for origin r
  for $1 \leq j < |N|$ **do**
    for $1 \leq m < |N|$ **do**
      if $m = j$ **then**
        set $m = m + 1$
      else
        set $n = 1$
      end if
      if $n = j$ or $n = m$ **then**
        set $n = n + 1$
      else
        set $u_r(m, n) = u_r(m, n) + u_r(m, j) u_r(j, n)$
      end if
      if $n < |N|$ **then**
        set $n = n + 1$
      end if
    end for
    for $1 \leq n \leq |N|$ **do**
      set $x(r, n) = u_r(j, n)$, where $x(r, n)$ is the number of reasonable routes between O-D pair (r, n)
    end for
  end for
end for
calculate the route diversity index $\mathfrak{R}$ using equation (2)
outputs
  route diversity index $\mathfrak{R}$

A. Simple Network

In this example, we use a simple network, shown in Fig. 4, to demonstrate the feasibility of the route diversity index and the correctness of the solution algorithm. This simple network has 6 nodes, 11 links with one-way and two-way connections, and 30 O-D pairs. Based on the simplification principle (see Fig. 2), there is only one station in a zone. Therefore, for this simple network, the simplified network is the same as the original network.

By performing the solution algorithm, we obtain the number of reasonable routes between each O-D pair presented in Table II. The route diversity index is 0.7667, indicating that some O-D pairs are not accessible (i.e., no feasible routes between the O-D pairs). To be specific, 13 O-D pairs have no feasible routes, 13 O-D pairs have only one route, 2 O-D pairs have two routes, and 2 O-D pairs have three routes. For this simple network, it can be seen that route diversity is very low as 26 O-D pairs have either no feasible routes or only one route to access from the origin to the destination, and only 4 O-D pairs have one or more alternatives.

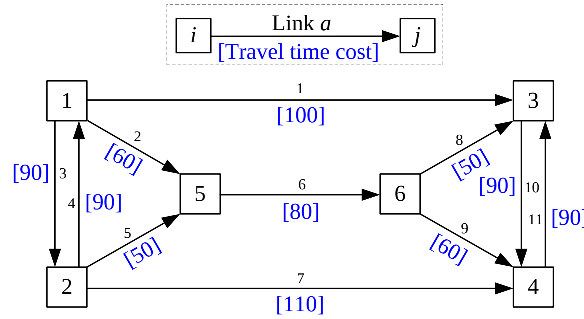


Fig. 4. Simple network.

Next, we present the detailed results between O-D pair (1, 4) to show the correctness of solution algorithm. Based on the first phase of the algorithm, we remove the unreasonable links for origin 1 (i.e., links 4, 5, 8, 11 that do not satisfy the definition of reasonable routes). The sub-network without unreasonable links for origin 1 is shown in Fig. 5a. Based on the second phase of the algorithm, we obtain 3 reasonable routes between O-D pair (1, 4) shown in Fig. 5b.

B. Beijing Metro Network

In this example, we use the Beijing Metro network obtained on October 15, 2014 to demonstrate the application of the route diversity index to a real-world metro network.

(1) Basic Data

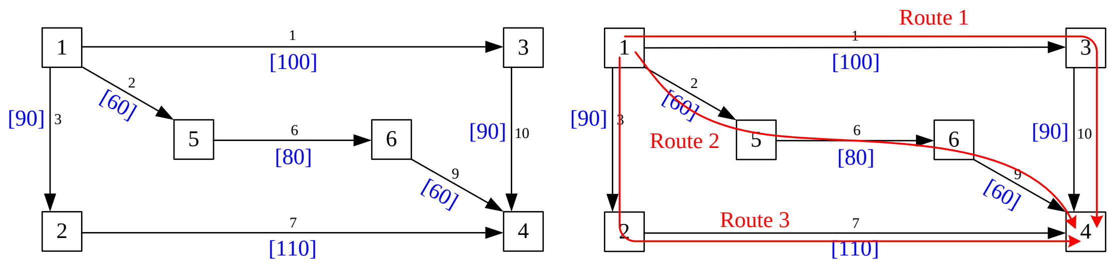
The original Beijing Metro network shown in Fig. 6 consists of 233 stations (Wangjing East station, Andelibeijie station and Yizhuang Railway station are not included, as they are not yet opened by Oct. 15, 2014), 526 links, and 54,056 O-D pairs. The data are from the official website of the Beijing Mass Transit Railway Operation Corporation Limited [33].

Based on the simplification principle (see Fig. 2), we circle the successive stations of the same line divided by transfer stations as a zone shown in Fig. 6. The original network is divided into 61 zones and 38 transfer stations. Therefore, the simplified network shown in Fig. 7 consists of 99 nodes, 254 links and 9702 O-D pairs.

In what follows, we use the route diversity index to address the two questions stated in the abstract: (1) "*how many reasonable routes are there for passengers between any two stations in normal operations or the event of a disruption?*" and (2) "*which stations are most vulnerable?*" This can be achieved by analyzing the distribution of numbers of reasonable routes and the reduction of route diversity based on the simplified network.

TABLE II
NUMBER OF REASONABLE ROUTES BETWEEN EACH O-D PAIR FOR THE SIMPLE NETWORK

	Destination 1	Destination 2	Destination 3	Destination 4	Destination 5	Destination 6
Origin 1	0	1	1	3	1	1
Origin 2	1	0	3	1	1	1
Origin 3	0	0	0	1	0	0
Origin 4	0	0	1	0	0	0
Origin 5	0	0	1	2	0	1
Origin 6	0	0	1	2	0	0



(a) Sub-network without unreasonable links for origin 1

(b) Obtained 3 reasonable routes for O-D pair (1, 4)

Fig. 5. Detailed found reasonable routes between O-D pair (1, 4).

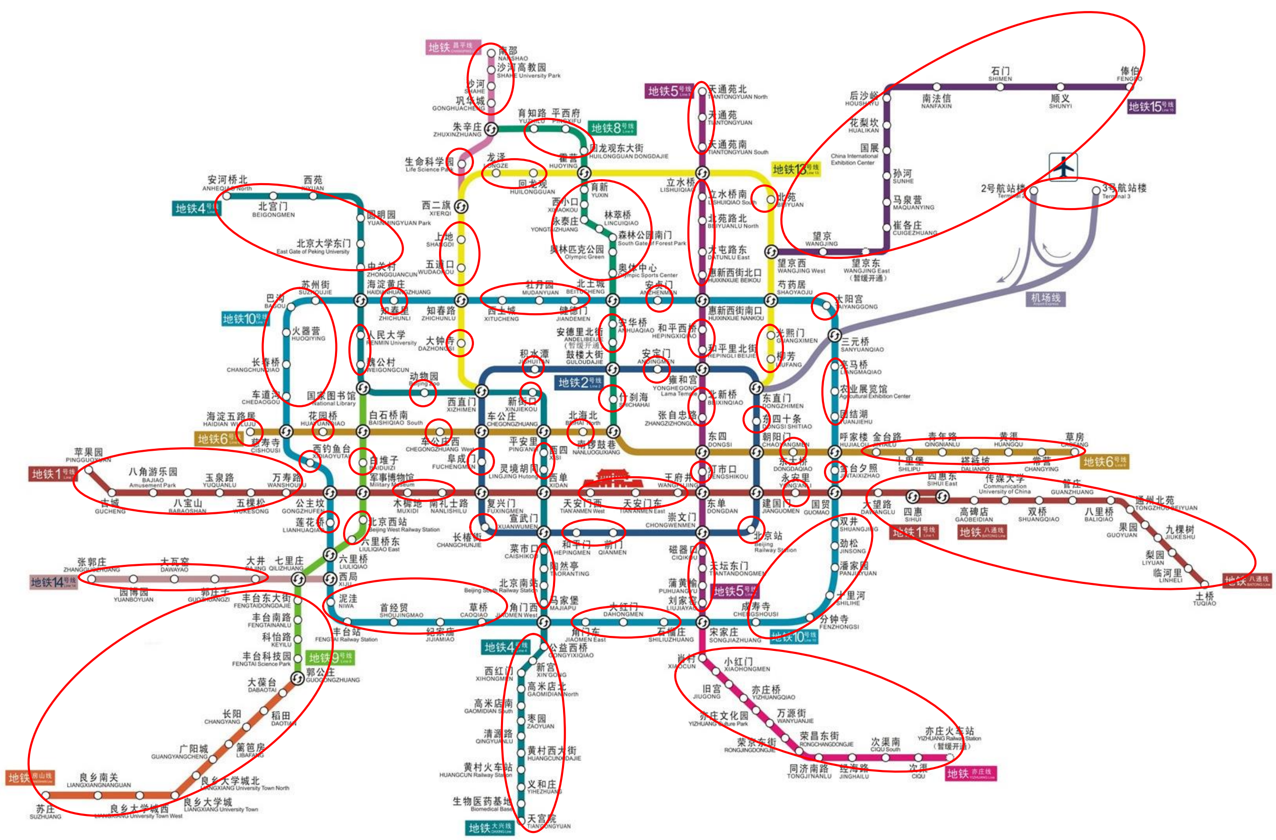


Fig. 6. Display map of the original Beijing Metro network (Oct. 15, 2014).

(2) Distribution of Numbers of Reasonable Routes

By performing the solution algorithm to the simplified network, we obtain the distribution of numbers of reasonable routes for all O-D pairs shown in Fig. 8. All O-D pairs have at least 1 reasonable route and at most 43 reasonable routes. The O-D pairs (13, 9), (9, 13), (77, 9) and (9, 77) have the largest reasonable routes, where node 9 denotes the Beijingzhan station (transfer station), node 13 denotes the Anheqiaobei

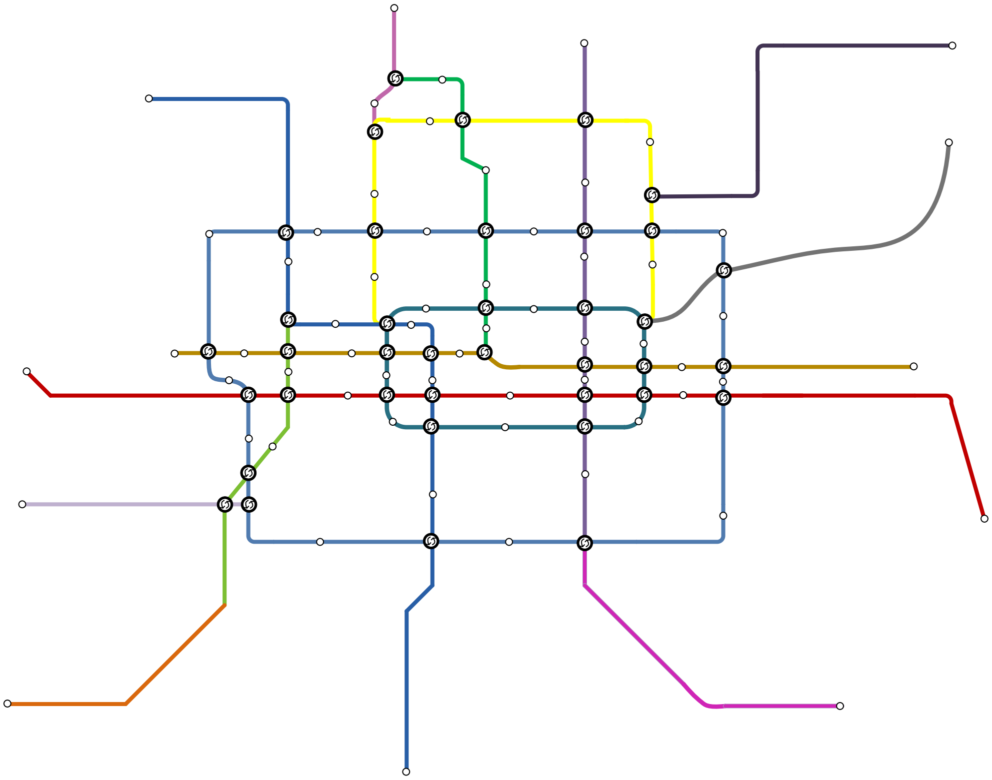


Fig. 7. Display map of the simplified Beijing Metro network.

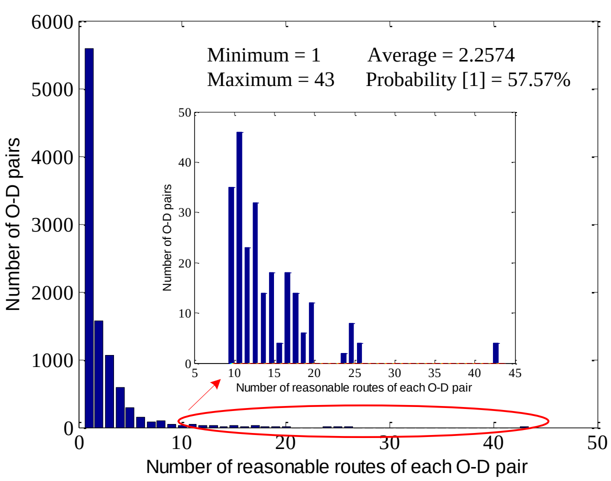


Fig. 8. Number of reasonable routes in the simplified network.

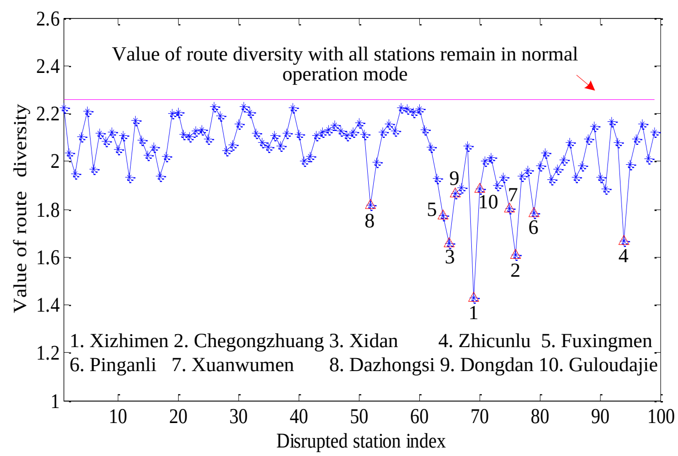


Fig. 9. Value of the route diversity index with the station disrupted in turn.

zone, and node 77 denotes the Haidianhuangzhuang zone. The percent of all O-D pairs connected by only 1 reasonable route is 57.57%, and the route diversity index is 2.2574.

(3) Most Vulnerable Stations

We want to find which stations have the largest impacts to the overall metro network, i.e., which station will decrease the route diversity index the most when it is disrupted? Note that a zone is considered disrupted if any station in this zone is disrupted.

There are 99 nodes in the simplified Beijing Metro network, and we sort all nodes as $\{1, 2, \cdots, 99\}$. We assume that the node is disrupted in turn, and the route diversity index in each case is shown in Fig. 9. The top ten most vulnerable stations are marked in Fig. 10. The value of the route diversity index with top ten most vulnerable stations disrupted in turn comparing to all stations remain in normal operation mode are presented in Table III. Take Xizhimen station as an example, when Xizhimen station is disrupted and other stations keep in normal operation, the value of route diversity is 1.4281 or the route diversity index is reduced by $(2.2574 - 1.4281)/2.2574 = 36.74\%$ in comparison with all stations in normal operation.

The obtained results show that Xizhimen station,

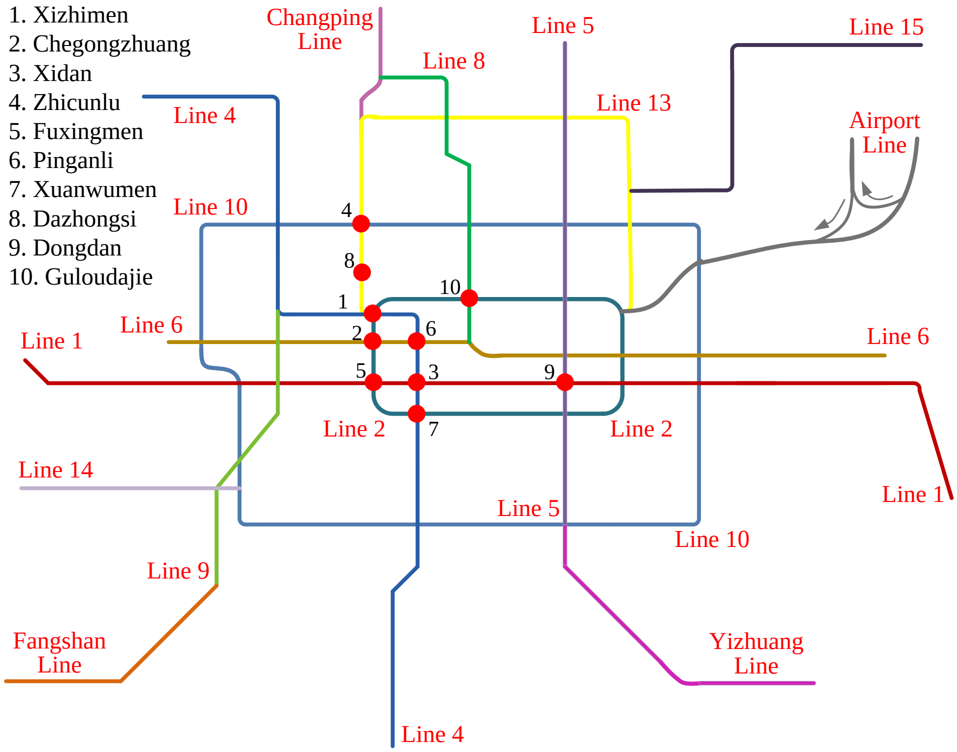


Fig. 10. Top ten most vulnerable stations.

TABLE III
VALUE OF THE ROUTE DIVERSITY INDEX WITH TOP TEN MOST VULNERABLE STATIONS

Station	Xizhimen	Chegongzhuang	Xidan	Zhicunlu	Fuxingmen
Route diversity index	1.4281	1.6066	1.6569	1.6654	1.7733
% decrease	-36.74%	-28.28%	-26.60%	-26.22%	-21.45%
Station	Pinganli	Xuanwumen	Dazhongsi	Dongdan	Guloudajie
Route diversity index	1.7808	1.8036	1.8181	1.8661	1.8825
% decrease	-21.11%	-20.11%	-19.46%	-17.33%	-16.61%

Chegongzhuang station, etc., have the largest impact to the overall metro network. Metro managers should preferentially protect these stations. At a first glance, the results of the top ten vulnerable stations appear to be straight-forward since stations in the center (or inner ring) with more lines passing through are typically considered as important hubs. However, as shown in the top ten vulnerable stations in the Beijing metro network, not all stations in the inner ring are vulnerable according to the decrease in route diversity index. This suggests station locations may play a role in addition to connectivity in a metro network. In addition, some stations outside of the inner ring (e.g., station Zhicunlu ranked 4^{th} and station Dazhongsi ranked 8^{th}) are considered vulnerable from the route diversity measure. Take Dazhongsi station as an example, which is adjacent to the most vulnerable station (i.e., Xizhimen station). It is an important node connecting to the upper part and the lower part of the network. Therefore, it is considered vulnerable even if it is located outside of the inner ring. However, it should be noted that the most vulnerable stations were evaluated based on the route diversity concept defined in this paper. It does not mean other stations are not vulnerable from other perspectives. For example, our current study does not consider passenger demand data in assessing the impacts of disruption. It would be interesting to examine how the vulnerable stations would change with the addition of passenger demand data.

V. CONCLUSION

The main contribution of this paper is to develop the route diversity index for measuring passenger route choice and network vulnerability. The route diversity index is based on the concept of reasonable routes proposed by Dial to account for travelers' imperfect perceptions of all available routes in the network as well as not being able to always select the shortest route. First, some definitions related to route diversity are provided. The unique feature of metro networks is also described. Finally, the solution algorithm for calculating the number of reasonable routes and the route diversity index is presented.

The example based on the Beijing Metro network with real-world infrastructure data is conducted, which can answer two fundamental questions for passengers and metro managers: "*How many reasonable routes for passengers between any* *two stations in normal operation or the event of a disruption?*" and "*which stations are most vulnerable, i.e., have the largest impacts to the overall metro networks when they are disrupted?*".

For future research, we will use not only the network topological data but also network O-D demand data to analyze the vulnerability and resiliency of real-world metro networks, which more accurately represent the behavior the passenger route choices. We also plan to replace the current Dijkstra's algorithm with state-of-the-art shortest route algorithms (e.g., the A* algorithm), and conduct a comparative study among the algorithms for finding shortest routes in large-scale metro networks. In addition, it would be a good idea to test the route diversity concept and the algorithm developed in this paper to other large-scale metro networks such as the Tokyo metro, the Paris metro, the New York metro, and the London Underground.

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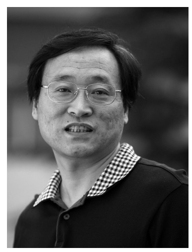

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