# Dynamic Response and Stability Analysis with Newton Harmonic Balance Method for Nonlinear Oscillating Hyperelastic Dielectric Elastomer Balloons

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## Abstract

Subject to various pressure and voltage values, the deformation of a hyperelastic dielectric elastomer (DE) membrane may attain different stable and unstable equilibria. In this paper, the neo-Hookean material model is adopted to describe the hyper-elastic behavior of a dielectric elastomer membrane. The effects of initial stretch ratio, pressure and voltage on nonlinear free vibration of a spherical dielectric elastomer balloon are investigated qualitatively and quantitatively. Through a linear stability analysis of the equilibrium states, the safe regime of initial stretch ratio for the deformation of dielectric elastomer balloon is confined. Under specific static driving pressure and voltage, the system oscillates about the stable equilibrium and there is no oscillation in the neighborhood of the unstable equilibrium. Besides, the critical pressure and voltage are determined. Beyond the critical values, there is no periodic oscillation. Along with the stability analysis, complex dynamical behavior such as drastic change of output regime, sporadic instability and sudden bifurcations can be predicted. Applying the Newton Harmonic Balance method for quantitative analysis, the frequency response can be readily predicted. It is found that the nonlinear free vibration frequency decreases with increasing initial stretch ratio and control parameters (pressure and voltage).

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## 1. Introduction

Because of its high efficiency on transforming electrical energy to mechanical work, dielectric elastomers are prosperous materials used for actuators and transducers [1, 2]. When subjected to a voltage, the thickness of a dielectric membrane reduces while its area expands, possibly straining over 100% [2]. The material behavior of dielectric elastomers can be represented by a hyper-elastic model (e.g., neo-Hookean model [3], Gent model [4] and Ogden model [5]) or a viscoelastic model where the viscoelasticity is mostly represented by a rheological model [6, 7]. Due to their superior material properties like extreme flexibility, excessive deformation, lightweight, low cost as well as chemical and biological compatibilities, there are many potential applications of dielectric elastomers in engineering.

Most studies focused on the quasi-static behavior of large deformation [8]. Applications exploiting the quasi-static characteristics include soft robots, adaptive optics, energy harvesting, and programmable haptic surfaces [9-12]. However, dielectric elastomers may deform over a wide range of frequency when inertia is considered. The dynamic behavior of dielectric elastomers is exploited in applications include loudspeakers [13,14], pump [15], and frequency tuning [16]. Based on these works, much work has been done to delve into the dynamic characteristics of dielectric elastomers [3,4,17-24].

Based on the thermodynamic principles [1], the deformation of various configurations of dielectric elastomers has been reported [3, 4, 18-20, 25-27]. Zhu et al. [3] investigated the inflation of dielectric elastomer balloons under the effects of pressure and applied voltage. For a static pressure and a constant voltage, the membrane may reach a stable equilibrium that corresponds to the stretch on the rising branch of the pressure-stretch curves or the voltage-stretch curves. A sinusoidal voltage may excite subharmonic, harmonic and superharmonic resonance effects to the membrane [18]. In addition, the influence of instability and bifurcation related to the dynamic response of dielectric elastomers is investigated [5, 26, 28-33]. There are also research studies in this respect to avoid the failure of dielectric elastomer materials in practical product fabrication, thus enhancing the design reliability and stability [30] and energy harvesting efficiency [34].

Many previous research efforts on the dynamic characteristics of dielectric elastomers are mostly investigated by numerical methods [3, 8, 17, 18, 35, 36]. Following the development of stability theorems to evaluate the existence of periodic solutions, the priority for research into nonlinear oscillations was placed on the simplification of computational methods to solve nonlinear problems. Recently, Tang et al. [37] constructed analytical approximations for solving the nonlinear vibration of dielectric elastomer balloons by means of the Newton Harmonic Balance (NHB) method [38-40]. This is a flexible and accurate analytical approach that has been extended to solve a variety of complicated nonlinear problems in physical science and engineering [40-45]. The NHB method is valid for both small and large amplitudes of oscillation. It is not only constricted to systems that have small parameters. Through explicit expression of analytical solutions, the influence of initial conditions and other governing parameters of interest to the system response can be characterized directly. Moreover, the lower-order analytical approximations of this analytical approach can achieve very good approximate results for various nonlinear problems. The computational effort can also be significantly reduced.

This paper is different from the previous work in [33] that only focuses on constructing analytical approximate solutions for the deformation of a dielectric elastomer balloons governed by a specific case of static pressure and voltage. The major investigation here is on the stability analysis of the nonlinear free vibration of a hyperelastic dielectric elastomer spherical balloon subject to various static pressure and voltage values. To consider the hyper-elasticity of dielectric elastomer materials using the neo-Hookean model, a conservative system is modeled as an autonomous second-order differential equation with general nonlinearity and negatively powered terms [3, 37]. When the driving pressure and voltage are both static, the system may reach an equilibrium. There are periodic motions around the stable equilibrium. The safe regime for the governing parameters corresponding to the stable equilibrium is studied. Within the safe region where periodic oscillation occurs, the system contains a stable equilibrium and an unstable equilibrium [3]. The deformation is periodic around the stable equilibrium before it enters electromechanical instability [17].

Although relevant works on bifurcation and stability for various dynamical models of dielectric elastomer spherical balloons have been conducted (e.g., [3], [5], [17], [18], [46],[22] and [23]), it still draws intensive research in bifurcation analysis due to significant interaction of the governing parameters (pressure and voltage). The change in bifurcation parameters can lead to a collision and disappearance of two equilibria in the dynamical system. In particular, the range of periodic oscillations is determined from stability analysis. The relationship between the system nonlinear frequency and the control parameters can be readily obtained by the NHB method [33-36]. Based on the present study, the influence of the initial stretch ratio on the nonlinear frequency is evaluated with respect to applied pressure and voltage. Besides, the impact of applied pressure and voltage on the electromechanical response (nonlinear frequency) is also reported.

## 2. Problem definition and formulation

The deformation of a spherical dielectric elastomer balloon subject to both pressure and voltage is illustrated in Fig. 1. The original state with an initial radius $R$ and thickness $H$ is shown in Fig. 1 (a) while Fig. 1 (b) shows the deformed state with radius $r$ and thickness $h$.

Based on the assumption of incompressibility, homogeneity of dielectric elastomer and isothermal condition during deformation, and by using the virtual work principle [3], the dimensionless governing equation for the inflation of the spherical dielectric elastomer balloon can be expressed as a single conservative system,

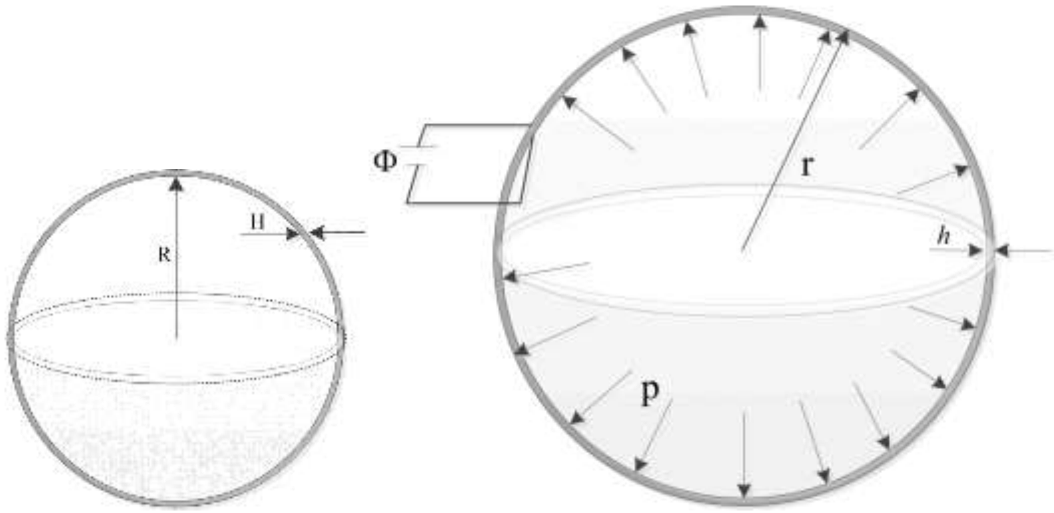
$$\frac{d^2\lambda}{dT^2} + g\left(\lambda, p, \Phi\right) = 0, \lambda(0) = \mathrm{A}, \lambda'(0) = 0 \tag{1}$$

with

$$g\left(\lambda, p, \Phi\right) = -2\lambda^{-5} + 2\lambda - \frac{pR}{\mu H}\lambda^2 - 2\frac{\varepsilon\Phi^2}{\mu H^2}\lambda^3 \tag{2}$$

in which $\lambda = r/R$ is the stretch ratio of with deformed radius $r$ and original radius $R$, the dimensionless time is $T = t\big/\left(R\sqrt{\rho/\mu}\right)$, dimensionless pressure $\frac{pR}{\mu H}$ and dimensionless voltage $\Phi\sqrt{\varepsilon/\mu}/H$. During the modelling process, the neo-Hookean material model is used to express hyperelasticity of the dielectric elastomers. It should be noted that the square of the normalized voltage $\frac{\varepsilon\Phi^2}{\mu H^2}$ is used exclusively as the representation of voltage in this paper.

In the equation above, it involves a negatively powered variable and non-classical non-odd nonlinearity.




(a) (b)

Fig. 1. Deformation of a dielectric elastomer balloon under pressure and voltage: (a) original state and (b) deformed state [3].

## 3. Linear stability analysis

The dynamics of a spherical dielectric elastomer balloon is governed by Eqs. (1) and (2) which is a set of two first-order differential equations as follows

$$\begin{cases} \dot{\lambda} = v \\ \dot{v} = -(-2\lambda^{-5} + 2\lambda - \dfrac{pR}{\mu H}\lambda^2 - 2\dfrac{\varepsilon\Phi^2}{\mu H^2}\lambda^3) \end{cases} \tag{3}$$

with the following initial conditions,

$$\lambda(0) = \mathrm{A}, v(0) = 0 \tag{4}$$

where a dot of $\lambda$ and $v$ denotes differentiation with respect to $T$.

For static $p$ and $\Phi$, the inflation of spherical dielectric balloon will reach an equilibrium state. At equilibrium, Eq. (3) satisfies the $\dot{\lambda} = 0$ and $\dot{v} = 0$ simultaneously, implying

$$g(\lambda_{eq}, p, \Phi) = 0 \tag{5}$$

In order to determine the stability of each equilibrium state, we consider the Jacobian matrix of the system in Eq. (3) at the equilibrium point $(\lambda_{eq}, 0)$

$$[J]_{\lambda=\lambda_{eq},\nu=0} = \begin{bmatrix} 0 & 1 \\ \Omega & 0 \end{bmatrix} \tag{6}$$

The characteristic equation is

$$\Lambda^2 - \Omega = 0 \tag{7}$$

where

$$\Omega\Big|_{\lambda=\lambda_{eq}} = -\frac{\partial g}{\partial \lambda}\Big|_{\lambda=\lambda_{eq}} \tag{8}$$

and $\frac{\partial g}{\partial \lambda} = 10\lambda^{-6} + 2 - 2\lambda\frac{pR}{\mu H} - 6\frac{\varepsilon\Phi^2}{\mu H^2}\lambda^2$, $\Lambda_1$ and $\Lambda_2$ are the eigenvalues of the problem. They dictate the approximate forms of $\lambda$ and $\nu$ in the neighborhood of $(\lambda_{eq}, 0)$.

The eigenvalues $\Lambda_1$ and $\Lambda_2$ depend on the sign of $\Omega$. If $\Omega > 0$ (i.e., $\frac{\partial g}{\partial \lambda} < 0$) the two roots are real and of opposite signs $\Lambda_1 = \sqrt{\Omega}$ and $\Lambda_2 = -\sqrt{\Omega}$. The equilibrium point is unstable and it is a saddle point. The asymptotic trajectories in the neighborhood reduce to open curves and no oscillation will occur. If $\Omega < 0$ (i.e., $\frac{\partial g}{\partial \lambda} > 0$), the two roots are purely imaginary, $\Lambda_1 = i\sqrt{-\Omega}$ and $\Lambda_2 = -i\sqrt{-\Omega}$. Hence, the equilibrium point is a center and the oscillation is periodic around the center. If $\Omega = 0$ and $\Lambda_1 = \Lambda_2 = 0$, it is a degenerated case and no oscillation will occur.

## 4. Safe regime for control parameters

According to the theory of static bifurcation [47, 48], a saddle-node bifurcation occurs as the number of equilibria changes due to the control parameters (e.g., pressure, voltage or both). In the present problem, the bifurcation points satisfy,

$$\frac{\partial g(\lambda, \mathrm{p}, \Phi)}{\partial \lambda}\Big|_{\lambda=\lambda_{eq}} = 0 \tag{9}$$

At the bifurcation state, the deformation of a spherical dielectric elastomer balloon will break down [17, 18, 28].

Solving Eqs. (5) and (9) simultaneously, the equilibrium state $\lambda_{eq}$ with respect to pressure and voltage due to the occurrence of bifurcation can be written as follows

$$\frac{pR}{\mu H}|_c = -16\lambda_{eq}^{-7} + 4\lambda_{eq}^{-1} \tag{10}$$

$$\frac{\varepsilon\Phi^2}{\mu H^2}|_c = 7\lambda_{eq}^{-8} - \lambda_{eq}^{-2} \tag{11}$$

The subscript '$c$' refers to the critical control parameters. Besides, we observe that the equilibrium bifurcation occurs when the natural frequency of the small amplitude oscillation evolves to zero[3]. When either pressure $\frac{pR}{\mu H}$ or voltage $\frac{\varepsilon\Phi^2}{\mu H^2}$ is prescribed, the bifurcation for voltage or pressure can thus be obtained from Eqs. (10) and (11).

Integrating Eq. (3) from the initial state $(\lambda(0), \nu(0))$ to the current state $(\lambda, \nu)$, the energy conservation equation for the inflation of spherical balloon is,

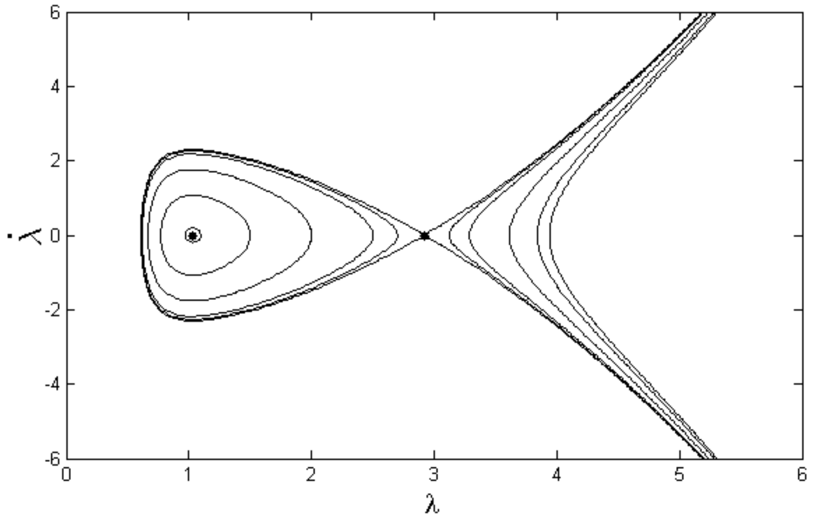
$$\begin{aligned}\frac{1}{2}\nu^2 - \frac{1}{2}\nu(0)^2 = (\frac{1}{3}\frac{pR}{\mu H}\lambda^3 + \frac{1}{2}\frac{\varepsilon\Phi^2}{\mu H^2}\lambda^4 - \frac{1}{2}\lambda^{-4} - \lambda^2) - \\ (\frac{1}{3}\frac{pR}{\mu H}\lambda(0)^3 + \frac{1}{2}\frac{\varepsilon\Phi^2}{\mu H^2}\lambda(0)^4 - \frac{1}{2}\lambda(0)^{-4} - \lambda(0)^2)\end{aligned} \tag{12}$$

The equation can be used to plot trajectories directly (i.e., $\nu$ versus $\lambda$) as shown in Fig. 2 and Fig. 3 for various initial conditions [49].

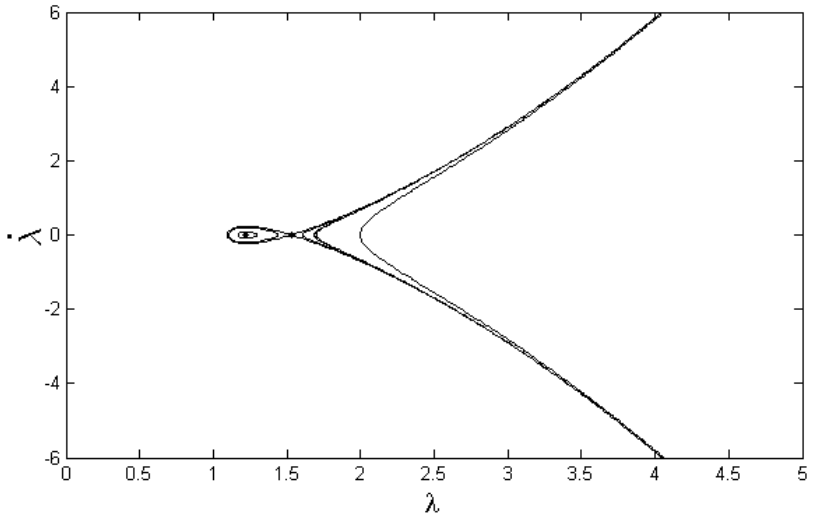
### 4.1 Effect of pressure

The critical pressure $\frac{pR}{\mu H}|_c$ at the saddle-node bifurcation [48, 49] is the maximum value that the dielectric membrane can withstand during deformation. Under this critical value, the nonlinear system can reach two equilibria, a stable one on the ascending branch with $\frac{\partial g}{\partial \lambda} > 0$ and an unstable one on the descending branch with $\frac{\partial g}{\partial \lambda} < 0$. In the neighborhood of the stable equilibrium, the deformation is a periodic oscillation. The periodic oscillation exists until it reaches the neighborhood of the unstable equilibrium, where there is a homoclinic orbit with an infinite period. Beyond this critical value, the nonlinear system will not reach any equilibrium and the deformation becomes unstable.

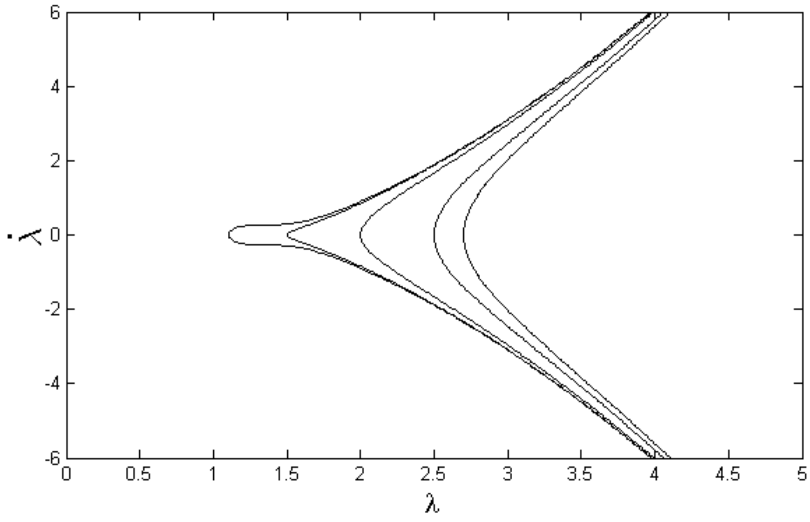
A constant squared voltage $\frac{\varepsilon\Phi^2}{\mu H^2}=0.1$ is taken as an example. Following Eqs. (10) and (11), the critical normalized pressure is $\frac{pR}{\mu H}|_c = 0.967$ with the degenerate equilibrium $\lambda_{eq} = 1.345$. Fig. 2 displays the phase planes before bifurcation, near bifurcation and after bifurcation at three different pressure values. On each phase plane, the left solid point denotes a center, and the right solid point denotes a saddle point. By changing the pressure from a small value to a large one, these three figures provide good illustration as the system evolves. In Figs. 2(a) and 2(b), when the normalized pressure (i.e., $\frac{pR}{\mu H}=0.1$ or $\frac{pR}{\mu H}=0.9$) is smaller than the critical value, both the center and saddle point co-exist. The oscillation is periodic around the center until it reaches a homoclinic orbit starting from the saddle point. The center and saddle point move towards each other until they collide at $\lambda_{eq} = 1.345$ when $\frac{pR}{\mu H}|_c = 0.967$. Meanwhile, the region for the periodic oscillation, which is confined by the homoclinic orbit, is decreasing. After collision, the deformation of this dielectric membrane loses its stability, as is shown in Fig. 2(c). The normalized pressure (i.e., $\frac{pR}{\mu H}=0.97$) is larger than the critical value, and there are no equilibrium and oscillation.




(a) $\frac{pR}{\mu H} = 0.1$




(b) $\frac{pR}{\mu H} = 0.90$

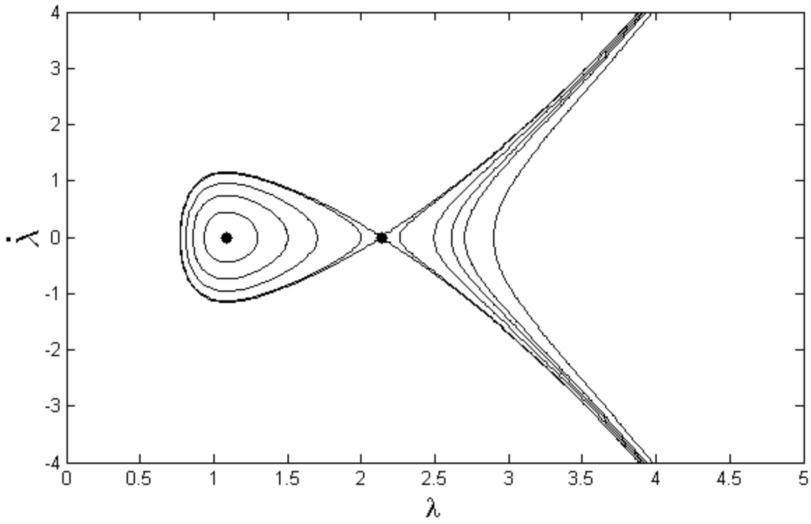



(c) $\frac{pR}{\mu H}=0.97$

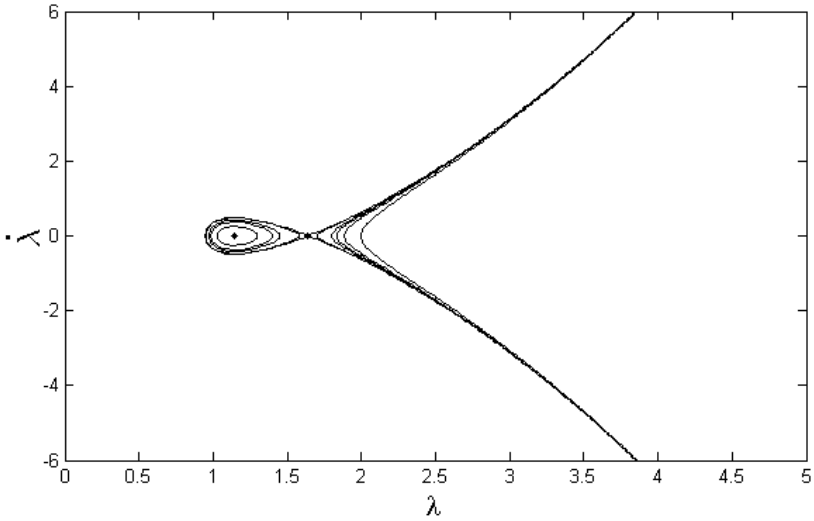
Fig.2 Phase planes for $\frac{\varepsilon\Phi^2}{\mu H^2}=0.1$ and (a) $\frac{pR}{\mu H}=0.1$ (b) $\frac{pR}{\mu H}=0.9$ (c) $\frac{pR}{\mu H}=0.97$.

**4.2 Effect of electric potential difference**

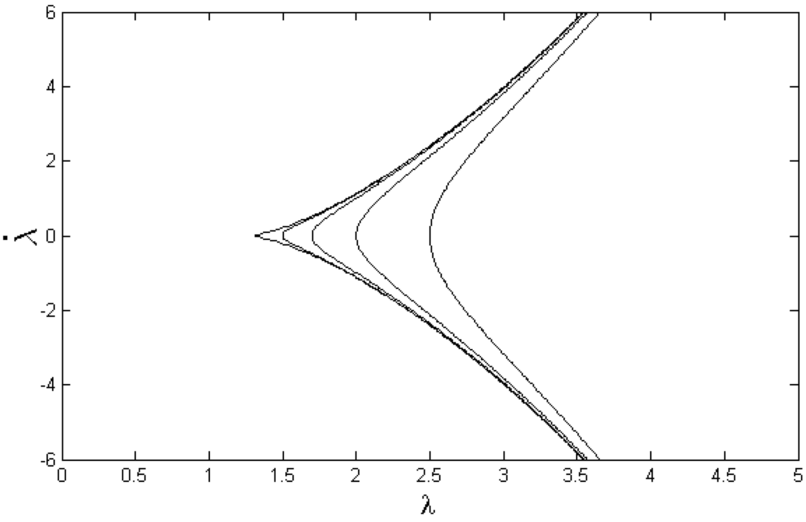
To illustrate the effect of electrical potential difference, the normalized pressure $\frac{pR}{\mu H}=0.5$ is taken as an example. The critical voltage is $\frac{\varepsilon\Phi^2}{\mu H^2}\big|_c=0.277$, with the degenerate equilibrium $\lambda_{eq}=1.298$. The phase planes for $\frac{\varepsilon\Phi^2}{\mu H^2}=0.1$, $\frac{\varepsilon\Phi^2}{\mu H^2}=0.2$ and $\frac{\varepsilon\Phi^2}{\mu H^2}=0.28$ are presented in Fig.3. When the driving voltage is smaller than the critical value, the center and saddle point can co-exist on the phase planes, as shown in Figs. 3(a) and 3(b). After bifurcation at the critical value, there is no oscillation due to electromechanical instability (EMI). EMI, a significant phenomenon in dielectric elastomers, has been well studied [28]. Physically, when a voltage is applied to a dielectric elastomer, the oppositely charged electrodes squeeze the membrane in the thickness, resulting a higher electric field. This positive feedback may lead the elastomer to deform dramatically, and then to be electrically broken down. Mathematically, EMI happens with $\frac{\partial g(\lambda,\Phi)}{\partial\lambda}\big|_{\lambda=\lambda_{eq}}=0$, corresponding to the peak of the voltage-stretch curve [3].




(a) $\frac{\varepsilon\Phi^2}{\mu H^2} = 0.1$




(b) $\frac{\varepsilon\Phi^2}{\mu H^2} = 0.20$




(c) $\frac{\varepsilon\Phi^2}{\mu H^2} = 0.28$

Fig.3 Phase planes for $\frac{pR}{\mu H} = 0.5$ and (a) $\frac{\varepsilon\Phi^2}{\mu H^2} = 0.1$ (b) $\frac{\varepsilon\Phi^2}{\mu H^2} = 0.2$ (c) $\frac{\varepsilon\Phi^2}{\mu H^2} = 0.28$.

## 5. Periodic oscillation analysis

For many nonlinear systems, the primary research focus is on the availability of accurate analytical approximations because exact solutions to many complicated nonlinear problems are not frequently available. To this end, the authors have extensive experience in deriving the Newton Harmonic Balance (NHB) method [37] to obtain the first two orders of the analytical approximations for the dynamics of a spherical dielectric elastomer balloon as shown in Eq. (1) that is governed by pressure and voltage parameters ( $pR/\mu H = \varepsilon\Phi^2/\mu H^2 = 0.1$). Zhu et al. [3] discussed the steady-state solutions under the parametric effect for the variation of pressure and voltage parameters. To explore a better understanding of the relationship between nonlinear frequency and control parameters (i.e., pressure and voltage), the analytical approximation derived by the NHB method are presented in the current paper. For brevity, the analytical approximation procedure of the NHB method is not repeated here, the readers are referred to Tang et al. [37] for further details.

Following the previous work [33] for the nonlinear oscillating system of Eq. (1), the first-order and the second-order approximate analytical frequencies for specific pressure, voltage and initial stretch ratio can be obtained readily. By comparing with the Runge-Kutta solutions, the second-order analytical approximation is much more accurate than the first-order analytical approximation, even when the initial stretch ratio is a large value. Hence, only second-order analytical approximations of the NHB method are exclusively presented in this paper.

To study the influence of the initial stretch ratio on the nonlinear frequency response ($\omega$), the pressure and voltage are specified as $\frac{pR}{\mu H}=0.1$ and $\frac{\varepsilon\Phi^2}{\mu H^2}=0.1$ [33]. For these parameters, there is a center at $\lambda_{eq}=1.029$ and another saddle point at $\lambda_{eq}=2.920$. The initial stretch ratio $A$ should not be in the neighborhood of the unstable equilibrium state because the solution may grow exponentially away from the saddle point and there is a homoclinic orbit with an infinite period. In Fig. 4, the relationship of the initial stretch ratio and the angular frequency is presented. With increasing initial stretch ratio, the nonlinear free vibration frequency decreases from the corresponding natural frequency and the membrane behaves like a soft spring [4]. It is inferred that the negative coefficient before the cubic term in Eq. (2) contributes to soft spring nonlinearity [24]. The angular frequencies obtained by the NHB method are presented and denoted by a dashed line. To observe this decreasing trend more clearly, the natural frequency for the small amplitude oscillation around the stable equilibrium state, is also presented and represented by the horizontal solid line.

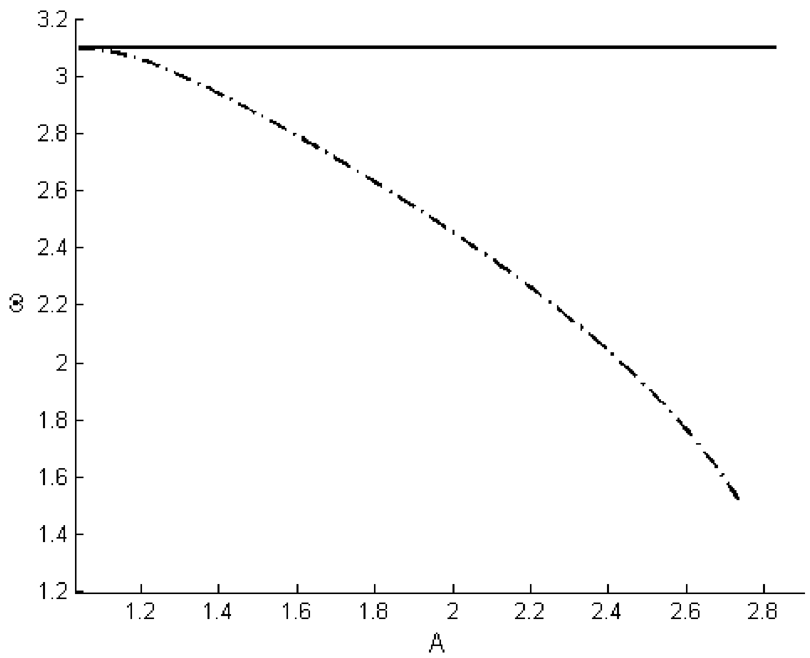


Fig.4 Relationship of the initial stretch ratio-frequency curve for $\frac{pR}{\mu H}=0.1$ and $\frac{\varepsilon\Phi^2}{\mu H^2}=0.1$.

To study the influence of the control parameters on the nonlinear free vibration frequency response, an initial stretch ratio $A=1.3$ that is within the safe regime for periodic oscillation is prescribed. Under this condition, the nonlinear frequency curves for three normalized pressure values $\frac{pR}{\mu H}=0.1$, $\frac{pR}{\mu H}=0.3$ and $\frac{pR}{\mu H}=0.5$ are presented in Fig. 5. Considering the onset conditions of periodic oscillation, there is a critical voltage [17, 18, 28] for each normalized pressure, i.e., $\frac{\varepsilon\Phi^2}{\mu H^2}\Big|_c=0.433$ for $\frac{pR}{\mu H}=0.1$, $\frac{\varepsilon\Phi^2}{\mu H^2}\Big|_c=0.354$ for $\frac{pR}{\mu H}=0.3$, and $\frac{\varepsilon\Phi^2}{\mu H^2}\Big|_c=0.277$ for $\frac{pR}{\mu H}=0.5$. In this case, increasing the pressure will reduce the critical value of the voltage. For each specified pressure, by increasing the normalized voltage with a constant step until the neighborhood of the bifurcation value, the nonlinear vibration frequencies calculated from the NHB method are plotted in Fig. 5.

For a specific pressure, the nonlinear frequency tends to decrease monotonically as the voltage increases within the safe region. For a specific voltage, the nonlinear frequency decreases as the pressure increases. This decreasing trend can be seen clearer by a three-dimensional diagram shown in Fig. 6. It is a more intuitionistic description on the relationship between the nonlinear frequency and the control parameters for the deformation of spherical dielectric elastomer balloon when the initial stretch ratio is prescribed as $A=1.3$. It is thus remarked that the vibration frequency decreases as either the driving pressure or the voltage increases. It is expected understanding the effect of an applied load on the response of dielectric elastomers will be an aid to frequency tuning.

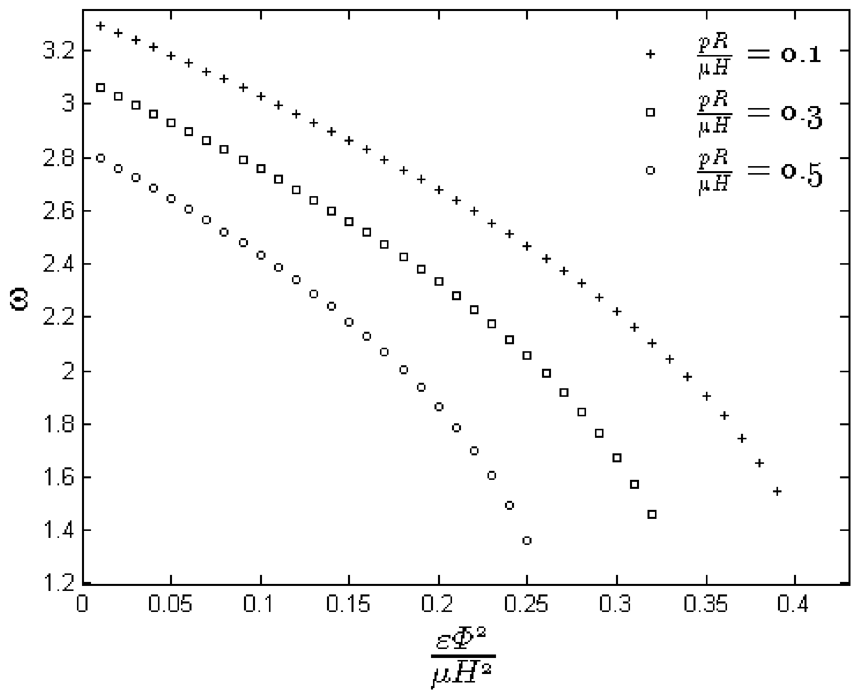



Fig. 5 Relationship of the voltage-frequency curves for $A$ =1.3

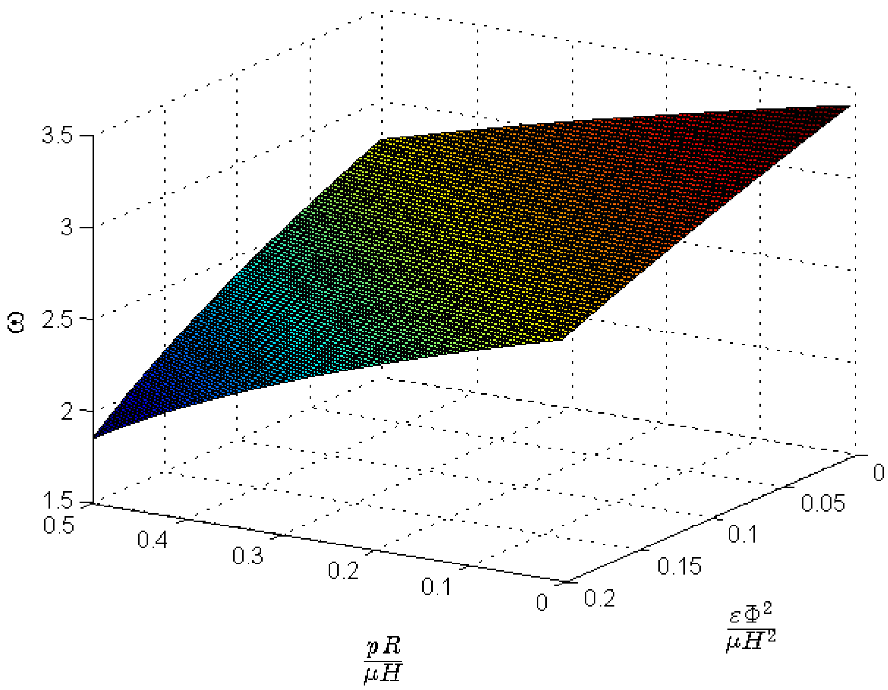



Fig. 6 Relationship between the nonlinear frequency ($\omega$) and the control parameters (pressure and voltage) for $A$ = 1.3

## 6. Conclusions

Based on the neo-Hookean model, the nonlinear free vibration of a dielectric elastomer balloon subject to both pressure and voltage has been investigated qualitatively and quantitatively. The deformation of the spherical balloon is modeled as a general non-odd nonlinear differential model that is associated with a rather high negatively-powered restoring force term. The system may reach an equilibrium state if all the control parameters are prescribed. The equilibrium stability is analyzed to identify the safe regime of the initial stretch ratio. Besides, there exist critical values for the control parameters. Theoretically, the critical points correspond to the state where the saddle-node bifurcation occurs. Below the critical value, both center and saddle point co-exist. The center and saddle move towards each other until they collide as one of the control parameters (i.e., the pressure or the voltage) increases. Beyond the critical value, all the equilibria vanish. Physically, the membrane deforms below the critical values and the deformation is periodic around the stable equilibrium. At the critical values, it corresponds to zero natural frequency. Beyond the critical applied pressure or voltage, the balloon and deformation becomes unstable. Besides, the critical voltage is an indication for EMI. For a specified voltage, the oppositely charged electrodes squeeze the membrane in the thickness of membrane will increase the electric field. This positive feedback leads to EMI.

The angular frequency for nonlinear oscillation of this spherical dielectric elastomer balloon is determined by the initial stretch ratio, the applied pressure and the voltage. Applying the Newton Harmonic Balance method, we are able to derive explicit and accurate approximate nonlinear frequency solutions for periodic oscillations represented by these three factors. By increasing the initial stretch ratio, with applied pressure and potential difference determined, the nonlinear vibration frequency will be reduced and it results in the soft-spring behavior. For a specific initial stretch ratio, a parametric analysis for both pressure and voltage is also presented. It is concluded that the nonlinear frequency decreases as the pressure or increases the voltage. Understanding of dynamic response of dielectric elastomers under electromechanical loading is expected to promote the potential applications as an actuator.

In this paper, a theoretical dynamic analysis for deformation of a hyperelastic dielectric elastomer balloon is presented. There are some modelling simplifications. First, the material model is neo-Hookean without considering the strain-stiffening effect [46]. It is assumed that during deformation, the material strain is far from its limiting strength [4] (?? What is strength limit??). Considering the limit stretch, the Gent material is applied and the governing equation is a bi-stable ordinary differential equation that results in a much more complex dynamic system. Next, the viscoelastic factors such as material damping and the current leakage during loading and unloading [21] that cause the dynamic response unpredictably are not considered. Further, the applied pressure or the voltage cannot always be static. The parametric excitation may exhibit harmonic, sub-harmonic and super-harmonic resonance [3,4,18]. Although these limitations have been extensively studied, only numerical or semi-analytical methods have been developed. The Newton Harmonic Balance method derived here has been extensively applied to the conservative system with one or two degrees. The extension of this method to the unsolved issues governed by non-autonomous or dissipative ordinary differential equations remain the future works.

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