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Time-Dependent Reliability and Redundancy of Corroded Prestressed Concrete Bridges at Material, Component, and System Levels

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Abstract:

Due to structural deterioration, performance of concrete bridges degrades with time, which may cause catastrophic failure events, especially for non-redundant structures. This paper develops a methodology for assessing time-variant reliability and redundancy of multi-girder prestressed concrete bridges at material, component, and system levels. Nonlinear analysis is performed to obtain constitutive relationship of reinforced sections and girder components, in which adverse effects of reinforcement corrosion on structural capacity and ductility are considered. Subsequently, the nonlinear finite element analysis is conducted to capture the time-dependent probabilistic resistance of bridge system considering different failure criteria (e.g., serviceability and ultimate limit states). Given the component- and system-level performance indicator, structural capacity-, reliability- and risk-informed redundancy is assessed. The feasibility and capability of proposed approach are illustrated on an existing prestressed concrete bridge. Results demonstrate that the integrated damage mechanism between different levels has a significant effect on the structural ultimate capacity, reliability, and redundancy of corroded structures.

Keywords: System-level performance; Reinforcement corrosion; Ductility; Redundancy; Reliability; Risk; Nonlinear finite element

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28 **Introduction**

29 Bridges act as a critical component between communities for transportation of goods and services; and
30 bridge failure could directly affect people's lives. Therefore, safety and performance assessment of
31 bridges receives significant attention. During service life, bridges may be subjected to a variety of
32 stressors, such as aggressive chloride environmental conditions, material aging, and fatigue loading,
33 among others, which could cause a significant reduction of structural performance and even structural
34 failure (Enright and Frangopol 1998; Kassir and Ghosn 2002; Wang *et al.* 2016; Lim *et al.* 2016;
35 Frangopol *et al.* 2017; Akiyama *et al.* 2017; Deng *et al.* 2019). Recently, disastrous events, such as
36 collapse of I-35W Mississippi River bridge (Hao 2009), the I-5 Skagit River bridge (Stark *et al.* 2016),
37 and the Morandi Bridge, have brought new challenges and requirements for assessment and
38 management of aging bridges at both component and system levels. As structures are usually designed
39 to ensure performance and safety at component level, studies on performance of deteriorating bridges
40 at system-level are of vital importance. The system-level reliability and redundancy assessment are
41 conducted in this study to ensure long-term performance of aging bridges at different levels (e.g.,
42 material, component, and system).

43 Structural reliability, accounting for uncertainties of material properties, geometric dimensions,
44 loads, and environmental conditions, is one of the fundamental indicators for performance assessment
45 of bridges (Ditlevsen and Madsen 1996; Thoft-Cristensen and Baker 2012; Sabatino *et al.* 2015; Feng
46 *et al.* 2018; Zheng and Dong 2019). Current design guidelines of bridges are generally established to
47 guarantee a rational reliability index of the load-carrying components (AQSIQ and MOHURD 1999;
48 AASHTO 2007). The component-reliability-oriented design and assessment methodology is

49 associated with several limitations. For instance, load effects of each component are computed using
50 a time-invariant live load distribution factor (LLDF), based on the initial elastic stage (Barr *et al.* 2001;
51 Huo *et al.* 2004; Hughs and Idriss 2006; Yousif and Hindi 2007). Load redistribution among different
52 components within plastic stage is ignored. Additionally, it may neglect the contribution of non-
53 functional transverse connection elements, such as transverse diaphragms and flange wet seams
54 (Eamon and Nowak 2002, 2004). Thus, it is needed to incorporate system reliability with bridge design
55 and assessment process. With respect to system reliability assessment, Estes and Frangopol (1999)
56 developed a series-parallel model for reliability analysis of multi-girder bridges, assuming that failure
57 of any three adjacent girders can cause failure of superstructure system. The proposed model is
58 relatively subjective, and the live load redistribution is not considered. Nowak and Zhou (1990)
59 developed a procedure for resistance and reliability analysis of multi-girder bridges using finite
60 element (FE) modeling. The procedure was adopted within the analysis of several types of bridges,
61 such as steel girder (Czarnecki and Nowak 2008), composite steel girder (Eamon and Nowak 2004;
62 Nowak 2004), reinforced concrete (RC) T-beam (Tabsh and Nowak 1991), and prestressed concrete
63 (PSC) girder (Nowak and Zhou 1990). Though these studies pioneered on the system reliability
64 analysis, more studies should be conducted on the time-dependent reliability incorporating
65 reinforcement corrosion and the effects of structural secondary elements (e.g., diaphragms, barriers)
66 at a multi-scale procedure (e.g., sectional, component, and system).

67 The system redundancy is another essential indicator for system-level structural performance
68 evaluation and for understanding the collapse mechanism of multi-component structures (Ghosn and
69 Moses 1998; Ghosn *et al.* 2010). Redundancy is affected by many factors, such as member behaviour

(e.g., ductility, ultimate to yield strength ratio) and configuration characteristics (e.g., member number, connecting strength and stiffness between different members) (Frangopol and Curley 1987). A number of measurements, either deterministic or probabilistic, have been proposed to quantify the redundancy. Most of these quantifications are based on comparisons regarding load-carry capacity or failure probability/reliability indices between intact and damaged system or the most vulnerable component within a system (Moses 1986; Frangopol and Curley 1987; Fu and Frangopol 1990). A risk-informed redundancy/robustness indicator was proposed by Baker *et al.* (2008), in which the probabilities and associated consequences are both considered. Recently, the evolution of system redundancy with time considering structural deterioration effects has attracted research interests. For instance, Okasha and Frangopol (2010) investigated time-variant redundancy of an idealized two parallel bar system subjected to increasing loading and continuous cross-area loss; Saydam and Frangopol (2011) suggested a computational framework for time-variant redundancy index of multi-girder steel bridges; and Biondini and Frangopol (2017) developed a probabilistic procedure for time-dependent redundancy of concrete structures incorporating local damage and ultimate system failure. It should be noted that, corrosion would lead to reduction in strength and transformations of constitutive relationship for reinforcement rebar and prestressed tendons (e.g., from bilinear elastic-plastic to linear elastic) (Barton *et al.* 2000; Darmawan and Stewart 2007; Li *et al.* 2009; Vu *et al.* 2009; Zeng *et al.* 2010; Zhang *et al.* 2012; Wang *et al.* 2017), which could affect the girder ductility and system redundancy. However, to the best knowledge of the authors, there are no studies available on the time-dependent redundancy assessment of multi-girder PSC bridges considering the relevant adverse effects induced by corrosion at different levels (e.g., material, section, component, and system) in a

91 comprehensive manner.

92 Overall, this paper aims to provide a comprehensive framework to assess structural performance
93 of multi-girder PSC bridges at different levels and analyze the effect of reinforcement corrosion on
94 system performance in terms of reliability and redundancy. A multi-stage nonlinear finite element (FE)
95 analysis-based incremental procedure is proposed to capture the time-dependent structural behavior
96 and system resistance associated with different failure criteria in a life-cycle context. Given the time-
97 dependent probabilistic resistances of individual girder and bridge system associated with both ULS
98 and SLS, the component- and system-level reliability of the bridge can be computed incorporating the
99 probabilistic live load model. Subsequently, by quantifying the differences of nominal resistance,
100 reliability indices, and failure risk between bridge system and the most vulnerable component within
101 the system, the relevant redundancy indicators can be computed. Additionally, an indicator is proposed
102 to quantify the extent of live-load redistribution between different girder components and to enhance
103 the definition of system redundancy. The developed methodology is applied to an existing PSC T-
104 girder bridge.

105 **Time-variant reliability analysis**

106 **Limit state function considering corrosion effect**

107 To compute system reliability of a deteriorating bridge, probabilistic models of capacity (resistance)
108 and load effects (demand) at each time point should be established firstly. There are basically two limit
109 states for reinforced and prestressed concrete structures, namely ultimate limit state (*ULS*) and
110 serviceability limit state (*SLS*). *ULS* generally refers to flexural or shear failure of the critical section
111 within a girder component, whereas *SLS* is usually associated with the unacceptable deflection or crack
112 size (Ministry of Transport of the People's Republic of China 2004a). In this paper, resistance of bridge
113 is assessed at both component and system levels considering *ULS* as well as *SLS*. *ULS* and *SLS* of a
114 multi-girder bridge system can be defined as the flexure/shear failure and excessive vertical deflection

115 of the most vulnerable girder within a bridge (Eamon and Nowak 2004), respectively. The system
116 resistance can be defined as the maximum live load (effects) that meets the prescribed limit states. The
117 performance functions of bridge system at time t , $G(t)$, can be identified as

$$118 \qquad \qquad \qquad G(t) = R(t) - S(t) \qquad \qquad \qquad (1)$$

119 where $R(t)$ and $S(t)$ = system resistance and live load effects at time t , respectively, and both can be
120 expressed in terms of multiples associated with nominal live load model (Frangopol *et al.* 2017). The
121 multiples associated with system resistance refers to the ratio of weight of the maximum live load that
122 induces the predefined limit state (i.e., ULS or SLS) to that of the nominal live load model.

123 Given limit state functions, system reliability index can be computed using First Order Reliability
124 Method (FORM), Second Order Reliability Method (SORM), and Monte Carlo Simulation (MCS)
125 (Ditlevsen and Madsen 1996).

126 **Corrosion model of reinforcement steel**

127 Corrosion of reinforcement steel is one of the primary sources responsible for deterioration of concrete
128 bridges (Enright and Frangopol 1998; Tu *et al.* 2017; Wang *et al.* 2018). Corrosion can introduce
129 section loss, ultimate/yield strength reduction, and stress concentration of reinforcement steel; thus, in
130 turn, this would reduce the structural resistance of individual components as well as the whole system.
131 There are different categories of steel corrosion, among them the most common ones are the
132 carbonization-induced uniform corrosion and pitting corrosion caused by chloride erosion. According
133 to González *et al.* (1995), pitting corrosion results in a much larger penetration depth (e.g., 4 to 8 times)
134 than that of uniform corrosion. Therefore, area loss and decrease of yield/ultimate strength of

reinforcement steel under pitting corrosion are accounted for herein. The corrosion initiation time, t_i (years), can be predicted using Fick's law as (Enright and Frangopol 1998)

$$t_i = \frac{d^2}{4D_c} \frac{1}{\left[\operatorname{erf}^{-1} \left(1 - \frac{C_{cr}}{C_0} \right) \right]^2} \quad (2)$$

where d = thickness of concrete cover (cm); D_c = diffusion coefficient of chloride ions (cm^2/year); C_0 = constant concentration of chloride ions on concrete surface (% weight of concrete); C_{cr} = threshold value of chloride ions concentration on reinforcement surface (% weight of concrete); and erf^{-1} represents inverse of error function.

For normal reinforcement bar, radius of corrosion pit at time t (years), $p(t)$ (mm), can be computed as (Val and Melchers 1997)

$$p(t) = 0.0116(t - t_i) i_{corr} R \quad (3)$$

where i_{corr} = corrosion current density ($\mu\text{A}/\text{cm}^2$) and R = the ratio between maximum and average penetration.

Generally, the corrosion in pre-stressing tendons could be induced by inappropriate construction. If pre-stressing tendons are well-installed, corrosion in the tendons is relatively rare. In this paper, in order to illustrate and quantify the potential effects of corrosion on both component- and system-level performance, the deterioration scenario associated with pre-stressing tendons is considered for the investigated PSC bridge.

It should be noted that Eq. (2) is suitable for normal reinforcement bars and pre-tensioned tendons that are directly contacted with concrete. For post-tensioned tendon protected by grouted duct, the

154 corrosion propagation process is much more complicated. With respect to post-tensioned tendon with
 155 metal duct, Guo et al. (2016) has developed an approximate procedure for corrosion initiation
 156 prediction. Accordingly, the time of corrosion initiation of post-tensioned tendons consists of three
 157 parts, namely (1) diffusion time of chloride ions from concrete surface to the metal duct; (2) time for
 158 a pit to form on the metallic duct with a depth equal to the thickness of the duct; and (3) diffusion time
 159 in grout. The first and third time-counterparts can be computed using Eq. (2), while the second time
 160 counterparts can be calculated using Eq. (3) incorporating the duct depth. However, since the duct
 161 depth is very thin compared with concrete cover, the second time-counterpart is normally much smaller
 162 than the other two time-counterparts. Given more information of the corrosion model associated with
 163 post-tensioned tendons, it could be easily updated within the proposed framework for corrosion
 164 assessment.

165 The geometric model proposed by Val and Melchers (1997) can be used to compute loss of
 166 effective cross-sectional area under pitting corrosion. Accordingly, net area of reinforcement bar at
 167 time t , $A_r(t)$, is (Val and Melchers 1997)

$$168 \quad A_r(t) = \begin{cases} \frac{\pi D_0^2}{4} - A_1 - A_2, & p(t) \leq \frac{\sqrt{2}}{2} D_0 \\ A_1 - A_2, & \frac{\sqrt{2}}{2} D_0 < p(t) \leq D_0 \\ 0, & p(t) > D_0 \end{cases} \quad (4)$$

169 where $\alpha = 2p(t)\sqrt{1 - \left[\frac{p(t)}{D_0}\right]^2}$; $\theta_1 = 2\arcsin\left(\frac{\alpha}{D_0}\right)$; $\theta_2 = 2\arcsin\left[\frac{\alpha}{2p(t)}\right]$;
 170 $A_1 = \frac{1}{2}\left[\theta_1\left(\frac{D_0}{2}\right)^2 - \alpha\left|\frac{D_0}{2} - \frac{p(t)^2}{D_0}\right|\right]$; $A_2 = \frac{1}{2}\left[\theta_2 p(t)^2 - \alpha\frac{p(t)^2}{D_0}\right]$; and D_0 is the initial diameter of the
 171 reinforcement bar.

172 In addition to the loss of net area, existing laboratory results indicate that corrosion can reduce the
 173 yield stress of normal reinforcement bar (Du *et al.* 2005)

$$174 \quad f_y(t) = (1 - 100\alpha_{corr}P_{corr})f_{y0} \quad (5)$$

175 where $f_y(t)$ = yield stress at time t ; f_{y0} = initial yield stress; P_{corr} = percentage of area loss that caused
 176 by corrosion (%); and α_{corr} = a coefficient.

177 With respect to the corrosion of prestressed tendons, the model proposed by Darmawan and
 178 Stewart (2007) can be used. Given the corrosion density i_{corr} ($\mu A/cm^2$), wire length l (mm), and initial
 179 corrosion time t_i (years), the probability distribution function (PDF) of $p(t)$ (mm) of single wire is
 180 (Darmawan and Stewart 2007)

$$181 \quad f_{p(t)}(t, i_{corr}, l) = \frac{\alpha}{\lambda^{0.54}} \exp\left[-\alpha\left(\frac{p(t)}{\lambda^{0.54}} - \mu\right)\right] \exp\left\{-\exp\left[-\alpha\left(\frac{p(t)}{\lambda^{0.54}} - \mu\right)\right]\right\}, t > t_i \quad (6)$$

182 where

$$183 \quad \lambda = \frac{D_0^2 - [D_0 - 0.0232i_{corr}(t - t_i)]^2}{D_0^2 - (D_0 - 0.0232i_{corr-exp}T_{0-exp})^2} \quad (7)$$

$$184 \quad \mu = \mu_{0-exp} + \frac{1}{\alpha_{0-exp}} \ln\left(\frac{1}{l_{0-exp}}\right), \quad \alpha = \alpha_{0-exp} \quad (8)$$

185 Herein, $T_{0\text{-exp}} = 0.03836$ years, $\mu_{0\text{-exp}} = 0.84$, $\alpha_{0\text{-exp}} = 8.10$, $i_{\text{corr-exp}} = 186 \mu\text{A}/\text{cm}^2$, $l_{0\text{-exp}} = 650$ mm,
 186 and these values are obtained from statistical analysis of the accelerated corrosion results.

187 For prestressed strands consisting of seven wires, it is assumed that the pitting only forms on the
 188 six outer wires (Darmawan and Stewart 2007). Therefore, the remaining net area of the entire strand
 189 $A_{sr}(t)$ is $6A_{wr}(t) + A_{w0}$, where $A_{wr}(t)$ is the remain net area of outer wire at time t , and A_{w0} denotes the
 190 time-invariant net area of the inner wire.

191 Experimental studies show that the ultimate tension strain and stress of prestressed tendons both
 192 decrease dramatically with the increase of corrosion extent (Barton *et al.* 2000; Darmawan and Stewart
 193 2007; Li *et al.* 2009; Vu *et al.* 2009; Zeng *et al.* 2010; Zhang *et al.* 2012; Wang *et al.* 2017). In addition,
 194 the hardening portion within the constitutive model of prestressed tendons also reduces as the increase
 195 of corrosion, which could make the prestressed tendon behave from a bilinear elastic to a brittle elastic
 196 (Zeng *et al.* 2010, Wang *et al.* 2017). The evolution of constitutive relationship of corroded prestressed
 197 tendon subjected to tension is described by Eqs. (9) and (10) (Zeng *et al.* 2010), as depicted in Fig. 1.

198 For $P_{\text{corr}} \leq P_{\text{corr},0}$

$$199 \quad \sigma_{pc} = \begin{cases} E_{pc} \varepsilon_{pc}; & \varepsilon_{pc} \leq \frac{0.85 f_{puc}}{E_{pc}} \\ 0.85 f_{puc} + \left(\varepsilon_{pc} - \frac{0.85 f_{puc}}{E_{pc}} \right) \left(\frac{0.15 f_{puc}}{\varepsilon_{puc} - \frac{0.85 f_{puc}}{E_{pc}}} \right); & \frac{0.85 f_{puc}}{E_{pc}} < \varepsilon_{pc} \leq \varepsilon_{puc} \end{cases} \quad (9)$$

200 For $P_{\text{corr}} > P_{\text{corr},0}$

$$201 \quad \sigma_{pc} = E_{pc} \varepsilon_{pc} \quad (10)$$

202 where $P_{corr,0}$ = threshold value of corrosion extent; σ_{pc} and ε_{pc} = tension stress and strain, respectively;
 203 E_{pc} , ε_{puc} , and f_{puc} = elastic modulus, ultimate tension strain, and ultimate tension stress of corroded
 204 prestressed tendons, respectively, and can be computed as (Zeng *et al.* 2010)

$$205 \quad E_{pc} = (1 - 0.848P_{corr})E_p \quad (11)$$

$$206 \quad \varepsilon_{puc} = (1 - 9.387P_{corr})\varepsilon_{pu} \quad (12)$$

$$207 \quad f_{puc} = \frac{1 - 2.683P_{corr}}{1 - P_{corr}}f_{pu} \quad (13)$$

208 where E_p , ε_{pu} , and f_{pu} = the elastic modulus, ultimate tension strain, and stress of un-corroded
 209 prestressed tendons, respectively.

210 **Load model**

211 Bridges are subjected to dead and live loads. Dead load is associated with self-weight of girder, deck,
 212 barriers, wearing surface, etc. With respect to live load, the semitrailer truck specified in the Chinese
 213 General Code for Design of Highway Bridges and Culverts (Ministry of Transport of the People's
 214 Republic of China 2004b) can be selected as the nominal load model. Considering uncertainties of live
 215 load, the maximum static live load effect through the lifetime period can be assumed to follow an
 216 extreme value distribution (type I) (AQSIQ and MOHURD 1999). Dynamic amplification effect of
 217 live load due to deck surface roughness, bridge dynamics, and vehicle dynamics is also considered
 218 using a dynamic load amplification factor, η_{DLA} (Ministry of Transport of the People's Republic of
 219 China 2004b). Given the load model, the system resistance is expressed in terms of multiples
 220 associated with nominal live load model, i.e., ratio between weight of the maximum live load that

induces the predefined limit state (i.e., *ULS* or *SLS*) to that of the nominal live load model. This aspect is discussed in the following section.

System performance considering ductility and redundancy

The system performance is evaluated using FE method in a probabilistic manner. Fig. 2 shows the general procedure for system resistance evaluation considering uncertainties. The detailed information of different modules is indicated in the following sections.

Sectional nonlinear analysis for Moment-Curvature relationship (MCR)

FE method is adopted to compute the resistance of bridge at system level. In order to account for effects of structural ductility and redundancy, material nonlinearity is considered in the FE model. Typically, this can be achieved by introducing the nonlinear constitutive relationships of concrete, reinforcement bar, and prestressed tendons (Guo *et al.* 2010). However, resistance analysis using such an exhaustive model is time-consuming considering numerous elements and incremental analysis procedure (see Fig. 2), not to mention that numerous simulations are required within reliability assessment. Therefore, a simplified and efficient FE methodology, grillage model, is selected for system resistance analysis (Hambly 2014). The grillage method has been demonstrated to have considerable accuracy for girder bridges with simple geometries and load conditions (Jaeger and Bakht 1982; Zokaie *et al.* 1991; Keogh and O'Brien 1996). Additionally, the grillage model has been widely used within the structural resistance analysis process in previous studies (Amer *et al.* 1999; Enright and Frangopol 1999; Al-Saidy *et al.* 2008; Théoret *et al.* 2011; Tu *et al.* 2017). In order to improve the computational efficiency, the grillage model is adopted to compute the probabilistic structural resistance. In grillage models, the

241 RC or PSC girder is modelled by discrete beam elements, for which the constitutive relationship is
242 determined at the sectional level. Therefore, sectional nonlinear analysis should be conducted firstly
243 to obtain sectional constitutive relationship based on the basic material constitutive models. In addition,
244 as the considered limit states of bridge system are associated with flexure failure and excessive
245 deflection, the nonlinear moment-curvature relationship (MCR) is assessed and then introduced into
246 the grillage model.

247 Given geometric sizes and the material constitutive relationships of the PSC section, a numerical
248 incremental algorithm is proposed to determine the MCR, as shown in Fig. 3. With respect to the
249 algorithm, the investigated section is firstly meshed into a number of discrete strips along its height.
250 The initial stress properties as well as the sectional curvature φ_0 of the section caused by prestressed
251 force P_0 is firstly determined based on the on the hypothesis of plane remaining plane and equilibrium
252 regarding axial force. Subsequently, the sectional curvature is increased gradually, and under each
253 given curvature φ_i , based on the hypothesis of plane remaining plane and equilibrium equations
254 regarding axial force, a bisection method-based iterative procedure is conducted to get the critical
255 height of the compression area within the section. Then, the stress properties as well as the integral
256 flexural moment of the section at this stage are obtained. The procedure is terminated when sectional
257 failure happens, corresponding to crushing of concrete at the top fiber or tensile fracture of prestressed
258 tendons. The behavior of concrete under compression is based on the Rusch model (Kupfer *et al.* 1969),
259 in which the ultimate compressive strain is 0.0033 for normal strength concrete. The effects of
260 corrosion on the MCR are considered within the computational process. Based on Eq. (12) the ultimate
261 tension strain of corroded prestressed tendons ε_{puc} is time-dependent and is reduced with time

262 considering deterioration effects.

263 **Grillage model for resistance analysis**

264 The grillage model, which has been approved to have a good balance between accuracy and
265 computational cost, is adopted herein for assessing the resistance of bridge systems (Hambly 2014).
266 For illustrative purpose, Fig. 4 shows a typical grillage model of a multi-girder bridge. The model
267 consists of five longitudinal members representing the girders. Seven transverse members, labeled as
268 *a, b, c, d, e, f*, and *g*, represent the transverse diaphragms, which are uniformly distributed along the
269 span. The remaining transverse members are virtual diaphragms, accounting for connection provided
270 by the girder flange. To make it possible to apply load at arbitrary locations in the horizontal plane,
271 shell elements are set upon the longitudinal and transverse beam grid to simulate pressure distribution
272 effect from the deck. The aforementioned nonlinear moment-curvature constitutive relationships are
273 assigned to the longitudinal girder members.

274 **Time-dependent redundancy assessment**

275 Redundancy, a quantitative relationship between component and system safety, is a critical indicator
276 within the performance assessment of structural systems (Ghosn *et al.* 2010; Frangopol *et al.* 2017).
277 Redundancy depends on several factors, for instance, degree of static indeterminacy of the structural
278 system and ductility of the components. For a static determinate system, failure of any component (no
279 matter brittle or ductile) would lead to a structure failure. For a static indeterminate system, the
280 remaining components usually cannot survive once brittle failure of any component happens, as they
281 need to take over the force that is released by the failure component (Frangopol and Curley 1987). In

other words, the component ductility could increase the possibility and extent of force redistribution among components after failure of the weakest component and then contribute the system redundancy (Frangopol and Curley 1987). Therefore, the ductility should be accounted for within the assessment of redundancy.

Generally, ductility is the ability of material, section, component, or system to sustain plastic deformation without significantly reducing its strength (Fajfar 1992). Fig. 5 presents the generalized ‘load-to-deformation’ relationships associated with different structural constitutes, i.e., ductile material, section, component, and structure, where d_u is the ultimate ‘deformation’ at failure of ductile material, section, component, and structure system; and d_y is the corresponding ‘deformation’ at elastic limit. Based on the ‘load-to-deformation’ relationships, ductility is usually measured using a ductility factor μ , which is the ratio between ultimate ‘deformation’ at failure d_u to the elastic limit ‘deformation’ d_y . Obviously, a larger ductility factor indicates a better ductile performance of the investigated structural constitute. Considering the deterioration effects (e.g., steel corrosion), the ductility factors of the structural constitutes should be time-dependent variables.

Different from ductility, redundancy of a structural system is assessed by combing the overall structural performance and its components. Generally, redundancy is defined as the capability of a structure to continue to carry loads after the ductile failure of the most critical component (Hendawi and Frangopol 1994, Ghosn and Moses 1998, Ghosn *et al.* 2010). The conceptual representation for structural behavior of a multi-girder bridge subjected to live load at different time points is shown in Fig. 6. The structural response can be represented by vertical deflection and quantified using the average mid-span displacement of all girders or the maximum displacement among girders. The live

load is generalized with respect to the nominal live load model and expressed as a load factor, LF . It can be observed that the most vulnerable girder fails (ductile failure) firstly under a load factor of $LF_1(t)$ at time t , while the bridge system can still sustain extra live load until the ultimate limit state (ULS) is approached at $LF_u(t)$, which usually corresponds to the flexural failure of one or several girders that disables the functionality of the bridge system. In addition, there may exist another feature point below $LF_u(t)$ representing another system limit state, e.g., SLS , labeled as $LF_s(t)$. It should be noted that the specific structural stages (i.e., $LF_1(t)$, $LF_s(t)$, and $LF_u(t)$) should be characterized based on the structure type, structure configuration, and component behavior, among others. For multi-girder steel and reinforced concrete bridges, ductile failure of the most vulnerable girder generally corresponds to yielding of steel and yielding of rebar within the girder (Hendawi and Frangopol 1994; Ghosn and Moses 1998; Ghosn *et al.* 2010). Therefore, for the investigated PSC bridges in this paper, $LF_1(t)$, $LF_s(t)$, and $LF_u(t)$ refer to the first occurring of yielding, excessive live load deflection ($l/100$, l is the bridge span), and flexural failure of any girder within the bridge system, respectively. In addition, deterioration effects are considered within the redundancy assessment process.

Once the performance curve of the bridge system is obtained, the ratios between the load factor at system limit state ($LF_s(t)$ and $LF_u(t)$) and load factor of the first girder failure ($LF_1(t)$) can be used to quantify the system redundancy, and expressed as (Ghosn and Moses 1998)

$$R_u(t) = \frac{LF_u(t)}{LF_1(t)}, \quad R_s(t) = \frac{LF_s(t)}{LF_1(t)} \quad (14)$$

where $R_u(t)$ and $R_s(t)$ = system reserve ratio at time t with respect to ULS and SLS , respectively. Ghosn *et al.* (2010) suggest a minimum value of 1.3 and 1.1 for R_u and R_s to ensure a redundant superstructure

323 system.

324 To account for the uncertainties inherent with the structure properties and loads, the difference in
325 reliability index between the system and the most vulnerable girder is used as the probabilistic measure
326 of redundancy (Ghosn and Moses 1998)

$$327 \quad \Delta\beta_u(t) = \beta_u(t) - \beta_1(t), \quad \Delta\beta_s(t) = \beta_s(t) - \beta_1(t) \quad (15)$$

328 where $\Delta\beta_u(t)$ and $\Delta\beta_s(t)$ = probabilistic redundancy measures with respect to *ULS* and *SLS*,
329 respectively; $\beta_u(t)$ and $\beta_s(t)$ = reliability indices of the structure system with respect to *ULS* and *SLS*,
330 respectively; $\beta_1(t)$ = reliability index of the most vulnerable girder and should be computed using $LF_1(t)$
331 as the capacity. It is suggested that the threshold value of $\Delta\beta_u(t)$ and $\Delta\beta_s(t)$ are 0.85 and 0.25,
332 respectively (Ghosn *et al.* 2010).

333 To account for the consequences of structural failure, it is necessary to integrate the consequences
334 into the redundancy evaluation. By combining occurrence probabilities of different failure scenarios
335 and corresponding consequences, Baker *et al.* (2008) proposed risk-informed redundancy/robustness
336 measures as

$$337 \quad R_{r,u}(t) = \frac{R_{Dir}(t)}{R_{Dir}(t) + R_{Idir,u}(t)}, \quad R_{r,s}(t) = \frac{R_{Dir}(t)}{R_{Dir}(t) + R_{Idir,s}(t)} \quad (16)$$

338 where $R_{r,u}(t)$ and $R_{r,s}(t)$ = risk-informed redundancy measures with respect to *ULS* and *SLS*,
339 respectively; $R_{Dir}(t)$ = direct risk due to the first girder failure; and $R_{Idir,u}(t)$ and $R_{Idir,s}(t)$ = indirect risk
340 due system failure under *ULS* and *SLS*, respectively. The three risk items involved in Eq. (16) can be
341 computed as

$$R_{Dir}(t) = P_{f,1}(t)C_{Dir}(t) \quad (17)$$

$$R_{Idir,u}(t) = P_{f,u}(t)C_{Idir,u}(t) \quad (18)$$

$$R_{Idir,s}(t) = P_{f,s}(t)C_{Idir,s}(t) \quad (19)$$

where $P_{f,1}$, $P_{f,u}$, and $P_{f,s}$ = probabilities of first girder failure, system failure under *ULS* and system failure under *SLS*, respectively; $C_{Dir}(t)$ = direct consequence due to the first girder failure; $C_{Idir,u}(t)$ and $C_{Idir,s}(t)$ = indirect consequences due to system failure under *ULS* and *SLS*, respectively. The direct consequence generally corresponds to potential repair cost of the damaged component. The indirect consequence includes the social and environmental impacts resulting from a failed system (Dong *et al.* 2013).

The risk-informed redundancy measures defined by Eq. (16) varies between zero and one, and a larger value indicates a smaller ratio between the indirect risk brought by system failure and the direct risk of component failure.

Illustrative example

The proposed methodology is applied to a PSC bridge with a span of 25 *m* as shown in Fig. 7(a). The bridge consists of five 1.4 *m*-height T-shape PSC girders to support a two-lane traffic. The geometric properties and arrangement of reinforcement steel of the girder are presented in Fig. 7(b). Adjacent girders are connected by continuous flange wet seams and seven uniformly-arranged transverse diaphragms (Fig. 7(c)). The bridge was designed based on the Chinese Code for Design of Reinforced Concrete and Prestressed Concrete Highway Bridges and Culverts (Ministry of Transport of the People's Republic of China 2004a). The FE grillage model of investigated bridge is constructed using

362 ANSYS (Ansys 2017).

363 **Component analysis**

364 MCR of the girder (section) is the prerequisites for system resistance analysis and is affected by several
365 parameters. The uncertainties associated with the parameters (e.g., geometric parameters, initial
366 material properties, modeling uncertainty factor, and corrosion-related parameters) are considered
367 within the analysis process, as presented in Table 1.

368 Given the relevant random variables, Latin Hypercube Sampling (LHS), a high-efficiency
369 sampling method (McKay *et al.* 1979), is used to evaluate the statistical properties of MCR of the
370 investigated section (e.g., the mid-span section of each girder) incorporating the incremental numerical
371 algorithm as shown in Fig. 3. At each time step, the time-dependent residual net area and strength of
372 reinforcement steel are computed according to the corrosion model. For instance, given $t = 40$ years,
373 a portion of the generated MCR curves are shown in Fig. 8(a). It should be noted that moment and
374 curvature presented are the incremental items under external loading, i.e., the initial effects introduced
375 by prestressed tendons are excluded. As indicated, the integral moment of the section increases
376 dramatically with the increase of curvature until prestressed tendons ‘yields’ (approaches its elastic
377 limit). Afterwards, the sectional moment increases gradually accompanied with a rapid growth of
378 curvature before sectional failure, which corresponds to the crushing of concrete at the top fiber or
379 tensile fracture of prestressed tendons. Therefore, an idealized bilinear model is used to represent the
380 original integral MCRs, as shown in Fig. 8(b), which is characterized by two feature points: yielding
381 point (φ_y, M_y) and failure point (φ_u, M_u) .

Distributions of the characteristic parameters of MCR (i.e., M_y , M_u , φ_y and φ_u) at different time points are obtained. To reveal the corrosion effects, expected values of M_y , M_u and φ_y , φ_u as a function of time are shown in Figs. 9(a) and (b), respectively. It can be found that expected values of M_y and M_u both start to decrease at $t = 40$ years when corrosion of prestressed tendons initiates. The decrease of M_y and M_u is almost synchronous, resulting in a reduction of 15.1% and 13.5% at $t = 100$ years for M_y and M_u , respectively. The expected value of ultimate curvature (φ_u) also decreases dramatically once the corrosion starts, with a reduction of 47.8% at the end of service time. This is due to the fact that ductility of prestressed tendons reduces dramatically as the corrosion propagates (Zeng *et al.* 2010), making the failure criterion of investigated section transforms from the crushing of top fiber concrete to the tensile fracture of prestressed tendons, while the latter criterion is much more brittle. Comparatively, the expected value of yielding curvature (φ_y) almost keeps constantly within the investigated time interval. The inconsistent deterioration between φ_u and φ_y results in a continuous reduction of sectional ductility. The sectional ductility factor μ_s decreases from 17.8 at $t = 0$ year to 9.0 at 100 years. Eventually, the moment-curvature slope (i.e., sectional stiffness) of second portion within constitutive model associated with the investigated PSC section (see Fig. 8(b)) increases dramatically under the propagation of steel corrosion.

System resistance and reliability analysis

The system resistance is evaluated using the FE method in a probabilistic manner. Given the associated random variables, the grillage model as indicated in Fig. 4 is implemented with ANSYS to capture the system resistance. Dead load is firstly applied to the FE model, then two side-by-side semitrailer trucks (i.e., the nominal live load model specified in the Chinese General Code for Design of Highway

403 Bridges and Culverts (Ministry of Transport of the People's Republic of China 2004b)) are gradually
404 applied to the model until the predefined limit states are reached. Specifically, the gross vehicle weight
405 (GVW) of the truck is proportionally increased while the distribution factors of per axle are kept
406 constant. The apply locations of the semitrailer trucks are shown in Fig. 4, in which the nominal trucks
407 are also presented. The longitudinal position of the truck load is determined using influence line
408 analysis, and the worst scenario for the flexural moment in mid-span is associated with the case that
409 the front wheel is placed 1.1 m away from the left support. With respect to the transverse direction, the
410 minimum clearance between the two trucks is 1.3 m, and the minimum clearance between the extreme-
411 exterior wheel and the barrier edge is 0.5 m. The system resistance is defined using a load factor, LF ,
412 which is the ratio between the GVW of the applied live load at failure (i.e., defined limit state) and the
413 GVW of the nominal live load model. A sample of load-deflection curve and load-moment curve at t
414 = 40 years are shown in Figs. 10(a) and (b), respectively. The presented vertical deflection and flexural
415 moment are the values of the mid-span section of each girder. As indicated in Figs. 10(a), the load-
416 deflection relationship of individual girder shows obvious nonlinearity under relatively large load due
417 to sectional nonlinearity. The deflection difference among girders is not obvious. This is due to the fact
418 that the wheel loads are nearly uniformly and symmetrically distributed along the bridge deck in
419 transverse direction due to the limited deck width, resulting in that the girders deform nearly parallel
420 in the vertical direction. Fig. 10(b) presents a brief diagram regarding the failure mechanism of the
421 bridge system from component to system level. The flexural moment of all girders increases linearly
422 with the increase of live load at the beginning. Then, girders 3 and 2, which have the lowest yielding
423 strength, yield successively, leading to a decreased moment growth rate of these two girders.

424 Simultaneously, the other three girders (i.e., girders 1, 4, and 5) take over the loadings released by
425 girders 2 and 3. This is reflected in the load-moment curves through an increased moment-load gradient
426 of these three girders. In the end, the bridge system fails as girder 1 reaches its moment capacity.

427 Random variables considered in the probabilistic analysis of system resistance include the
428 magnitude of dead load, transverse loading position of live load, and time-dependent characteristic
429 parameters of MCRs (ϕ_y , ϕ_u , M_y , and M_u). Herein, dead load is assumed to follow a normal distribution
430 with a bias factor of 1.0148 and a COV of 0.0431 (AQSIQ and MOHURD 1999). The transverse
431 loading position is generated using the probabilistic model proposed by Al-Zaid (1986), in which the
432 distance between the extreme-exterior wheel and the barrier edge (d_1 in Fig. 4) follows a lognormal
433 distribution. The expected values of system resistances as a function of service time are presented in
434 Fig. 11(a) based on the nonlinear finite element modelling. As LF_u and LF_1 are related to the time-
435 dependent ultimate capacity (M_u) and yielding capacity (M_y) of the most critical girder, respectively,
436 the trend of LF_u and LF_1 is in accordance with that of the component resistances (M_u and M_y). Therefore,
437 mean values of LF_u and LF_1 also deteriorate synchronously with time. For instance, the mean value of
438 LF_u and LF_1 at $t = 100$ years is about 79% and 81% of that at $t = 40$ years, respectively. Since the
439 criteria of *SLS* is deterministic ($l/100$) and sectional stiffness of post-yielding stage of the investigated
440 PSC section increases dramatically under the propagation of steel corrosion, the degradation of LF_s
441 due to corrosion is slighter than that of LF_u and LF_1 .

442 Once the time-variant probabilistic models of system resistances are obtained, the corresponding
443 system reliability at each time is calculated using the first order-second moment method (FORM), as
444 shown in Fig. 11(b). The reliability index of the most critical girder component (Girder 1) is also shown

445 in the figure, which is computed using the traditional time-invariant live load distribution factor
446 (LLDF). As indicated, the reliability indices of bridge system are much larger than the reliability index
447 of individual girder, indicating the benefit of redundancy and ductility on structural safety. Specifically,
448 considering corrosion effects, the reliability index of the most critical girder degrades to the threshold
449 (i.e., 4.2) at about 80 years, whereas the two system reliability indices remain above the threshold
450 value through the design service life.

451 **System redundancy analysis**

452 The time-dependent deterministic and probabilistic system redundancy measures (i.e., $R_u(t)$, $R_s(t)$ and
453 $\Delta\beta_u(t)$, $\Delta\beta_s(t)$) are computed using Eqs. (14) and (15), respectively. Fig. 12(a) presents the results of
454 the time-dependent system reserve ratio $R_u(t)$ and $R_s(t)$. As indicated, the system reserve ratios
455 regarding to *ULS* and *SLS* (i.e., $R_u(t)$ and $R_s(t)$) are larger than 1.30 and 1.10 through the service life,
456 indicating a satisfactory redundancy performance. Additionally, considering the load redistribution
457 effects, the value of system reserve ratio under *ULS*, $R_u(t)$, is much larger than the value of component
458 resistance reserve ratio $R_c(t)$ (i.e., $M_u(t)/M_y(t)$).

459 As the system resistance factor associated with *SLS*, LF_s , degrades slighter than LF_1 under
460 corrosion (see Fig. 11(a)), the system reserve ratio associated with *SLS*, $R_s(t)$, increases slightly with
461 time. On the other hand, the system redundancy indicator $R_u(t)$ regarding *ULS* decreases slightly in
462 accordance with the corrosion process (from 1.51 at $t = 40$ years to 1.46 at $t = 100$ years), though the
463 component reserve ratio, $R_c(t)$, increases marginally within the time interval. This opposite trend
464 between system reserve ratio $R_u(t)$ and component reserve ratio $R_c(t)$ mainly attributes to the negative
465 effects of corrosion on load redistribution effects at bridge system level. Specifically, as corrosion

466 aggravates, the ductility of girder component μ_c degrades significantly, as shown in Fig. 9(b). While
467 the component ductility is one of the most important preconditions for the occurring of load
468 redistribution among different girders within the bridge system. Therefore, the contribution of
469 component capacity reserve to the system capacity reserve, i.e., system redundancy, is impaired by
470 steel corrosion.

471 To validate the above illustrations regarding the negative effects of steel corrosion on system
472 redundancy, the investigated bridge is re-analyzed by reducing the height of transverse diaphragms
473 from 1.26 m to 0.05 m. In this way, the transverse connection stiffness of the bridge superstructure as
474 well as the load redistribution effects can be negligible. $R_u(t)$ of the loosened bridge system is shown
475 in Fig. 12(a). As indicated, $R_u(t)$ of the revised bridge system is significantly smaller than that of the
476 original bridge, verifying the fact that transverse diaphragms play an important role within the
477 redundancy of multi-girder bridge. In addition, evolution trend of the updated $R_u(t)$ turns to be
478 consistent with that of the component reserve ratio $R_c(t)$ when the effect of load redistribution is
479 negligible. It should be noted that as the dead load items are extracted within the computation of the
480 load factor LF_u , LF_1 as well as $R_u(t)$, the value of $R_u(t)$ of the loosened bridge is still larger than that of
481 component reserve ratio $R_c(t)$.

482 Considering the importance of load redistribution effect in the mechanism of system redundancy,
483 a new indicator η_{red} is proposed to quantify it and therefore to enhance the definition of redundancy.
484 η_{red} equals to the ratio between system reserve ratio $R_u(t)$ and component reserve ratio $R_c(t)$, as
485 expressed in Eq. (20). Obviously, the value of η_{red} should no less than 1, and a larger value of η_{red}
486 indicates a more profound load redistribution among different components.

$$\eta_{red} = \frac{R_u(t)}{R_c(t)} \quad (20)$$

The time-dependent η_{red} for the initial and loosened bridges are shown in Fig. 12(b). As indicated, the load redistribution indicator η_{red} of the initial bridge system decreases gradually with time due to the component brittleness induced by corrosion (i.e., decrease in component ductility). The values of η_{red} at $t = 40$ and 100 years are approximate 1.29 and 1.23, respectively. As the load redistribution effect is negligible in the loosened bridge, the redistribution indicator η_{red} of this system fluctuates marginally through the service life. Overall, the formulation mechanism of system redundancy of multi-component structures and the associated effects of steel corrosion can be briefly illustrated in Fig. 13(a).

The probabilistic system redundancy indices $\Delta\beta_u(t)$ and $\Delta\beta_s(t)$ as a function of time are shown in Fig. 13(b). As indicated, the redundancy indices under *ULS* and *SLS* are larger than the threshold value of 0.85 and 0.25 suggested by Ghosn *et. al* (2010). In addition, $\Delta\beta_u(t)$ and $\Delta\beta_s(t)$ gradually decreases and increases with time as the corrosion propagates, respectively, which is similar with $R_u(t)$ and $R_s(t)$. The variation of $\Delta\beta_u(t)$ and $\Delta\beta_s(t)$ is much larger than that of $R_u(t)$ and $R_s(t)$. For instance, the value of $\Delta\beta_u(t)$ at $t = 100$ years is about 91% of that at $t = 40$ years, while the reduction of $R_u(t)$ within this time interval is 3%.

The risk-informed redundancy $R_{r,u}$ and $R_{r,s}$ as defined by Eq. (16) is also computed. In the computation, the occurring probabilities associated with different failure scenarios are transformed from the corresponding reliability indices. The consequences of system failure considering *ULS* and *SLS* are assumed to be 100 and 50 times of the consequences of component failure, respectively. It is found that both $R_{r,u}$ and $R_{r,s}$ are varied marginally through the service life of the bridge and almost kept

508 as 1. This is because the design of the bridge is quite conservative, which is reflected by the relative
509 high reliability indices of both individual girder and the whole system (see Fig. 11(b)).

510 **Conclusions**

511 The paper presents a computational framework for time-dependent system reliability and redundancy
512 assessment of multi-girder PSC bridges. The system-level resistance of the bridge under different
513 failure criteria is evaluated using a multi-stage incremental procedure supported by nonlinear FE
514 method. The effect of reinforcement corrosion on girder capacity and ductility as well as system-level
515 live load redistribution is integrated into the procedure.

516 The following conclusions are drawn:

517 (1) At each time step, the moment-curvature relationship (MCR) of the investigated representative
518 PSC section can be idealized by a bilinear model, which could significantly increase the
519 computation efficiency. The expected values of ultimate moment and yielding moment of the
520 section decrease moderately with time. The expected value of ultimate curvature decreases
521 dramatically due to the brittleness brought by corrosion of prestressed tendons, leading to
522 significant reduction in sectional and component ductility.

523 (2) The reliability indices of bridge system subjected to *ULS* and *SLS* are much larger than the
524 reliability index of individual girder, indicating the benefit of system redundancy and ductility
525 on structural safety. Specifically, considering corrosion effects, the reliability index of single
526 girder degrades to the index threshold (i.e., 4.2) at about 80 years, whereas the two system
527 reliability indices remain above the threshold value through the service life, e.g., 100 years.

(3) The deterministic redundancy regarding *SLS* increases slightly with time as corrosion propagates. In the meantime, since reinforcement corrosion impairs component ductility and then introduces adverse influence on live-load redistribution effect, the system redundancy regarding *ULS* decreases though the component reserve ratio increases marginally through this time interval.

(4) An indicator that equals to the ratio between system reserve ratio and component reserve ratio is proposed to quantify the extent of live-load redistribution between different girder components. It is found that the indicator mainly depends on the stiffness of the structural connection among components (e.g., transverse diaphragms) and structure configurations.

(5) Comparing with the deterministic and risk-informed redundancy measures, the variation of the probabilistic redundancy is much more profound within the service life of the bridge. Therefore, the probabilistic measures may be a better indicator for reflecting the negative effect of reinforcement corrosion on the redundancy property of multi-girder bridges. The propose general framework could also be applied to other types of multi-component structures. The framework can aid in understanding essential parameters as well as formulation mechanism within the computation of system reliability and redundancy, and eventually promote effective and economic design of redundant structure systems in a life-cycle context.

(6) The negative effects of steel corrosion on system reliability and redundancy are emphasized in this paper. The concrete shrinkage, creep and pre-stressing loss could also have some adverse influences on the structural long-term performance. Given more information, these effects can be incorporated within the evaluation process integrating damage mechanism

among different structural levels.

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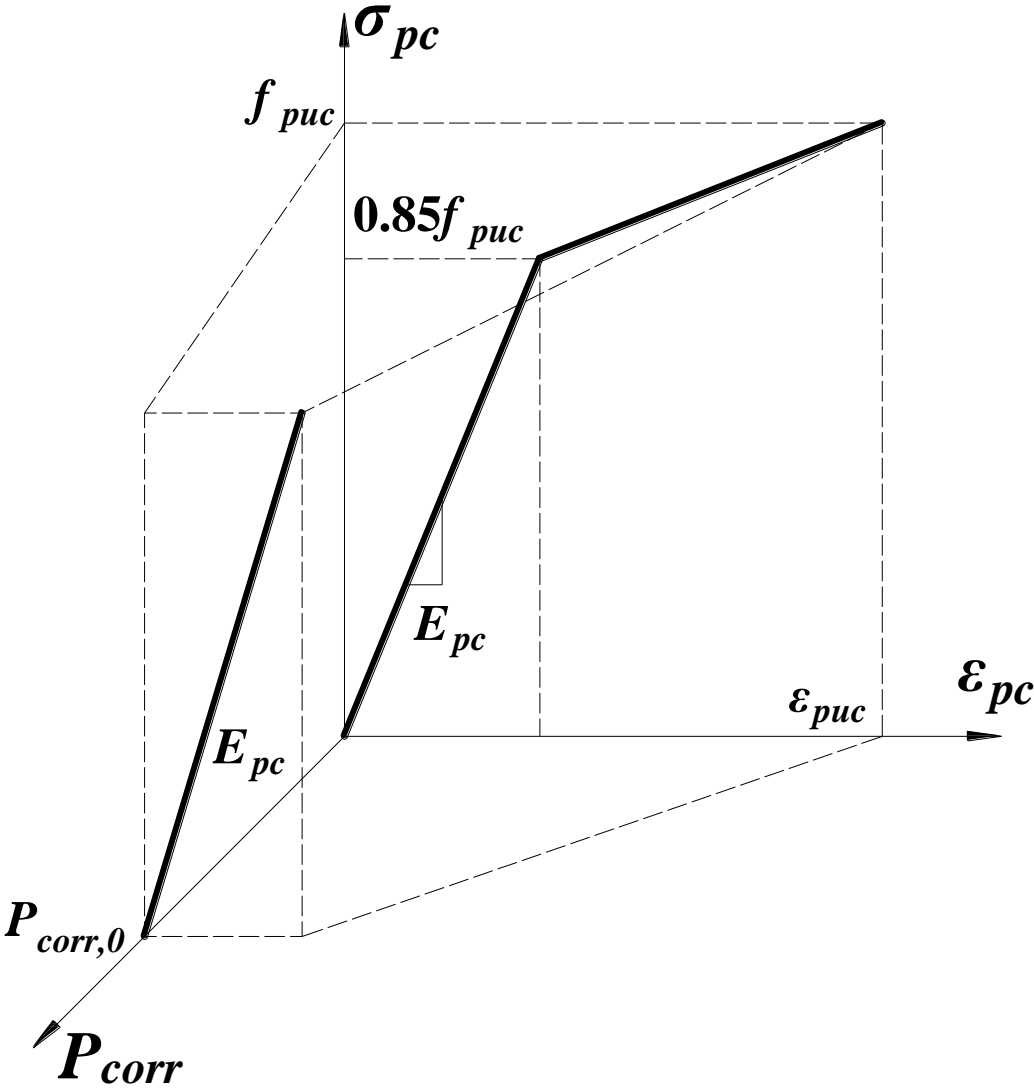
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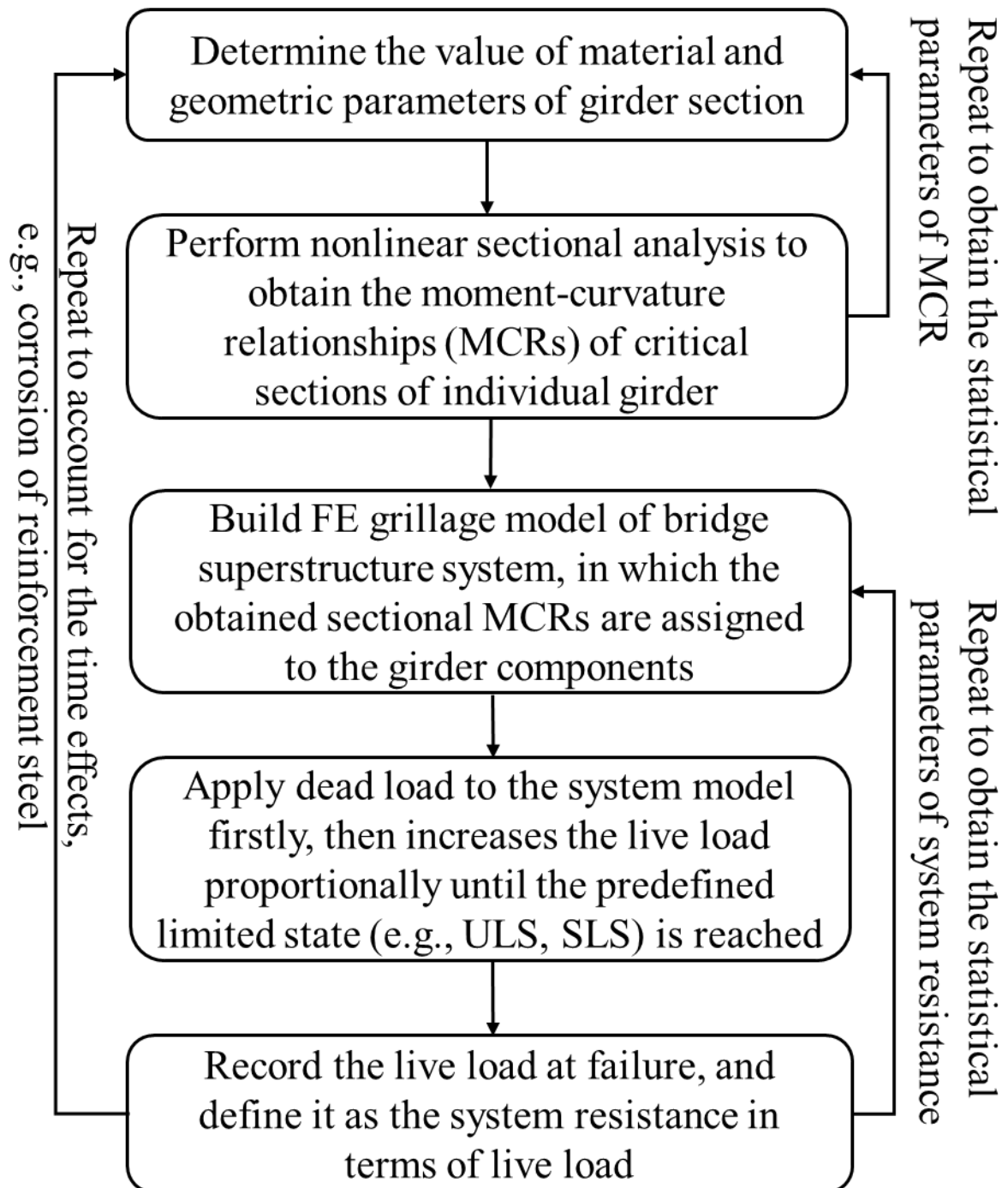
List of Tables

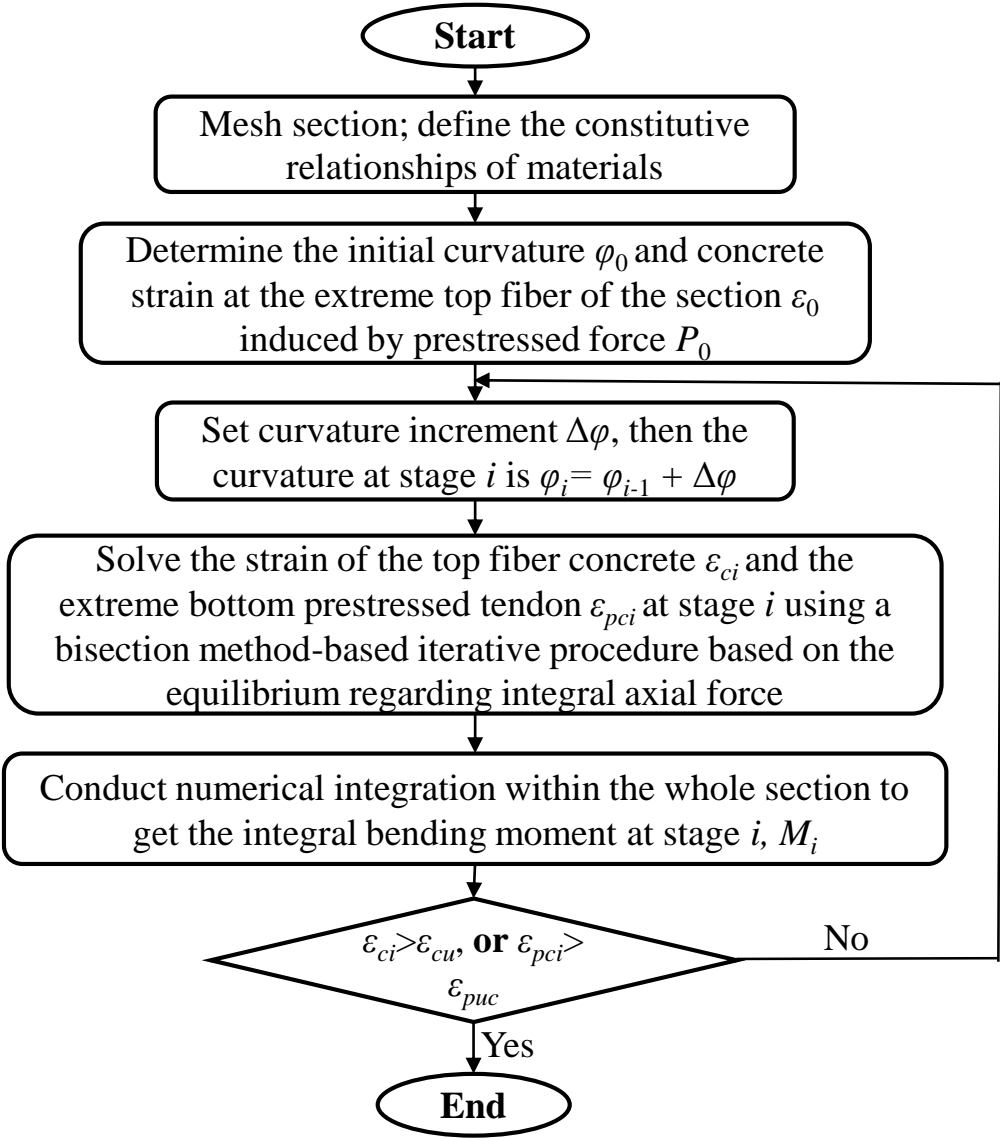
Table 1. Random variables involved in component analysis

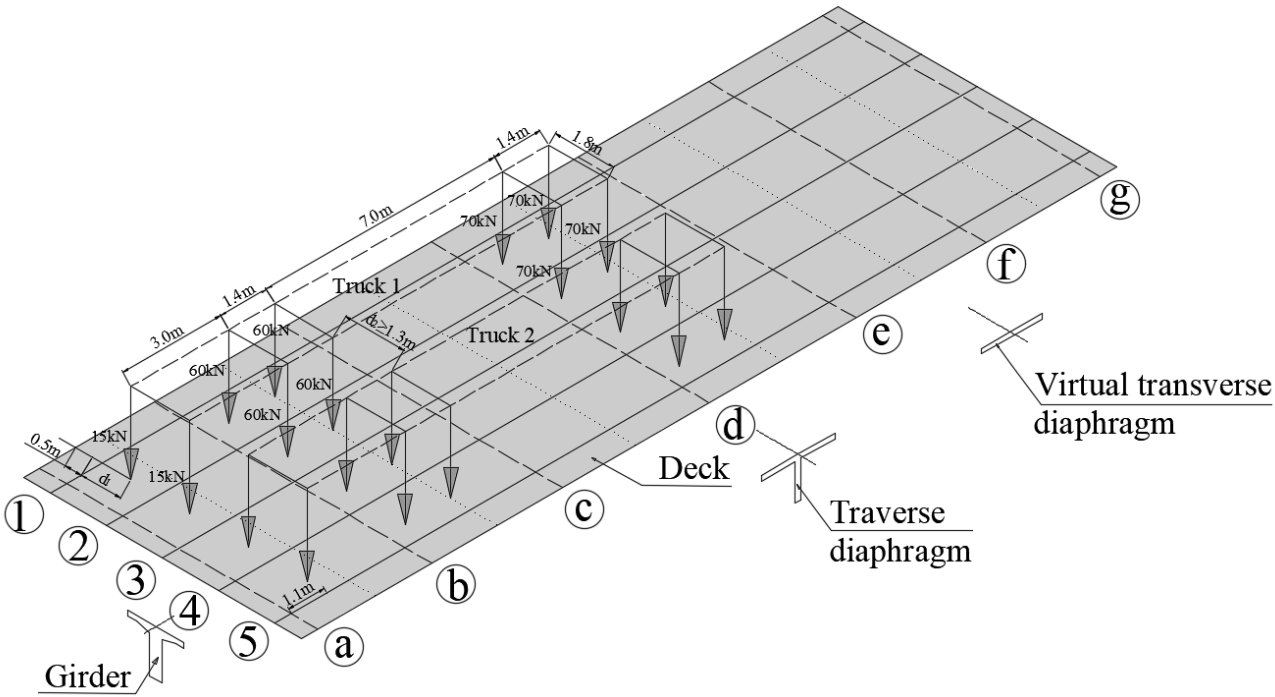
Variable	Properties	Mean	COV	Distribution type	References
b'_f	Effective width of flange	1.0013 ^a	0.0081	Normal	AQSIQ and MOHURD 1999
b	Width of web	1.032 ^a	0.1019	Normal	AQSIQ and MOHURD 1999
h'_f	Height of flange	1.032 ^a	0.1019	Normal	AQSIQ and MOHURD 1999
h_0	Effective height of section	1.0124 ^a	0.0229	Normal	AQSIQ and MOHURD 1999
a'_s	Distance from the top fiber to the centroid of the compressive reinforcement	1.0178 ^a	0.0496	Normal	AQSIQ and MOHURD 1999
h	Nominal height of the section	1.032 ^a	0.1019	Normal	AQSIQ and MOHURD 1999
d	Thickness of concrete cover	1.0178 ^a	0.0496	Normal	AQSIQ and MOHURD 1999
A_s, A'_s, A_p	Initial area of tensile reinforcement, compressive reinforcement and prestressed tendon	1.000 ^a	0.0350	Normal	AQSIQ and MOHURD 1999
f_c (MPa)	Compressive strength of concrete	39.9	0.15	Lognormal	Attard and Stewart 1998
f_{y0} (MPa)	Initial yield strength of reinforcement bar	369	0.0719	Normal	AQSIQ and MOHURD 1999, Ministry of Transport of the People's Republic of China 2004a
f_{pu} (MPa)	Initial ultimate strength of prestressed tendons	1932	0.0142	Normal	Mirza <i>et al.</i> 1979
k_{pM}	Modeling uncertainty factor for flexural moment capacity	1.110	0.062	Normal	Li <i>et al.</i> 1997
D_c (cm ² /year)	Diffusion coefficient	0.631	0.2	Lognormal	Val and Trapper 2008
C_0 (kg/m ³)	Surface chloride content	15	0.2	Normal	Val and Trapper 2008
C_{cr} (kg/m ³)	Threshold chloride concentration	2.0	0.2	Normal	Val and Trapper 2008
i_{corr} (μA/cm ²)	Corrosion current density	1.0	0.2	Normal	González <i>et al.</i> 1995
R	Penetration ratio	3.0	0.33	Normal	Stewart and Rosowsky 1998

Note: ^aThe normalized value divided by nominal value.









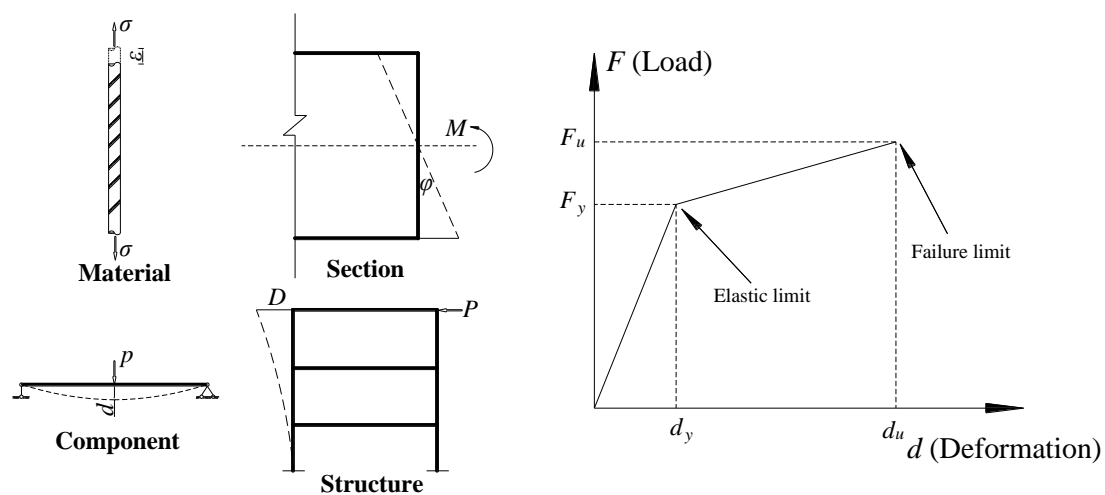
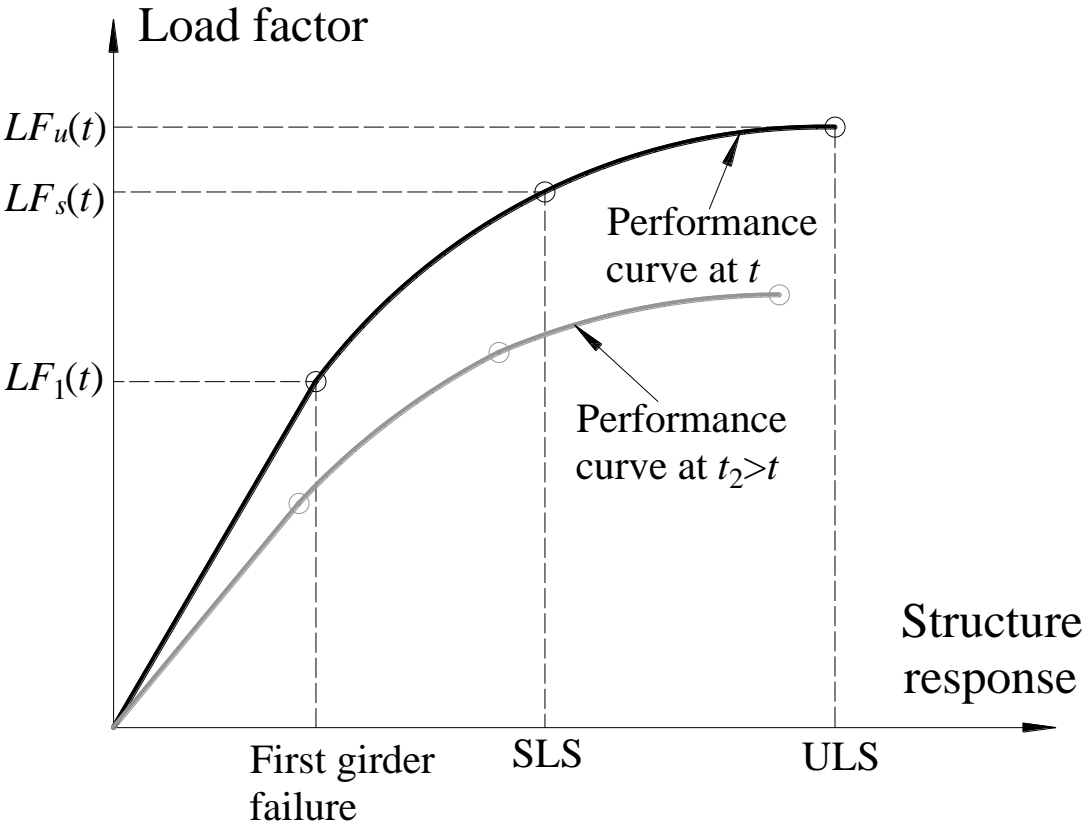
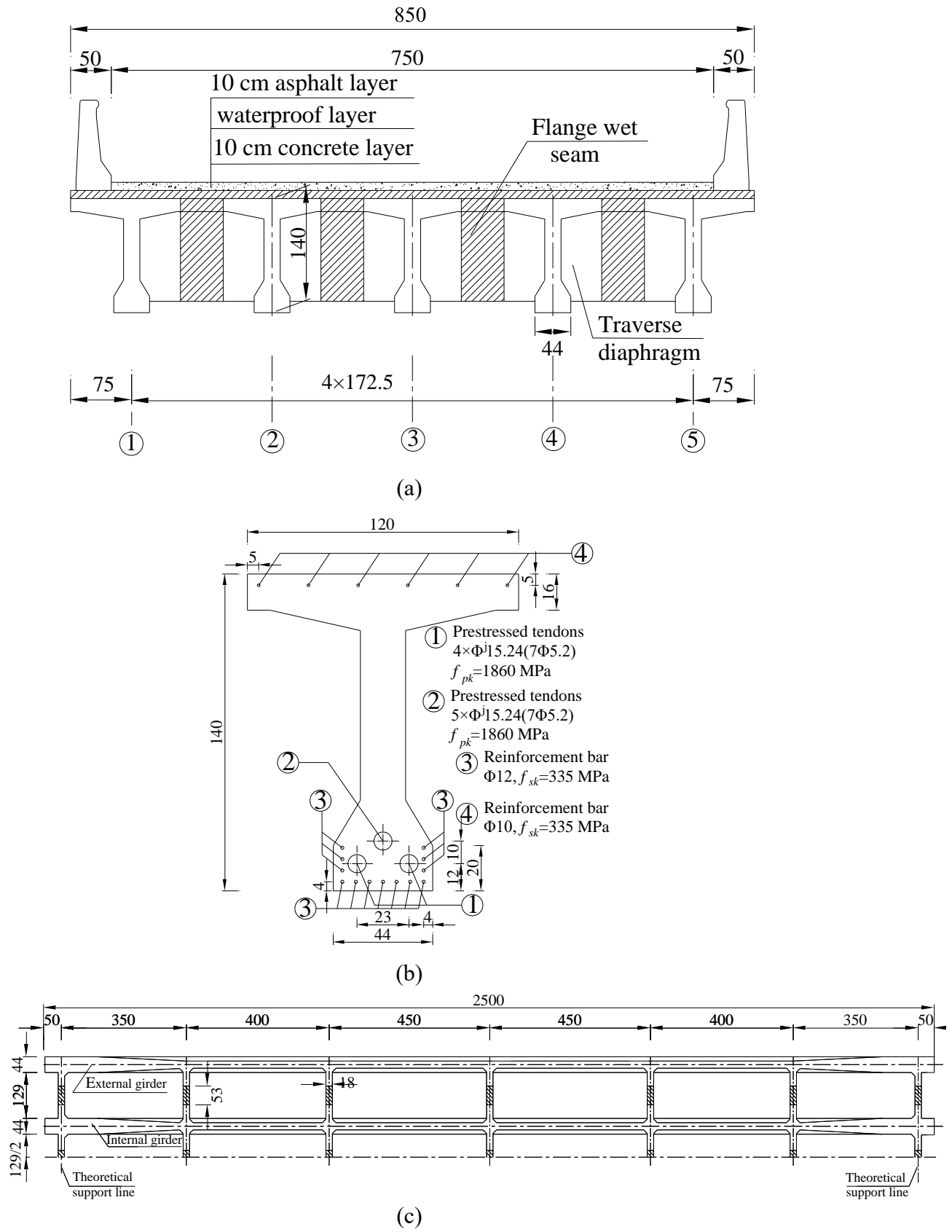
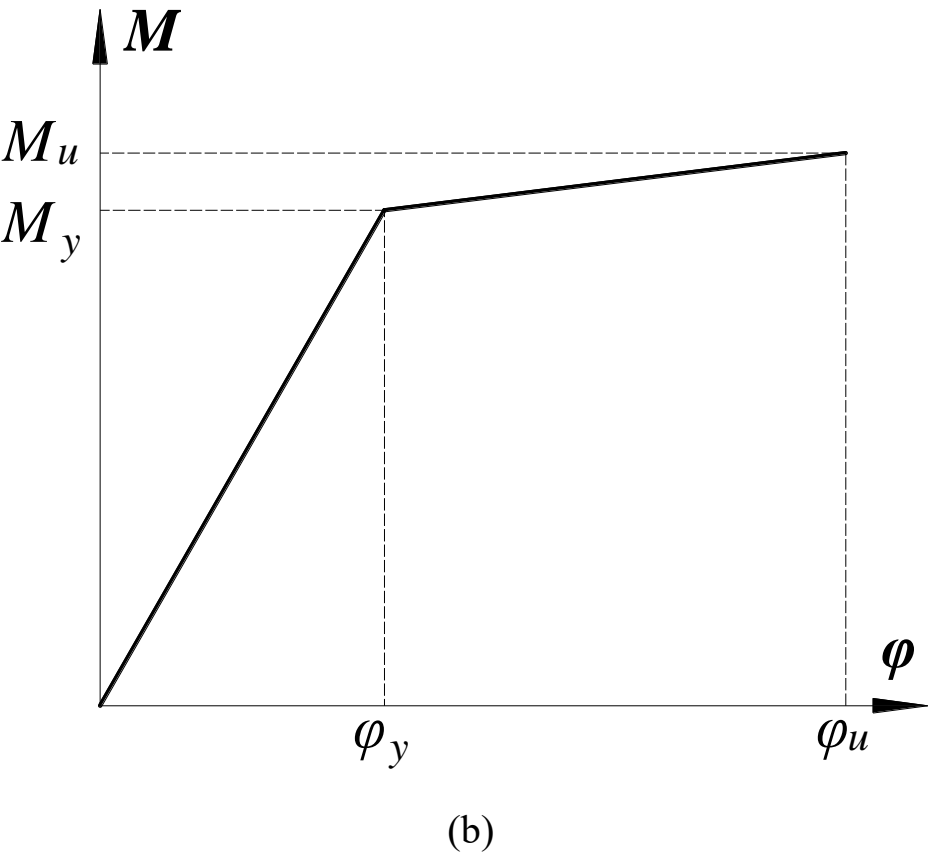
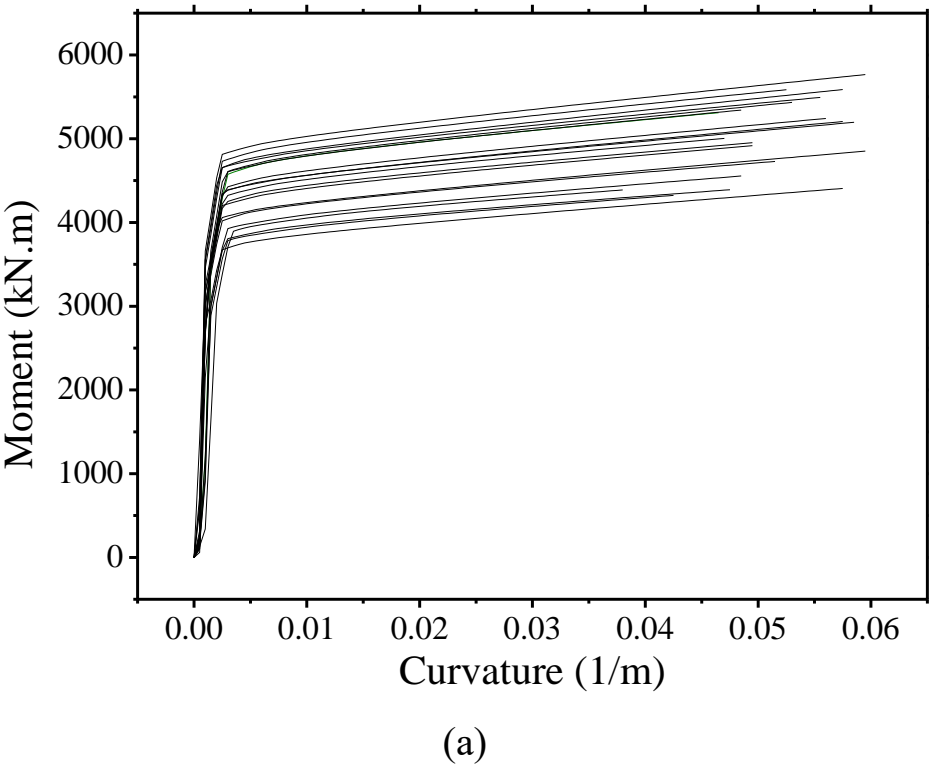
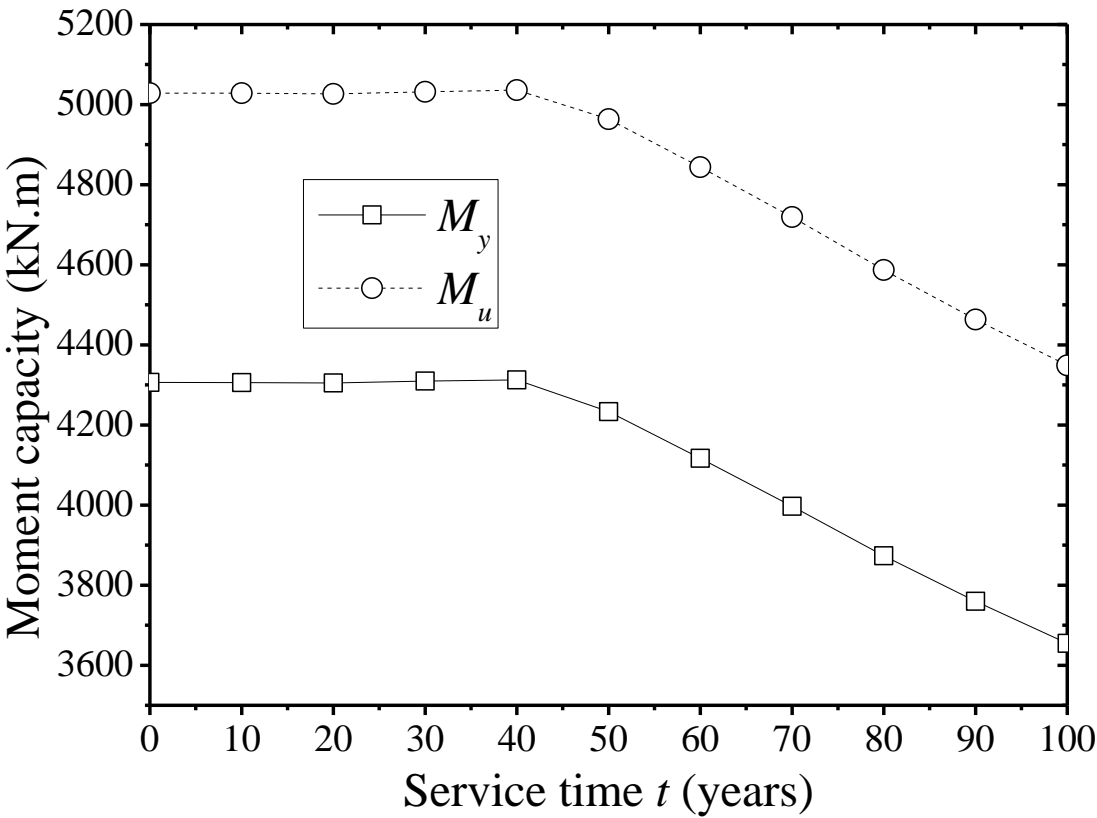


Figure 6

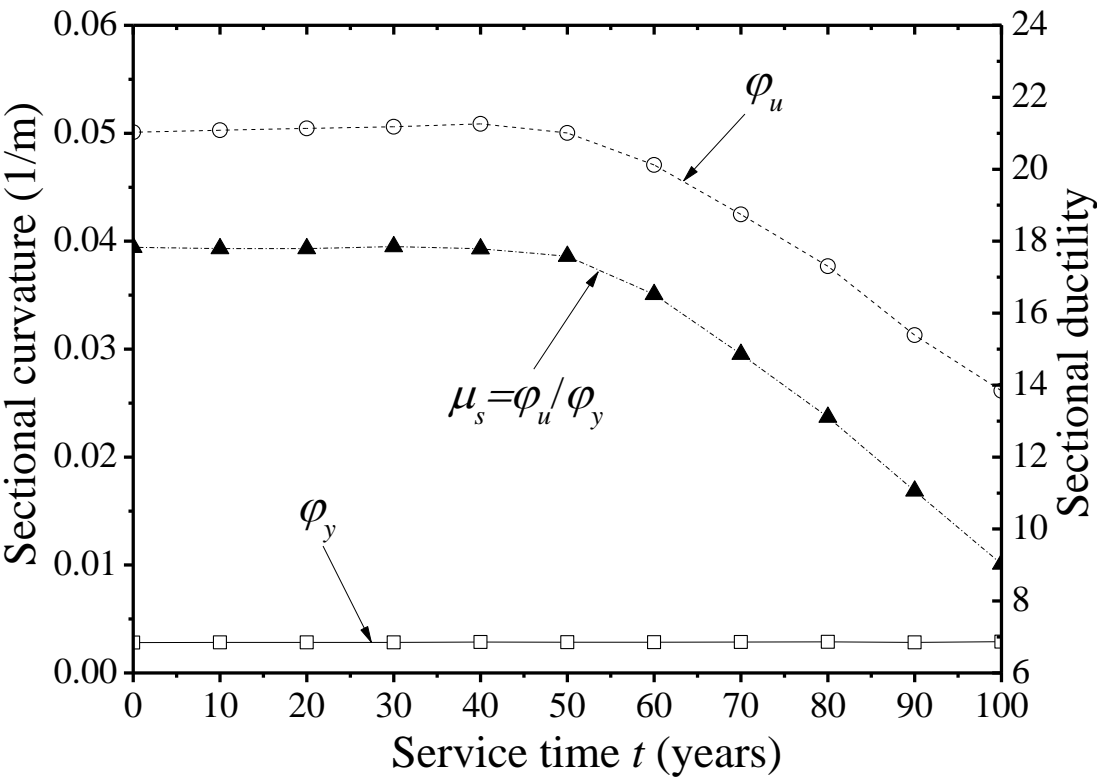




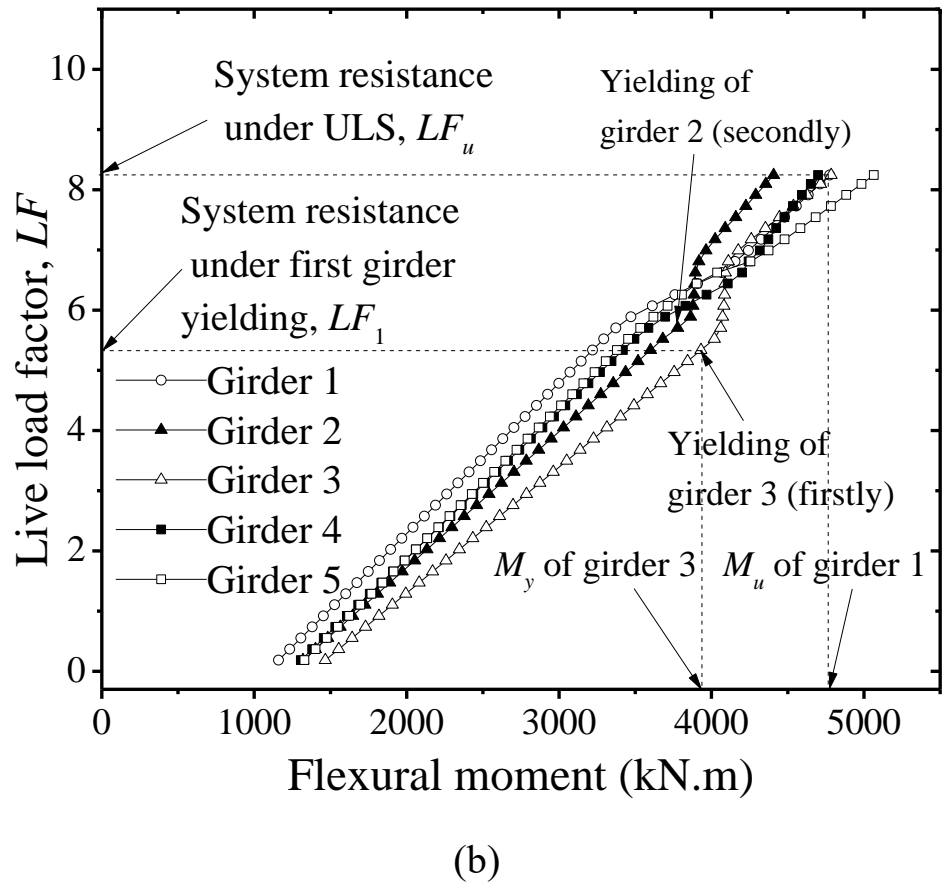
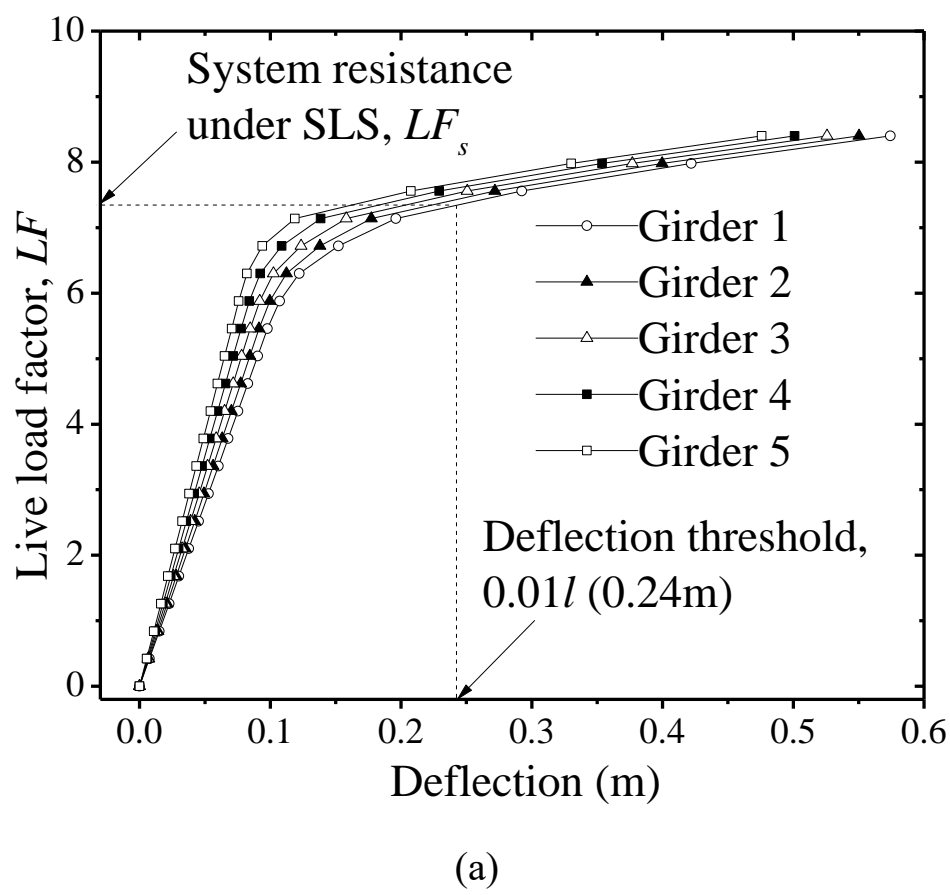


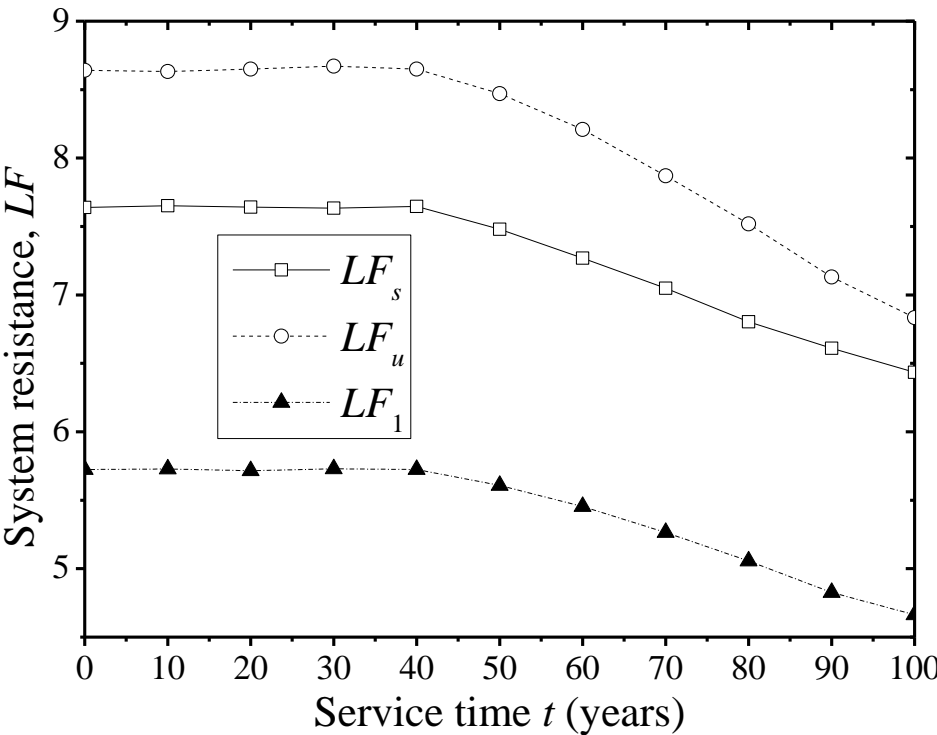


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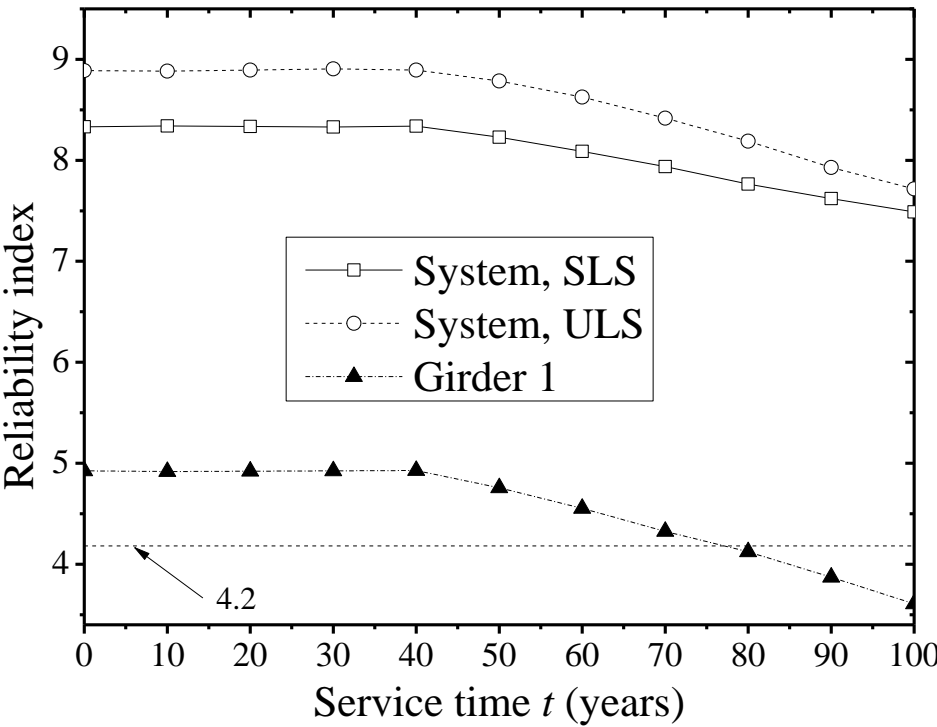


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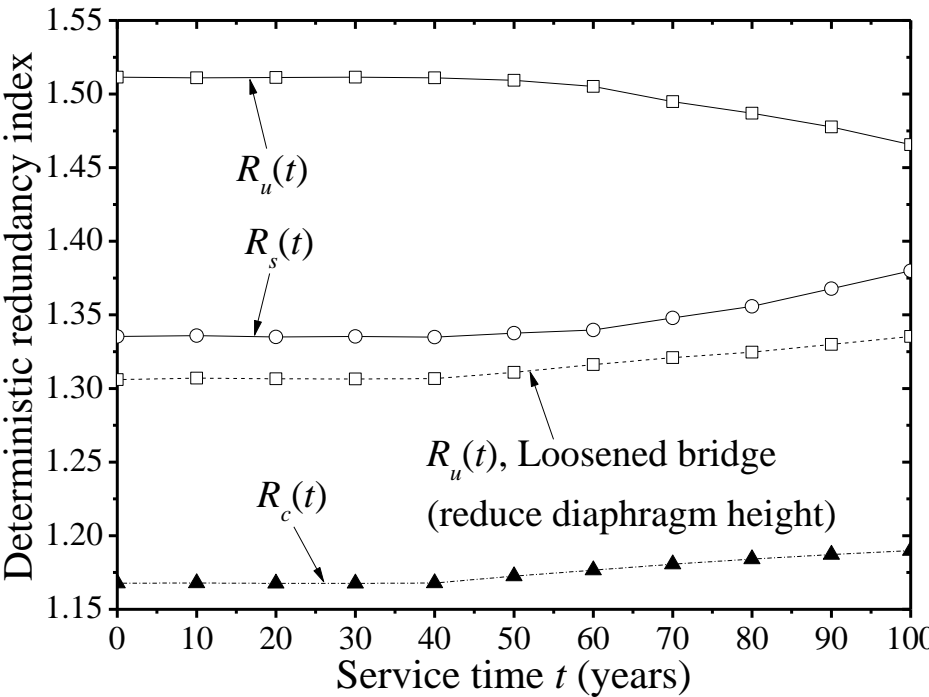




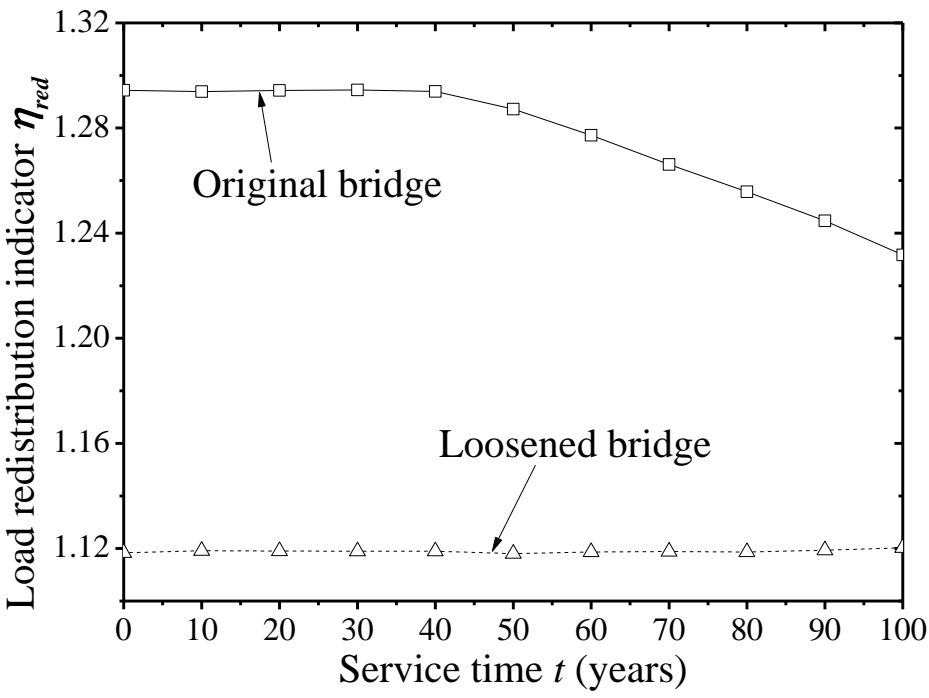
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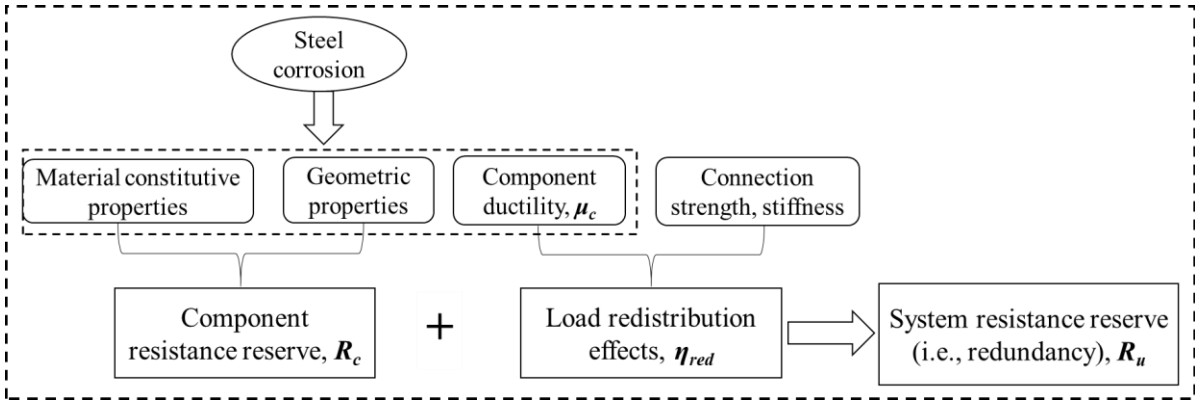
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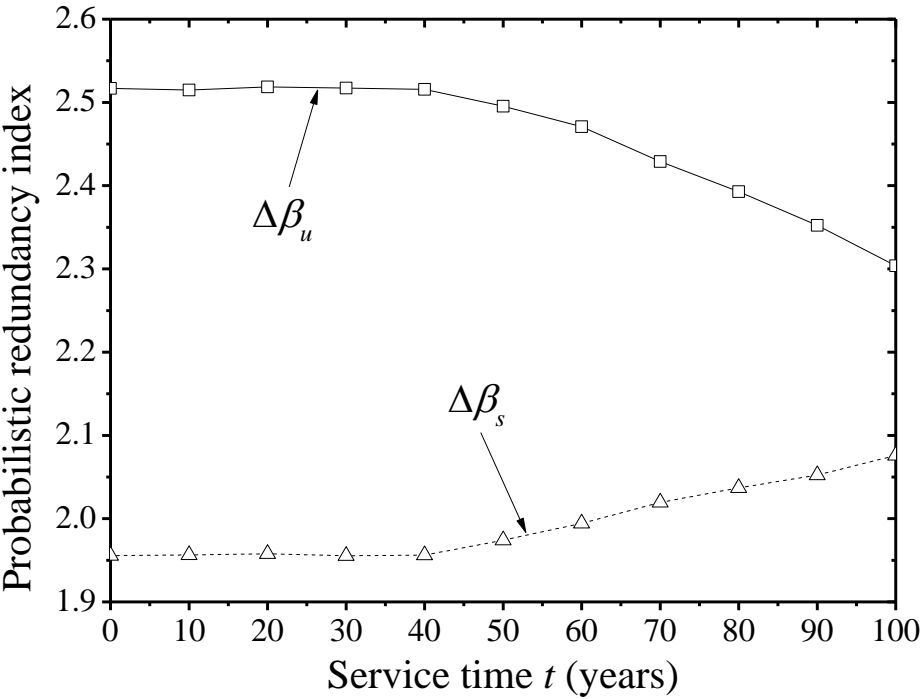
(a)



(b)



(a)



(b)

Figure Caption List

Fig. 1. Constitutive model of corroded prestressed tendons under tension

Fig. 2. General procedure for computation of system resistance

Fig. 3. Flowchart of the numerical incremental algorithm for sectional Moment-Curvature relationship

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