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Maximizing space-time accessibility in multi-modal transit networks: an activity-based approach

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Over the past decades, the impact of transport operating strategy improvements on space-time accessibility, which is an important research area for network design problem, has not been explicitly investigated particularly with the use of activity-based approach. In this paper, a novel activity-based space-time accessibility measure is introduced for considering individuals' accessibility to various activities and travels in a unified super-network framework. A bi-level programming model is proposed for optimizing time-dependent transit line headways and fares in a multi-modal transit network from the activity-based space-time accessibility perspective. In the upper level, transit line headways and fares are optimized by time of day to maximize the network-wide activity-based space-time accessibility. At the lower level, an activity-based network equilibrium model is adapted to provide the resultant activity-travel patterns as reactions to the upper level decision. A simplified network in Hong Kong selected area is used to illustrate the application of the proposed model.

Keywords: space-time accessibility; activity-based approach; daily activity-travel pattern; bi-level programming model

1. Introduction

Individuals conduct activities and travels under constraints of time and space. Space-time accessibility, which is a core concept in time geography literature, is one of the key concerns in individuals' daily activity-travel pattern (DATP) scheduling. It reflects the ease of conducting desired activities/travels and thus reflects characteristics of both the land use system and the multi-modal transportation system (Kwan 1998; Wu and Hine 2003; Miller 1999, 2005; Kwan and Weber 2008). A high level of space-time accessibility to activities is crucial to meet individuals' various needs in daily life.

In the literature, many studies were devoted to developing different measurements of space-time accessibility (Geurs and van Wee 2004; Dong et al. 2006; Chen et al. 2007; Miller 2007; Neutens et al. 2012; Kwan 2013). With the measurements of space-time accessibility, many applications can be conducted such as appraising access to job opportunities and various facilities, investigating social equity and segregation, and planning accessibility-oriented transport systems (Wang and Chai 2009; Nurlaela and Curtis 2012; Weber and Kwan 2013; Wang et al. 2018).

In recent years, the accessibility-oriented planning for transport systems is receiving much attention with the aim of increasing the network-wide space-time accessibility (Cervero 1997). Several accessibility-oriented transport network design models were recently developed (refer to Table 1). Antunes et al. (2002, 2003) and Tong et al. (2015) investigated road network design problem with the aim of maximizing space-time accessibility. With consideration of individuals' travel behaviour responses to network improvement, Gulhan et al. (2018) studied the effects of accessibility measures in decision-making process of bus network design, and Di et al. (2018) and Chen et al. (2019) proposed bi-level programming models for accessibility-based road network design. However, individuals' activity behaviour responses to network improvement were not considered in these studies because they conducted travel behaviour analysis using a trip-based approach. In the trip-based approach, the underlying motivation of trip making is ignored and the linkages between activities and travels cannot be reflected. For long-term transport planning, individuals' activity and travel choice behaviours should be simultaneously modelled and the crowding effects in vehicles and at activity locations should be considered. Hence a need is likely to exist for using an activity-based approach in accessibility-oriented network design instead of conventional trip-based approach.

Table 1. Comparison of accessibility-oriented network design models

Publication	Behaviour analysis approach	Network	Decision variables	Travel demand	Behavioural responses
Antunes et al. (2002, 2003)	Trip -based	Road network	Road link construction/ improvement	Not considered	Not considered
Tong et al. (2015)	Trip -based	Road network	Road link construction	Not considered	Not considered
Gulhan et al. (2018)	Trip -based	Bus Network	Bus routes	Considered	Travel choice behaviour
Di et al. (2018)	Trip -based	Road network	Road link construction	Considered	Travel choice behaviour
Chen et al. (2019)	Trip -based	Road network	Road link construction	Considered	Travel choice behaviour
This study	Activity -based	Multi-modal transit network	Time-dependent transit line headways and fares	Considered	Activity and travel choice behaviour with crowding effects

Unlike the trip-based approach, the activity-based approach covers another class of models which perceives the underlying motivation of trip making (Axhausen and Gärling, 1992; Yamamoto and Kitamura 1999; Roorda and Miller 2005; Arentze and Timmermans 2009; Ruiz and Roorda 2011; Chow and Recker 2012; Liao et al. 2013; Rasouli and Timmermans 2014). The activity-based approach enables an integrated investigation into the DATP scheduling mechanism with time and space constraints, i.e. what activities to be conducted, in what sequence, when and for how long, when each trip starts, which transport mode/route is to be used, and how the activities and travels interrelate in congested transport networks. Over the past decade, several activity-based network equilibrium or network design models were proposed (Ramadurai and Ukkusuri 2010; Kang et al. 2013; Chow and Djavadian 2015; Fu and Lam 2018; Li et al. 2018; Khoa et al. 2020; Liu et al. 2020), however, few studies considered space-time accessibility in the activity-based models. To realize the accessibility-oriented network design, there is a practical need to propose an activity-based space-time accessibility measure which can assess individuals' all-day accessibility to all activity locations in the network, incorporating both scheduling constraints and trip chaining. The activity-based space-time accessibility measure should combine various types of trips and activities into a unified framework, and reflect the time-dependent activity/travel (dis-)utilities with considering crowding effect in transport networks.

From Table 1, it can be found that most existing accessibility-oriented network design models are for road networks. In many Asian cities with densely populated development such as Hong Kong, most daily travel is conducted using various public transit modes (over 90 % in Hong Kong) rather than privately owned cars. To realize the accessibility-oriented planning of multi-modal transit networks, various transit operating strategies can be implemented to manage travel demand and improve the overall performance of transit system. Among all strategies, time-dependent transit line headways and fares have been proposed as effective ones across the world (Li et al. 2010; Lovric et al. 2016). Headways and fares of transit services are two major concerns for individuals in transit networks, which reflect the service level of transit system and jointly influence individuals' DATP choices. It is necessary to jointly optimize headways and fares of transit services, as transit fares should be able to cover the operating cost of the transit lines and the cost is directly related to headways of transit services. Well-designed time-dependent headway and fare strategies may contribute to the flexibility of individuals' DATPs, so that the DATP utilities can be improved and the space-time accessibility of activity locations can achieve a high level, particularly in transit-oriented cities in Asia. Therefore, some intriguing and important issues are raised: how to determine the optimal transit line headways and fares for accessibility-oriented transit network planning? What are the effects of time-dependent headway and fare strategies on the individuals' DATPs? In the literature, the impact of improvements of headway and fare strategies on space-time accessibility, which is an important research area for transit network design, has not been explicitly investigated particularly with the use of activity-based approach.

On the basis of the above, in this paper, a super-network platform proposed by Fu and Lam (2014, 2018) is adapted to simultaneously consider individuals' activity choices and travel choices so that we can measure individuals' accessibility to various activities and travels in a unified framework. A novel activity-based space-time accessibility measure is introduced to measure the all-day and network-wide accessibility, incorporating activity-travel scheduling constraints.

With the activity-based space-time accessibility measure, we propose a bi-level programming model for maximizing the activity-based space-time accessibility of activity locations. An activity-based network equilibrium model is used to model individuals' DATP choice behaviour at the lower level of the proposed bi-level model. The lower level model comprehensively investigates individuals' choice behaviour in multi-modal transit networks, including both activity choices (e.g. activity start time and duration, activity sequence, and activity location) and travel choices (e.g. departure time, route and mode). Crowding discomforts both in vehicles and at activity locations are taken into account for long-term transit planning. Considering the network equilibrium results as reactions to the upper level decision, in the upper level, time-dependent transit line headways and fares are optimized so as to maximize the all-day and network-wide activity-based space-time accessibility. The operating cost of transit services is considered and the net profit of transit operators is adopted as a constraint of the upper level model.

The structure of this paper is as follows. Assumptions, super-network representation and link (dis-)utilities are firstly given in Section 2. Section 3 presents a novel activity-based space-time accessibility measure. Section 4 describes the proposed bi-level programming model and the solution algorithm. A numerical example is provided in Section 5 for illustrating the application of the proposed model and solution algorithm. Conclusions are drawn in Section 6, together with suggestions for further research.

2. Assumptions and network representation

Model assumptions are given in the following section. A new super-network platform is introduced and utilities/dis-utilities of different links on the super-network platform are discussed in the subsequent sections. The notations used in this paper are listed in Table 2.

Table 2. Notations

K	equally spaced time intervals
k	time of day; $k=1,2,...,K,K+1$
M	conventional multi-modal transport network; $M=\left(I,V\right)$
I	the set of physical nodes; $I=\left\{i\right\}$
V	the set of physical links; $V=\left\{v\right\}$
B	the set of transport modes; $B=\{b\}$
M_b	sub-network of mode b; $M_b=(I_b,V_b)$, $I_b\subseteq I$, $V_b\subseteq V$
G	the ATS-SAM super-network; $G=(N,A)$
N	the set of nodes in ATS-SAM super-network
A	the set of links in ATS-SAM super-network; $A=A_{\mathrm{a}}\cup A_{\mathrm{t}}\cup A_{\mathrm{d}}$
A_{a}	the set of activity links; $A_{\mathrm{a}}=\{a_{\mathrm{a}}\}$
A_{t}	the set of transfer links; $A_{\mathrm{t}}=\{a_{\mathrm{t}}\}$
A_{d}	the set of direct in-vehicle links made up of physical links; $A_{\mathrm{d}}=\{a_{\mathrm{d}}\}$
a_{xn}	a direct in-vehicle link from ATS-SAM node $x\in N$ to ATS-SAM node $n\in N$
s	transfer state; $s\in S$
$\eta(s)$	associated transport mode of transfer state s; $\eta(s)\in B$
$\xi(s)$	the set of probable transfers from state s; $\xi(s)\subseteq S$
l	alight or aboard indicator;
u_{a}	utility of activity link a_{a}
$\bar{u}_{a_{\mathrm{a}}}(k)$	marginal utility of activity link a_{a}
$f_{a_{\mathrm{a}}}$	passenger flow on activity link a_{a}
$\kappa_{a_{\mathrm{a}}}$	capacity of the activity location
$\beta'_{a_{\mathrm{a}}}$, $\theta'_{a_{\mathrm{a}}}$	parameters by activity type in activity link utility function
$disu_v(k)$	dis-utility of physical link v with start time interval k
t_v^0	free-flow travel time of physical link v
κ_b	vehicle capacity of mode b

g_b^k	headway of mode b at time interval k
$\mathbf{g}$	vector of transit line headway; $\mathbf{g} = (..., g_b^k, ...)^T$
vot	value of time
β_b, θ_b	parameters by mode b in physical link utility function
$f_v(k)$	passenger flow on the physical link v at time interval k
$\delta(a_{\mathrm{d}}, v)$	incidence relationship between in-vehicle link a_{d} and physical link v
$disu_{a_{\mathrm{d}}}$	dis-utility of in-vehicle link a_{d}
$h_{a_{\mathrm{d}}}$	original transit fare with respect to the direct in-vehicle link a_{d}
$\rho_{a_{\mathrm{d}}}^k$	discount rate of transit fare with respect to the transit mode of in-vehicle link a_{d} and the time period k
$\boldsymbol{\rho}$	vector of transit fare's discount rate; $\boldsymbol{\rho} = (..., \rho_{a_{\mathrm{d}}}^k, ...)^T$
$disu_{a_{\mathrm{t}}}$	dis-utility of transfer link a_{t}
$\bar{c}(\mathbf{g})$	total operating cost of all transit modes
c_b^0	fixed cost per time unit of transit mode b
T_b	cycle journey time of a transit vehicle of mode b
ζ_b	operating cost per vehicle per time unit on transit lines of mode b
$\tilde{c}(\boldsymbol{\rho}, \mathbf{g})$	net profit of transit operators
P	route set in ATS-SAM super-network (i.e. DATP set); $P = \{p\}$
φ_p	the utility of DATP p
$\delta(p, a)$	incidence relationship between DATP p and link a
$SUSR(n(i,s,l,k))$	space-utility service region of node $n(i,s,l,k) \in N$
$u_{\min}$	a utility threshold in defining $SUSR(n(i,s,l,k))$
$t_{\min}$	a given travel time threshold
$\bar{U}_n(x, \rho_{a_{\mathrm{d}}}^k, g_b^k)$	utility of individuals at node *x* going to node *n*
χ	utility increasing parameter
N_x	ATS-SAM node choice set for serving individuals at ATS-SAM node x
$Acc(x(i,s,l,k), \rho_{a_{\mathrm{d}}}^k, g_b^k)$	accessibility of node x
I_{a}	the set of activity locations
Z	network-wide activity-based space-time accessibility of all activity locations
f_p	flow on DATP p
q	total population in the network
Ω	feasible set of DATP flows

τ	a given limit of the net profit of transit operator
$u_{a_a}^{\max}$	maximum accumulated utility of activity a_a in bell-shaped marginal utility function
α_{a_a}, β_{a_a}, γ_{a_a}	activity-specific parameters in bell-shaped marginal utility function

2.1. Model assumptions

In order to facilitate essential ideas without loss of generality, the following assumptions are adopted in this study.

A1: The DATP is considered in a fixed study horizon, divided into K equally spaced time intervals (Lam and Yin 2001; Zhang et al. 2005; Li et al. 2010; Tong et al. 2015).

A2: The proposed model falls within the static model category for long-term planning at the strategic level. Therefore, it is assumed that individuals have perfect knowledge of traffic conditions throughout the whole network (Di et al. 2018).

A3: The utility maximization approach is employed to formulate the individuals' DATP choices (Li et al. 2010). Individuals are assumed to make their decisions to maximize the DATP utility (Fu and Lam 2014, 2018; Fu et al. 2014).

A4: In this study, only one behaviourally homogeneous group is considered to facilitate the presentation of the essential ideas without loss of generality. Activity interdependency of household members is not considered (Lam and Yin 2001; Gulhan et al. 2018).

A5: Crowding discomforts in vehicles and at activity locations are modelled. All individuals can get on the buses or subways, i.e. vehicle capacity constraint is not considered (Lo et al. 2003; Fu and Lam 2018).

Three types of activities are investigated and described in this study: work, shopping, and home activities. Home and work are considered as compulsory activities, while shopping is non-compulsory activities. Activity choices (including activity sequence, activity location, activity start time and duration) and travel choices (including departure time, travel mode and route) are not fixed.

2.2. An ATS-SAM super-network

To simultaneously consider time and space coordination, and the relationship between activity and route/mode choices, super-network representations have been adapted in several studies to model the activity-travel scheduling (Liao et al. 2013; Liao 2016, 2019a, 2019b; Liu et al. 2015, 2016). A super-network platform named the ATS-SAM super-network developed by Fu and Lam (2014, 2018) is adapted in this study. The ATS-SAM super-network is an integration and expansion of the activity-time-space (ATS) network and the state-augmented multi-modal (SAM) transport network. The ATS network is an expanded network in which activity links are introduced into the conventional space-time network. As a result, both the individuals' activity and travel choices can be captured in the ATS super-network. The SAM network can be used to eliminate the unrealistic transfers and tackle the non-linear fare structures of transit systems, such as the system in Hong Kong. By constructing the ATS-SAM super-network, the merits of these two networks are achieved simultaneously as a consequence.

Consider a multi-modal transport network $M = (I, V)$, where $I = \{i\}$ and $V = \{v\}$ are, respectively, the set of physical nodes and the set of physical links. The multi-modal transport network M can be divided into several sub-networks $M_b = (I_b, V_b)$, $b \in B$, $I_b \subseteq I$, $V_b \subseteq V$, where $b \in B$ is a specified transport mode, and I_b and V_b, respectively, are the set of nodes and the set of links associated with the sub-network M_b. The sub-networks are combined and represented by a strongly connected graph $G = (N, A)$ through a state-augmentation approach (Bertsekas 1995), where N is a set of nodes and A is a set of links. The resultant network G is further developed by incorporating time-space coordinates and activity links. This augmentation produces the ATS-SAM super-network (refer to Figure 1 as an example). For more details of the ATS-SAM super-network, please refer to Fu and Lam (2014, 2018).

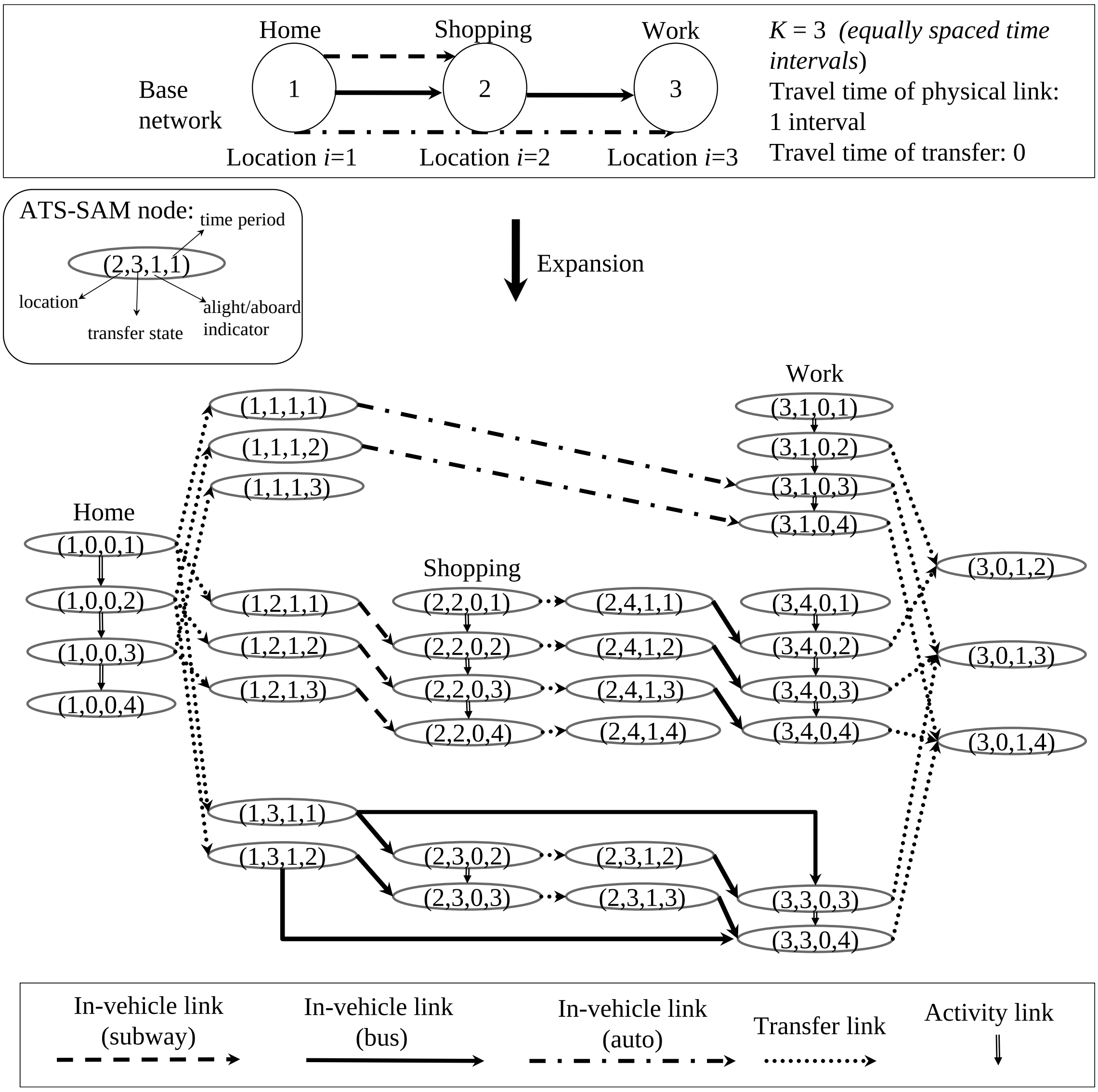


Figure 1. An illustrative example of the ATS-SAM super-network

The study horizon is divided into K equally spaced time intervals. Let $k = 1, 2, ..., K, K+1$ be the start time of a node or link. Each node in ATS-SAM super-network is described as (i,s,l,k), where i is the physical location of the node and s is the transfer state used to model probable transfers. l is the alight or aboard indicator. The value of l is equal to 1 (0) indicating that the individual is at the beginning (end) of an in-vehicle link. Each transfer state $s \in S$ associates with the use of a particular transport mode $\eta(s) \in B$ and a set of probable transfers $\xi(s) \subseteq S$. The probable transfer states in Figure 1 follow the ones proposed in Lo et al. (2003). As only transit modes are considered in this study, the merit of the ATS-SAM network regarding eliminating unrealistic transfers cannot be demonstrated. However, auto mode can be considered in further studies and the probable transfer state can play its role. Links in ATS-SAM super-network are classified into three categories, i.e. $A = A_{\mathrm{t}} \cup A_{\mathrm{d}} \cup A_{\mathrm{a}}$, where A_{t} is the set of transfer links, A_{d} is the set of direct in-vehicle links made up of physical links, and $A_{\mathrm{a}} = \{a_{\mathrm{a}}\}$ is the set of activity links. Each in-vehicle link $a_{xn} \in A_{\mathrm{d}}$ represents a direct in-vehicle movement from ATS-SAM node $x \in N$ to ATS-SAM node $n \in N$.

The activity time windows such as opening hours of facilities are not required as the activity utility by time of day is adopted in this study (Fu and Lam 2018). The process of route searching in the ATS-SAM super-network can lead to realistic and more generalized results regarding the times to perform activities during the study period.

2.3. Link utility/dis-utility in the ATS-SAM super-network

In the utility maximization context (see Assumption A3), the total utility obtained from a DATP is the summation of the utility gained from activities and the dis-utility resulting from travels. In the ATS-SAM super-network, activity links are associated with positive utility while in-vehicle/transfer links are associated with dis-utility of travel.

Previous studies have indicated that crowding discomfort has a significant impact on individuals' choice of transit service for long-term planning. Thus, the crowding effects both at activity locations and within transit vehicles are explicitly considered in the proposed activity-based model particularly for congested transit networks in Asia. The concept of activity utility is widely used in the activity-based approach (Adler and Ben-Akiva 1979; Kitamura 1984). Individuals gain activity utility as they can obtain satisfaction or meeting some of their needs. Considering the crowding discomfort at activity locations, the utility of activity link a_{a} (from start time k for one interval) is expressed as

$$u_{a_{\mathrm{a}}} = (1 + \beta'_{a_{\mathrm{a}}} (\frac{f_{a_{\mathrm{a}}}}{\kappa_{a_{\mathrm{a}}}})^{\theta'_{a_{\mathrm{a}}}}) \int_k^{k+1} \bar{u}_{a_{\mathrm{a}}}(\omega) d\omega. \qquad a_{\mathrm{a}} \in A_{\mathrm{a}} \qquad (1)$$

In this equation, $\int_k^{k+1} \bar{u}_{a_{\mathrm{a}}}(\omega) d\omega$ expresses the utility of performing activity link a_{a} from start time k to end time $k+1$. Suppose that $\bar{u}_{a_{\mathrm{a}}}(k)$ is the marginal utility of activity link a_{a}. In this study, a bell-shaped marginal utility function is adopted which will be introduced in the numerical example. In this study, the passenger flow on activity link (i.e. $f_{a_{\mathrm{a}}}$) is considered to describe the crowding discomfort at activity locations similar to the Bureau of Public Roads (BPR) function adopted for road traffic. Let $\kappa_{a_{\mathrm{a}}}$ be the capacity of the activity location. $\beta'_{a_{\mathrm{a}}}$ and $\theta'_{a_{\mathrm{a}}}$ are model parameters by activity type. β'_{a_a} is equal to 0 for compulsory activities (e.g. home and work) as the utility of compulsory activities is assumed not affected by the crowding at activity locations (Fu and Lam 2018). The reality of positive bandwagon effect (e.g. a restaurant where more people present may attract more people going there) is not considered in this study. Both negative (crowding) and positive (bandwagon) effects should be explicitly formulated in further studies. Different conditions can be given to indicate whether positive or negative effect is dominant on the activity link (Kang et al. 2013; Liu et al. 2020).

The dis-utility of physical link v with start time interval k (denoted as $disu_v(k)$) is expressed using a BPR type of function to quantify the increasing discomfort when the number of passengers increases (Nielsen 2000; Lo et al. 2003):

$$disu_v(k) = -\text{vot} \cdot t_v^0 \left(1 + \beta_b \left(\frac{f_v(k) \cdot g_b^k}{\kappa_b}\right)^{\theta_b}\right), \qquad v \in V_b \tag{2}$$

where t_v^0 is the free-flow travel time of physical link v; κ_b is the vehicle capacity of mode b; g_b^k denotes the headway of mode b at time interval k; vot is the value of time; β_b and θ_b are model parameters by mode b. Let $f_v(k)$ be the passenger flow on the physical link v at time interval k, which can be expressed as the summation of passenger flows on all in-vehicle links consisting of this physical link:

$$f_v(k) = \sum_{a_\text{d} \in A_\text{d}} f_{a_\text{d}} \cdot \delta(a_\text{d}, v), \tag{3}$$

where $\delta(a_\text{d}, v)$ is equal to 1 if physical link v is in direct in-vehicle link a_d; 0 otherwise.

The in-vehicle link dis-utility can be obtained by the summation of related physical link dis-utilities and the transit fare:

$$disu_{a_\text{d}} = \sum_{v \in V} disu_v(k) \cdot \delta(a_\text{d}, v) - \rho_{a_\text{d}}^k \cdot h_{a_\text{d}}, \tag{4}$$

where h_{a_d} denotes the original transit fare with respect to the direct in-vehicle link a_d with which the non-linear fares can be directly represented by node-to-node basis. $\rho_{a_\text{d}}^k$ is a discount rate of transit fare with respect to the transit mode of in-vehicle link a_d and the time period k. In this way, the time-dependent transit fare can be specified. Denote $\boldsymbol{\rho}$ as the vector of transit fare's discount rate, $\boldsymbol{\rho} = (..., \rho_{a_\text{d}}^k, ...)^T$, and $\mathbf{g}$ as the vector of transit line headway, $\mathbf{g} = (..., g_b^k, ...)^T$. In this study, $\boldsymbol{\rho}$ and $\mathbf{g}$ are the design variables in the upper level of the proposed bi-level programming model.

As regards transfer links by mode, the link dis-utility can be expressed as

$$disu_{a_\text{t}} = -\text{vot} \cdot \frac{g_b^k}{2}, \qquad a_\text{t} \in A_\text{t} \tag{5}$$

where g_b^k denotes the time-dependent headway of the transit line to which individuals transfer on the transfer link concerned.

Following Li et al. (2011), the total operating cost of all transit modes (denoted by $\bar{c}(\mathbf{g})$) can be expressed as fixed costs plus transit route operating costs:

$$\bar{c}(\mathbf{g}) = \sum_{b \in B} c_b^0 + \sum_{b \in B} \zeta_b T_b / g_b^k, \tag{6}$$

where c_b^0 denotes the fixed cost per time unit of transit mode b, and T_b is the cycle journey time of a transit vehicle of mode b, and ζ_b is the operating cost per vehicle per time unit on transit lines of mode b. In this study, the net profit of transit operators (denoted as $\tilde{c}(\boldsymbol{\rho},\mathbf{g})$) is defined as the difference between the total revenue that is generated from transit fares and the total operating cost (Li et al. 2011):

$$\tilde{c}(\boldsymbol{\rho},\mathbf{g}) = \sum_{a_{\mathrm{d}}\in A_{\mathrm{d}}} \sum_{k\in K} \rho_{a_{\mathrm{d}}}^{k} \cdot h_{a_{\mathrm{d}}} - \bar{c}(\mathbf{g}). \tag{7}$$

Let $P = \{p\}$ be the route set in the ATS-SAM super-network (i.e. DATP set). The daily utility gain, i.e. the utility of DATP $p \in P$ (denoted as φ_p), can be obtained by summing the dis-utilities of in-vehicle links, dis-utilities of transfer links and utilities of activity links:

$$\varphi_p = \sum_{a_{\mathrm{d}}\in A_{\mathrm{d}}} disu_{a_{\mathrm{d}}} \delta(p,a_{\mathrm{d}}) + \sum_{a_{\mathrm{t}}\in A_{\mathrm{t}}} disu_{a_{\mathrm{t}}} \delta(p,a_{\mathrm{t}}) + \sum_{a_{\mathrm{a}}\in A_{\mathrm{a}}} u_{a_{\mathrm{a}}} \delta(p,a_{\mathrm{a}}), \tag{8}$$

where $\delta(p,a)$ is the incidence relationship between DATP and link; $\delta(p,a)$ equals 1 if the link is used in the DATP, 0 otherwise.

3. Activity-based space-time accessibility measure

In multi-modal transit networks, individuals schedule their daily activity and travel choices on the basis of utility gain or loss. Both activity utility and travel dis-utility affect the accessibility of activity locations. The conventional trip-based accessibility measures normally do not consider potential travel demand and cannot demonstrate the impacts of activity (such as activity utility, activity location and activity time) on the accessibility. Thus, it is meaningful to propose a novel activity-based accessibility measure and solve the transit network optimization problem with the new measure. In this section, a new concept of service region is developed and a novel activity-based space-time accessibility measure is then proposed.

3.1. Space-utility service region

The service area of an activity location is generally defined by a geographical region, in which individuals can reach the location within a given distance or time threshold (e.g., a 30-min service area of the activity location). This service area concept has been widely used in previous studies (Kim and Kwan 2003; Wan et al. 2012; Chen et al. 2017; Wang et al. 2018). However, such service area concept cannot represent the utility gain or loss in individuals' activity and travel scheduling. In this study, a concept of space-utility service region (SUSR) is defined as below to take activity choice behaviour into account and consider flow-dependent utility.

Definition 1. Given a node $n(i,s,l,k) \in N$ in the ATS-SAM super-network and a utility threshold $u_{\min}$, the node *n*' SUSR can be expressed as

$$\begin{aligned} SUSR(n(i,s,l,k)) = \{x(i',s',l',k') | a_{xn} \in A_{\mathrm{d}}, u_{a_{xn}} \geq u_{\min}, i' \neq i, s' = s, l' \neq l, k' \leq k\} \\ \bigcup \{x(i',s',l',k') | a_{xn} \in A_{\mathrm{t}}, u_{a_{xn}} \geq u_{\min}, i' = i, s \in \xi(s'), l' \neq l, k' = k\} \\ \bigcup \{x(i',s',l',k') | a_{xn} \in A_{\mathrm{a}}\}. \end{aligned} \tag{9}$$

With this definition, some ATS-SAM nodes *x* are included in node *n*' SUSR, which implies that node *n*'s service region takes into account some relationships between *x* and

n, such as their locations (i), probable transfer (s), alight/aboard relationship (l) and time periods (k). There are three parts in the right side of Equation (9). The three parts include the nodes connecting n respectively with in-vehicle links, transfer links and activity links.

The ATS-SAM link a_{xn} is regarded as an accessible link for node n. The utility threshold $u_{\min}$ is determined by a given travel time threshold $t_{\min}$ and individuals' value of time:

$$u_{\min} = -t_{\min} \cdot \text{vot}. \tag{10}$$

$t_{\min}$ is following the work of many previous studies on accessibility measures. It is a given time threshold within which individuals can travel from another location to the activity location, so that we can define the service area of the activity location (Chen et al. 2017; Wang et al. 2018).

Figure 2 shows a simple example to illustrate the SUSR concept proposed in this study compared to space-time service region (STSR) used in previous studies (Miller 2005; Chen et al. 2017). The conventional space-time coordinates is extended to space-utility coordinates to represent individuals' utility gain or loss. Individuals gain utility through conducting activities and lose utility because of travels or transfers. In this example, three activity locations (i.e. i_1, i_2, i') are considered and the origin location is i'. Dividing the study time period into four intervals (i.e. $K = 4$), each location is augmented to nodes from $(...,...,...,k_0)$ to $(...,...,...,k_4)$ using the ATS-SAM expansion approach introduced in Section 2.2. Activity links, in-vehicle links and transfer links are built between the ATS-SAM nodes. The SUSR of each ATS-SAM node can be built based on Equation (9).

Figure 2(a) shows two examples of SUSR (i.e. SUSRs of ATS-SAM node $(i_1, s, 0, k_2)$ and ATS-SAM node $(i_2, s, 0, k_2)$). It can be found that the node $(i', s, 1, k_1)$ is in the SUSR of $(i_1, s, 0, k_2)$ but not in the SUSR of $(i_2, s, 0, k_2)$. However, if the STSR is adopted (refer to Chen et al. (2017)), it can be seen from Figure 2(b) that $(i', s, 1, k_1)$ is accessible to both location i_1 and location i_2. Thus, it can be seen that with the use of the new SUSR concept, the accessibility of location i' is reduced compared to conventional methods. This is due to that a utility threshold $u_{\min}$ is adopted instead of travel time threshold $t_{\min}$ in the proposed SUSR concept to consider the flow-dependent link (dis-)utility.

3.2. Activity-based space-time accessibility

To combine travels, transfers and activities into a unified measure of accessibility, following the space-time utility accessibility approach (Miller 1999), the utility of individuals at node x going to node n in ATS-SAM super-network can be modified using exponential function as

$$\overline{U}_n(x, \rho_{a_\text{d}}^k, g_b^k) = \exp(\chi \cdot u_{a_{xn}}(\rho_{a_\text{d}}^k, g_b^k)), \quad a_{xn} \in A_\text{a} \cup A_\text{d} \cup A_\text{t} \tag{11}$$

where χ is the utility increasing parameter. $u_{a_{xn}}(\rho_{a_\text{d}}^k, g_b^k)$ refers to activity link utility u_{a_a} if $a_{xn} \in A_\text{a}$ (see Equation (1)), or in-vehicle link dis-utility $disu_{a_\text{d}}$ if $a_{xn} \in A_\text{d}$ (see Equations (2)-(4)), or transfer link dis-utility $disu_{a_\text{t}}$ if $a_{xn} \in A_\text{t}$ (see Equation (5)). From Equations (1)-(5), it can be seen that passenger flows are considered in the proposed activity-based space-time accessibility measure.

Definition 2. In this study, an activity-based space-time accessibility measure is proposed to evaluate accessibility to all activity locations in the study area. A node x can be covered by the SUSRs of several nodes in ATS-SAM super-network (refer to **Definition 1**). These nodes constitute the ATS-SAM node choice set for serving individuals at ATS-SAM node x (denoted by N_x) as

$$N_x = \{x \in SUSR(n), \forall n \in N\} \quad (12)$$

The accessibility of node x is measured by the sum of the utility from node $x(i,s,l,k)$ to all nodes in N_x:

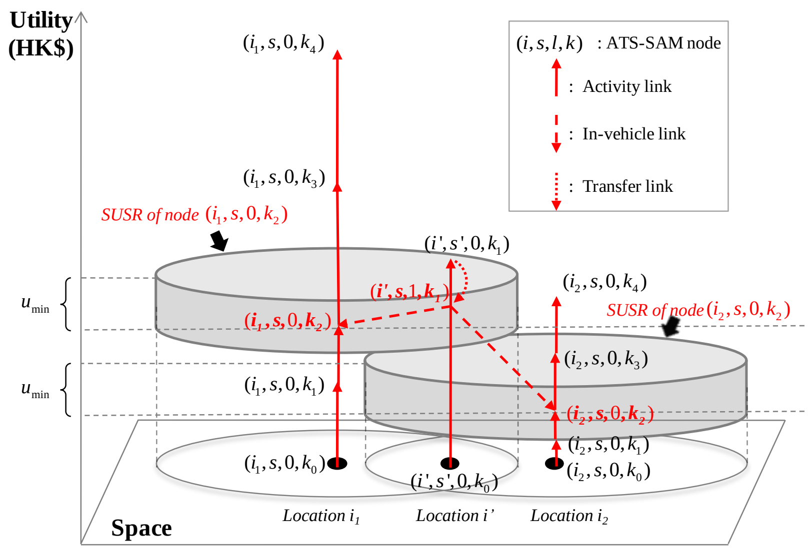
$$Acc(x(i,s,l,k), \rho_{a_d}^k, g_b^k) = \sum_{\forall n \in N_x} \overline{U}_n(x, \rho_{a_d}^k, g_b^k). \quad (13)$$

Definition 3. Let I_a be the set of activity locations (including Home (H), Work (W), Shopping (S), etc.) in the study area. The network-wide activity-based space-time accessibility of all activity locations for time period k' to k'' (denoted as Z) can be expressed as

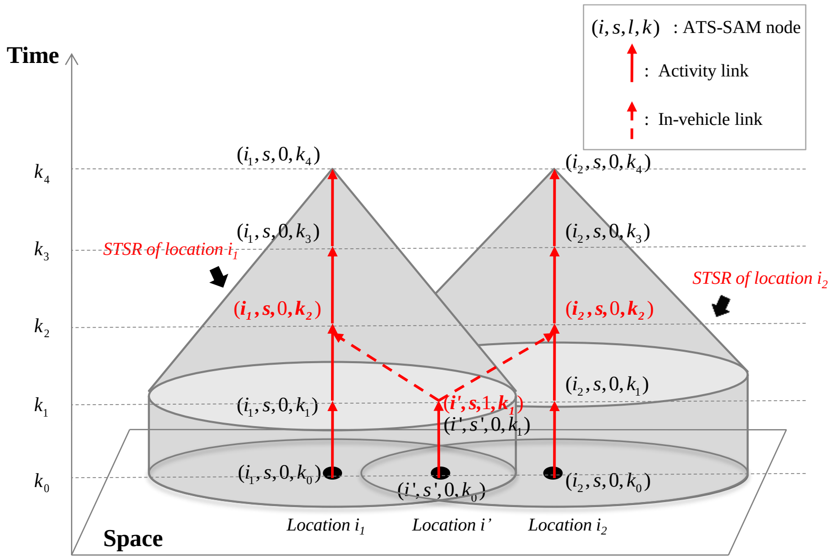
$$Z = \sum_{i \in I_a} \sum_{k' \le k \le k''} Acc(x(i,s,l,k), \rho_{a_d}^k, g_b^k), \quad (14)$$

where $Acc(x(i,s,l,k), \rho_{a_d}^k, g_b^k)$ denotes the accessibility of ATS-SAM node $x(i,s,l,k)$, which can be obtained on the basis of **Definition 2**.

The proposed activity-based space-time accessibility measure is successful in (a) considering the crowding effect (i.e. flow-dependent link (dis-)utility) in multi-modal transit networks, (b) combining various types of trips and activities into a unified measure of accessibility (not possible with trip-based approach), (c) modelling the impact of all feasible activity-travel patterns on accessibility by using the ATS-SAM super-network (not possible with trip-based approach).



(a) Example of SUSR of ATS-SAM node (with utility threshold)



(b) Example of STSR of activity location (with travel time threshold)

Figure 2. Comparison of SUSR and STSR

4. Model formulation and solution algorithm

In this section, the transit network optimization problem considered in this study is modelled by a bi-level framework. The formations of lower level and upper level are elaborated.

4.1. Activity-based network equilibrium

With the use of the ATS-SAM super-network, individuals' activity choices (i.e. activity locations, sequence and durations) and travel choices (i.e. route and mode choices, transfers) are explicitly represented by different links in the proposed super-network platform. Activities with different start times are constructed as different activity links. In this way, the time-dependent activity utility, in this study, can be modelled in terms of static values. The relationships between activity and travel choices are reflected by the ATS-SAM super-network topology. Each route from origin to destination in the super-network represents a feasible DATP. Therefore, by using the ATS-SAM super-network platform, the time-dependent DATP scheduling problem can be formulated as a static activity-based network equilibrium model (Fu and Lam 2014, 2018).

The proposed activity-based network equilibrium model falls into the category of static model in nature for long-term planning at the strategic level, thus it can be postulated that all individuals would have a user equilibrium (UE) activity-travel choice pattern: for each day, the utilities of all used DATPs are largest and equal, and all unused DATPs have smaller utilities. Denote π as the optimal route (i.e. the optimal DATP) with the largest utility in the ATS-SAM super-network. The UE condition can be expressed as

$$f_p(\varphi_\pi - \varphi_p) = 0, \tag{15}$$

$$q = \sum_{p \in P} f_p, \tag{16}$$

$$\varphi_\pi - \varphi_p \geq 0, \tag{17}$$

$$f_p \geq 0, \tag{18}$$

where f_p denotes the flow on DATP p and q denotes the total population in the study network.

The above UE condition can also be formulated as a variational inequality (VI) problem:

Find $f_p^* \in \Omega$ such that

$$\sum_{p \in P} \varphi_p^* (f_p^* - f_p) \geq 0, \qquad \forall f_p \in \Omega \tag{19}$$

where Ω denotes the feasible set of DATP flows defined by (17) and (19).

Theorem 1. The solution of the VI problem (19) is equivalent to the UE condition (15)-(18).

Proof For the proof, readers are referred to Smith (1979) on the route VI formulation for the static traffic assignment problem.

Theorem 2. At least one solution of the VI problem (19) exists.

Proof According to Facchinei and Pang (2003), the proof can be completed by the following two properties: (a) The DATP utility is continuous; (b) the feasible set Ω is compact and convex.

4.2. A bi-level programming model

A bi-level programming model is formulated in this study for maximizing network-wide activity-based space-time accessibility. The bi-level model framework can reflect different choice behaviour of network planners and individuals. The network planners produce transit operating strategies with the objective of optimizing network performance such as accessibility, while individuals schedule their activities and travels aiming at obtain maximum utility.

The structure of the proposed bi-level programming model is demonstrated by Figure 3. In the proposed bi-level model, the upper level aims to measure the space-time accessibility of all activity locations under equilibrium DATPs, and generate optimal time-dependent transit line headways and fares. The DATPs resulting from the lower level reflect the quality of upper level decisions. In this way, the upper level and the lower level form a feedback loop for optimization of transit networks. Both the upper and lower levels are in the framework of ATS-SAM super-network to simultaneously model activity choices and travel choices. The detailed formation of the bi-level model is given as below.

With the aim of maximizing the network-wide activity-based space-time accessibility (refer to **Definition 3**), the upper level model can be formulated as

$$\underset{\boldsymbol{\rho},\mathbf{g}}{Max}\, Z = \sum_{i \in I_\mathrm{a}} \sum_{k' \leq k \leq k''} Acc(x(i,s,l,k), \rho_{a_\mathrm{d}}^k, g_b^k), \tag{20}$$

subject to

$$\tilde{c}(\boldsymbol{\rho}, \mathbf{g}) \geq \tau, \tag{21}$$

$$\rho_{\min} \leq \rho_{a_\mathrm{d}}^k \leq \rho_{\max}, \tag{22}$$

$$g_{\min} \leq g_b^k \leq g_{\max}. \tag{23}$$

In the constraint (21), τ is a given limit of the net profit of transit operator. Constraint (22) is the feasibility constraint which defines the minimum and maximum discount rates of transit fares. Constraint (23) is the feasibility constraint which defines the minimum and maximum headways of transit lines. It is difficult to find a global optimum for the bi-level mathematical programming problem with constraints (21)-(23) and the VI equilibrium constraint (19). A heuristic algorithm is presented to solve the proposed model in the next section.

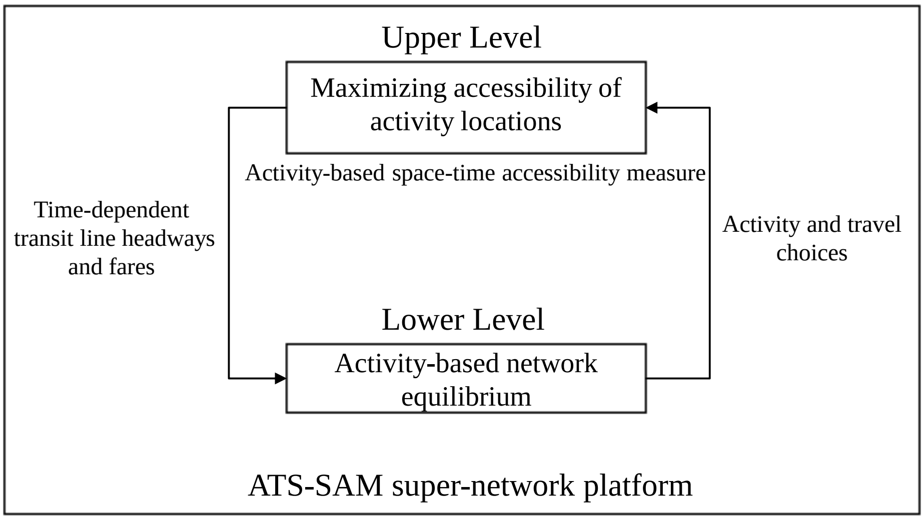


Figure 3. Structure of the proposed bi-level model

4.3. Solution algorithm

Considering the computational efficiency in practice, heuristic algorithms are more suitable than exact algorithms in this study. In the literature, many heuristic solution algorithms have been developed for solving bi-level programming models (Luo et al. 1996). In this study, the artificial bee colony (ABC) algorithm (Szeto et al. 2011; Szeto and Jiang 2012, 2014; Chen et al. 2015; Huang et al. 2016) is adapted for solving the proposed model. The ABC algorithm is inspired by the intelligent food finding behaviour of honey bees, which belongs to a class of evolutionary algorithms. The ABC algorithm outperforms some other heuristic algorithms (such as the generic algorithm) by its inherent local search mechanism so that the solution quality may be enhanced. The solution algorithm for solving the bi-level programming model is depicted with Figure 4 and outlined as follows.

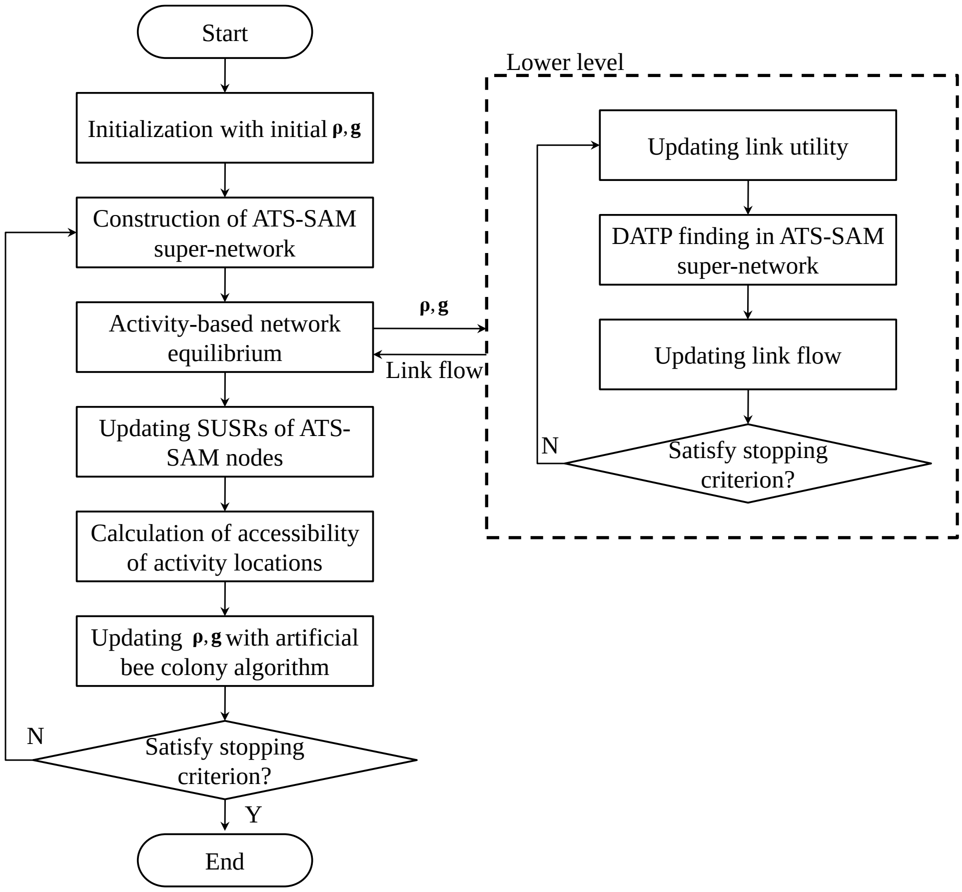


Figure 4. Solution algorithm

Step 1. Choose initial $\mathbf{\rho}$ and $\mathbf{g}$. Transform the traditional multi-modal transit network into the ATS-SAM super-network.

Step 2. Initialization of input parameters. Set the colony size, the numbers of employed bees, onlooker bees and the limit counter. Set the iteration counter to 0, and set the maximum number of iteration.

Step 3. Solve the activity-based network equilibrium problem. Update the SUSRs of ATS-SAM nodes by Equation (9), then calculate the accessibility objective function Z as Equation (14).

Step 4. Update $\mathbf{\rho}$ and $\mathbf{g}$ with ABC algorithm. The ABC algorithm includes employed bee phase, onlooker bee phase and scout bee phase. The details of ABC algorithm can be found in Szeto et al. (2011).

Step 5. Increase the iteration counter by one. If the number exceeds the maximum number of iteration, stop; otherwise, go to *Step 3*.

In the above *Step 3*, the activity-based network equilibrium problem can be solved by the following steps (i.e. *Steps 3.1*-*3.5*). In the *Steps 3.1*-*3.5*, the route set (i.e. DATP set) is generated by the column generation technique. This avoids the burden of enumerating a pre-defined set of DATPs. A non-empty DATP set is created by path finding algorithm. The network equilibrium problem is then solved on the DATP set, upon which the new optimal DATP will be found and evaluated if being added to the DATP set. This process is repeated until no more new DATP can be found (Leventhal et al. 1973; Chen et al. 2001; Chen et al. 2011; Lu et al. 2016; Wang et al. 2019).

Step 3.1. Initialization. Let $n = 0$. Call the shortest path faster algorithm (SPFA) (Duan 1994) to find the optimal DATP $\pi \in P$ in the ATS-SAM super-network (i.e.

DATP) with the largest utility. Assign all individuals on π.

Step 3.2. Obtain route and link flows $\mathbf{f}^n$. Update link (dis-)utilities.

Step 3.3. Column generation. Call the SPFA algorithm to find the optimal DATP $\pi \in P$. If φ_π is larger than any φ_p in P, add π into P.

Step 3.4. Flow update. Perform an all-or-nothing assignment based on DATP utilities, and yield auxiliary DATP flows. Obtain updated DATP flows using a method of successive average process. Update link flows and link (dis-)utilities.

Step 3.5. Convergence test. For an acceptable convergence level υ, if $\frac{\sum_{p\in P} f_p(\varphi_\pi - \varphi_p)}{\sum_{p\in P} f_p \varphi_p} \le \upsilon$, stop. Otherwise let $n = n+1$ and go back to *Step 3.3*.

5. Numerical example

To give a better presentation of the essential ideas and show the contributions of this study, the proposed model and solution algorithm are applied to a simplified multi-modal transit network in Hong Kong selected area. Optimal transit line headways and discount rates of transit fares for different time periods can be generated by the proposed model. For a better illustration of commuters' DATP choice behaviour, we compare transit line headways and fares in peak periods (i.e. 7 a.m. - 9 a.m. and 5 p.m. - 7 p.m.) and off-peak periods as examples. The space-time accessibility is examined by comparing activity-based approach and trip-based approach. Sensitivity analyses of the parameter in accessibility measure and the level of potential population are also carried out.

Hong Kong's Tuen Mun-Yuen Long-Kowloon network (extended from the one used by Shao et al. (2018)) is adopted in this example, shown in Figure 5. Details of the transit network are given by Table 3. The study time period is from 6:00 a.m. to 24:00 p.m. Three types of activity are considered, i.e. Home (H), Work (W) and Shopping (S). A new shopping mall is expected to be located at S1 or S2 in the study network, considered as Scenario 1 and Scenario 2 respectively in this numerical example, so that different land use plans can be explored and compared.

For each activity, a marginal utility expresses the utility gained from a time unit of participation. Different functions can be used to describe the marginal utility. It is believed that individuals have different preferred times for various activity participations. Activity participation usually starts with a warming up phase in which the marginal utility increases. After reaching a maximum point, the marginal utility decreases. In this study, the bell-shaped marginal utility function proposed by Ettema and Timmermans (2003) is adopted:

$$\bar{u}_{a_a}(k) = \frac{\gamma_{a_a}\beta_{a_a}u_{a_a}^{\max}}{\exp[\beta_{a_a}(k-\alpha_{a_a})]\left\{1+\exp[-\beta_{a_a}(k-\alpha_{a_a})]\right\}^{\gamma_{a_a}+1}}, \tag{24}$$

where k is the time of day; $u_{a_a}^{\max}$ is the maximum accumulated utility of activity a_a, and α_{a_a}, β_{a_a}, γ_{a_a} are the activity-specific parameters which can be estimated on the basis of survey data. This function captures not only activity characteristics but also activity participation time. Many related studies have adopted this type of function for modelling the marginal utility of activity (Zhang et al. 2005; Li et al. 2010). Table 4 shows the given parameters in the marginal utility function for this numerical example.

The value of time was HK$ 60.00/hour. In the ABC algorithm, the colony size was 8. The numbers of employed bees, onlooker bees and the limit counter were respectively 4, 4 and 5, and the maximum number of iteration was 20. Other parameters were set as $q = 2000$, $\beta'_{a_a} = 0.1$, $\beta_b = 0.1$, $\theta'_{a_a} = 2$, $\theta_b = 2$, $\chi = 0.1$, $\rho_{\min} = 0.5$, $\rho_{\max} = 1.5$, $g_{\min} = 3$ min , $g_{\max} = 16$ min , $\kappa_{a_a} = 1500$, $\kappa_{subway} = 1000$, $\kappa_{bus} = 150$, $c^0_{bus} = 500$ HK$/h , $c^0_{subway} = 1500$ HK$/h , $u_{\min} = $ -30 HK$, ζ_{bus} =10 HK$/h-veh , ζ_{subway} =50 HK$/h-veh .

Table 3. Basic input data for the example transit network

	Bus line 1		Bus line 2		Subway line	
Link	1-2-3	3-2-4	1-3	3-4	1-5	5-4
Travel time	15 min	35 min	25 min	50 min	15 min	35 min
Fare	HK$ 12.8	HK$ 12.8	HK$ 12.8	HK$ 9.6	HK$ 7.0	HK$ 21.0
Headway	10 min	10 min	10 min	10 min	4 min	4 min

Table 4. Given parameters in the marginal utility function

	Work (6:00–12:00)	Work (12:00–24:00)	Home (6:00–12:00)	Home (12:00–24:00)	Shopping
$u^{\max}_{a_a}$ (HK$)	1800	1700	1600	2500	800
α_{a_a}	660	900	360	1320	1140
β_{a_a}	0.021	0.021	0.0048	0.0048	0.018
γ_{a_a}	0.8	0.8	1.8	1.8	1

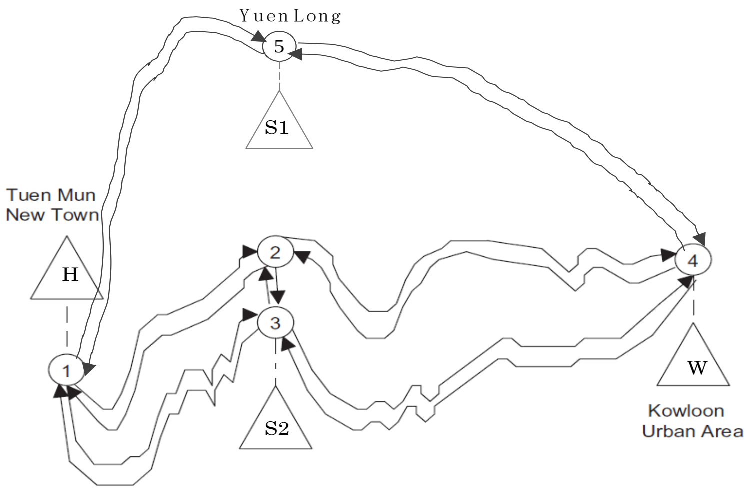
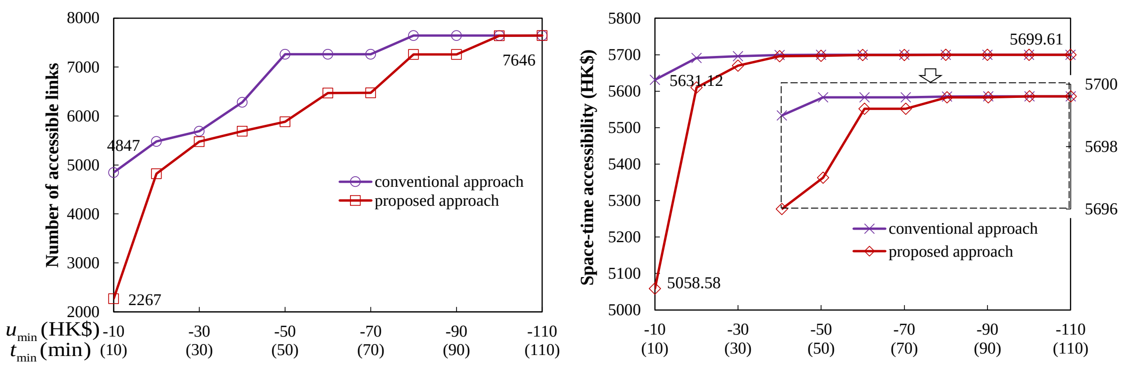


Figure 5. Hong Kong's Tuen Mun-Yuen Long-Kowloon network

With **Definition 1**, the SUSR of each node in the ATS-SAM super-network can be obtained. On the basis of SUSRs, accessible links of each node in the super-network can be found. The accessible links in ATS-SAM super-network represent individuals' feasible activity/travel choices with consideration of probable transfers, space-time constraint and utility constraint. Figure 6 shows the number of accessible links and the corresponding space-time accessibility using the proposed approach in this study (i.e. with different utility thresholds) compared to those using conventional approach (i.e. with different travel time thresholds).

From Figure 6(a), it can be found that with the increase of travel time threshold (or with the decrease of utility threshold), the number of accessible links raises. This is because a large travel time (or dis-utility) threshold ensures a large number of nodes in SUSRs. It can also be found that the number of accessible links with conventional travel time threshold is larger than that with our adopted utility threshold. Conventional accessibility measures use single travel times as input, and time-dependent activity utilities and travel/transfer dis-utilities considering passenger flows are not taken into account. By using the generalized utility threshold instead of travel time threshold, we can explicitly consider individuals' activity/travel choice behaviour by considering their utility gain/loss, and exclude some overcrowded links. In this way, the space-time nodes individuals can reach are reduced, and more realistic accessibility can be estimated.

It can be seen from Figure 6(a) that the number of accessible links with 10 minutes travel time threshold is 4847 and the number of accessible links with HK$ -10 is only 2267. When the threshold is as large as 110 min (HK$ -110 correspondingly), with either type of threshold all links in the super-network will be accessible (7646 links in total). Figure 6(b) shows the resultant space-time accessibilities with the two types of threshold. The accessibilities increase from HK$ 5058.58 and HK$ 5631.12 to HK$ 5699.61 respectively. Thus, it can be concluded that the space-time accessibility of activity locations would be overestimated if we do not consider the flow-based link (dis-)utility.



(a) (b)

Figure 6. Accessibility characteristics using the proposed approach and conventional approach: (a) number of accessible links in ATS-SAM super-network, (b) space-time accessibility

The activity-based space-time accessibility values associated with two scenarios (i.e. shopping mall located at S1 and shopping mall located at S2) are provided in Figure 7, which shows the convergence characteristics of the proposed solution algorithm. It can be seen that for both two scenarios the optimization has been achieved after 28 iterations. This result indicates that the proposed algorithm can solve the bi-level programming problem for this typical network. To further illustrate the proposed model, the optimal solutions will be presented in the following discussions.

The activity-travel distributions associated with the different scenarios and a sensitivity analysis of population level will also be given.

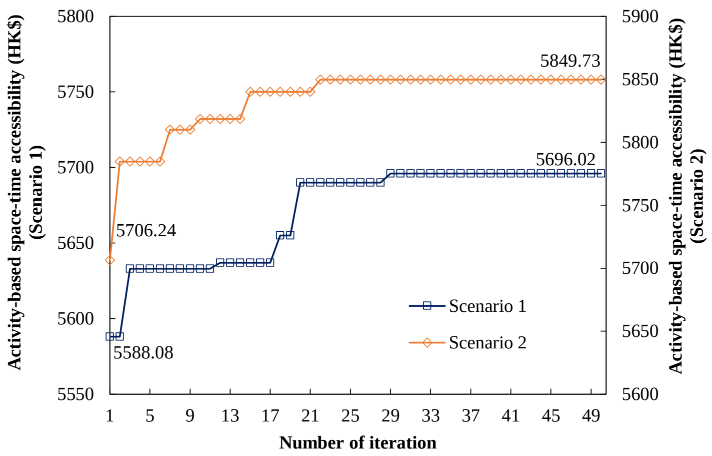


Figure 7. Convergence characteristics of the solution algorithm

Table 5 shows the optimal solutions of time-dependent transit line headways and fares with the proposed model. The accessibility of all activity locations are investigated with the proposed activity-based approach compared to conventional trip-based approach. Individuals' time spent on travels and different activities are also presented. By comparing the model results, it is found that the network-wide accessibility measured by activity-based approach is substantially larger than the one measured by trip-based approach. This is because the activity-based space-time accessibility measure can reflect individuals' utility gain from activity participation. When the new shopping mall is located at S2 (i.e. Scenario 2) and the study area is with a relatively low population level, the activity-based accessibility is the largest (HK$ 5849.73). However, when the population level is high (e.g. the population tripled), it can be found that the accessibility of activity locations decreases for all scenarios (e.g. decreased to HK$ 5635.57 for Scenario 2) because of the increased travel dis-utility and decreased activity utility in the crowded network.

Optimal transit lines headways and fares are presented in Table 5 for different scenarios under different population levels. The optimal headways and fares by different time periods are investigated (taking peak and off-peak period as examples). As regards the trip-based approach, headways and fare discount rates cannot be optimized by time of day as the trip-based approach focuses only on independent trips and the linkages between trips are ignored. However, if we use the activity-based approach, transit line headways and fares can be optimized for different time periods. It can be seen from Table 5 that transit fares are increased for peak periods and decreased for off-peak periods. Taking Scenario 2 at a low population level as an example, the optimal discount rate of transit fare for peak periods is 1.3 and the rate for off-peak period is 0.7. The optimal headways of transit service during peak period (i.e. 5 min for bus and 3 min for subway) are shorter than those during off-peak period (i.e. 8 min for bus and 6 min for subway). From the results, it can be concluded that the transit line headways and fares can be optimized using the proposed bi-level programming model. For maximizing the network-wide accessibility of activity locations, we can adopt different headway and fare strategies during different time periods.

Table 5 also shows some results regarding individuals' DATPs. Individuals'

DATP utility under equilibrium status can be found. It is interesting to see that the equilibrium DATP utility of Scenario 1 (i.e. constructing a new shopping mall at S1) is larger (e.g. HK$ 3952.13) than that of Scenario 2 while the network-wide accessibility for Scenario 2 is larger. This result indicates that a new activity location may exert different effects on network-wide accessibility and individuals' daily utility gain.

Individuals' activity and travel choice behaviours can also be investigated by the proposed model. For example, from Table 5, it can found that with a low population level, the average daily duration for Home, Work and Shopping are respectively around 5 hours, 8 hours and 2 hours. However, when the network is very crowded (i.e. with a high population level), individuals tend not to conduct non-compulsory Shopping to avoid the crowding in vehicle and at activity location. In addition, we found that individuals' mode choice is affected by population level and activity choice. For example, when the population level is low, the average duration on subway for Scenario 1 is much longer than that for Scenario 2 (i.e. 1.25 h compared to 0.58 h). This is due to that, in the example network, individuals can only take subway to the Shopping location S1. However, when the population level is high, individuals use subway more than bus for both scenarios (i.e. 1.29 h and 1.11 h) as they cancel Shopping activity and tend to choose subway to alleviate in-vehicle crowding discomfort.

For comparing the objective of maximizing activity-based accessibility with other conventional objectives of transit network optimization, Table 6 shows the optimal solutions for two different objectives (i.e. maximizing activity-based accessibility and minimizing total travel dis-utility). It can be found that the optimal solutions of transit line headways and fares are different under the two objectives. If the objective is to minimize the total travel dis-utility in the transit network, constructing a shopping mall at location S1 (i.e. Scenario 1) results in the smallest travel dis-utility (i.e. HK$ -171.67). However, the corresponding accessibility of all activity locations is only HK$ 5557.18, which is smaller than the accessibility in Scenario 2 with objective of maximizing activity-based accessibility (i.e. HK$ 5849.73). Thus, transport authorities and/or transit operators need to weigh total travel dis-utility against network-wide space-time accessibility and choose suitable transit operating strategies for different objectives.

Figure 8 depicts the results of activity-based space-time accessibility by time of day using the proposed model. Figure 8(a) shows the network-wide space-time accessibility and Figures 8(b-d) show the space-time accessibility of three activity locations (i.e. Work, Home and Shopping). The accessibility for two cases (i.e. an optimal case with time-dependent headways and fares, and an original case with even headways and time-independent fares) under different population levels are depicted. It can be seen that the activity-based space-time accessibility varies by time of day. There are two peaks of network-wide activity-based accessibility (i.e. HK$ 922.00 in the morning and HK$ 744.96 in the afternoon) over a whole day. This is resulted from the large amount of utilities gained from compulsory Work activity in the morning and in the afternoon (refer to Figure 8(b)). For Home activity (see Figure 8(c)), there is a decrease of accessibility (i.e. decreased to HK$ 84.41 for the optimal case and HK$ 80.09 for the original case) after work in the afternoon (around 17:00) under a high population level. It is due to the crowding effect in vehicle are considered and the increased travel dis-utility are taken into account in the proposed space-time accessibility measure. With the optimal time-dependent headways and fares, the decrease of accessibility is less compared to the original case with even headways and time-independent fares, as some individuals adjust their departure time to Home. As regards the accessibility of Shopping location (see Figure 8(d)), as individuals cancel Shopping activity under high population level (refer to Table 5), the accessibility is much fewer than that under low population level (i.e. HK$ 12.87 compared to HK$ 57.12). The results indicate that the space-time accessibility of activity location is dependent on individuals' activity and travel choice behaviours, rather than constant throughout the whole day.

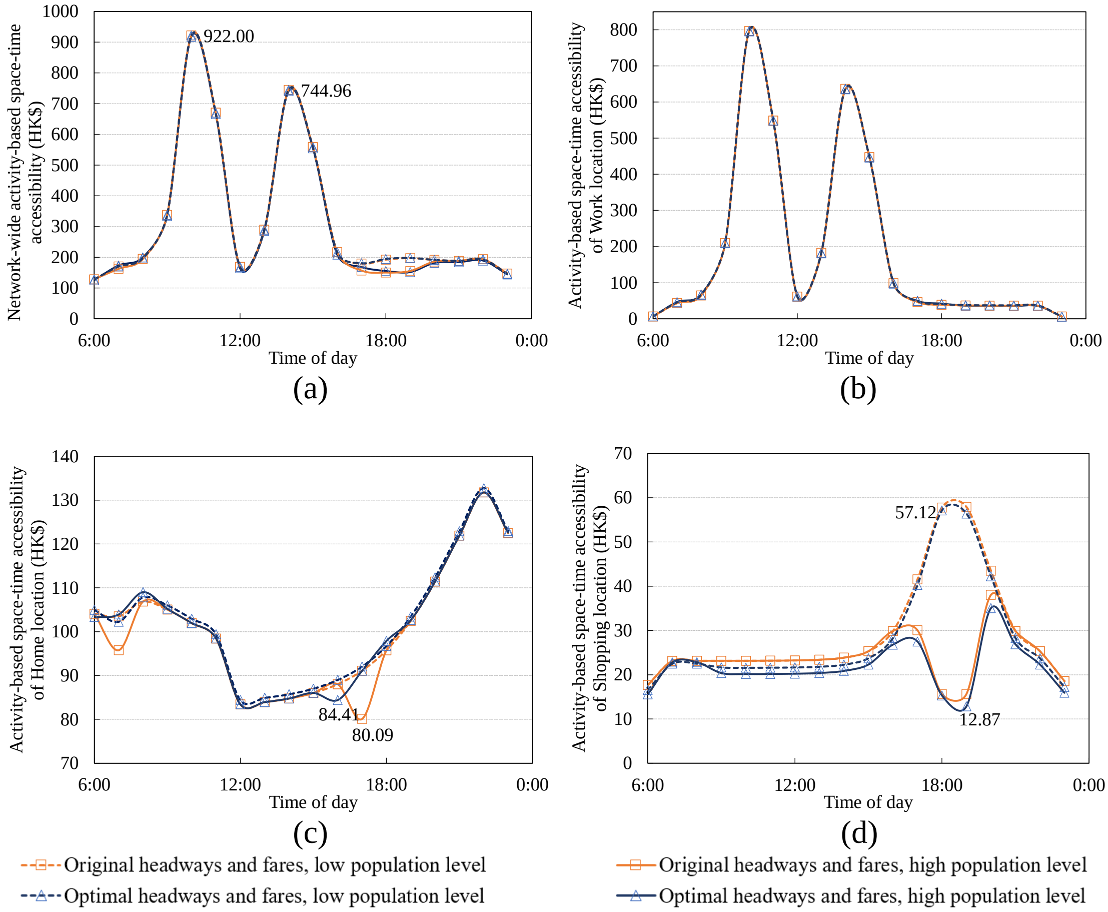


Figure 8. Activity-based space-time accessibility by time of day: (a) network-wide accessibility, (b) accessibility of Work location, (c) accessibility of Home location, (d) accessibility of Shopping location

Figure 9 presents the temporal population distribution for travels and different activities under different population levels with original and optimal headways and fares. By comparing Figures 9(b) and 9(d) to Figures 9(a) and 9(c), it can be found that with the optimized time-dependent headways and fares, some individuals have different departure time choices compared to the original case. This is due to the fact that the optimal headways and fares of transit services are time-dependent in our proposed model, compared to even headways and time-independent fares in the original case. It is also noted that if the population level in the study area is high (see Figures 9(c) and 9(d)), individuals do not conduct non-compulsory Shopping activity to avoid crowding discomfort both in vehicle and at Shopping location. Thus, with the proposed bi-level programming model, time-dependent transit line headways and fares can be optimized with consideration of maximizing activity-based space-time accessibility, and the resultant changes of individuals' activity and travel choice behaviours (including activity type, activity start time and duration, departure time of each trip, route/mode choice, etc.) can be explicitly investigated.

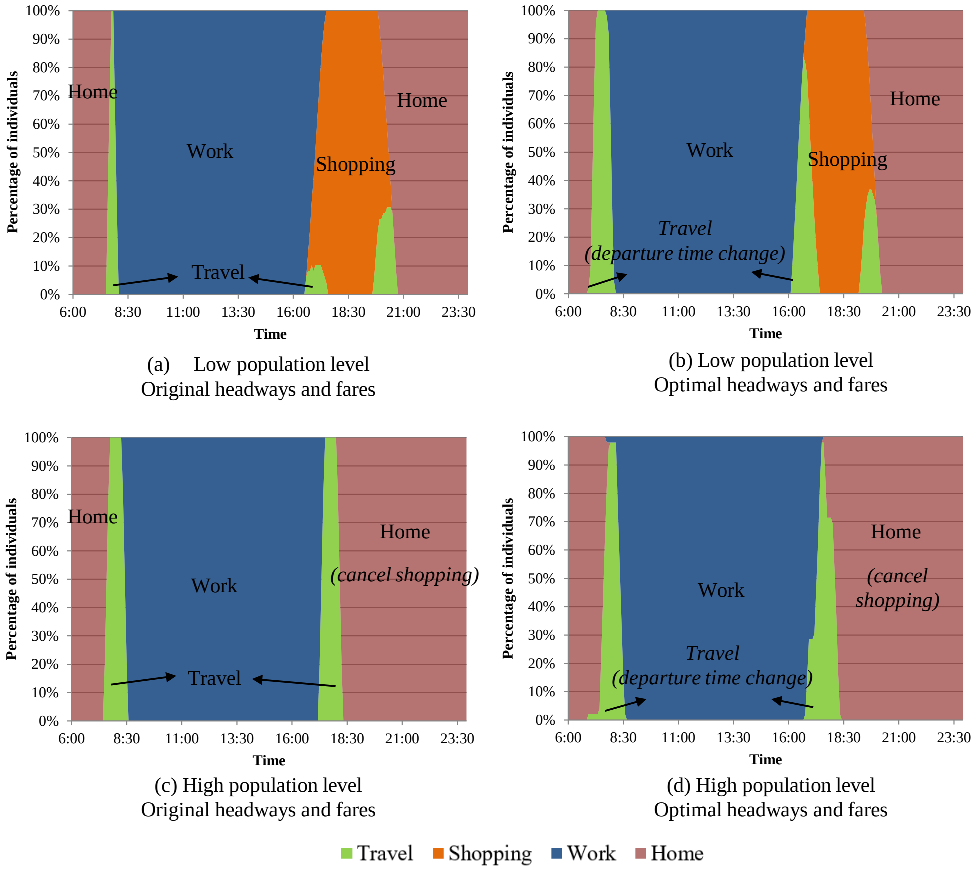


Figure 9. Temporal population distributions under different population levels with original and optimal transit line headways and fares

Table 5. Optimal solutions with trip-based and activity-based approaches

		Optimal discount rates of transit fare		Optimal line headways (min)				Space-time accessibility of activity locations (HK$)	Equilibrium DATP utility (HK$)	Average in-vehicle duration (h)		Average activity duration (h)			Total time (h)
				Bus		Subway									
		Peak	Off-peak	Peak	Off-peak	Peak	Off-peak			Bus	Subway	H	W	S	
Low population level	Trip-based approach	1		15		6		1649.32	/	0.41	1.25	/	/	/	/
	Activity-based approach (Scenario 1)	1.2	0.8	7	7	5	8	5696.02	**3952.13**	0.42	1.25	5.30	8.41	2.62	18
	Activity-based approach (Scenario 2)	**1.3**	**0.7**	**5**	**8**	**3**	**6**	**5849.73**	3939.48	1.09	**0.58**	5.30	8.52	2.51	18
High population level	Trip-based approach	1.3		14		7		1618.92	/	0.45	1.21	/	/	/	/
	Activity-based approach (Scenario 1)	1.4	0.5	6	7	4	13	5544.93	3817.06	0.37	**1.29**	7.58	8.76	**0**	18
	Activity-based approach (Scenario 2)	1.4	0.8	8	10	3	6	**5635.57**	3737.45	0.56	**1.11**	7.53	8.80	**0**	18

Table 6. Optimal solutions for different objectives (i.e. maximizing activity-based space-time accessibility and minimizing total travel dis-utility)

Objective		Optimal discount rates of transit fare		Optimal line headways (min)				Space-time accessibility of activity locations (HK$)	Travel dis-utility (HK$/individual)
				Bus		Subway			
		Peak	Off-peak	Peak	Off -peak	Peak	Off -peak		
Maximizing accessibility	Scenario 1	1.2	0.8	7	7	5	8	5696.02	-180.11
	Scenario 2	1.3	0.7	5	8	3	6	**5849.73**	-187.64
Minimizing total travel dis-utility	Scenario 1	1.2	0.7	10	13	3	10	**5557.18**	**-171.67**
	Scenario 2	1.1	1	4	9	3	7	5811.05	-185.48

6. Conclusions

In the literature, little attention has been given to maximizing the space-time accessibility for improvements of multi-modal transit network. It is of note that the current development of accessibility measures lacks a rigid and comprehensive modelling framework to consider individuals' all choice dimensions including both activities and travels. The work presented in this paper extends existing theories by proposing a novel activity-based space-time accessibility measure and developing a bi-level programming model for optimizing time-dependent transit line headways and fares in multi-modal transit network with the objective of maximizing network-wide activity-based space-time accessibility.

At the lower level of the proposed bi-level model, individuals' DATP patterns are investigated using an activity-based network equilibrium approach on an ATS-SAM super-network platform. Compared to previous related studies, activities are treated endogenously by specifying time-dependent utilities of activity participation. At the upper level, transit line headways and fares are optimized by time of day so as to maximize the proposed activity-based space-time accessibility. The novel aspect of the proposed activity-based accessibility measure is that it can assess individuals' all-day accessibility to all activity locations in the network, incorporating both scheduling constraints and trip chaining. Utility gain/loss from activities/travels for different time periods at different locations is reflected in the novel measure. In this paper, a solution algorithm without prior enumeration of DATPs is proposed for solving the DATP scheduling problem on the ATS-SAM super-network, and a heuristic solution algorithm is adapted to solve the proposed bi-level programming model. The transport authorities and/or transit operators can make use of the proposed model to optimize transit line headways and fares by time of day. In the proposed model, individuals' various choice behaviour in their DATPs (including activity type, activity start time and duration, departure time, travel mode and route, etc.) can be taken into account.

A case study is carried out to highlight the performance of the proposed activity-based space-time accessibility measure and the bi-level programming model. Some insightful findings are presented. It is found that the proposed activity-based accessibility measure outperforms the conventional approaches in incorporating flow-based activity/travel (dis-)utility, thus results in a more realistic accessibility to all activity locations in the network. The activity-based space-time accessibility is found to be varied by time of day, which overcomes the disadvantage of conventional approaches that they cannot obtain different levels of accessibility by different time periods. The solutions of the optimization problem using trip-based approach and activity-based approach are compared to distinguish the merits of the proposed model. It is demonstrated that the activity-based space-time accessibility measure can reflect individuals' utility gain/loss from activity/travel participation. Using the activity-based approach, maximum space-time accessibility can be achieved with a joint implementation of time-dependent transit line headways and time-dependent transit fares, compared to even headways and time-independent fares. Individuals' attitudes toward various activities is varied under different population levels. In addition, it is found that the location of a new facility exerts different effects on network-wide activity-based accessibility and individuals' daily utility gain. The proposed model provides a useful tool for estimating the time-varying profile of the activity-based space-time accessibility in multi-modal transit networks over time of day, and can be used to evaluate various transit operating schemes at the strategic planning level.

On the basis of the model proposed in this paper, some extensions can be envisaged. The proposed accessibility measure does not incorporate individuals' taste heterogeneity, so it cannot reflect the impacts of socio-economic factors (such as car ownership and household structure) on the DATP choice and the accessibility by different groups of individual. In further research, there is a need to examine different levels of accessibility considering socio-economic factors so as to answer the questions such as who is impacted by different transit operating strategies. Extra benefit from joint activity/travel can also be explicitly considered using the activity-based approach. In addition, travel time uncertainty in the multi-modal transport network should be considered in modelling activity and travel choice behaviour and measuring space-time accessibility. Individuals' attitudes toward various transit modes under different on-time arrival probabilities can be investigated.

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