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# Condition Analysis of Expansion Joints of a Long-Span Suspension Bridge through Metamodel-based Model Updating Considering Thermal Effect

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## Summary

Expansion joints of bridges are vulnerable to damage due to the thermal expansion and contraction, vehicle traffic, etc. Currently, the temperature-displacement relationship model may be only qualitative method for condition evaluation of bridge expansion joints using the field monitoring data. The quantitative assessment based on the finite element model (FEM) updating techniques is heavy computational burden and time-consuming. Therefore, a Gaussian process (GP) metamodel-based model updating method is proposed in this study, and performed for the quantitative identification on the boundary condition of the expansion joints of Jiangyin Suspension Bridge using the long-term displacement and temperature monitoring data. At first, the relationship between the longitudinal boundary stiffness (LBS) and structural temperature is formulated on the basis of thermal effects of the bridge deck. The range of SBS is approximately estimated by the regression coefficients from one-year monitoring data and is used as initial bounds for the subsequent model updating procedure. The GP metamodel is formulated to map the relationship between the SBS and the longitudinal

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displacements under the thermal effects. The LBS identification of the Jiangyin Suspension Bridge is performed within the fast-running GP metamodel. The results show that the longitudinal displacements using the updated LBS are in good agreement with the measurements, which verifies the effectiveness of GP metamodel-based model updating method in identifying the LBS of the long-span suspension bridge.

**KEYWORDS:** expansion joints, boundary stiffness, thermal effect, Gaussian process metamodel, structural health monitoring

## 1 Introduction

Bridge expansion joints are designed to accommodate expansion and contraction of the bridge resulting from a variety of factors such as thermal effect, concrete shrinkage, creep, live loading, settlement of the foundation and substructure, and environmental variations<sup>1-7</sup>. Effective functioning of expansion joints is not only required for smooth traffic but also for ensuring bridge performance in extreme events, such as earthquakes and hurricanes. In practice, expansion joints are more vulnerable to damage than other bridge components. Damage in expansion joints may cause variation in structural dynamic and static properties. For example, Fu and DeWolf<sup>8</sup> pointed out that the natural frequencies of a two-span skewed composite bridge changed due to the damaged bearings. Xu et al.<sup>9</sup> found that the measured first natural frequency was 10% higher than the theoretical counterpart and explained that this difference was caused by the unclear understanding of the boundary conditions. Moreover, severe damage of expansion joints may restrain the movement of the bridge, generating large internal forces and causing damage to the bridge<sup>10</sup>. Kromanis et al.<sup>11</sup> found that the transversal temperature gradient of the Cleddau bridge caused plan-bending of the box girder and led to

45 degradation of the bearing.

46 The development of structural health monitoring (SHM) enables to gain insight into the  
47 performance of expansion joints during operation using the long-term monitoring data. Xia et al.<sup>12</sup>  
48 studied the thermal effects on a large-span suspension bridge using one-year monitoring data and  
49 found that the damaged expansion joints produced restraining internal forces on the main girder. Ni  
50 et al.<sup>13</sup> established the normal correlation pattern between the effective temperature and the  
51 expansion joints movement of the Ting Kau Bridge using the long-term displacement and  
52 temperature measurement, allowing for condition assessment of the bridge expansion joints. Ding  
53 and Li<sup>14</sup> found that temperature variation was the major factor of the expansion joint displacement.  
54 In addition, they used a mean value control chart to characterize the residual displacement for  
55 performance alarming of bridge expansion joints. Guo et al.<sup>15</sup> decomposed expansion joint  
56 displacement measurements into various frequency bands and revealed that the temperature-induced  
57 movements mainly contributed to displacement fluctuation whereas the wind and vehicle loads  
58 largely contributed to total cumulative displacement. Li and Sun<sup>16</sup>, Huang et al.<sup>17</sup> and Winkler et al.  
59<sup>18</sup> formulated the relation between temperature and displacement of bridge expansion joints and used  
60 it for assessing the performance of the expansion joints.

61 The above studies of the temperature-displacement relationship model are the qualitative  
62 methods for condition evaluation of bridge expansion joints. In recent years, the quantitative  
63 assessment of the boundary condition (e.g., restraint stiffness/force) of bridges has received an  
64 increasing interest. Yarnold et al.<sup>19</sup> identified the boundary stiffness of the expansion bearings of a  
65 steel arch bridge using a temperature-based model updating procedure. Murphy and Yarnold<sup>20</sup>  
66 demonstrated the capability of the temperature-based structural identification scheme in calibrating

67 the boundary condition parameters of a Route 61 Bridge. Zhou and Song <sup>21</sup> proposed a FEM  
68 updating method for structural identification of a pedestrian bridge with consideration of  
69 environmental effects. However, these studies focused on the simple structures. Jesus et al. <sup>22, 23</sup>  
70 applied a modular Bayesian model updating method to identify the boundary condition of a scaled  
71 aluminum bridge and a suspension bridge subjected to either thermal effect only or both temperature  
72 and traffic load effects. The importance of the definition reasonable boundaries is pointed out in this  
73 study. Xia et al. <sup>24</sup> theoretically derived the LBS identification method for a long-span suspension  
74 bridge based on temperature-induced effects. This study can be used to determine the initial bounds  
75 based on the long-term monitoring data.

76 Although the model updating technique has been developed to identify the boundary stiffness of  
77 bridges, some issues, such as initial boundary stiffness bounds, accurate thermal analysis, and  
78 efficient model updating method, still exist, especially for the long-span bridges. First, unreasonable  
79 initial bounds may cause the obtained results to lose physical meaning. However, these initial bounds  
80 are not strictly defined in the above literatures on the model updating. Second, large-scale bridges  
81 such as long-span suspension bridges generally have a complicated geometric configuration. The  
82 temperature at each differently oriented member may vary at different times. It is very difficult to  
83 accurately predict the detailed temperature distribution over the entire bridge using the simplified or  
84 scaled model. The refined 3D FEMs are essential for performing accurate thermal analysis. Third,  
85 the traditional FEM updating technique cannot be well performed under the thermal analysis of the  
86 refined 3D FEM because it requires a lot of calculation and time-consuming. So, the GP metamodel  
87 is utilized in this paper.

88 In this study, Jiangyin Suspension Bridge is used as a test example for the boundary condition  
89 identification. This paper is organized as follows: In Section 2, the SHM system of Jiangyin  
90 Suspension Bridge and its sensor layout are briefly introduced. The regression analysis between the  
91 longitudinal displacements of the expansion joints and the structural temperature is performed. In  
92 Section 3, the LBS of the expansion joints is formulated on the basis of the thermal effects of the  
93 bridge deck. In Section 4, the details on LBS identification using GP metamodel-based model  
94 updating are given. In Section 5, the initial LBS bounds are determined from the regression  
95 coefficients which are obtained using one-year monitoring data. The bounds are then used for the GP  
96 model updating <sup>25, 26</sup> to identify the LBS of Jiangyin Suspension Bridge. Finally, the conclusions of  
97 this work are drawn in Section 6.

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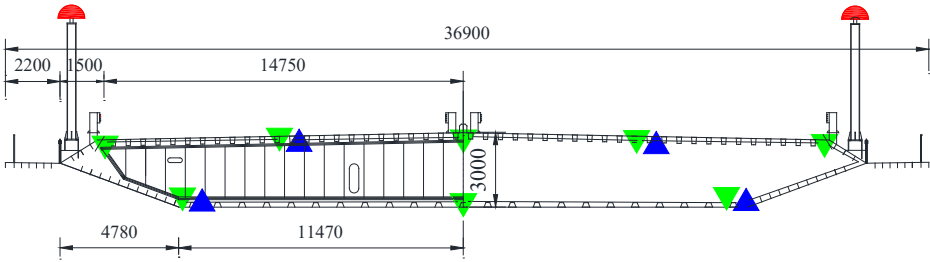
## 99 **2 Jiangyin Suspension Bridge and statistical analysis**

### 100 **2.1 Jiangyin Suspension Bridge**

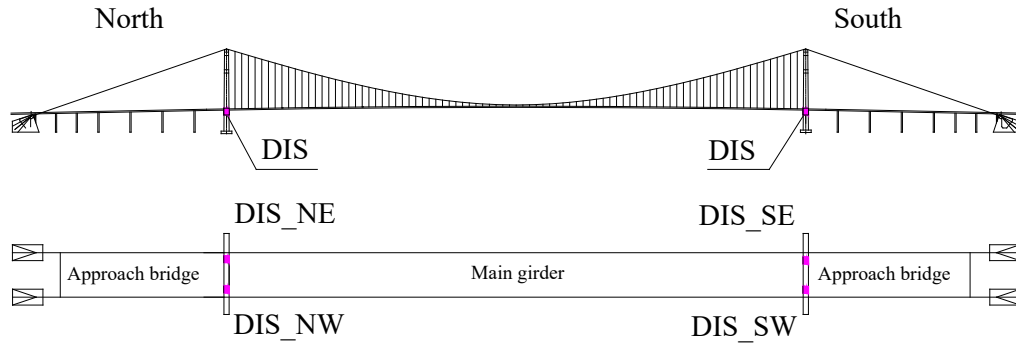
101 The Jiangyin Suspension Bridge has a main span of 1385 m striding the Yangtze River. The  
102 main span has a welded streamlined constant depth steel box girder of 3 m high and 36.9 m wide,  
103 which utilizes the asphalt concrete pavement and has a navigation clearance of 50 m. The bridge has  
104 two reinforced concrete towers of 190 m high and two main cables anchored in the gravity  
105 anchorages. The Jiangyin Bridge is of single-span and the bearings in the ends of the bridge deck  
106 provide vertical support only. Therefore, the bridge deck can move freely in the longitudinal  
107 direction.

108 A SHM system was designed and installed on the bridge in 2005, which consists of about 170  
109 sensors including 20 accelerometers (AS), 116 Fiber Bragg grating sensors (FBG), 4 displacement

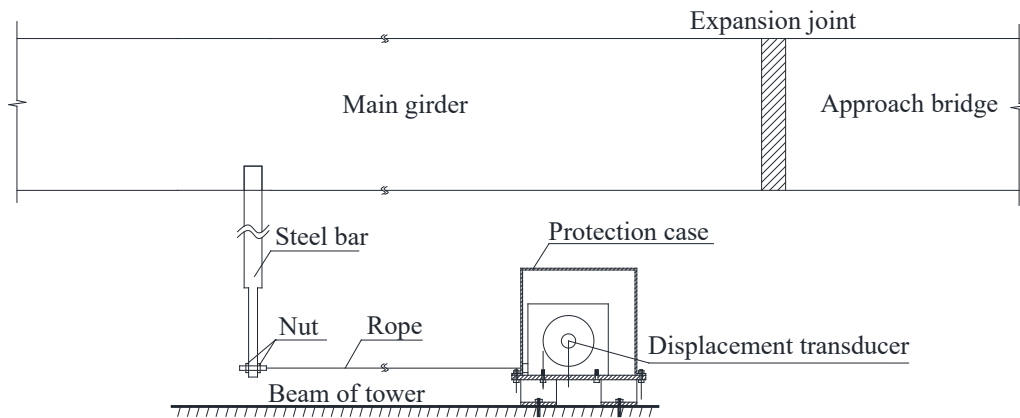
110 sensors (DIS), 9 global positioning system (GPS) receivers, and a number of shear pins, ultrasonic  
 111 anemometers, etc. The sensors are distributed in nine equidistant cross sections (Nos. 1 ~ 9), as  
 112 shown in Figure 1. The 4 draw-wire DIS (denoted as DIS\_NE, DIS\_NW, DIS\_SE, and DIS\_SW in  
 113 Figure 2) were installed on both ends of the bridge deck to monitor the longitudinal displacements.  
 114 One end of the DIS is fixed on the beam of the tower, and the other end on the main beam of the  
 115 deck. The sampling frequencies of the DIS and the FBGT are 50 Hz and 1.1 Hz, respectively.  
 116



**Figure 1.** Sensor layout of the Jiangyin Suspension Bridge.



(a) Location of displacement transducers



(b) Configuration of displacement transducers (DIS\_SW)

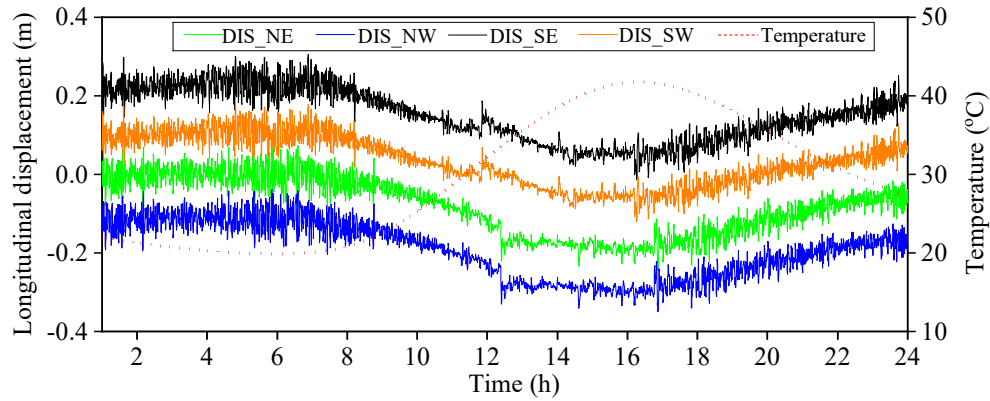
**Figure 2.** Displacement transducer at the ends of bridge deck.

## 2.2 Statistical analysis of temperature-longitudinal displacement relationship

One-year monitoring data during 1 Dec 2005 ~ 30 Nov 2006 is available for investigating the relationship between temperature and longitudinal displacement. The data on 20 May 2006 is shown in Figure 3, in which the effective temperature of the bridge deck is used. In the early morning of 0:00 ~ 7:00, the structural temperature and longitudinal displacement of the beam end had no significant change. During the period of 7:00 ~ 16:00, due to the solar radiation effect, the effective temperature of the structure raised from 19.9 °C to 41.7 °C, and the longitudinal displacement of the 4 expansion joints decreased by 0.19 m, indicating that the bridge deck expanded and the gap of the expansion joints became smaller. From 16:00 to 24:00, the decrease in temperature caused the bridge

128 deck to contract and the gap of the expansion joints became larger. It is concluded from Figure 3 that  
 129 the longitudinal deformation of the expansion joints is mainly caused by the temperature change.

130



**Figure 3.** Longitudinal displacement of expansion joints with respect to temperature on one day  
 (20 May 2006).

131

132 The hourly mean temperature and longitudinal displacement monitoring data in one-month  
 133 period (20 May ~ 19 June 2006) are illustrated in Figure 4. When the daily temperature change was  
 134 small (for example, may be a cloudy day), the displacement variations were small as well. When the  
 135 daily temperature change was relatively large, the displacement variations were significant.  
 136 Moreover, the longitudinal displacements of the four DISs changed with temperature consistently.

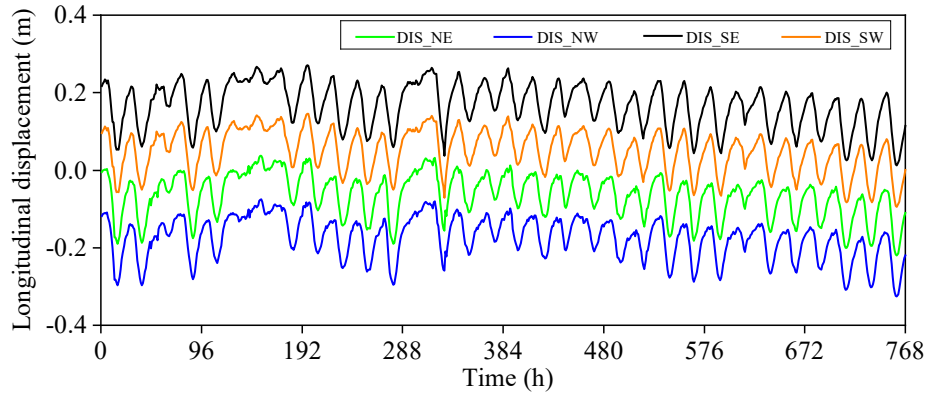
137 Furthermore, the seasonal variation in the hourly mean temperature and longitudinal  
 138 displacement data are shown in Figure 5. The longitudinal displacements have a similar variation  
 139 pattern as the seasonal temperature. The maximum longitudinal displacement and maximum  
 140 temperature occurred in July and August. The longitudinal displacement caused by the seasonal  
 141 temperature change can reach up to 0.5 m, more than that caused by the daily temperature variation.

142 The regression analysis (one-month monitoring data from 20 May to 19 June 2006 for  
 143 demonstration) is then adopted to qualitatively determine the relationship between temperature and

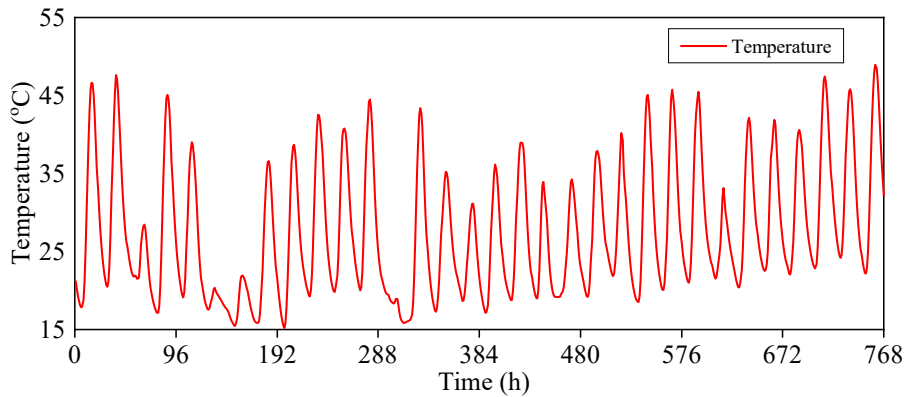


144 longitudinal displacement. In order to better compare the correlation of the four displacement meters  
 145 with temperature, the initial value of the displacement is normalized to zero. The results of the four  
 146 joints are presented in Figure 6. The temperature and longitudinal displacement have a linear  
 147 relationship and the sensitivities of the four longitudinal displacements to the temperature,  
 148 determined by the slope of the lines, are almost the same. The slopes of the lines are not only  
 149 associated with the structural configuration, but also the stiffness of the expansion joints, which is  
 150 equivalent to the LBS and will be investigated in following sections.

151

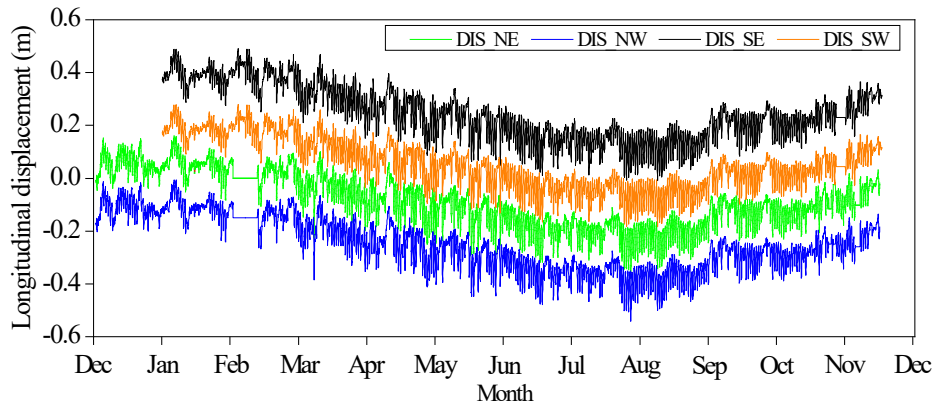


(a) Four longitudinal displacement curves

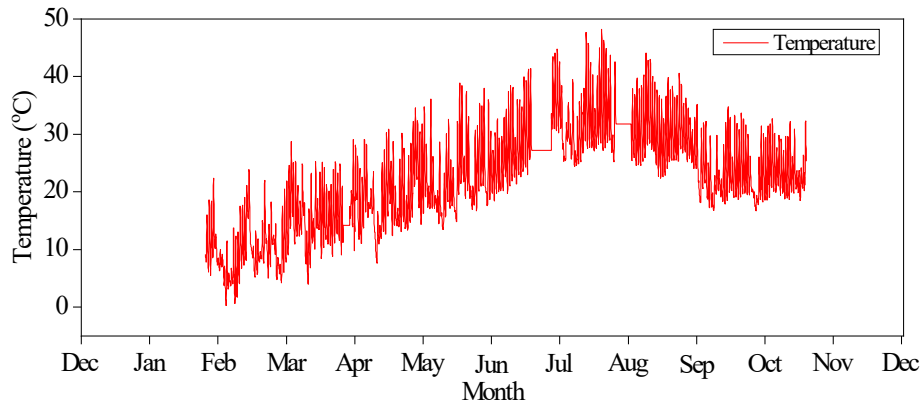


(b) Temperature curve

**Figure 4.** Longitudinal displacement of expansion joints with respect to temperature in one-month period (20 May ~ 19 June 2006).

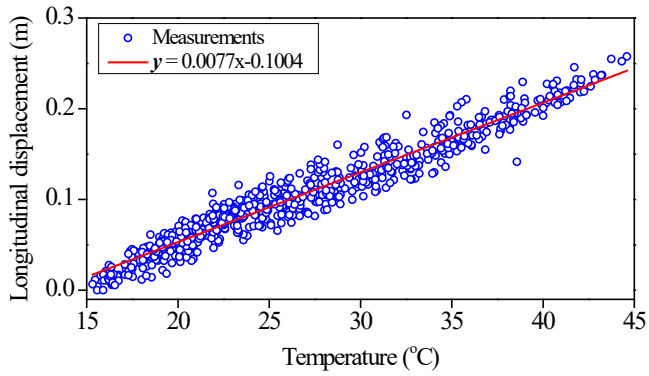


(a) Four longitudinal displacement curves

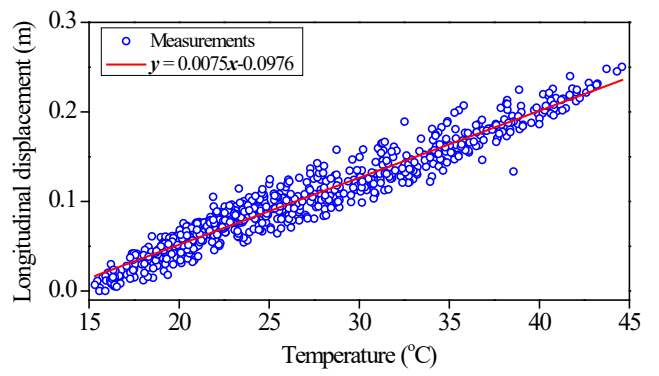


(b) Temperature curve

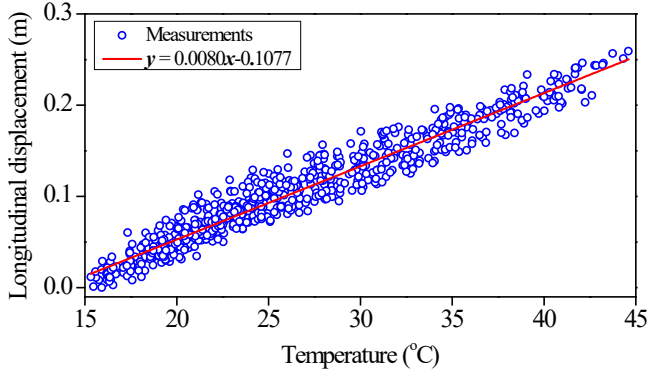
**Figure 5.** Longitudinal displacement of expansion joints with respect to temperature in one-year period (1 Dec 2005 ~ 30 Nov 2006).



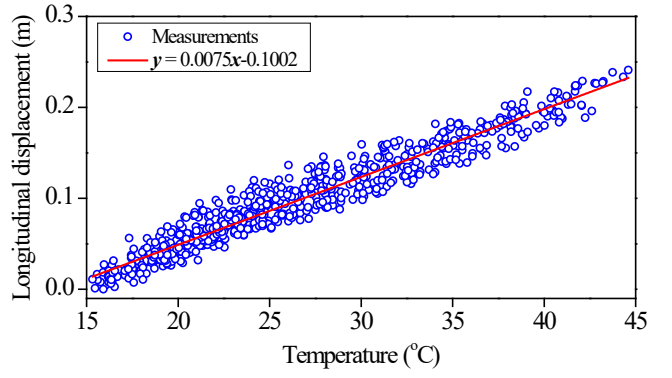
(a) North-east expansion joint



(b) North-west expansion joint



(c) South-east expansion joint



(d) South-west expansion joint

**Figure 6.** Linear regress analysis of temperature and longitudinal displacement of expansion joints.

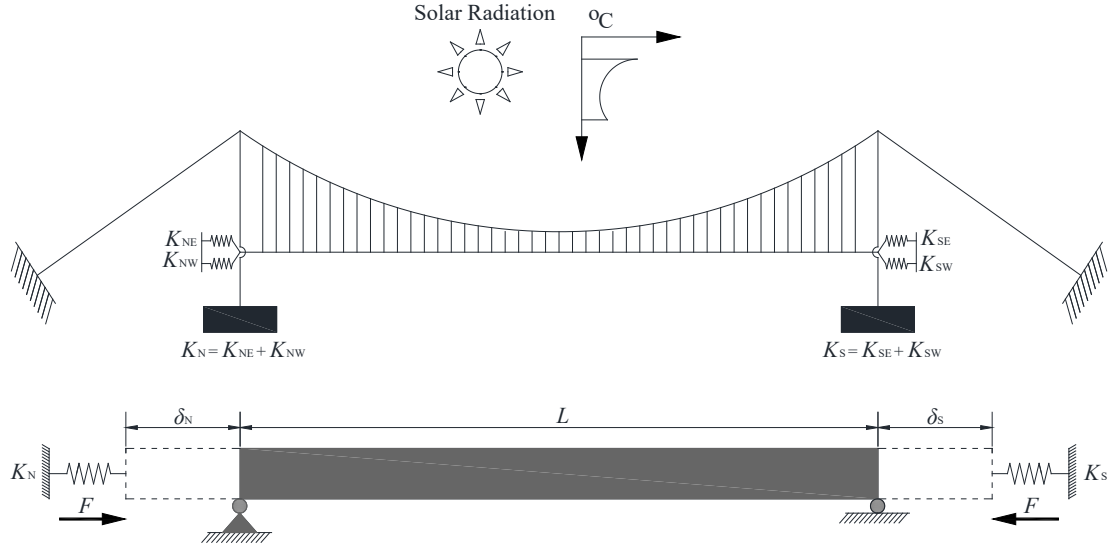
152

### 153 3 LBS formula derivation and calculation

154 The thermal behavior of the bridge deck can be schematically represented as shown in Figure 7,  
 155 in which the bridge deck is simplified as a simple beam with two roller supports and the longitudinal  
 156 constraints simulated by 4 springs at the ends. The longitudinal deformation of the bridge deck  
 157 satisfies the Hooke's law, that is,

$$F = \delta_N K_N = \delta_S K_S \quad (1)$$

158 where  $F$  is the restrained force exerted at the bridge deck;  $\delta_N$  and  $\delta_S$  are the displacements of the  
 159 north and south ends of the bridge deck, respectively; and  $K_N$  and  $K_S$  are the LBS of the north  
 160 and south ends of the bridge deck, respectively.



**Figure 7.** Schematic thermal behavior of the bridge deck.

The relationship between temperature and displacement satisfies,

$$\alpha \Delta T L = \varepsilon L + \delta \quad (2)$$

where  $\alpha$  is the thermal expansion coefficient;  $\Delta T$  is the variation in the effective temperature of the structure;  $L$  is the length of the main span (1385 m);  $\delta = \delta_N + \delta_S$  is the total longitudinal displacement at the expansion joints; and  $\varepsilon$  is elastic strain of the bridge deck and can be calculated from the restrained force as,

$$\varepsilon = \frac{F}{EA} \quad (3)$$

where  $E$  is the elastic modulus; and  $A$  is the cross-sectional area of the deck.

Substituting Equation (3) into Equation (2) yields,

$$\alpha \Delta T L = \frac{F}{EA} L + (\delta_N + \delta_S) \quad (4)$$

According to Equations (1) and (4), the LBS of the north and south ends of the bridge deck can be obtained as,

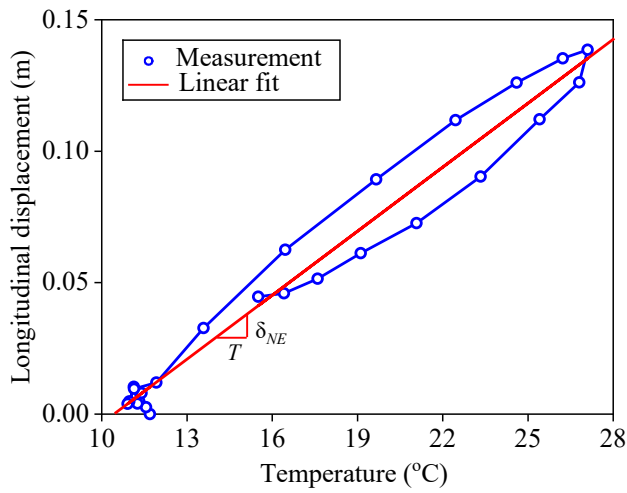
$$K_N = \frac{EA}{L} \left[ \alpha L \left( \frac{\Delta T}{\delta_N} \right) - \left( \frac{\delta_S}{\delta_N} + 1 \right) \right] \quad (5)$$

$$K_S = \frac{EA}{L} \left[ \alpha L \left( \frac{\Delta T}{\delta_S} \right) - \left( \frac{\delta_N}{\delta_S} + 1 \right) \right] \quad (6)$$

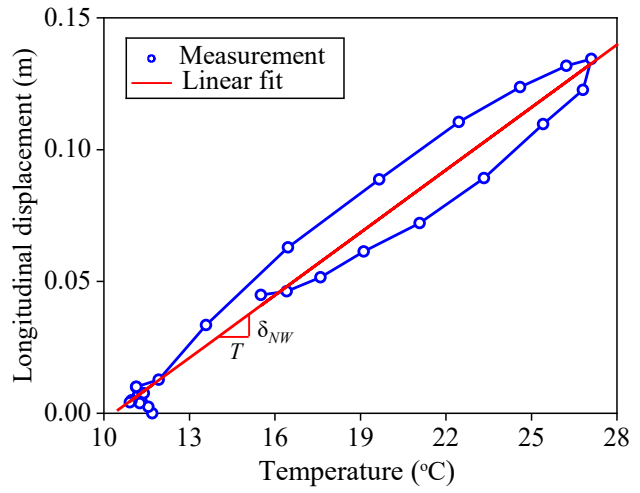
From these formulas above, the LBS can be calculated using the monitoring data  $\Delta T$ ,  $\delta_S$  and  $\delta_N$ .

Furthermore, the calculated LBS need to be calculated and verified.

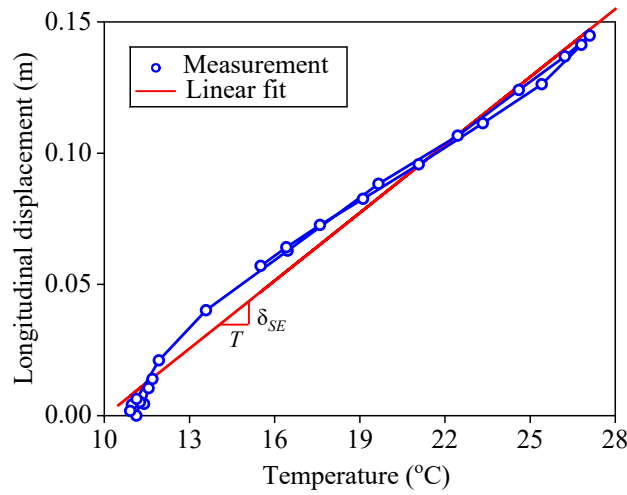
The day 15 April 2006 as an example is selected for the LBS calculation. According to Equations (5) and (6), the two main values should be determined, one is  $\Delta T/\delta$ , the other is  $\delta_S/\delta_N$  (or  $\delta_N/\delta_S$ ). The former is shown in Figure 8. Figures 8(a) and (b) show that the relationship between the longitudinal displacement and temperature at the north side presents an elliptic curve in daily variation. The  $\Delta T/\delta$  values at the downstream and upstream of the north side are 141.1 and 138.3, respectively. Figures 8(c) and (d) show that the relationship between the longitudinal displacement and temperature at the south side displays an overlap curve in daily variation. The  $\Delta T/\delta$  values at the downstream and upstream of the south side are 135.1 and 138.9, respectively.  $\delta_{SE}/\delta_{NE}$ ,  $\delta_{SW}/\delta_{NW}$ ,  $\delta_{NE}/\delta_{SE}$  and  $\delta_{NW}/\delta_{SW}$  are 1.045, 1.0006, 0.9569, and 0.9994, respectively. Based on Equations (5) and (6), the LBS values are calculated as  $k_{NE} = 6.54 \times 10^6 \text{ N/m}$ ,  $k_{NW} = 7.02 \times 10^6 \text{ N/m}$ ,  $k_{SE} = 6.26 \times 10^6 \text{ N/m}$ ,  $k_{SW} = 7 \times 10^6 \text{ N/m}$ . The above analysis indicates that the LBS of four locations are slightly different. And then, the estimated LBSs are applied to the 3D FEM for temperature-induced longitudinal calculation.



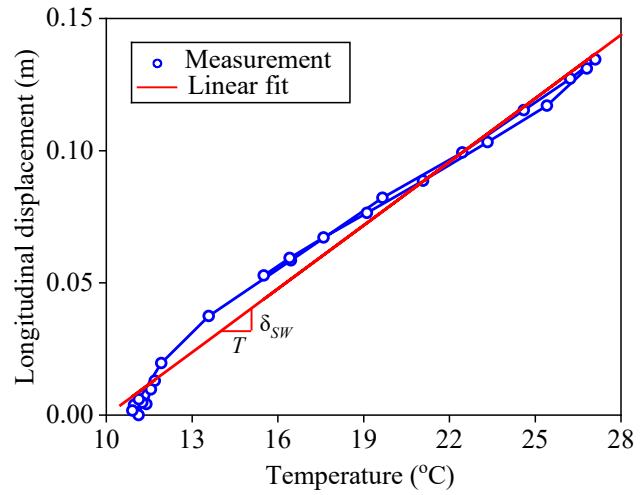
(a) North-east expansion joint



(b) North-west expansion joint



(c) South-east expansion joint



(d) South-west expansion joint

**Figure 8.** Displacement vs. temperature of the bridge deck on 15 April 2006.

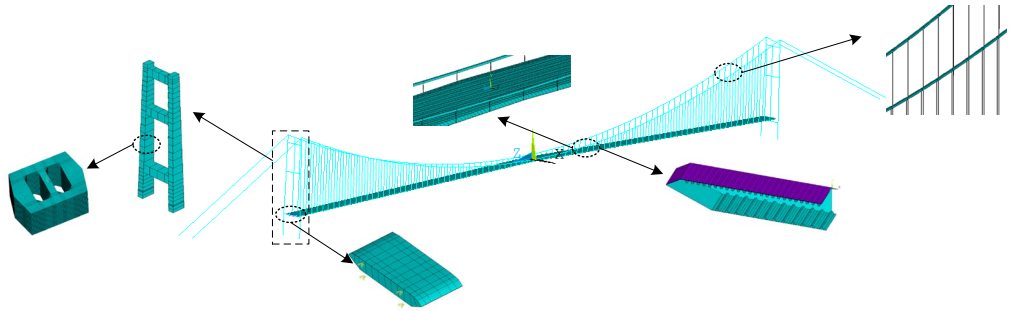
187

188 The FEM of Jiangyin Suspension Bridge is constructed in ANSYS <sup>27</sup> software. The main cables,  
 189 hangers, and towers are modeled using link elements and the steel-box girder is simulated using  
 190 elastic shell elements. The ends of main cables and the bottom of towers are completely restrained.  
 191 The resulting FEM consists of a total of 37,276 nodes and 23,070 elements, as shown in Figure 9(a).  
 192 The mass density and elastic modulus of the steel and concrete is 7800 kg/m<sup>3</sup> , 2550 kg/m<sup>3</sup> , 2.1×10<sup>5</sup>  
 193 MPa and 3.3×10<sup>4</sup> MPa, respectively.

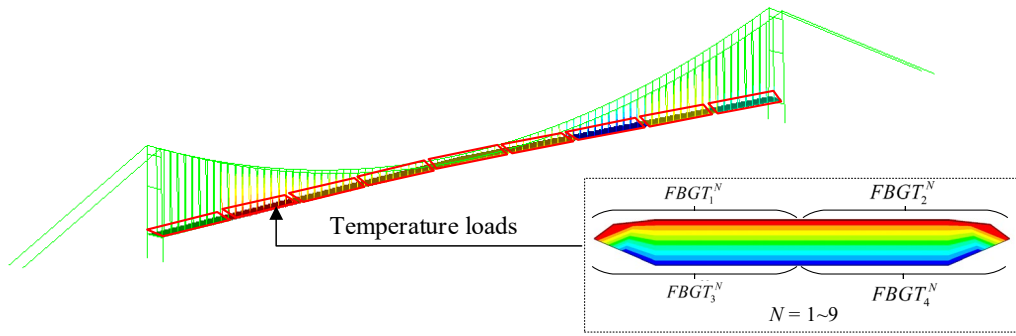
194 The boundary conditions at the end of the expansion joints are simulated using the 3D linear  
 195 spring element (Combin14 element). One end of the spring is connected to the bridge deck, and the  
 196 other end is connected to the reference points, which are fixed. The calculated LBSs of  $k_{NE} =$   
 197  $6.54 \times 10^6 \text{ N/m}$ ,  $k_{NW} = 7.02 \times 10^6 \text{ N/m}$ ,  $k_{SE} = 6.26 \times 10^6 \text{ N/m}$ ,  $k_{SW} = 7 \times 10^6 \text{ N/m}$  are input to the Combin14  
 198 elements in end of bridge deck. Since the temperature distribution along the longitudinal direction of  
 199 the deck is not uniform as shown in Figure 9(c), the main girder is divided into 9 segments, each  
 200 being loading different measured temperature. On the cross section of each segment, the temperature  
 201 loading is represented by the 4 distributed FBGT (Figure 9b). The rest of the members are subjected  
 202 to ambient temperature because they have little effect on the longitudinal displacement of the main  
 203 beam. During the solution process, 3600 seconds are defined as a substep, resulting in a total of 24  
 204 sets for one day. It takes about 20 minutes to solve the 24-hour temperature field change. The  
 205 computer configuration is Intel(R) Core(TM) i7-4790 CPU @3.60 GHz and RAM 24.0 GB.  
 206 Obviously, this calculation process is extremely time-consuming if the substep size becomes smaller.  
 207 Finally, the thermal analysis<sup>28</sup> is conducted by applying the measured temperature data on the  
 208 refined 3D FEM. The associated longitudinal displacements of the expansion joints are calculated  
 209 and will be compared with measured values.

210 The FEM-calculated longitudinal displacements using estimated LBS mentioned above are  
 211 shown in Figure 10, in which the measurements are also presented for comparison purpose. Although  
 212 the two sets of displacement have similar variation pattern, their magnitudes are different, especially  
 213 the peaks. The discrepancy may be due to: one is that the temperature distribution of the suspension  
 214 bridge is complicated, and a limited number of sensors cannot capture the temperature behavior of  
 215 the whole bridge; the other is that the LBS are calculated based on a simplified model. Therefore, it

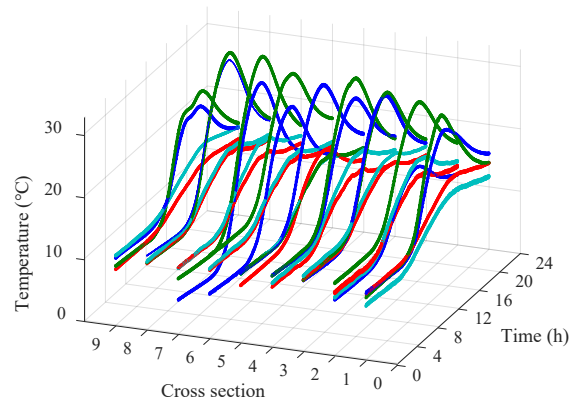
216 is necessary to identify the accurate LBS values by minimizing the discrepancy between the  
 217 FEM-calculated and measured longitudinal displacements. The LBS identification can be achieved  
 218 by FEM updating procedure detailed in the subsequent section.



(a) A refined FEM



(b) Temperature field analysis

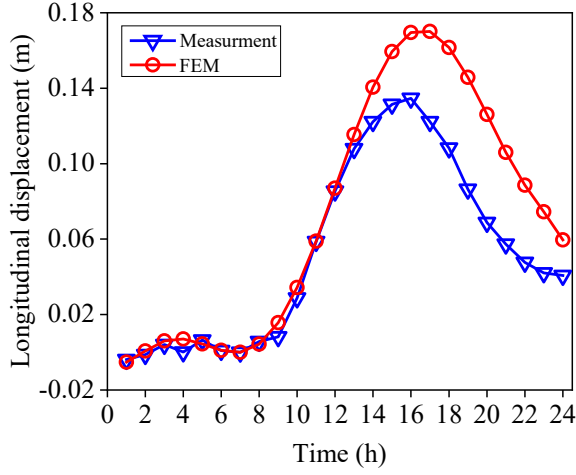


(c) Measured temperature on each cross section

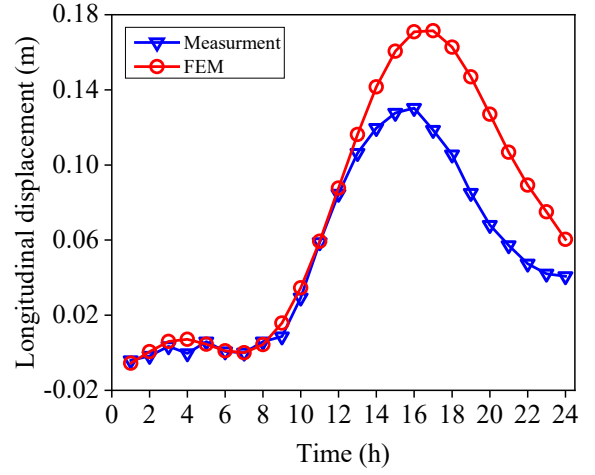
**Figure 9.** Finite element modeling and analysis of the Jiangyin Suspension Bridge.

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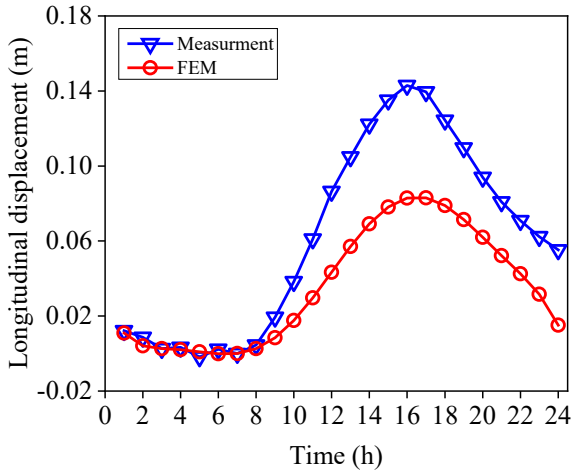




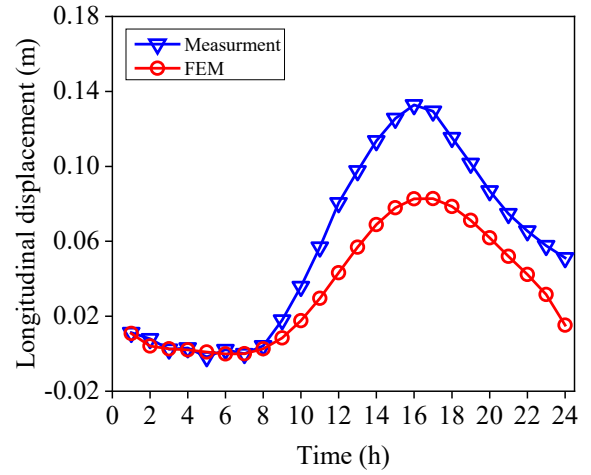
(a) DIS\_NE



(b) DIS\_NW



(c) DIS\_SE



(d) DIS\_SW

**Figure 10.** Calculated and measured longitudinal displacements due to temperature variation.

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## 221 4 LBS identification using metamodel-based model updating

### 222 4.1 Gaussian process metamodel

223 The LBS identification of Jiangyin Suspension Bridge based on the FEM updating suffers from  
 224 the high computational cost. To reduce the computational burden, the GP metamodel is used to map  
 225 the relationship between the LBS and the longitudinal displacements under the thermal effects. GP  
 226 metamodel<sup>29-31</sup> is fully characterized by the mean and covariance functions. In the following, it

considers the zero-mean function and squared exponential covariance function<sup>32</sup> expressed by

$$C(\mathbf{x}, \mathbf{x}') = \eta^2 \exp \left[ -\frac{1}{2} \sum_{k=1}^d \left( \frac{x_k - x'_k}{\ell_k} \right)^2 \right] \quad (7)$$

where  $x_k$  and  $x'_k$  are the  $k$ th component of  $\mathbf{x}$  and  $\mathbf{x}'$ , respectively;  $d$  is the length of  $\mathbf{x}$ ;  $\eta^2$  is the signal variance; and  $\ell$  is the characteristic length scale.

Consider an  $n$ -pair training data set  $\mathcal{D} = \{\mathbf{X}, \mathbf{Y}\}$ , where  $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^n$  and  $\mathbf{Y} = \{y_i\}_{i=1}^n$ , and  $y_*$  denotes the latent function realization to be predicted at an untried point  $\mathbf{x}_*$ . Using the Gaussian prior assumption that the function realizations follow the Gaussian distribution, one has

$$\mathbf{Y} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}) \quad (8)$$

$$\begin{bmatrix} \mathbf{Y} \\ y_* \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{bmatrix} \mathbf{C} & \mathbf{C}_* \\ \mathbf{C}_*^\top & \tilde{C} \end{bmatrix} \right) \quad (9)$$

where  $\mathbf{C} = C(\mathbf{X}, \mathbf{X})$ ;  $\mathbf{C}_* = C(\mathbf{x}_*, \mathbf{X})$ ;  $\tilde{C} = C(\mathbf{x}_*, \mathbf{x}_*)$ ; and  $(\bullet)^\top$  represent the transpose operator.

Applying Bayes' rule, the joint posterior distribution of  $y_*$  conditioned on the training data set  $\mathcal{D}$  is also Gaussian, in the expression of

$$y_* \sim \mathcal{N}(\mu_{y_*}, \sigma_{y_*}^2) \quad (10)$$

with the mean and variance given by

$$\mu_{y_*} = \mathbf{C}_* \mathbf{C}^{-1} \mathbf{Y} \quad (11)$$

$$\sigma_{y_*}^2 = \tilde{C} - \mathbf{C}_*^\top \mathbf{C}^{-1} \mathbf{C}_* \quad (12)$$

Due to the adoption of the zero-mean function, the covariance function parameters  $\Theta = \{\ell_1, \dots, \ell_d, \eta^2\}$ , usually termed as hyperparameters in the machine learning community, completely govern the GP metamodel. In the Bayesian context, the hyperparameters are commonly estimated by maximizing the marginal likelihood of the training data. For computational convenience, the estimation of the hyperparameters is converted to minimization of the negative

242 logarithmic marginal likelihood (NLML). The NLML,  $\mathcal{L}(\Theta)$ , and its partial derivatives with respect  
 243 to the hyperparameters  $\Theta$  have closed-form expressions as follow

$$\mathcal{L}(\Theta) = \frac{1}{2} \mathbf{Y}^\top \mathbf{C}^{-1} \mathbf{Y} + \frac{1}{2} \log |\mathbf{C}| + \frac{n}{2} \log(2\pi) \quad (13)$$

$$\frac{\partial \mathcal{L}(\Theta)}{\partial \Theta_i} = \frac{1}{2} \text{tr} \left( \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \Theta_i} \right) - \frac{1}{2} \mathbf{Y}^\top \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \Theta_i} \mathbf{C}^{-1} \mathbf{Y} \quad (14)$$

244 where  $|\bullet|$ ,  $\text{tr}(\bullet)$ , and  $(\bullet)^\top$  represent the determinant, trace, and transpose operators, respectively.

245

## 246 4.2 Objective function and optimization

247 The LBS identification is performed through the GP metamodel-based model updating. Model  
 248 updating is a process of calibrating structural parameters, so that the calibrated FEM can better  
 249 reflect the observed data <sup>33</sup>. It is essentially a mathematical optimization problem of finding the  
 250 global minimum of an objective function that defines the difference between the analytical and  
 251 experimental responses. In this study, the objective function is formulated as a sum of the relative  
 252 differences between the analytical and measured longitudinal displacements,

$$\Pi(\mathbf{x}) = \sum_{j=1}^k \frac{\|\hat{\mathbf{y}}_j(\mathbf{x}) - \mathbf{y}_j\|_2^2}{\|\mathbf{y}_j\|_2^2} \quad (15)$$

253 where  $\mathbf{x}$  is the LBS to be updated,  $\hat{\mathbf{y}}_j(\mathbf{x})$  represents the GP predicted longitudinal displacement at  
 254 the  $i$ th position;  $\mathbf{y}_j$  is the corresponding measurement; and  $\|\bullet\|_2$  denotes 2-norm operator.

255 Subsequently, the LBS identification can be posed as a constrained minimization problem of  
 256 objective function  $\Pi(\mathbf{x})$ , such that

$$\begin{aligned} \hat{\mathbf{x}} &= \arg \min_{\mathbf{x}} \Pi(\mathbf{x}) \\ \text{s.t. } \underline{\mathbf{x}} &\leq \mathbf{x} \leq \bar{\mathbf{x}} \end{aligned} \quad (16)$$

257 where  $\underline{\mathbf{x}}$  and  $\bar{\mathbf{x}}$  stand for the lower and upper bounds of the LBS, respectively. The whole

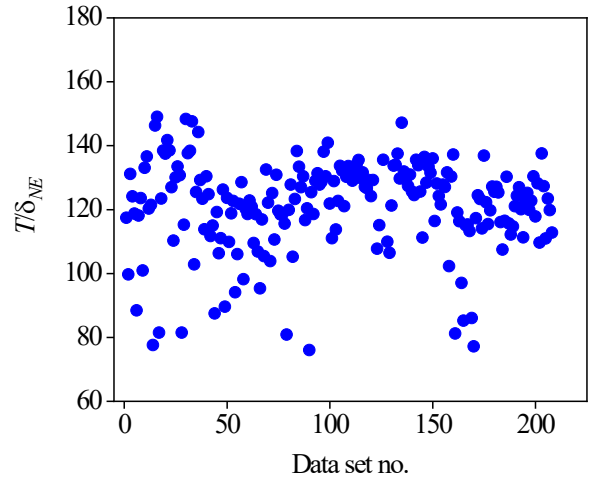
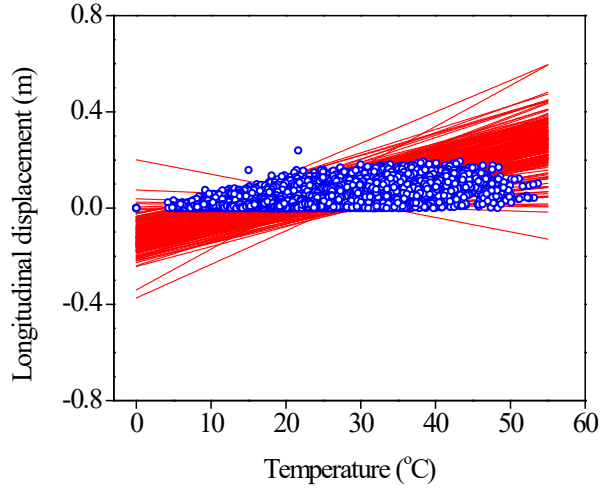
258 optimization process does not involve the computationally demanding FEM owing to the use of the  
259 GP metamodel. As such, the above optimization problem can be readily solved using the available  
260 optimization techniques. Here the numerical software MATLAB is adopted, and *fmincon* function is  
261 used as the optimization solver to search the minimum of the objective function. The resultant  
262 optimal parameters are considered as the LBS of the Jiangyin Bridge.

263

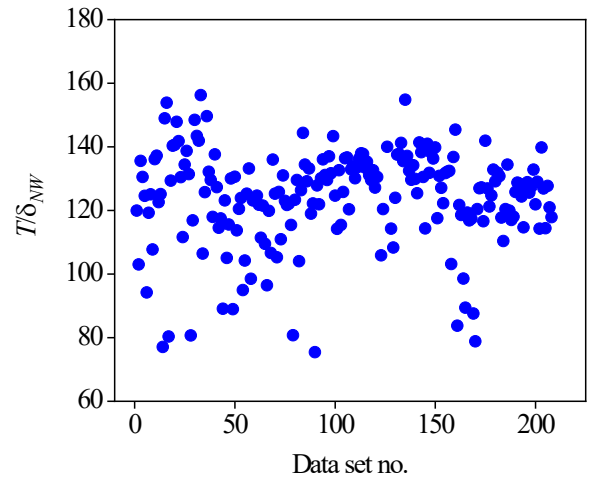
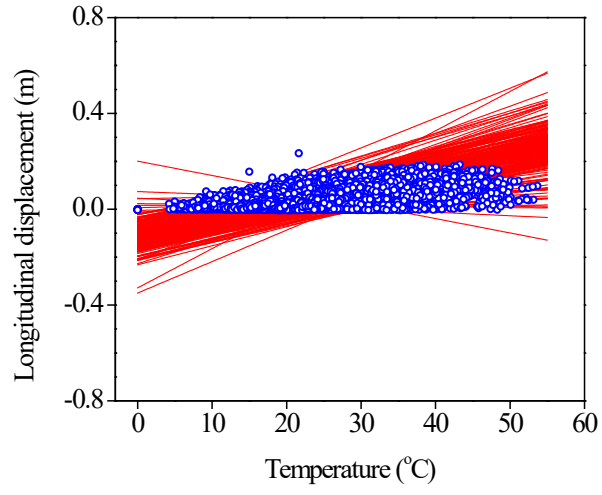
## 264 **5 LBS identification of the Jiangyin Suspension Bridge**

### 265 **5.1 Initial LBS bounds**

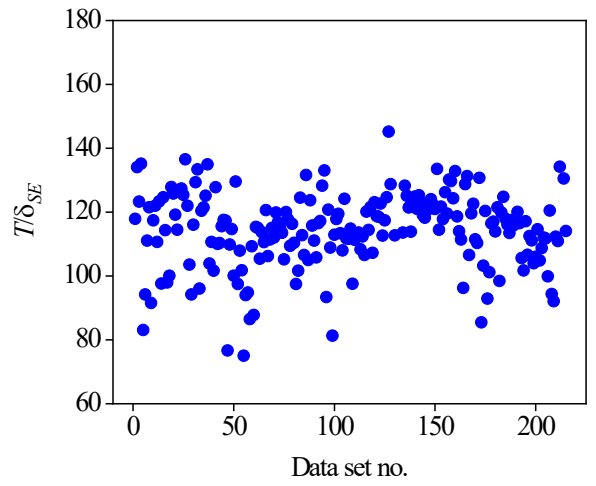
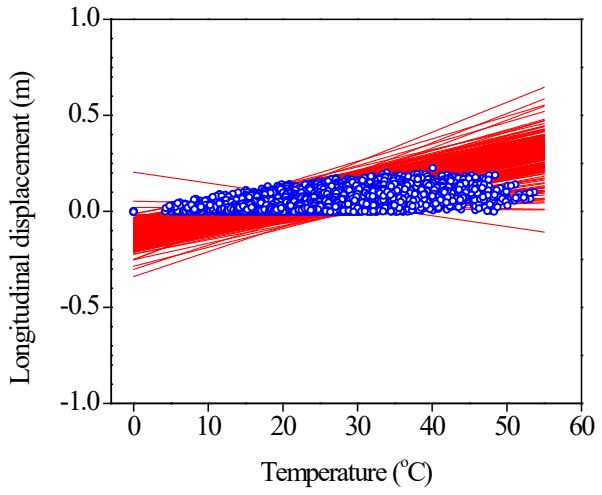
266 One-year monitoring data from 1 Dec 2005 to 30 Nov 2006 is used to determine the initial LBS  
267 bounds. Due to lack of data, only 244 days of data can be used for regression analysis. A total of 244  
268 regression lines are obtained and plotted in the left column of Figure 11. To have a clear picture of all  
269  $\Delta T / \delta$  values, they are plotted in the right plane of Figure 11. From Equations (5) and (6), the  
270 approximate LBS values averaged over 244 values are estimated as  $k_{NE} = 3.5 \times 10^6$  N/m,  
271  $k_{NW} = 7.1 \times 10^6$  N/m,  $k_{SE} = 2.1 \times 10^6$  N/m, and  $k_{SW} = 6.1 \times 10^6$  N/m, respectively. The range bounds of  
272 LBS are determined as  $k_{NE} \in [0, 6.7 \times 10^7]$  ,  $k_{NW} \in [0, 8 \times 10^7]$  ,  $k_{SE} \in [0, 5.2 \times 10^7]$  , and  
273  $k_{SW} \in [0, 6 \times 10^7]$  , respectively. The determined range bounds of LBS will used be set as the lower  
274 and upper bounds of  $\mathbf{x}$  in Equation (16).



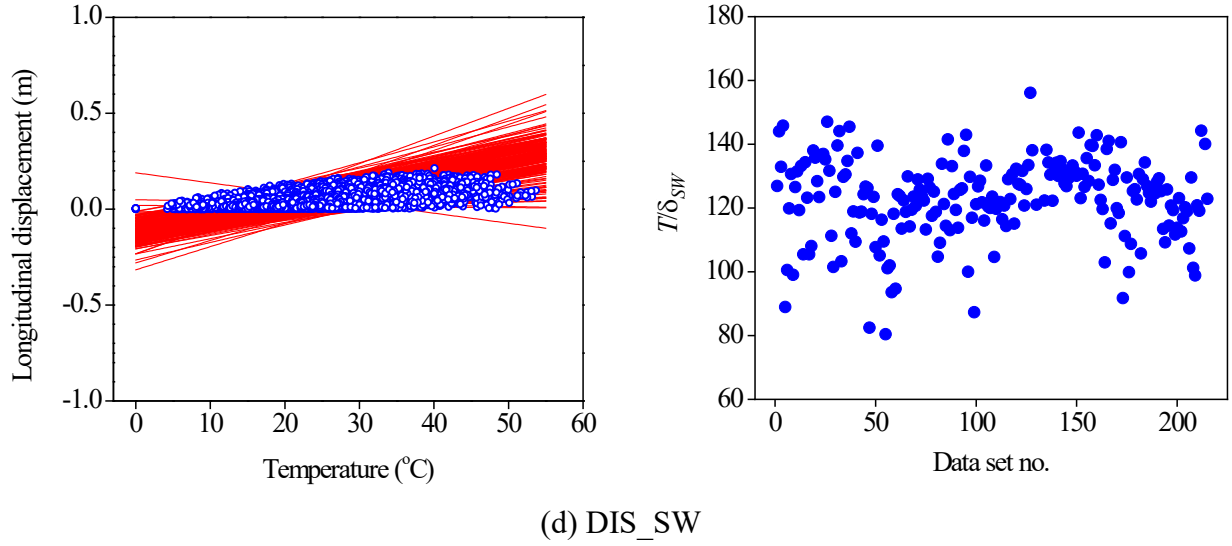
(a) DIS\_NE



(b) DIS\_NW



(c) DIS\_SE



**Figure 11.** Displacement vs. temperature of the bridge deck from 1 Dec 2005 to 30 Nov 2006.

## 5.2 Construction of the GP metamodel

Construction of the GP metamodel consists of training data generation, hyperparameter estimation, and model validation. The specific implementation process is given as follows:

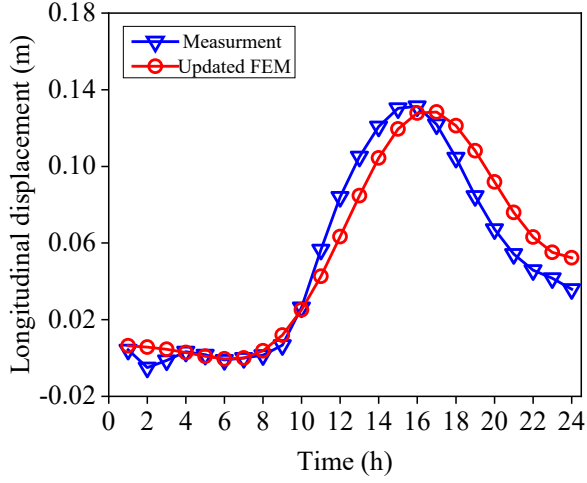
1. The Sobol sequence sampling is adopted for the experimental design to generate input samples since it owns an attractive low-discrepancy feature<sup>34</sup>. The generated Sobol points fall in a unit hypercube and thus need to be transformed into the real physical space of the LBS, that is, the initial bounds of  $k_{NE}$ ,  $k_{NW}$ ,  $k_{SE}$ ,  $k_{SW}$  determined in the above section.
2. The hourly mean temperature monitoring data is then selected as the input loads on the refined FEM. The temperature data are applied to the 3D FEM for thermal analysis to calculate the associated longitudinal displacements. The thermal analysis is repeated for each time instant. Then the longitudinal displacement array is obtained. Then, the generation of training data, which is composed of the LBSs and the obtained longitudinal displacement, is completed.
3. The obtained input-output (LBS and longitudinal displacement) pairs are regarded as training data for construction of the GP metamodel. As mentioned earlier, the hyperparameters

completely determine the GP metamodel. The hyperparameters are estimated from the training data by minimizing the NLML. To avoid getting stuck in the local minima during the optimization, the multi-starting point strategy-assisted optimization solver is utilized to infer the optimal hyperparameters.

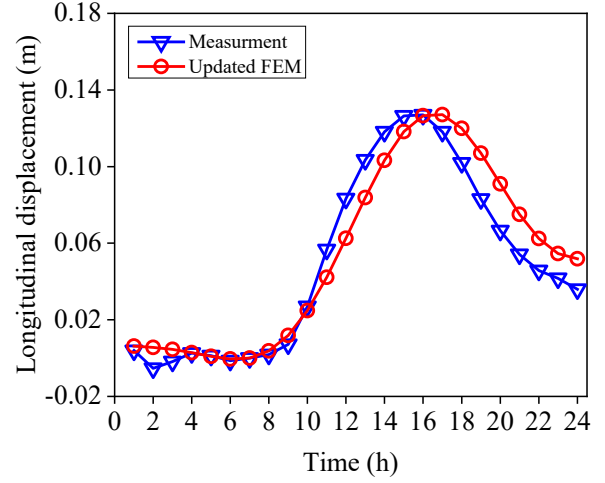
4. The model validation process is conducted to check the accuracy of the constructed GP metamodel using the leave-one-out cross-validation (LOOCV) technique. The validated GP metamodel is used as the surrogate of the FEM for the subsequent LBS identification.

### 5.3 LBS identification

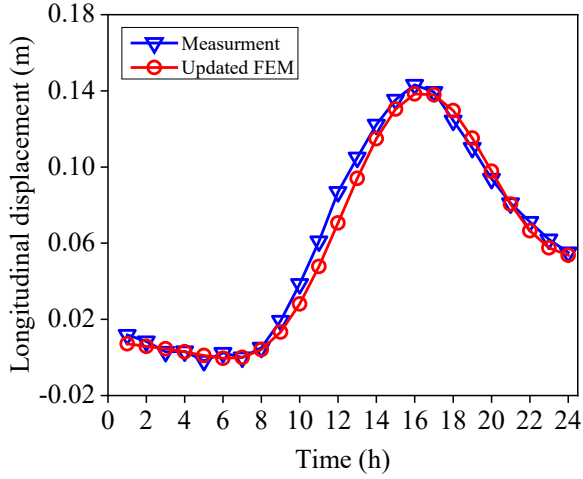
Since the FEM-derived longitudinal displacements are approximated by the constructed GP metamodel, the optimization procedure for identification of the LBS is computationally efficient. Subsequently, the metamodel-based model updating is adopted. Following the implementation procedure detailed in Section 4.2, the optimal LBS is identified by minimizing the difference between the analytical and measured longitudinal displacements defined by Equation (16). The results are  $k_{NE} = 3.37 \times 10^6$  N/m,  $k_{NW} = 5.12 \times 10^6$  N/m,  $k_{SE} = 3.37 \times 10^6$  N/m, and  $k_{SW} = 4.25 \times 10^6$  N/m. The longitudinal displacements on 25 May 2006 are selected to verify the LBS identification results. The comparison of updated FEM-derived and measured longitudinal displacements is shown Figure 12, exhibiting a good agreement.



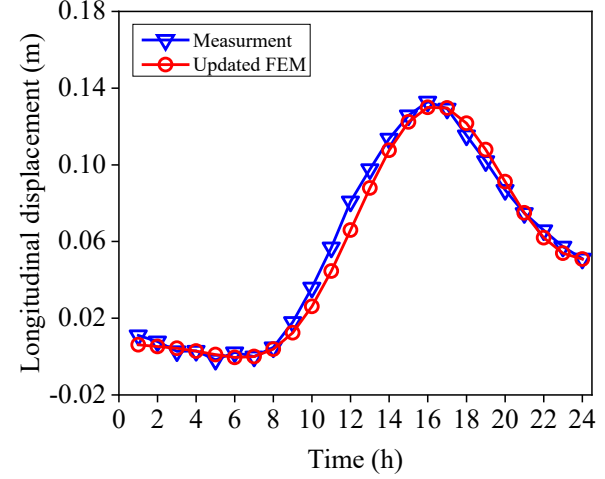
(a) DIS\_NE



(b) DIS\_NW



(c) DIS\_SE



(d) DIS\_SW

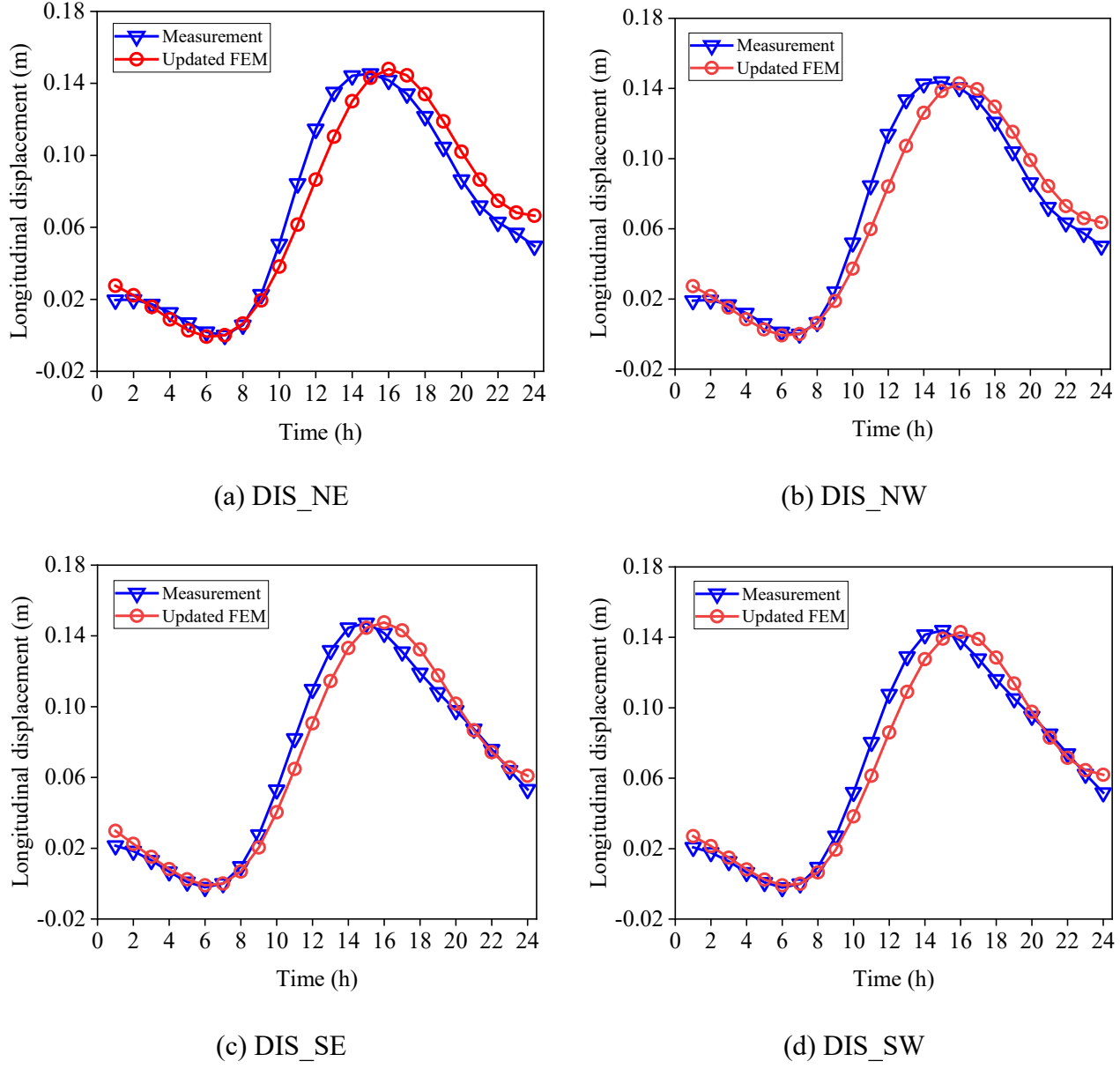
**Figure 12.** FEM-derived and measured longitudinal displacements under temperature loading after calibration.

309

310 Further, the updated LBS values are also applied to one day (20 April 2007) beyond the used  
 311 one-year period. The predicted and measured longitudinal displacements on the day are compared in  
 312 Figure 13, also showing a good agreement. It should be noted that lag between the updated FEM  
 313 lines and the measurement lines shown in Figures 12 and 13 may be because the time delay may  
 314 occur on the temperature and displacement signals. The lag phenomenon was also reported by Guo et



315 al.<sup>15</sup>. The calculated displacement has a little error when temperature rises. This may be because the  
 316 FEM does not consider the wind and traffic loads. In line with above observations, it can be  
 317 concluded that the identified LBS is able to reflect the real condition of the bridge expansion joints.  
 318



**Figure 13.** Assessment of prediction performance of the calibrated FEM using identified LBS.

319  
 320 **6 Conclusions**

321 The bridge expansion joints suffer damage more frequently than other structural components,

322 especially for a long-span bridge. Destructive bridge expansion joints not only cause discomfort in  
323 driving, but also change the mechanical characteristics of the bridge structure. So, it is important to  
324 assess the condition of the bridge expansion joints. However, traditional FEM updating technology  
325 requires a large computational cost and time-consuming. Therefore, this study presents an efficient  
326 metamodel-based model updating method for the LBS identification of a long-span suspension  
327 bridge using the long-term displacement and temperature measurements. The main conclusions of  
328 this study are summarized as follows:

329 (1) Jiangyin Suspension Bridge as an example for LBS research. The temperature and longitudinal  
330 displacement have a linear relationship and the sensitivities of the four longitudinal  
331 displacements to the temperature, determined by the slope of the lines, are almost the same. The  
332 slopes of the lines are not only associated with the structural configuration, but also the stiffness  
333 of the expansion joints, which is equivalent to the LBS.

334 (2) The relationship among the boundary stiffness, structural temperature and longitudinal  
335 displacement is formulated using the thermal effect mechanism. The reasonable LBS bounds are  
336 determined on the one-year monitoring data. A GP metamodel as a surrogate of the refined 3D  
337 FEM for model updating and LBS identification. The calibration results show that there is  
338 improved match between the analytical and measured longitudinal displacements. Moreover, the  
339 prediction performance of the calibrated FEM using the accurate LBS is explored. The predicted  
340 longitudinal displacements also exhibit a good agreement with the measurements. Therefore, it is  
341 concluded that the identified LBS is sufficiently accurate and can reflect the real condition of the  
342 bridge expansion joints. It is verified that the GP metamodel-based model updating method is  
343 effective for LBS identification of long-span bridges.

344

345 **Acknowledgement**

346       This research was financially supported by the National Natural Science Foundation of China  
347 (51878235 and 51778204) and General Research Found (Project No. PolyU 152125/17E) of RGC,  
348 Hong Kong. Dr. Yufeng Zhang of Jiangsu Transportation Institute (JSTI) is greatly appreciated for  
349 providing the monitoring data of the Jiangyin Suspension Bridge.

350

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