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Optimal SVC placement for Maximizing Photovoltaic Hosting Capacity in Distribution Network

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Abstract: It has become a challenging task to accommodate the proliferation of distributed photovoltaic (PV) generation in distribution networks (DN). In order to enhance the PV generation hosting capacity (HC) in DN, we propose a novel optimal static VAR compensator (SVC) placement model in this paper. The objective is to maximize the PV admissibility and minimize investment cost of SVC while maintaining required levels of power quality and system stability. To reduce the complexity of the problem, linearization is utilized to transform the original nonlinear problem into a mix-integer linear program. Finally, the effectiveness of the proposed model is validated using a modified IEEE 37-bus distribution system with excessive PVs.

Keywords: PV generation, hosting capacity, distribution networks, SVC placement, linearization.

NOMENCLATURE

Sets and Indices

i / N	Index/set of distribution nodes.
m	Index of photovoltaic generation units.
$\beta(i)$	Set of child nodes of the node i .
t / T	Index/set of time slots.

Variables

$P_{i,t}$ / $Q_{i,t}$	Active/Reactive power flow through the branch between node $i-1$ and node i .
$q_{i,t}^{SVC}$	Compensation rate of SVC of node i at t .
a_i^{SVC}	Binary decision variable flagging SVC placement of node i .
$V_{i,t}$	Bus voltage of node i at t .
E_m^{PV}	Installed PV generation capacity of unit m .

Parameters

w^{PV}	Weight factor of Installed PV generation capacity.
w^{SVC}	Weight factor of SVC investment cost.
$w^{Penalty}$	Weight factor of penalty cost of slack variables for voltage magnitude.
C_{inv}^{SVC}	Objective function coefficient associated to the SVC investment cost.
M_{inv}^{SVC}	Maximum allowed total SVC investment cost.
δ_t^{PV}	PV power output level (ratio of installled PV generation capacity) at time t .
r_i / x_i	Resistance/Reactance of the branch between node $i-1$ and node i .
$p_{i,t}^d$ / $q_{i,t}^d$	Active/Reactive net load at node i at t .
$\overline{P}_i$ / $\overline{Q}_i$ / $\overline{S}_i$	Upper bound of active/reactive/apparent power flow through the branch between node $i-1$ and node i .
$\underline{V}_i$ / $\overline{V}_i$	Lower/Upper bound of voltage at node i .
$\underline{V}_i^d$ / $\overline{V}_i^d$	Lower/Upper bound of bus voltage deviation at node i .
$\underline{q}_i^{SVC}$ / $\overline{q}_i^{SVC}$	Lower/Upper bound of SVC compensation level.

1. INTRODUCTION

The proliferation of renewable distributed generation (RDG), especially PV generation, is a promising strategy to address the worldwide energy and environmental concerns. The widespread use of PV generation technologies has a lot of benefits such as reducing energy cost and emission, deferring upgrade of transmission network, relieving reliance on fossil fuels (Walling, Saint, Dugan, Burke, & Kojovic, 2008). On the other hand, the overuse of PV generation may disrupt normal operating conditions of the power system, like violations of thermal limits and voltage constraints, due to the lack of advanced control schemes (Agüero, 2012).

PV hosting capacity is defined as the maximum total PV capacity that a distribution network can accommodate without violating operational constraints, especially bus voltage constraints. Various factors could impact PV hosting like PV type, DN characteristics, limiting criteria defined by the DN operator (Ayres, Freitas, De Almeida, & Da Silva, 2010; Baran, Hooshyar, Shen, & Huang, 2012; Smith & Rylander, 2012). Consequently, it is challenging to assess the PV hosting capacity of a distribution network. Simulation based method and optimization based method are most used approaches to quantify the PV hosting capacity. References (Dent, Ochoa, & Harrison, 2010; Dubey, Santoso, & Maitra, 2015; Shayani & de Oliveira, 2011) have presented some approaches for determining maximum PV penetration limit in the DN. To efficiently integrate PV into DN, several methods have been proposed. Reference (Wang, Chen, Ge, & Wu, 2016) evaluates the ability of a distribution system to accommodate DGs and proposes an maximum hosting capacity (MHC) method by modifying operation parameters of existing components. In (Tant, Geth, Six, Tant, & Driesen, 2013), the potential of using battery energy storage systems in the public low-voltage distribution system is investigated to postpone upgrades of what to increase the PV penetration level. In (Morren & de Haan, 2008), a significant increase in allowable integration of DG can be achieved by controlling reactive power compensation level of the DG unit itself. Reference (Capitanescu, Ochoa, Margossian, & Hatziargyriou, 2015) explores how the DG hosting capacity can be improved by means of static and dynamic network reconfiguration and multi-period optimal power flow (MP-OPF)-based method is presented to maximize the DG hosting capacity.

The installation of SVC in the DN is an interesting alternative for solving the aforementioned technical problems. Classical SVC placement studies (Mansour, Xu, Alvarado, & Rinzin, 1993; Mínguez, Milano, Zárate-Miñano, & Conejo, 2007; J. Singh, Singh, & Srivastava, 2007) do not consider the PV penetration in DN. High level of PV integration may bring some problems to the system, especially bus overvoltage problem. The SVC is mainly installed for voltage regulation, which is widely used in traditional distribution system and has an influence on MHC. Therefore, in this work, we endeavor to enhance the PV hosting capacity by optimally plan the SVC location. The latest advance in SVC manufacture and installation renders this approach a promising solution for integrating higher level of PV generation.

Therefore, in this paper, an optimal SVC planning model is proposed for maximizing PV generation hosting capacity in distribution system and minimizing SVC investment cost subject to bus voltage constraints. We formulate a placement model where optimal SVC placement decisions can be acquired by maximizing the installed PV generation capacity with minimum SVC investment cost subject to power flow constraints and voltage constraints. Linearization techniques are employed to transform the original nonlinear program into a mixed integer linear program (MILP). The expected operation scenarios of PV outputs and load demand are developed based on the historical data. Finally, the proposed model is tested on the modified 37-node distribution system to verify its effectiveness.

The rest of this paper is organized as follows. Section 2 gives the mathematical formulation of the deterministic optimization based optimal SVC planning model. Section 3 describes the case study to evaluate the effectiveness of the proposed approach and then followed by the detailed analyses and discussion of results. Finally, concluding remarks are included in Section 4.

2. MATHEMATICAL FORMULATION

In this section, we propose the optimal SVC placement model for maximizing PV hosting capacity. Then, the mathematical formulation is transformed to be a mixed integer linear program to maximize hosting capacity of PV generation.

2.1 Distribution Network Model

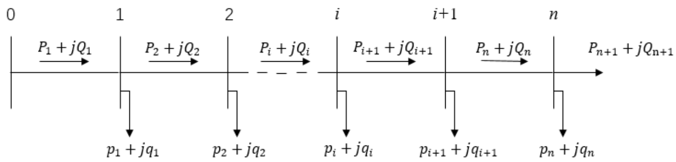

Fig. 1. Diagram of a radial distribution system.

Considering a distribution system as shown in Fig. 1, there are n+1 buses that are indexed by $i = 0,1,2,...,n$. DistFlow equations can be used to model the power flow in a radial distribution network (M. Baran & F. F. Wu, 1989; M. E. Baran & F. F. Wu, 1989b) as shown below.

$$P_{i+1} = P_i - r_i \frac{P_i^2 + Q_i^2}{V_i^2} - p_i, \forall i \in N \tag{1a}$$

$$Q_{i+1} = Q_i - x_i \frac{P_i^2 + Q_i^2}{V_i^2} - q_i, \forall i \in N \tag{1b}$$

$$V_{i+1}^2 = V_i^2 - 2(r_{i+1}P_{i+1} + x_{i+1}Q_{i+1}) + (r_{i+1}^2 + x_{i+1}^2)\frac{P_{i+1}^2 + Q_{i+1}^2}{V_i^2}, \forall i \in N \tag{1c}$$

$$p_i = p_i^d - p_i^g, \forall i \in N \tag{1d}$$

$$q_i = q_i^d - q_i^g, \forall i \in N \tag{1e}$$

where (1a) and (1b) describe the active and reactive power balance at bus i, respectively; (1c) describes the voltage transmit along the branch. Generally, Distflow model (1) is nonconvex. To reduce the complexity, linearized DistFlow model (M. Baran & F. F. Wu, 1989; M. E. Baran & F. F. Wu, 1989a) is employed in this paper. Under normal operation, linearized DistFlow maintains high accuracy and can be formulated as follows,

$$P_{i+1} = P_i - p_i, \forall i \in N \tag{2a}$$

$$Q_{i+1} = Q_i - q_i, \forall i \in N \tag{2b}$$

$$V_{i+1} = V_i - \frac{r_{i+1}P_{i+1} + x_{i+1}Q_{i+1}}{V_0}, \forall i \in N \tag{2c}$$

$$p_i = p_i^d - p_i^g, \forall i \in N \tag{1d}$$

$$q_i = q_i^d - q_i^g, \forall i \in N \tag{1e}$$

2.2 Mathematical Formulation of SVC Placement Problem

In this subsection, the framework of planning model is given. The detailed planning model can be described as follows,

1) Objective function

The focus of this paper is on maximizing the PV hosting capacity of distribution system through optimal SVC placement. The objective is to determine the optimal locations and sizes of SVCs to achieve maximum allowable installed PV generation capacity, and meanwhile satisfy the technical constraints of DN, defined as follows,

$$\text{Max} \sum_{m} E_m^{PV} \tag{3a}$$

$$\text{Min} \sum_{i} C_{inv}^{SVC} a_i^{SVC} \tag{3b}$$

where objective function (3a) is to maximize the installed capacity of PV generation, and objective function (3b) is to minimize the investment cost of SVC.

The maximization problem in (3a) is changed to a minimization problem, and the penalty cost of slack variables for voltage magnitude are added, then combined with (3b) to form a new sum-weight objective function (3c), defined as follows,

$$\begin{aligned}\text{Min} - w^{PV}\sum_{m} E_m^{PV} + w^{SVC}\sum_{i} C_{inv}^{SVC} a_i^{SVC} \\ + w^{Penalty}\sum_{i} C^{Penalty}(\underline{s}_i + \bar{s})\end{aligned} \tag{3c}$$

2) Constraints

$$\begin{aligned}P_{i+1,t} = P_{i,t} - p_{i,t}^{d} + p_{m,t}^{PV}, \forall i \in N, \forall m \in \beta(i), \forall t \in T \\ \text{where } p_{m,t}^{PV} = \delta_t^{PV} E_m^{PV}\end{aligned} \tag{4a}$$

$$Q_{i+1,t} = Q_{i,t} + a_i^{SVC} q_{i,t}^{SVC} - q_{i,t}^{d}, \forall i \in N, \forall t \in T \tag{4b}$$

$$V_{i+1,t} = V_{i,t} - \frac{r_{i+1}P_{i+1,t} + x_{i+1}Q_{i+1,t}}{V_0}, \forall i \in N, \forall t \in T \tag{4c}$$

$$P_{i,t} \le \left|\bar{P}_i\right|, \forall i \in N, \forall t \in T \tag{4d}$$

$$Q_{i,t} \le \left|\bar{Q}_i\right|, \forall i \in N, \forall t \in T \tag{4e}$$

$$\begin{aligned}\underline{V}_i - \underline{s}_i \le V_{i,t} \le \bar{V}_i + \bar{s}_i, \forall i \in N, \forall t \in T \\ \text{where } \underline{s}_i \ge 0, \bar{s}_i \ge 0\end{aligned} \tag{4f}$$

$$\begin{aligned}\underline{V}_i^{d} \le V_{i,t}^{d} \le \bar{V}_i^{d}, \forall i \in N, \forall t \in T \\ \text{where } V_{i,t}^{d} = |V_{i,t} - 1|\end{aligned} \tag{4g}$$

$$\underline{E}_m^{PV} \le E_m^{PV} \le \bar{E}_m^{PV}, \forall m \in \beta(i) \tag{4h}$$

$$\underline{q}_i^{SVC} \le q_{i,t}^{SVC} \le \bar{q}_i^{SVC}, \forall i \in N, \forall t \in T \tag{4i}$$

$$\sum_{i} C_{inv}^{SVC} a_i^{SVC} \le M_{inv}^{SVC}, \forall i \in N \tag{4j}$$

where constraints (4a) and (4c) are linearized DistFlow equations. denotes PV power output, which is the rate of installed capacity of PV generation. Constraint (4d) and (4e) describe the DN branch active and reactive power limitation. Constraints (4f) and (4g) ensure that the voltage magnitude and voltage deviation of each node is within a predefined range. Non-negative slack variables   and   are included in (4f) to relax bus voltage constraints. Constraint (4h) denotes the installed PV generation capacity limitation. Constraint (4i) denotes the installed SVC capacity limitation. Constraint (4j) describes that the total SVC investment cost cannot exceed a predefined limit from practical view.

2.3 Linearization Technique

There is the bilinear term $a_i^{SVC} q_{i,t}^{SVC}$ appearing in the constraint (4b), which renders problem (4) nonconvex. Nevertheless, it can be linearized by introducing an auxiliary variable $z_{i,t}^{SVC}$ since a_i^{SVC} is a binary variable and $q_{i,t}^{SVC}$ is a continuous variable. Hence, $a_i^{SVC} q_{i,t}^{SVC}$ can be replaced by $z_{i,t}^{SVC}$ with the following four inequalities.

$$-a_i^{SVC}\bar{q}_{i,t}^{SVC} + z_{i,t}^{SVC} \le 0, \forall i \in N, \forall t \in T \tag{5a}$$

$$a_i^{SVC}\underline{q}_{i,t}^{SVC} - z_{i,t}^{SVC} \le 0, \forall i \in N, \forall t \in T \tag{5b}$$

$$-a_i^{SVC}\underline{q}_{i,t}^{SVC} + z_{i,t}^{SVC} \le q_{i,t}^{SVC} - \underline{q}_{i,t}^{SVC}, \forall i \in N, \forall t \in T \tag{5c}$$

$$a_i^{SVC}\bar{q}_{i,t}^{SVC} - z_{i,t}^{SVC} \le -q_{i,t}^{SVC} + \bar{q}_{i,t}^{SVC}, \forall i \in N, \forall t \in T \tag{5d}$$

3. CASE STUDY

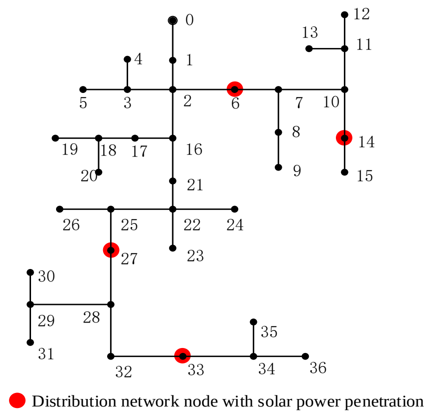

Fig. 2. Modified IEEE 37-bus test distribution system.

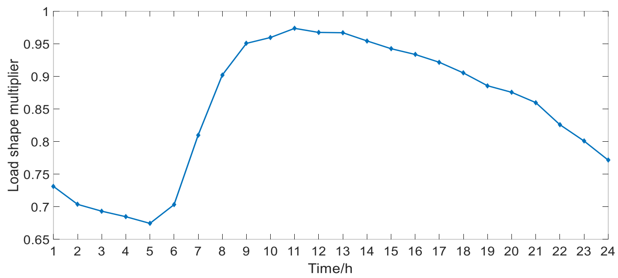

Fig. 3. Load shape multiplier.

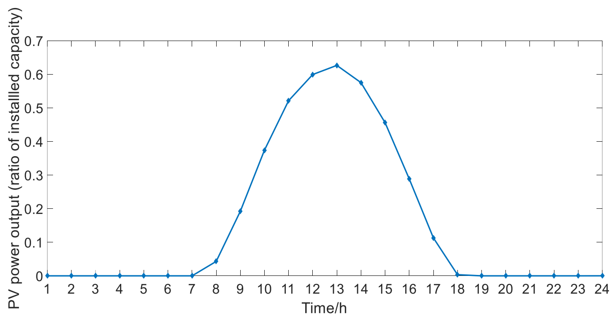

Fig. 4. Solar power output.

In this section, the proposed model is texted on the modified IEEE 37-node distribution system (D. Singh & Verma, 2009) to verity its effectiveness. Fig. 2 shows the test distribution system, which is modified by installing four PV distributed generators at node 4, 10, 20 and 29 respectively. Details about the test system can be found in (D. Singh & Verma, 2009). Fig.3 shows the 24-hour profiles of load shape multipliers for all nodes in this test modified DN. The mean load of each node can be calculated by multiplying the multipliers with the load consumptions of the IEEE 37-node distribution system. The peak load of the system is 2.457 MW and 1.201 MVAr and minimum load is 20% of the peak load. Fig. 4 shows the 24-hour profiles of mean values of solar power outputs for all PV generators located in this 37-node system. The proposed planning model is formulated as a mixed integer linear program (MILP), which is solved using CVX (Grant, Boyd, & Ye, 2008).

3.1 Optimization Results

The SVC placement results are: SVCs should be installed at nodes 6, 7, 8, 9, 10, 11, 23, 24, 26, 29, 33 and 34. Table 1 shows the results of maximum allocation of PV hosting capacity. As seen from the table, the MHC of PV generation in this test distribution network is 0.44 p.u..

Table 1. Results of maximum PV hosting capacity

Location (Node)	PV Size (p.u.)
6	0.012
14	0.006
27	0.013
33	0.013

3.2 Investigation of Voltage Profiles

Fig. 5 demonstrates the detailed information of voltage profiles in the test DN. The mean value of voltage profiles of 24 hours of all nodes is 0.9928 p.u. and the maximum voltage deviation is 0.05 p.u.. This means all voltage profiles are within the standard. Thus, with optimal placement of SVC, PV hosting capacity is maximized while satisfying steady state voltage limits. Fig. 6 gives the voltage profiles of the test distribution system under the condition of maximum PV capacity installation. There are four profiles shown in this figure: 1) voltage profile on node 14 without SVC; 2) voltage profile on node 27 without SVC; 3) voltage profile on node 14 with SVC; 4) voltage profile on node 27 with SVC. After maximum PV integration, the voltage deviation becomes large. With optimal SVC placement, as profiles 3) and 4) shown, the bus voltage magnitudes of 24 hours are within the standard and the largest voltage deviation of these two profiles is less than 0.05 p.u.. However, for profiles 1) and 2), both voltage magnitudes from 8 am to 17pm exceeds the limits and the maximum voltage deviation of these two profiles is quite large in this period. The reason is that PV output is high in the daytime, and without SVC to control voltage profiles, the voltage magnitude exceeds the limits and voltage deviation become quite large.

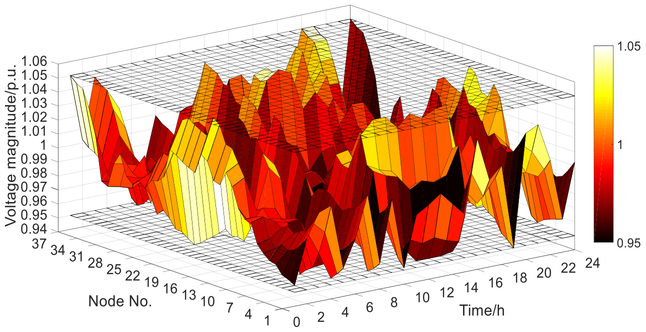

Fig. 5. Voltage magnitude.

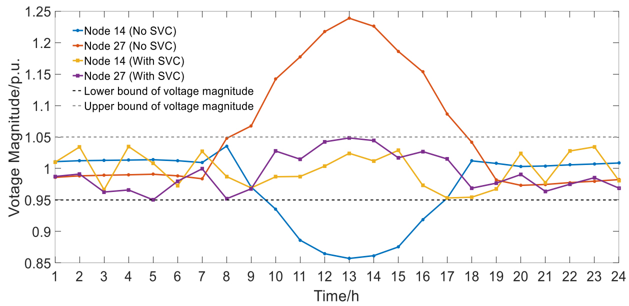

Fig. 6. Voltage profiles comparison.

3.3 Sensitivity Analysis

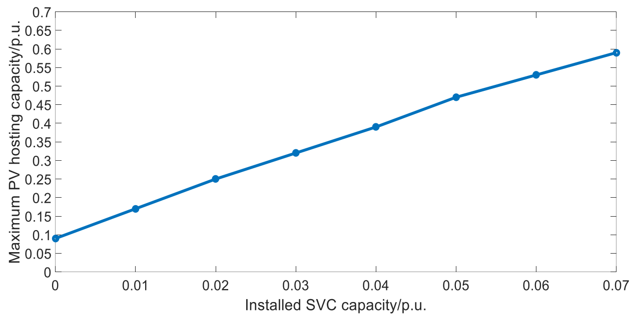

Fig. 7. Impact of installed SVC capacity on maximum PV hosting capacity enhancement (with same installed number).

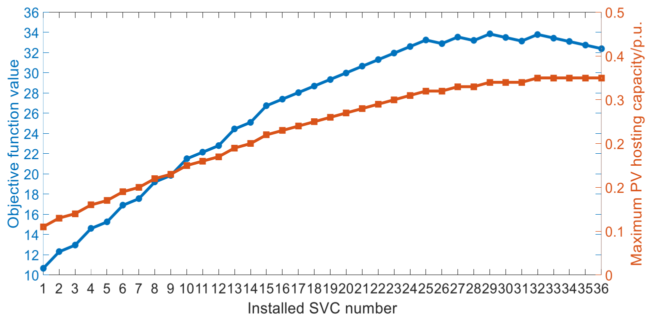

Fig. 8. Impact of installed SVC number on PV hosting capacity enhancement and objective function value.

In order to explore how MHC of PV generation is influenced by the capacity and the number of SVC devices in the DN, a sensitivity analysis of the SVC installation number is performed. Fig. 7 illustrates the impact of installed SVC capacity on the maximum PV hosting capacity. As seen in this figure, the enhancement of maximum PV hosting capacity is achieved with the increased installed capacity of SVC. It should be noted that there is some room for HC improvement even without SVC installed. Fig. 8 illustrates the impact of installed SVC number on the objective function and the maximum PV hosting capacity. It can be seen from this figure that changing the installed SVC number improve maximum PV hosting capacity significantly at first, but the improvement of MHC tends to level off after installing enough well-located SVC in test DN. It can be also observed that with increased SVC installation number, the objective function value shows a rapid increase at first and then a slight decrease as more SVC placed in the DN. This is because when the number of installed SVCs is larger enough, the MHC of PV generation cannot be enhanced significantly but the investment of SVC becomes high, resulting in the decline of objective function value.

4. CONCLUSION

This paper presents a SVC placement approach and a MILP-based solution method to obtain the most appropriate location-allocation of SVCs for maximum allowable PV hosting capacity, while maintaining steady state voltage limits. Given the objective function to maximize total installed distributed PV generation capacity and minimize investment cost of SVC, a proper penalty function is presented to take into account the limiting factor—voltage limits. Simulation results on a modified IEEE 37-bus distribution system verify the effectiveness of the proposed method by revealing that maximum PV hosting capacity can be achieved by optimal location-allocation of SVCs under voltage constraints. To explore the impact of installed SVC number and capacity on MHC, sensitive analysis is conducted, which offers a practical approach to optimally determine SVC placement. The proposed framework can be used to account for other RDG hosting capacity limiting factors, such as voltage deviation constraints. Besides, the results need further investigation by more operation scenarios.

REFERENCES

Agüero, J. R. (2012). *Improving the efficiency of power distribution systems through technical and non-technical losses reduction.* Paper presented at the Transmission and Distribution Conference and Exposition (T&D), 2012 IEEE PES.

Ayres, H., Freitas, W., De Almeida, M., & Da Silva, L. (2010). Method for determining the maximum allowable penetration level of distributed generation without steady-state voltage violations. *IET Generation, Transmission & Distribution, 4*(4), 495-508.

Baran, M., & Wu, F. F. (1989). Optimal sizing of capacitors placed on a radial distribution system. *IEEE Transactions on Power Delivery, 4*(1), 735-743. doi:10.1109/61.19266

Baran, M. E., Hooshyar, H., Shen, Z., & Huang, A. (2012). Accommodating high PV penetration on distribution feeders. *IEEE Transactions on Smart Grid, 3*(2), 1039-1046.

Baran, M. E., & Wu, F. F. (1989a). Network reconfiguration in distribution systems for loss reduction and load balancing. *IEEE Transactions on Power Delivery, 4*(2), 1401-1407. doi:10.1109/61.25627

Baran, M. E., & Wu, F. F. (1989b). Optimal capacitor placement on radial distribution systems. *IEEE Transactions on Power Delivery, 4*(1), 725-734. doi:10.1109/61.19265

Capitanescu, F., Ochoa, L. F., Margossian, H., & Hatziargyriou, N. D. (2015). Assessing the potential of network reconfiguration to improve distributed generation hosting capacity in active distribution systems. *IEEE Transactions on Power Systems, 30*(1), 346-356.

Dent, C. J., Ochoa, L. F., & Harrison, G. P. (2010). Network distributed generation capacity analysis using OPF with voltage step constraints. *IEEE Transactions on Power Systems, 25*(1), 296-304.

Dubey, A., Santoso, S., & Maitra, A. (2015). *Understanding photovoltaic hosting capacity of distribution circuits.* Paper presented at the Power & Energy Society General Meeting, 2015 IEEE.

Grant, M., Boyd, S., & Ye, Y. (2008). CVX: Matlab software for disciplined convex programming. In.

Mansour, Y., Xu, W., Alvarado, F., & Rinzin, C. (1993). *SVC placement using critical modes of voltage instability.* Paper presented at the Power Industry Computer Application Conference, 1993. Conference Proceedings.

Mínguez, R., Milano, F., Zárate-Miñano, R., & Conejo, A. J. (2007). Optimal network placement of SVC devices. *IEEE Transactions on Power Systems, 22*(4), 1851-1860.

Morren, J., & de Haan, S. W. (2008). Maximum penetration level of distributed generation without violating voltage limits.

Shayani, R. A., & de Oliveira, M. A. G. (2011). Photovoltaic generation penetration limits in radial distribution systems. *IEEE Transactions on Power Systems, 26*(3), 1625-1631.

Singh, D., & Verma, K. (2009). Multiobjective optimization for DG planning with load models. *IEEE Transactions on Power Systems, 24*(1), 427-436.

Singh, J., Singh, S., & Srivastava, S. (2007). An approach for optimal placement of static VAr compensators based on reactive power spot price. *IEEE Transactions on Power Systems, 22*(4), 2021-2029.

Smith, J., & Rylander, M. (2012). Stochastic analysis to determine feeder hosting capacity for distributed solar PV. *Electric Power Research Inst., Palo Alto, CA, Tech. Rep, 1026640*.

Tant, J., Geth, F., Six, D., Tant, P., & Driesen, J. (2013). Multiobjective battery storage to improve PV

integration in residential distribution grids. *IEEE Transactions on Sustainable Energy*, *4*(1), 182-191.
Walling, R., Saint, R., Dugan, R. C., Burke, J., & Kojovic, L. A. (2008). Summary of distributed resources impact on power delivery systems. *IEEE Transactions on Power Delivery*, *23*(3), 1636-1644.
Wang, S., Chen, S., Ge, L., & Wu, L. (2016). Distributed generation hosting capacity evaluation for distribution systems considering the robust optimal operation of oltc and svc. *IEEE Transactions on Sustainable Energy*, *7*(3), 1111-1123.