Simple Propagation Model for Nonlinear Fourier Transform (NFT) based Transmission

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Abstract: NFT-based approach for fibre optic transmission was recently proposed, aiming at achieving rate beyond the linear capacity limit. This paper simplifies the signal propagation model and characterises the effects of noises in NFT transmissions.

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1. NFT Transmission - Beyond Linear Capacity Limit

Global fibre optic communication system is reaching its transmission capacity limit. It is based on linear communications systems for which fibre nonlinearities are channel impairments causing interference among different frequency channels at high transmission rate and hence limiting transmission capacity. A new approach has been proposed [2,3], in which nonlinearity is incorporated in the channel model rather than treated as impairment. It is based on designing modulation schemes in Nonlinear Fourier Transform (NFT) domain where independent and non-interfering propagating modes can be defined. Capacity analysis in [4] showed that the approach does not suffer fibre nonlinearity-induced limitation as in conventional systems (where data rate will peak and decay as power increases), suggesting that NFT approach can cope with nonlinearities and has potential to achieve transmission capacity beyond linear capacity limit.

To design an efficient NFT transmission system, it is crucial to characterise the effect of noises in the NFT domain. However, only limited amount preliminary work of the noise effect on continuous and discrete part of the eigenvalues (for general signals) has been reported [1, 5]. In this paper, we address this important problem by deriving a simpler propagation and noise model, which makes analysis simpler. Specifically, we showed that 1) perturbation in NFT discrete eigenvalues can be accurately modelled as an accumulation of *independent* smaller perturbations (induced in each short fibre segment), showing also that the perturbations caused by noise-noise interaction is in fact negligible, 2) each smaller perturbation is significantly dominated by a noise component (whose power is significantly lower than the total noise power). As a result, effects of noise in the NFT domain can be approximated in a much simpler fashion.

Model for Signal Propagation – Signal propagation is often modeled as follows: A fibre is partitioned into M segments. When a signal propagates in a segment, it will undergo two phases: 1) noiseless propagation (subject to only fibre nonlinearity and dispersion), and 2) noise addition (to model the amplification noise). See Fig. 1. To illustrate, we will assume that the input signal is a multi-soliton (which only has discrete eigenvalues) and $\lambda^\dagger$ is one of the discrete eigenvalues. Let $q_m(t)$ be the output (and the input) of the m^{th} (and $m+1^{th}$) segment. Hence, $q_0(t)$ and $q_M(t)$ are respectively the inputs and the output of the channel. In the m^{th} segment, its input $q_{m-1}(t)$ will first propagate without noise and then a noise $n_m(t)$ will be added. The addition of noise will cause the discrete NFT eigenvalues to perturb. Let the discrete eigenvalues at the output of the m^{th} segment be $\lambda_m^\dagger$. This paper simplifies the above model which helps characterise the correlation between discrete eigenvalues of the propagated signal.

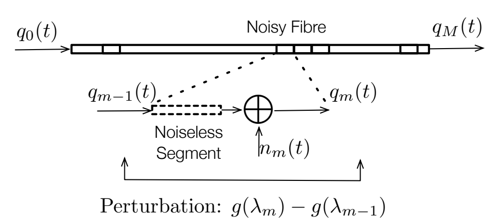

Fig. 1. Signal Propagation Model

Simplification – Strictly speaking, the perturbation of $\lambda_m^\dagger$ depends on both the noise $n_m(t)$ injected at the segment and the input of the segment $q_{m-1}(t)$ which also depends on noises added in all the previous segments (hence the noise-noise interaction). In this paper, we show that *the influence or the level of noise-noise interaction is negligible and hence be ignored*. More specifically, let $\bar{q}_m(t)$ be the output of the signal after propagating through m segments "noiselessly". Hence, $\bar{q}_m(t)$ is a deterministic signal. Without noise, its discrete eigenvalues stay invariant. Next, let $\hat{q}_m(t) = \bar{q}_m(t) + n_m(t)$. In other words, $\hat{q}_m(t)$ is obtained by propagating the input signal $q_0(t)$ for m segments, followed by adding the noise $n_m(t)$ in the m^{th} segment. Let $\hat{\lambda}_m^\dagger$ be the perturbed discrete eigenvalues of $\hat{q}_m(t)$. Clearly, the perturbation is only due to the noise $n_m(t)$ injected in the m^{th} segment, i.e., noise-noise interaction is absent here.

Main Result: *Perturbation of $\lambda_m^\dagger$ is well approximated by "accumulating" all independent small perturbations of eigenvalues in the segments*. More specifically, 1) $\lambda_m^\dagger - \lambda^\dagger \approx \sum_{k=1}^m (\hat{\lambda}_k^\dagger - \lambda^\dagger)$, for $m = 1,\ldots,M$; and 2) The set of random variables $\{\hat{\lambda}_m^\dagger - \lambda^\dagger, m = 1,\ldots,M\}$ is independent. As a consequence of our first main result, to characterise the perturbation of the discrete eigenvalues, it suffices to characterise each individual perturbations $\varepsilon_m^\dagger \triangleq \hat{\lambda}_m^\dagger - \lambda^\dagger$.

Note that perturbation $\varepsilon_m^\dagger$ is due to the addition of the noise $n_m(t)$. Strictly speaking, $n_m(t)$ is modeled as a white noise (which has infinite power) or a bandlimited noise (whose power is proportional to the noise bandwidth. Our next result shows that the noise $n_m(t)$ has a component dominating the perturbation $\varepsilon_m^\dagger$.

Implication: Decompose $n_m(t)$ as the sum $n_m(t) = n_m^{(1)}(t) + n_m^{(2)}(t)$ such that 1) $n_m^{(1)}(t)$ and $n_m^{(2)}(t)$ are orthogonal to each other, and 2) $n_m^{(1)}(t)$ is a scalar multiple of $q_m(t)$. We call $n_m^{(1)}(t)$ the *scaling noise* (as it is a scale multiple of $q_m(t)$) and call $n_m^{(2)}(t)$ the *residual noise*. We observe that 1) the signal power of $n_m^{(1)}(t)$ is significantly smaller than $n_m^{(2)}(t)$; and 2) $\tilde{\lambda}_m^\dagger$ is a good approximation for $\hat{\lambda}_m^\dagger$ where $\tilde{\lambda}_m^\dagger$ is the discrete eigenvalues of $\tilde{q}_m(t) \triangleq \bar{q}_m(t) + n_m^{(1)}(t)$. Consequently, one can approximate the perturbation $\varepsilon_m^\dagger$ as $\tilde{\lambda}_m^\dagger - \lambda^\dagger$, by assuming that the noise injected in each segment is only the scaling noise (which has a much smaller noise power and also lives in a 1 dimensional signal space). As a result, the characterisation of the perturbation in eigenvalues become simpler. In addition, this also explains the correlation between eigenvalues, as they are both significantly affected by the same scaling noise.

2. Simulation

In the simulation, we chose $q_0(t) = 2sech(t)$, which has two discrete eigenvalues at $0.5\mathbf{i}$ and $1.5\mathbf{i}$. The signal will be propagated along the fibre. We plot the (imaginary part of the) discrete eigenvalues (λ_1 and λ_2) of the propagated signal at 10 positions in the fibre to L=0.25π with a step of 0.025π. The scattering plots are shown in Fig. 2. The lower scatter plot (with red circles) corresponds the exact propagation with noise-noise interaction. The upper scatter plot (with blue circles) is obtained using our simplified model where eigenvalue perturbation is modeled as the sum of many small independent perturbations (and hence noise-noise interaction is ignored). Compare the two scatter plots, the perturbations of the eigenvalues look extremely similar with a relative difference of 0.027%, indicating that our model is accurate. Also, if we examine the scattering plots, we notice that the two discrete eigenvalues correlated with each other. This correlation can be largely explained by the fact that perturbations of the eigenvalues are due to the same scaling noise.

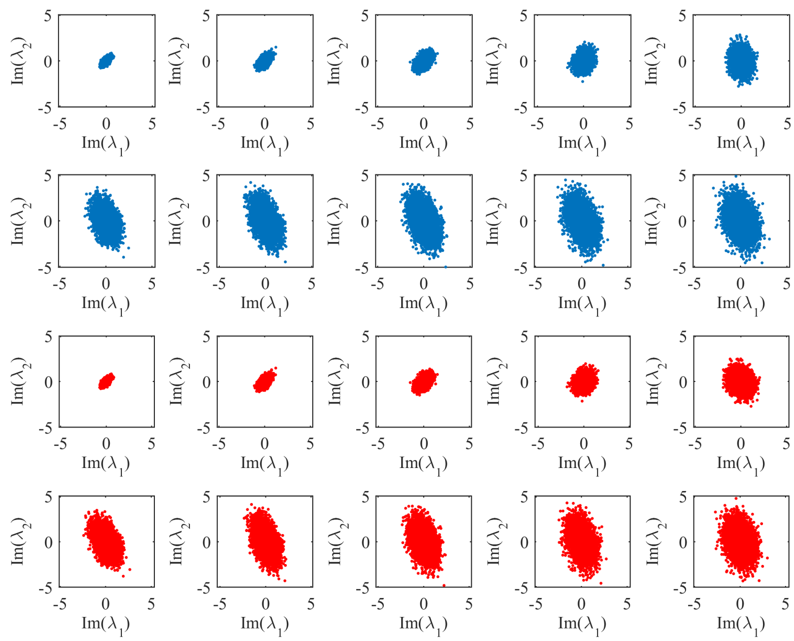

Fig. 2. Perturbation of discrete eigenvalues

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