Signal Processing Techniques for Nonlinear Fourier Transform Systems

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Abstract: Nonlinear fiber-optic transmissions based on Nonlinear Fourier Transform (NFT) is an emerging area in optical communications with more and more experiments demonstrating their potentials. We review recent developments in signal processing techniques for NFT systems.

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Introduction

Kerr nonlinearity is a fundamental capacity-limiting factor in fiber-optic communications. Yousefi and Kschischang [1-3] have revived the theoretical framework of Nonlinear Fourier Transform (NFT) and proposed nonlinear frequency division multiplexing (NFDM) as a new signaling scheme that incorporates soliton theory with communication theory. The NFT of a signal $q(t)$, supported in the interval $[T_1,T_2]$, is defined by solving the differential system

$$\frac{dv}{dt}=\begin{pmatrix}-j\lambda & q(t)\\ -q^*(t) & j\lambda\end{pmatrix}v,\quad v(T_1,\lambda)=\begin{pmatrix}v_1(T_1,\lambda)\\ v_2(T_1,\lambda)\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}e^{-j\lambda T_1} \tag{1}$$

where λ and $v(t,\lambda)$ are, respectively, the eigenvalue and eigenvector. Let

$$\begin{pmatrix}a(\lambda)\\ b(\lambda)\end{pmatrix}=\lim_{t\to\infty}\begin{pmatrix}v_1(t,\lambda)e^{j\lambda t}\\ v_2(t,\lambda)e^{-j\lambda t}\end{pmatrix}. \tag{2}$$

The NFT is a function of λ defined as $q(\lambda)=b(\lambda)/a(\lambda)$ for $\lambda\in\mathbb{R}$ and $q(\lambda)=b(\lambda)/a'(\lambda)$ for $\lambda\in S\subset\mathbb{C}^+$ where the prime denotes differentiation and S is the set of the (isolated) zeros of the analytic function $a(\lambda_{rt})=0$ in the upper half complex plane $\mathbb{C}^+$. Proof-of-concept experimental demonstrations [4-7], has already established a foundation for an increasing community of theoreticians and experimentalists to further improve the transmission performance of NFT systems. A key component driving NFT research is digital signal processing (DSP) techniques specific to NFT systems. In this paper, we will discuss such DSP advances as well as remaining challenges.

Signal Processing for discrete-spectrum modulation systems

Discrete-spectrum modulation systems is a generalization of the well-known soliton transmissions. Discrete-spectrum is composed by eigenvalue ($\lambda_n\in\mathbb{C}^+, n=1,2,\dots,N$) and their spectral amplitude $\hat{q}(\lambda_n)=b(\lambda_n)/a'(\lambda_n)$. For eigenvalue modulated signals, we found that information can also be well-recovered from $a(\lambda)$ of a set of given λ that is not necessarily the roots ($\lambda\neq\lambda_n$) [8] as shown in fig. 1. For certain choice of λ, this decoding method shows even better performance and avoids the cumbersome root-searching process.

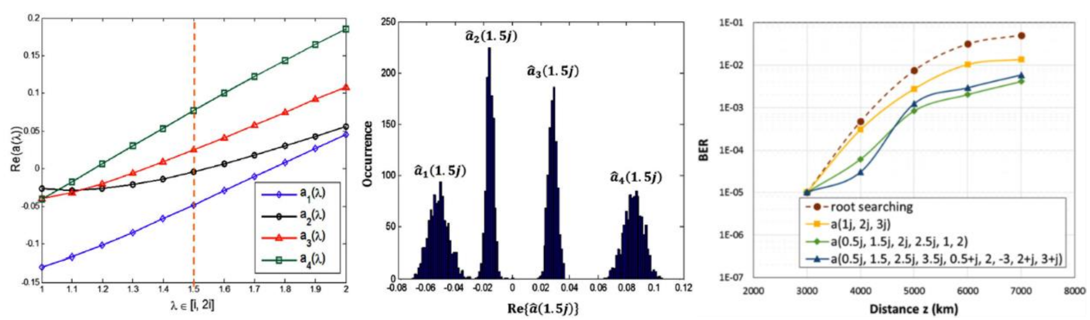

Fig. 1. (a) Re$\{\hat{a}_m(\lambda)\}$ vs. λ for m=1,2,3,4, illustrating the received $\hat{\boldsymbol{a}}(\boldsymbol{\lambda})$ can be used for symbol detection. (b) Empirical distributions of $\hat{a}_m(1.5j)$ from which the means and co-variances of $a_1(\lambda), a_2(\lambda), a_3(\lambda)$ and $a_4(\lambda)$ can be estimated and used for ML detection Signal detection using specific $a(\lambda)$ locations instead of roots [8].

For phase modulation on a single spectral amplitudes (q̂-modulation), a MMSE filter can be derived based on the well-established 1-soliton noise model [3] [9] is proposed to effectively reduce the noise [10]. Also, it can be shown that the information is only presented at $\hat{b}(\lambda)$ as shown in fig. 2, which means that b̂ detection is free from the noise of a′(λ). Therefore, one should just detect and process $\hat{b}(\lambda)$ at the receiver.

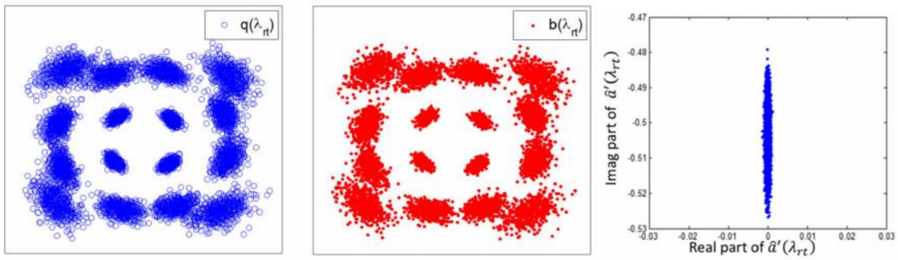

Fig. 2. For phase and amplitude modulation on spectral amplitudes, information is only contained in $\hat{b}(\lambda)$ and a′(λ) only contains noise [8].

Furthermore, we identify significant correlations between the noise of eigenvalue locations λ, $a'(\lambda)$ and $b(\lambda)$. We can make use the noise in eigenvalues $\Delta\lambda$ and $\Delta a'(\lambda)$ respectively and derive a linear minimum mean square error(LMMSE) filter $\mathbf{w}_{amp}^{\boldsymbol{H}}[\Delta\lambda\ \Delta a'(\lambda)]$ and $\mathbf{w}_{phase}^{\boldsymbol{H}}[\Delta\lambda\ \Delta a'(\lambda)]$ to minimize the magnitude and noise of $b(\lambda)$ and tremendously improve detection performance as shown in Fig. 3 [8].

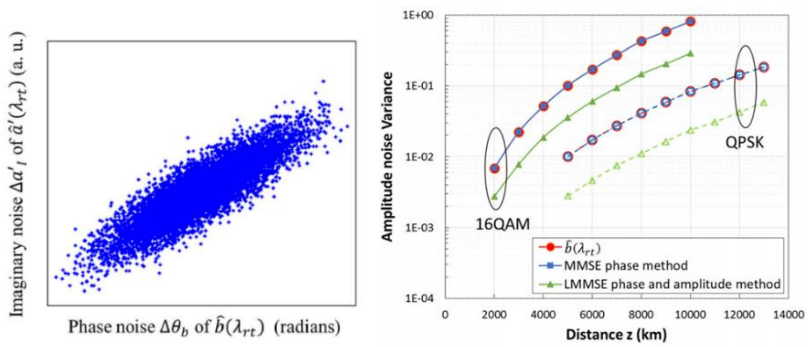

Fig. 3. Exploiting noise correlation between eigenvalue locations, $a'(\lambda)$ and $b(\lambda)$ can tremendously improve transmission performance [8].

We can also modulate both the eigenvalue and the spectral amplitude to achieve higher efficiency for 1-soliton transmissions [11]. In particular, the real part of the eigenvalues can be modulated jointly with spectral amplitude. We implemented a 4-eigenvalue modulated 1-soliton signals with 16-APSK modulation on the spectral amplitude. As the real part of eigenvalue roughly translate into the group velocity of the pulse, we can adjust the spectral magnitude of the signal to offset the pulse drifting and ensure the pulse remains within the NFT time interval at the receiver. The results are shown in Fig. 4.

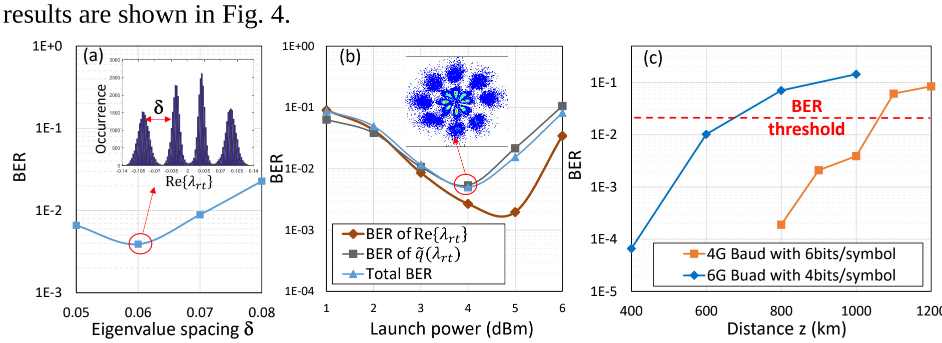

Fig. 4. 4 GBaud 16-APSK transmission over 1000 km with 2-bit eigenvalue modulation from the set $\lambda_{rt} = \{-\frac{3\delta}{2} + 0.3j,\ -\frac{\delta}{2} + 0.3j, \frac{\delta}{2} + 0.3j, \frac{3\delta}{2} + 0.3j\}$ (a) BER versus eigenvalue spacing δ between Re{λ_{rt}} (received eigenvalue distributions shown in inset); (b) signal launched power versus BER; (c) BER comparison between 6 GBaud 16-APSK and 4 GBaud APSK with 2-bit eigenvalue modulation (total 24 Gb/s) system as a function of transmission distance [11].

Signal processing for continuous-spectrum modulation systems

Continuous-spectrum $(q(\lambda) = b(\lambda)/a(\lambda), \lambda \in \mathbb{R})$ modulation has also seen tremendous transmission improvements over the years. A key improvement in DSP for continuous-spectrum systems is the b-modulation scheme proposed and demonstrated in [12-13]. In b-modulation, time-duration of signal is controllable by limiting the duration of inverse Fourier transform of $b(\lambda)$. It is achieved by specific design of $b(\lambda)$ carriers [12-13]. Then, $a(\lambda)$ is calculated from $b(\lambda)$ by

$$a(\lambda) = \exp\{1/2\pi j \int_{-\infty}^{\infty} \log(1 - |b(\lambda')|^2) / (\lambda' - \lambda) d\lambda'\}.$$

And the time-domain signal $q(t)$ can be recovered by INFT from $a(\lambda)$ and $b(\lambda)$. Recently, we have proposed an improved b-modulation method by using flat-top carriers and constellation shaping and the improvement is experimentally demonstrated [14].

It should be noted that, while there are still a lot of work on better DSP for NFT systems, detecting the received waveform based on minimum Euclidean distance and neutral networks are also attempted and it can outperform other NFT detection techniques for certain scenarios[4] [15]. New modulation schemes on GLM-kernel enable information recovery by efficient fast Fourier transform algorithm [16].

Finally, despite the tremendous progress in experimental demonstrations, there is still a large gap from conventional signaling techniques for linear systems as well as from information theoretic predictions [1-3]. This is because of a lack of analytical understanding of the statistical relations between noise of eigenvalue and spectral amplitudes. While the noise analysis for 1-eigenvalue systems are quite well understood, there is only very limited attempts on noise analysis for multi-eigenvalues and continuous spectrum. Some numerical studies have been developed to estimate the noise of scattering vectors [17] and multi-soliton pulses [18]. More complete analytical models on noise characterization for multi-eigenvalues are to be developed.

Conclusions

In this paper, we have provided an overview of developments in DSP techniques for NFT systems. As there is still a large gap between the theoretical promise of NFDM and current experimental results, much remains to be elucidated to fully realize the potentials of NFT systems.

Acknowledgements

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