Precommitments in Two-Sided Market Competition

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Problem definition: We consider a two-sided market competition problem where two platforms such as Uber and Lyft compete on both supply and demand sides and study the impact of precommitments in a variety of practically motivated instruments, on the equilibrium outcomes. ***Methodology/results:*** We investigate multi-stage competition games by starting with a sufficiently low demand uncertainty. First, we show that a precommitment made on the less competitive (demand or supply) side (on price or wage) has a less intense outcome than no commitment (i.e., spot-market price and wage competition). Then we show that somewhat surprisingly, if the competition intensities of both sides are sufficiently close, the commission precommitment, where the platforms first compete in setting their commission rates and then their prices, is less profitable than no precommitment at all, and vice versa. Furthermore, we show that the capacity precommitment, in which the platforms first commit to a matching capacity and then set price and wage simultaneously subject to the precommitted capacity, leads to the most profitable outcome of all competition modes, and extend the celebrated Kreps-Scheinkman equivalency to the two-sided market (without demand uncertainty). Then we extend the comparisons of various competition modes to account for a relatively high demand uncertainty. We show that the comparison between the spot-market price and wage competition and the commission precommitment stays the same as that with a sufficiently low demand uncertainty. In addition, the more flexible competition modes such as no commitment and commission precommitment benefit from higher demand uncertainty (with a fixed mean demand) due to their operational flexibility in response to the market changes. Further, a relatively high demand uncertainty may undermine or enhance the value of the wage precommitment, as opposed to no commitment. Finally, we also account for platforms with asymmetric parameters and matching friction, and find that our main insights tend to be robust. ***Managerial implications:*** Our results caution platforms that a precommitment to the wrong instrument can be worse than no commitment at all. Moreover, the regulation of classifying gig workers as employees, despite many of its benefits to workers, may lead to a less competitive market outcome and, surprisingly, hurt gig workers by paying them lower wages.

1. Introduction

Nowadays, platforms in two-sided markets such as ride-hailing and food delivery, bridging products/service providers and consumers, have more impact on our daily lives than ever. The increasing prevalence of two-sided markets prompts heated competition between platforms. For example, Uber and Lyft, the most popular ride-hailing firms, compete heavily on pricing, attempting to lure customers away from each other by constantly undercutting on price via promotions. Unlike the traditional market where competition focuses on the demand side, two-sided platforms also compete heavily on the supply side as those platforms crowdsource goods and services. For example, Uber and Lyft also compete in setting wages and offering promotions to attract independent contractors.

As a two-sided platform needs to make price and wage decisions on both demand and supply sides, it is natural to make those decisions contingently in the spot market, in order to better react to market conditions. Yet it may be puzzling to see that in a competitive market a two-sided platform may tie its hands by committing to prices or commissions. For instance, the flat-rate pricing strategy launched by Uber at one time allowed passengers to ride for a fixed price, e.g., as low as $2 per ride in San Francisco, regardless of the market conditions.[1] Time-invariant flat fee delivery is the practice in today's on-demand food delivery markets with crowdsourced couriers.[2] Moreover, it is common for platforms to preannounce a fixed commission rate, followed by pricing decisions in the spot market, e.g., Uber and Lyft charge its drivers a commission of 20-25% of the trip fare depending on the stage at which these platforms enter a specific market. DoorDash's commission rate is typically 10-11% of the order total, while that of Uber Eats runs as much as 15%.

On the supply side, platforms may be required by law to make commitments. For instance, in 2019, Assembly Bill 5 (AB5) was signed into law in California, basically requiring gig companies to reclassify their workers as employees, though later Uber, Lyft, and DoorDash got an exemption from the legislation. European countries such as the UK and the Netherlands have also ruled that gig workers must be classified as employees.[3] If classified as employees, gig workers will decide on whether to sign up as employees based on the wage each platform has committed to, and then the platform ccan contingently vary the price charged to consumers based on the spot-market condition.

[1] See https://money.cnn.com/2016/08/25/technology/uber-plus/index.html.

[2] The delivery fee of various food delivery service apps can depend on customers' distance from the restaurant but does not vary across time, see https://urbantastebud.com/best-food-delivery-service-apps/.

[3] See, e.g., https://www.bbc.com/news/business-56123668 and https://www.reuters.com/world/europe/dutch-court-rules-uber-drivers-are-employees-not-contractors-newspaper-2021-09-13/.

Further, in August 2019, New York City announced that a cap on for-hire vehicle licenses, first introduced in 2018, would be extended for one year, to combat congestion and low driver wages (Honan 2019). Yu et al. (2020) point out that the Chinese government also introduced regulatory measures to control the maximum number of registered drivers for DiDi in various tier-1 cities such as Beijing. These regulations suggest that the driver pool size can be adopted by platforms as a precommitment device (see Cachon et al. 2017 and Gurvich et al. 2019 for studies on such an endogenized decision, and Benjaafar et al. 2022 and Chen et al. 2022 for treating it as exogenously given in ride-hailing and food delivery markets respectively).

Given the various possible precommitment devices identified above, we will examine how those precommitments would fare in two-sided competition. We examine preferences from the perspective of the platforms; the drivers, riders, and the social planner would prefer the opposite measures.[4] More specifically, we study a two-sided market competition game in which two platforms compete on both demand and supply sides. We adopt a linear demand system of differentiated services that depends on both platforms' prices. Further, we adopt a linear supply system of differentiated employers that depends on both platforms' wages.[5] (If the platforms crowdsourced goods, the decision on the supply side would be the wholesale price.) The platforms set their own price and wage. Based on the demand and supply systems, the potential demand and supply emerge, and the matching quantity for each platform equals the minimum of its potential demand and supply, which is a unique and inherent feature in two-sided market competition at the operational level (see Hu 2021, Section 3 for justification). Each platform's profit is calculated by multiplying its profit margin per unit (price minus wage) by the matching quantity. The generality of our model makes the insights derived applicable to a general two-sided market, though we may frequently use the ride-hailing market as an illustrative example.

In the base model, we focus on the scenario when demand uncertainty is sufficiently low. This tends to be the case for markets with predictable conditions, such as ride-hailing for some tourist locations, e.g., Hawaii's Big Island during tourist seasons when pleasant weather remains constant. We first consider three types of competition: simultaneous price and wage competition (spot-market competition with no precommitment), wage precommitment competition (followed by price competition), and price precommitment competition (followed by wage competition), all involving

[4] In a competitive two-sided market, the lower the prices and the higher the wages, the better off the drivers, riders, and social welfare are, but the worse off the platforms are. That is, the competitive platforms' profitability goes against the welfare of the riders, drivers, and social planner.

[5] See `https://thecollegeinvestor.com/20641/ultimate-lyft-vs-uber-comparison-drivers-riders/` for differences between ride-hailing platforms Uber and Lyft in the eyes of riders and drivers.

price and wage decisions, but with different or no precommitment devices. In a clean-cut fashion, we show that the effect of precommitments on the equilibrium outcome depends on the comparison of the competition intensities of the two sides. If a precommitment is made on the less (more) competitive side with a smaller (larger) competition intensity, the sequential competition with the precommitment has a less (more) intense equilibrium outcome than simultaneous price and wage competition with no precommitment, i.e., leading to higher (lower) prices and platform profits and lower (higher) wages and matching quantities. In terms of insight, this result can be viewed as a nontrivial generalization of the celebrated Kreps-Scheinkman equivalency (Kreps and Scheinkman 1983), which says the precommitment to capacity (with no competition on the supply side) leads to a less intense equilibrium outcome than price competition. If the platform precommits to a matching quantity (through price or wage) on the less competitive side, such a precommitment alleviates the competition compared with simultaneous two-sided competition. However, if a precommitment is made on the more competitive side, the precommitment leads to a more intense outcome than simultaneous two-sided competition. One immediate implication is that if the labor market is less competitive than the consumer market, the regulation of classifying gig workers as employees so that the mode of the wage precommitment may be sustained, despite many of its benefits to workers, such as health insurance, paid sick days, paid vacation, and retirements, may lead to a less competitive market outcome and, surprisingly, hurt gig workers by paying them lower wages, than the status quo of spot-market competition.

Motivated by the prevalence of a platform having a fixed and preannounced commission rate, we analyze commission precommitment competition, a two-stage game where the platforms compete in setting the commission rate before the realization of the market size and then in setting their price contingent on the realization, and compare it with the types of competitions mentioned above. We find several intriguing results when the demand variance is sufficiently small. First, the leverage of the commission precommitment in influencing the price is indirect, and the commission precommitment is less profitable than precommitment to price or wage on the less competitive side, because the latter could directly lead to a precommitted matching quantity in the first stage, whereas the former fails to do so. On the contrary, the commission precommitment is more profitable than the precommitment to price or wage on the more competitive side. Second, the commission precommitment is more profitable than no precommitment when the competition intensity of one side is sufficiently higher than that of the other. These results may explain the prevalence of the commission precommitment observed in practice. Third, only when the two-sided competition intensities are sufficiently close, does the commission precommitment perform less profitably than no precommitment. This is because the intrinsic desire to balance supply and demand could already effectively impose a constraint on the two-sided competitions when the two sides are about to be equally competitive, whereas in this case, the commission precommitment tips off such a balance.

Then we analyze quantity competition that competes in directly setting the matching quantity before the realization of the market size, with the market-clearing price and wage resolved from the predetermined quantities contingent on the realization. We show that when the demand variance is sufficiently small, quantity competition, as an extension of Cournot competition in a one-sided market to two-sided competition, admits a higher equilibrium price than simultaneous price and wage competition which is the counterpart of Bertrand (price) competition in the one-sided market. This generalizes the well-known insight that Cournot competition is less intense than Bertrand competition (Singh and Vives 1984) to two-sided competition. Moreover, quantity competition beats both price and wage precommitment competitions in terms of profitability, indicating that the direct precommitment to quantity is more effective than the precommitment to either price or wage (thus the quantity commitment also beats the commission commitment). The intuition is that the indirect precommitment to quantity through price or wage is not as direct as the precommitment to quantity itself.

To see how the desirable outcome of quantity competition for the platforms can be achieved, we investigate a two-stage game where the platforms impose a cap on the matching quantity in the first stage before the realization of the market size and then set price and wage simultaneously in a subsequent stage contingent on the realization. We find that when there is no demand uncertainty, this two-stage game yields the same equilibrium outcome as quantity competition, confirming that Kreps-Scheinkman equivalency still holds under two-sided competition.[6] This result implies that the platforms can potentially achieve the *least* competitive market outcome by capping the size of their labor pool, which ironically seems to be the regulatory aim of many governmental agencies.

In an extension, we consider a relatively high demand uncertainty. We show that the comparison between the spot-market price and wage competition and commission precommitment competition stays the same as that with a sufficiently low uncertainty. In addition, we find that more flexible competition modes such as no commitment and the commission precommitment benefit from larger demand variability (with a fixed mean demand) due to their operational flexibility in response to market changes. Further, a larger demand uncertainty may undermine or enhance the (dis)value of

[6] When the demand variance is sufficiently small, the equilibrium of the two-stage game converges to that of quantity competition.

the wage precommitment, as opposed to no commitment. On the one hand, the wage precommitment can tie the hands of platforms in responding to market changes. On the other hand, the wage precommitment has the benefit of constraining the competition on the supply side which could be throat-cutting in the spot market. Finally, a larger market size uncertainty generally undermines the value of the price precommitment.

In the Online Appendix, we also account for platforms with asymmetric parameters and customers' wait time due to the mismatch of supply and demand at the operational level. In general, we find that our main insights tend to hold in those extensions.

2. Literature Review

Our work is closely related to the classic economics literature on oligopoly pricing. It is well established that Cournot (quantity) competition results in higher prices than Bertrand (price) competition for homogeneous or differentially substitutable products. With a linear demand system of duopoly like ours, Singh and Vives (1984) show that quantity competition leads to higher prices and profits than price competition for substitutable products. Vives (1985) extends the result to a setting where firms sell multiple differentiated products with a general demand structure. In contrast, we study price/wage and quantity competition in a two-sided market and show that the precommitment to the less competitive side can alleviate competition, which is neither a derivation nor a simple extension of the results in the economics literature.

Kreps-Scheinkman equivalency establishes a fundamental connection between quantity and price competition. Specifically, Kreps and Scheinkman (1983) show that a Bertrand (price) competition under precommitted capacity yields the same equilibrium outcome as Cournot (quantity) competition. That is, a two-stage game in which the firms first commit to a capacity and then play price competition with the predetermined capacity leads to the same outcome as in Cournot competition. Farahat et al. (2019) confirm that Kreps-Scheinkman equivalency still holds in a differentiated product setting with commonly used demand functions and general spillover models. To expand the whole spectrum from price competition to quantity competition, Vives (1986) considers the impact of flexible vs. inflexible technology; the former allows firms to produce more products than a precommitted capacity quantity. As the firms move from inflexible to flexible technology, the equilibrium price ranges from Cournot price to Bertrand price, because the power of precommitment is weakened by more flexible technology. With demand uncertainty, Afeche et al. (2021) demonstrate that the presence of reorder opportunity may yield larger initial precommitted orders and lower expected profit, thus mitigating the value of the original precommitment. Complementing the literature, we show that when the demand variance is sufficiently small, Kreps-Scheinkman equivalency holds in a two-sided market in which the platforms can commit to a cap on their matching quantity.

Competition in two-sided markets has attracted great interest from both economics and operations management communities. In the economics literature, Rochet and Tirole (2003) study the effect of platform governance on price allocation and end-user surplus in a two-sided market of competing platforms and compare the outcomes with socially optimal ones. Liu et al. (2021) generalize Rochet and Tirole's framework and show that the impact of increased platform competition on the equilibrium transaction fees charged by platforms depends heavily on whether buyers multihome. Caillaud and Jullien (2003) analyze competition between intermediaries who can use sophisticated pricing such as registration and transaction fees. Nikzad (2020) investigates the effects of the size of the labor pool on the equilibrium outcomes in ride-hailing markets. Tan and Zhou (2020) construct a general model (i.e., an arbitrary number of platforms, a more general form of product differentiation, and more than two sides) of price competition between platforms in multi-sided markets to study how the platforms compete for multiple sides of customers and the impact of competition on prices and welfare. In contrast, we follow the economics literature of Kreps-Scheinkman equivalency and study the effect of precommitments in a two-sided competitive market.

In the operations management literature on a competitive ride-hailing market, Cohen and Zhang (2021) study two competing platforms that cooperate and introduce a new joint service to the market. They show that this joint service can benefit all of platforms, riders, and drivers through a properly designed profit-sharing contract. Bernstein et al. (2021) consider a pricing game between two platforms that employ the commission-rate contract with the drivers, and assume both customers and drivers are sensitive to congestion in the system which affects their chance of being matched. The authors consider single-homing where each driver works through a single platform (corresponding to $\beta = 0$ in our setting) and multi-homing where each driver can work through both platforms (corresponding to $\beta > 0$ in our setting), and find that all players are worse off in the multi-homing setting. In order to see whether competing platforms can co-exist profitably, Bai and Tang (2020) examine a similar two-sided competition problem where the competition involves not only price and wage offered by platforms, but also the resulting wait time for customers and utilization for drivers. They identify under what conditions the race-to-the-bottom Bertrand equilibrium would persist and under what conditions both platforms could be profitable. Siddiq and Taylor (2022) examine the effect of the access to autonomous vehicles (AVs) on the platforms that compete on both demand and supply sides. They identify conditions under which platforms' access to AVs, which allows a platform to withdraw from the competitive labor market, reduces platform profit, agent welfare, and social welfare. Benjaafar et al. (2020) consider a model similar to Bernstein et al. (2021) where the two platforms decide simultaneously on the price and wage and then workers decide which platform to serve and simultaneously customers decide which platform to use. They find that whether workers benefit from the competition among on-demand service platforms (i.e., the equilibrium wage is greater than that in the monopoly case) depends on the size of the labor market, whereas the platforms are always worse off with the competition. As a follow-up, Wu et al. (2020) focus on comparing the simultaneous movements of workers and customers vs. their sequential movements, in response to the wages and prices announced by the competing platforms. Chen et al. (2020) investigate the platforms' bonus strategy by analyzing a model that incorporates platforms' two-sided competition in a multi-period setting, where each platform can offer a bonus to service providers who participate consistently. They identify cases in which the platforms will offer a bonus in equilibrium and analyze the impact of bonus offerings on platform profit and social welfare. Ahmadinejad et al. (2020) investigate the possibility of whether competition leads to market failure in the form of the "tragedy of the commons." Finally, Noh et al. (2021) and Daniels and Turcic (2021) study the competition between taxis and a ride-hailing platform. Unlike those papers that mostly examine one mode of competition between platforms or taxis, we analyze a set of competition modes between ride-hailing platforms with various commitment devices in which platforms make decisions sequentially, and focus on the impact of different commitment devices on the equilibrium outcomes.

3. Model

We study a two-sided market competition problem in which two platforms compete on both the supply and demand sides. Platform i ($i = 1, 2$) competes in offering a wage w_i to independent service providers and a price p_i to consumers. On the consumer side, each platform's demand increases in the competitor's price and decreases in its own price. Specifically, we assume the demand of platform i, denoted by $d_i(p_i, p_j)$, follows a (piecewise) linear form:

$$d_i(\boldsymbol{p}) = d_i(p_i, p_j) = [\Omega - p_i + \gamma p_j]^+, \quad j \neq i,\ i = 1, 2, \tag{1}$$

where Ω is a random market size. We assume that the two platforms face exactly the same consumer market size Ω. Such a perfectly positive correlation of the potential market sizes can be driven by weather or a special occasion, which applies equally to the same area where both platforms operate. The parameter $0 \leq \gamma < 1$ is the demand substitution factor that represents the level of service differentiation and competition intensity on the demand side. The extreme case of $\gamma = 0$ corresponds to a situation where the services provided by the two platforms are not substitutable and thus there is no competition on the demand side. On the service provider side, like the demand side, each platform's supply depends on both platforms' wages; in particular, it increases in its own wage and decreases in the competitor's wage. Specifically, we assume the supply of platform i, denoted by $s_i(w_i, w_j)$, also follows a (piecewise) linear form:

$$s_i(\boldsymbol{w}) = s_i(w_i, w_j) = [w_i - \beta w_j]^+, \quad j \neq i, \, i = 1, 2, \tag{2}$$

where $0 \leq \beta < 1$ is the supply substitution factor and measures the competition intensity of two platforms in the labor market. Likewise, $\beta = 0$ means there is no competition on the supply side.

Platform i earns a profit margin of $p_i - w_i$ for each unit of matching between supply and demand. Hence, its profit denoted by $\pi_i(\boldsymbol{w}, \boldsymbol{p})$ can be written as

$$\pi_i(\boldsymbol{w}, \boldsymbol{p}) = (p_i - w_i) \min\{d_i(p_i, p_j), s_i(w_i, w_j)\}, \quad j \neq i, \, i = 1, 2. \tag{3}$$

For all the types of competition analyzed in this paper, if supply is less than demand for a platform, the limited supply will be randomly allocated among those who demand it and the unsatisfied demand will be lost. Similarly, if demand is less than supply for a platform, the limited demand will be randomly allocated among those who supply it and the extra supply will be wasted. This is the same *proportional rationing* rule adopted in Kreps and Scheinkman (1983).

Next, we analyze and compare the following set of competition modes.

- Simultaneous price and wage competition in the spot market (mode P). This mode resembles the spot-market competition without any precommitment and extends the classic price competition in a one-sided (mostly the demand side) market, by letting platforms decide on both price and wage simultaneously after the realization of the market size.

- Wage precommitment (mode wp). By competing on wage in the first stage before the realization of the market size, the platforms precommit to a supply quantity generated according to the supply functions (2), which is the upper bound on how much each platform can ultimately sell to consumers. In the subsequent stage, with the knowledge of the market size realization and the supply quantity of each platform, the platforms compete on price and then the demand is realized according to (1). This mode could become prevalent soon, since many countries and jurisdictions are passing legislation reclassifying gig workers as employees (see Hu et al. 2022).

- Price precommitment (mode pw). The platforms compete on price in the first stage before the realization of the market size. In the subsequent stage, with the knowledge of the market size realization and thus the demand quantity of each platform generated according to the demand function (1), the platforms compete on wage and then the supply is realized according to (2). This mode is motivated by flat-rate pricing observed in the ride-hailing and on-demand food delivery markets.

- Commission precommitment (mode C). In this mode, the platforms compete on the commission rate first before the realization of the market size and then on the price in the second stage after the realization of the market size, as the wage can be derived from the price and the precommitted commission rate. This mode is motivated by the commonly observed flat commission rate, a unique feature of a two-sided market (see Hu and Zhou 2020).

Next, we first derive the equilibrium of different modes in Sections 3.1-3.4. For each mode $X \in \{P, wp, pw, C, Q\}$ (mode Q will be introduced in §4.1), we use $(p_X^*, w_X^*, d_X^*, s_X^*, z_X^*, \pi_X^*)$ to denote the symmetric equilibrium price, wage, demand quantity, supply quantity, matching quantity, and profit level for each firm, respectively. Then we analyze how the precommitments fare against the competition with no commitment by assuming a sufficiently low demand uncertainty in Section 4, and finally investigate the impact of a general and possibly high market size uncertainty on the performances of different precommitments in Section 5.

3.1. Simultaneous Price and Wage Competition: No Commitment

As described earlier, simultaneous price and wage competition is a one-shot game with no commitment on any instrument in which, both firms decide on the price and wage simultaneously contingent on the realization of the market size. In this mode, for any fixed price and wage decisions of its competitor, each firm will make its own decisions such that its demand equals its supply quantity; otherwise, the firm could improve its profit by either raising the price or lowering the wage. This offers the best response by a firm given the other firm's decisions on price and wage. Solving the set of best-response functions yields the following equilibrium.

Lemma 1. *For any realized market size x, the simultaneous price and wage competition admits a unique equilibrium of prices and wages, which is symmetric for any firm:*

$$p_P^* = \frac{3-2\beta}{4-3\beta-3\gamma+2\beta\gamma}x, \quad w_P^* = \frac{1}{4-3\beta-3\gamma+2\beta\gamma}x,$$

and the resulting equilibrium matching quantity and platform's profit level for any firm are:

$$z_P^* = \frac{1-\beta}{4-3\beta-3\gamma+2\beta\gamma}x, \quad \pi_P^* = \frac{2(1-\beta)^2}{(4-3\beta-3\gamma+2\beta\gamma)^2}x^2.$$

Therefore, the expected equilibrium profit in this mode is simply

$$E[\pi_P^*] = \frac{2(1-\beta)^2}{(4-3\beta-3\gamma+2\beta\gamma)^2}E[\Omega^2], \tag{4}$$

which holds regardless of the distribution of the market size.

It can be readily verified that $p_P^* > w_P^*$ and $d_P^* = s_P^* = z_P^* > 0$ for any fixed x. That is, the platforms will keep a positive profit margin for themselves, and not surprisingly, the platforms will set price and wage in the spot market such that demand and supply can be perfectly matched. Moreover, given that the expected profit $E[\pi_P^*]$ is proportional to the second moment of the potential market size $E[\Omega^2] = (E[\Omega])^2 + \text{Var}[\Omega]$, it is implied that the more variable the market is, given its fixed expectation, the higher a platform's expected profit is. The reason is that the platform's profit function given a market-size realization is convex in the market size, indicating that the profit gain from an increase in the market size is greater than the profit loss from the same amount of decrease in the market size.

3.2. Wage Precommitment Competition

For any pair of precommitted wages $\boldsymbol{w}$, after the market size is realized, the best-response price of each platform takes the form of either a supply-depletion price or a profit-maximizing price. For a low wage (the resulting small precommitted supply quantity) and a high realized market size, the platform would at best choose the supply-depletion price that clears its entire supply. For a high wage (the resulting ample precommitted supply quantity) and a low realized market size, the profit-maximizing price would be the best choice. To simplify the analysis, we assume that the two firms take symmetric actions. We find that in equilibrium, the wages $\boldsymbol{w}$ are set either low such that both platforms adopt the supply-depletion price for any market size or intermediate such that both adopt the profit-maximizing price for market-size realizations below a threshold, and the supply-depletion price otherwise. Intuitively, if high wages are set in the first stage such that both firms adopt the profit-maximizing price for any market size, then each firm has an incentive to reduce the wage and increase the profit margin. The detailed analysis is relegated to Online Supplement B.

Although the explicit form of the equilibrium under a general market size uncertainty is too complicate to derive, we find that when the variance of the market size is sufficiently small, the equilibrium wages $\boldsymbol{w}$ in mode wp are set such that both platforms adopt the supply-depletion price for any market size realization. Then, we are able to move back to the first stage, analyze the wage decisions, and derive the first-stage equilibrium.

Lemma 2. *Suppose the demand variance is sufficiently small. For any realized market size x, the wage precommitment competition admits a unique sub-game perfect equilibrium of price in the second stage and wage in the first stage, which is symmetric for any firm:*

$$p_{wp}^* = \frac{(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)x-(1-\beta)(1+\gamma)E[\Omega]}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)}, \tag{5}$$

$$w_{wp}^* = \frac{1+\gamma}{4-3\beta+\gamma-2\gamma^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2}E[\Omega], \tag{6}$$

such that the supply is equal to the demand. As a result, the expected equilibrium profit is

$$E[\pi_{wp}^*] = \frac{(1-\beta)^2(2-\gamma^2-\beta\gamma)(1+\gamma)}{(1-\gamma)(4-3\beta+\gamma-2\beta\gamma-2\gamma^2+\beta^2\gamma+\beta\gamma^2)^2}(E[\Omega])^2. \tag{7}$$

3.3. Price Precommitment Competition

For any pair of precommitted prices $\boldsymbol{p}$, after the market size is realized, the best-response wage of each platform takes the form of either a demand-depletion wage or a profit-maximizing wage. To simplify the analysis, we assume that the two firms take symmetric actions. We find that in equilibrium, the prices $\boldsymbol{p}$ are set either high such that both platforms adopt the demand-depletion wage for any market size or intermediate such that both adopt the profit-maximizing wage for market-size realizations above a threshold, and the demand-depletion wage otherwise. Intuitively, if low prices are set in the first stage such that both firms adopt the profit-maximizing wage for any market size, then each firm has an incentive to raise the price and increase the profit margin. The detailed analysis is relegated to Online Supplement C.

Similarly, when the variance of the market size is sufficiently small, the equilibrium prices $\boldsymbol{p}$ in mode *pw* are set such that both platforms adopt the demand-depletion wage for any market size realization. Then, we are able to move back to the first stage, analyze the price decisions, and derive the first-stage equilibrium.

Lemma 3. *Suppose the demand variance is sufficiently small. For any realized market size x, the price precommitment competition admits a unique sub-game perfect equilibrium of wage in the second stage and price in the first stage, which is symmetric for any firm:*

$$p_{pw}^* = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2}E[\Omega],$$

$$w_{pw}^* = \frac{(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)x-(1-\gamma)(3+\beta-\beta^2-\beta\gamma)E[\Omega]}{(1-\beta)(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)},$$

such that the supply is equal to the demand. As a result, the expected equilibrium profit is

$$E[\pi_{pw}^*] = \frac{(1-\beta^2)(2-\beta^2-\beta\gamma)}{(4-3\gamma+\beta-2\beta\gamma-2\beta^2+\beta^2\gamma+\beta\gamma^2)^2}(E[\Omega])^2 + \frac{(E[\Omega])^2-E[\Omega^2]}{1-\beta}. \tag{8}$$

3.4. Commission Precommitment Competition

The commission rate contract is widely used across many two-sided platforms. Uber, for instance, designates a commission rate of 20-25% and passes 75-80% of each ride fare to its drivers. Airbnb charges its hosts a host service fee of 3% for each reservation. The commission rate is a strategic decision to which the platform precommits at least for a period of time. We study the commission precommitment competition, a two-stage competition game, in which the platforms compete on the commission rate before the realization of the market size and then on the price in the second stage contingent on the realization (with the wage derived from the price and the precommitted payout ratio). If the payout ratio to the suppliers is denoted by α_i, the commission rate of the platform is $1-\alpha_i$. That is, for each unit of matching quantity, the service provider receives $w_i = \alpha_i p_i$ and the platform retains $(1-\alpha_i)p_i$. Denote by $\boldsymbol{\alpha}$ the payout ratio vector. Therefore, the profit of platform i can be written as

$$\pi_i(\boldsymbol{\alpha}, \boldsymbol{p}) = (1-\alpha_i)p_i \min\{d_i(p_i, p_j), s_i(\alpha_i p_i, \alpha_j p_j)\}, \quad j \neq i, i = 1, 2.$$

We first show that for any fixed commission rate $\boldsymbol{\alpha}$ and realized market size x, it is optimal for firms to set the equilibrium price such that the demand equals the supply quantity in the second stage (see Lemma D.1 in Online Appendix D). We also observe that the optimal price in the second stage decreases in $\boldsymbol{\alpha}$ committed by the platforms in the first stage. Given a lower commission rate in the first stage, suppose that a lower price is set in the second stage, which then induces a higher demand. However, the lower price and commission rate produce a lower wage, leading to a lower supply, which cannot match the higher demand. Hence, a higher price must be set in the second stage.

Given that the supply at optimality equates to demand, the price and thus the platform profit can be expressed as the function of $\boldsymbol{\alpha}$, from which we obtain the optimal commission rate as a function of the other firm's decision on its commission rate. Solving the set of equations yields the equilibrium. For the sake of tractability, we focus on the symmetric solution.

LEMMA 4. *For any realized market size x, the commission-rate precommitment competition admits a unique symmetric sub-game perfect equilibrium of price in the second stage and payout ratio in the first stage:*

$$p_C^* = \frac{1}{1-\gamma+(1-\beta)\alpha^*}x, \tag{9}$$

$$\alpha^* = \frac{1}{2(1-\beta)[3+(1-\gamma)\beta-\beta^2]}\Big\{\beta(3-\gamma^2) - \beta^2(\gamma+1) - (1-\gamma)(2+2\gamma+\beta) + \sqrt{(1+\gamma)[\beta^4(1+\gamma) - 2\beta^3\gamma(3-\gamma) + \beta^2\gamma(12-11\gamma+\gamma^2) - 4\beta(1-\gamma)(4+2\gamma-\gamma^2) + 4(4-4\gamma-\gamma^2+\gamma^3)]}\Big\}.$$

As a result, the expected equilibrium profit level for any firm is:

$$E[\pi_C^*] = \frac{\alpha^*(1-\beta)(1-\alpha^*)}{(1+\alpha^*-\gamma-\beta\alpha^*)^2}E[\Omega^2]. \tag{10}$$

Like the contingent simultaneous price and wage competition (see (4)), the expected profit is proportional to the second moment of the potential market size $E[\Omega^2]$. Hence, the firms' profitability increases in the variance of the market size uncertainty. This is because, although the commission rates are announced before the realization of the market size uncertainty, such a precommitment still gives the firms the flexibility to react to the market size uncertainty in the subsequent stage through their contingent pricing decisions.

4. Comparisons of Various Modes with a Sufficiently Low Demand Uncertainty

This section compares the various competition modes to examine the performance of precommitments in different instruments when there is a sufficiently low demand uncertainty. To simplify notation, we make the following definition.

DEFINITION 1. We say that the platforms prefer competition mode X over Y, denoted by $X \succeq Y$, if X results in higher expected prices and platform profits, i.e., $E[p_X^*] \geq E[p_Y^*]$, $E[\pi_X^*] \geq E[\pi_Y^*]$, and lower expected wages and matching quantities, i.e., $E[w_X^*] \leq E[w_Y^*]$, $E[z_X^*] \leq E[z_Y^*]$.

We note that the preference $\succeq$ is from the perspective of the platforms. With the linear demand and supply systems possibly derived from a representative consumer and driver maximizing a quadratic utility function, respectively, lower prices mean higher rider welfare, and higher wages mean higher driver surplus.

LEMMA 5. *For any realized market size, consumer surplus, service provider surplus, and social welfare increase in the total matching quantity.*

Lemma 5 implies that the opposite of $\succeq$ would be preferred by the riders, drivers, and the social planner with the objective of maximizing the total social welfare which moves in the same direction as the total matching quantity. In the rest of the paper, we stand in the platforms' perspective, but note that the opposite preference would hold for the riders, drivers, and the social planner. If the platforms have an alternative preference from maximizing profitability, such as maximizing social welfare, matching efficiency/quantity, consumer surplus, or driver surplus, their preferences in various modes would be the opposite of $\succeq$.

We first compare simultaneous price and wage competition, wage precommitment competition, and price precommitment competition, all of which involve price and wage decisions but in a different sequence of decision-making.

PROPOSITION 1 (COMPARISON OF MODES P, wp, pw).
Suppose the demand variance is sufficiently small.

(a) If the demand side is more competitive than the supply side (i.e., $\gamma \geq \beta$), the preference ranking by the platform is: $wp \succeq P \succeq pw$.

(b) If the supply side is more competitive than the demand side (i.e., $\gamma < \beta$), the preference ranking by the platform is: $pw \succeq P \succeq wp$.

Proposition 1 shows that the ranking of the three closely related competition modes depends on the comparison of the competition intensities of the two sides. If the demand side is more competitive, the wage precommitment competition leads to higher equilibrium prices and profit levels than simultaneous price and wage competition, which in turn leads to higher equilibrium prices and profit levels than the price precommitment competition. The rankings are reversed if the supply side is more competitive. In other words, if the decisions of the less competitive side are made earlier and those of the more competitive side are delayed to a later point, the sequential competition has a less intense market outcome than simultaneous price and wage competition.

The intuition can be explained as follows. When the platform precommits matching quantity through wage or price (when the demand variance is sufficiently small) on the less competitive side, the precommitment alleviates the competition compared with simultaneous two-sided competition. In the extreme case, on the supply side, there is no wage-sensitive supply and the wage is a constant, which can be viewed as no competition at all. By precommitting quantities in a sequential competition with price competition in the second stage, the equilibrium outcome of quantity competition is less intense compared with price competition, in view of the celebrated results of Kreps-Scheinkman equivalency and Singh and Vives (1984). In contrast, if the decisions on the more competitive side are made in the first stage, the precommitment leads to a more intense market outcome than simultaneous price and wage competition. When the price and wage are jointly determined, the less competitive side would constrain the competition intensity of the more competitive side, as both sides of supply and demand would be balanced in equilibrium. However, such a balancing constraint is absent in the first stage when the more intense decisions are being made.

Proposition 1 essentially generalizes the intuition from Kreps and Scheinkman (1983) and Singh and Vives (1984) to the setting where the supply is crowdsourced and elastic, and deepens our understanding of the two-sided market. However, Proposition 1 is neither a derivation nor a simple extension of the results in the economics and operations literature. As an immediate implication, because precommitment on the less competitive side benefits platforms, despite many of its benefits to workers such as health insurance, paid sick leaves, paid vacation, and retirements, the regulation that gig workers have to be classified as employees so as to sustain the mode of the wage precommitment away from spot market competition may alleviate the competition between platforms, and counterproductively, hurt gig workers by paying them lower wages, if the supply side is less competitive than the demand side.

Next, we compare the commission precommitment competition with others.

Proposition 2 (Comparison of Mode C with Others).
Suppose the demand variance is sufficiently small.

(a) (i) If the demand side is more competitive than the supply side (i.e., $\gamma \geq \beta$), the preference ranking by the platform is $wp \succeq C \succeq pw$.

(ii) If the supply side is more competitive than the demand side (i.e., $\gamma < \beta$), the preference ranking by the platform is $pw \succeq C \succeq wp$.

(b) (i) If $\gamma \geq \beta$ or $\gamma \ll \beta$, the preference ranking by the platform is $C \succeq P$.

(ii) If $\gamma \leq \beta$ and γ is sufficiently close to β, the preference ranking by the platform is $P \succeq C$.

Part (a) of Proposition 2 shows that like simultaneous price and wage competition, commission precommitment is less profitable than the direct precommitment to price or wage on the less competitive side. This is because, with precommitment to price (when the demand variance is sufficiently small) or wage, the matching quantity is precommitted as an outcome. However, with precommitment to commission, the matching quantity is not yet committed. For example, suppose the supply side is more competitive than the demand side. When the demand variance is sufficiently small, in the price precommitment competition where a higher price on the less competitive side is precommitted, the matching quantity is almost *directly* committed to be at a lower level. However, in a commission precommitment game, due to the intense competition on the supply side, the platforms need to commit to a higher commission to attract service providers, which results in a lower price, as the equilibrium price in the second stage decreases in the commission rate committed earlier. In essence, the leverage of the commission precommitment in influencing the price and resulting matching quantity is *indirect* and hence less profitable.

Nevertheless, Part (b) of Proposition 2 shows that commission precommitment performs better than no commitment (i.e., simultaneous price and wage competition) in most of the parameter space of two-sided competition intensities γ and β. This happens when the demand side is more competitive than the supply side (i.e., $\gamma \geq \beta$) or the supply side is sufficiently more competitive than the demand side (i.e., $\gamma \ll \beta$). For example, consider two extreme cases: either (1) $\beta = 0$ (i.e., no competition on the supply side) when simultaneous price and wage competition reduces to price precommitment competition, or (2) $\gamma = 0$ (i.e., no competition on the demand side) when simultaneous price and wage competition reduces to wage precommitment competition (see Lemma D.2 in Online Supplement D). Part (a) shows that direct precommitment to the price or wage decision on the more competitive side leads to more intense competition than indirect precommitment to commission. Therefore, for those extreme cases, simultaneous price and wage competition, reduced to price or wage precommitment competition on the more competitive side, is less profitable than commission precommitment competition. In general, although the commission commitment is indirect, compared to no commitment it can still alleviate the competition in the second stage and thus lead to a higher profit for platforms when the two-sided intensities are not close enough.

Only when the two-sided intensities γ and $\beta(\geq \gamma)$ are close enough in a narrow band, does the commission precommitment actually lead to a more intense market outcome than simultaneous price and wage competition, though the difference is not significant. That is, the precommitment to the commission may hurt platforms as opposed to no precommitment at all. This counterintuitive insight can be explained as follows. When two-sided intensities are close, there is little need to use the precommitted decision on the less competitive side to restrict the competition on the more competitive side, because the market clearing of supply and demand already imposes a constraint on the two-sided competition. However, the commitment to the less-effective device of the commission rate can tip over the balance, leading to a more competitive outcome than simultaneous price and wage competition.

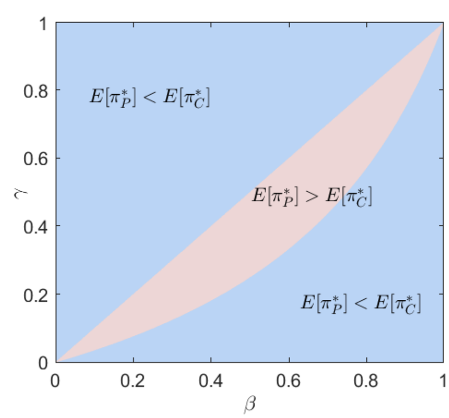


Figure 1 **Comparison between $E[\pi_C^*]$ and $E[\pi_P^*]$ under a low market size uncertainty**

Figure 1 compares commission precommitment competition with simultenous price and wage competition with no precommitment for each pair of (β, γ) by assuming Ω takes the values of 4.01

and 3.99 with equal probability. Figure 1 demonstrates that in most regions the commission precommitment is more profitable than simultaneous price and wage competition. Only in the narrow band with γ slightly smaller than β, is it less profitable, which is consistent with Proposition 2(b).

4.1. Quantity Competition: Capacity Precommitment

We observe from Section 4 that when the demand variance is sufficiently small, the effectiveness of precommitment devices depends on the extent to which the platforms can sustain a relatively small matching quantity and thus a high profit margin per matching unit. This observation motivates us to study the competitive decision directly on the matching quantity: Quantity competition (mode Q) in a two-sided market, analogous to Cournot competition in a one-sided market, competes on the matching quantity before the realization of the market size, with a market-clearing price and wage derived from the quantity decisions contingent on the realization. Note that Reynolds and Wilson (2000) study the stochastic version of the deterministic capacity precommitment game of Kreps and Scheinkman (1983) and find that the precommitment to capacity before market size uncertainty is realized may result in the non-existence of a symmetric equilibrium in pure strategy if the demand variance exceeds a threshold, so we study quantity competition only when the demand variance is sufficiently small. In the following, we will first confirm that indeed quantity competition is the most profitable of all the competition modes considered and then show how to practically achieve this outcome through a precommitment to supply capacity.

In quantity competition, given the matching quantity decisions and realized market size, price and wage are derived automatically such that demand and supply are equal to the matching quantity and the market clears. Hence, the platform's profit can be written as a function of the matching quantity of both platforms, from which we obtain each platform's optimal matching quantity as a function of the competitor's matching quantity. Solving this set of equations yields the equilibrium of quantity competition.

LEMMA 6. *Suppose the demand variance is sufficiently small. For any realized market size x, the quantity competition admits a unique equilibrium of matching quantities, which is symmetric for any firm:*

$$p_Q^* = \frac{(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)x-(1-\gamma^2)(1-\beta^2)E[\Omega]}{(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)},$$

$$w_Q^* = \frac{(1+\gamma)(1+\beta)}{4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2}E[\Omega],$$

$$z_Q^* = \frac{(1+\gamma)(1-\beta^2)}{4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2}E[\Omega].$$

The expected equilibrium profit level for any firm is:

$$E[\pi_Q^*] = \frac{(2-\gamma^2-\beta^2)(1+\gamma)(1-\beta^2)}{(1-\gamma)(4+\beta+\gamma-2\gamma^2-2\beta^2-\beta^2\gamma-\beta\gamma^2)^2}(E[\Omega])^2.$$

Now we compare quantity competition with all the other competition modes.

PROPOSITION 3 (DOMINANCE OF MODE Q). *Suppose the demand variance is sufficiently small. The platforms prefer Q over any of the competition modes among wp, P, pw, and C, i.e.,* $Q \succeq \max\{wp, P, pw, C\}$.

Proposition 3 shows that the quantity competition leads to higher prices and profits and lower wages and matching quantities than any of simultaneous price and wage competition and wage/price/commission precommitment competition. It generalizes the well-known result that quantity competition alleviates the competition and leads to higher prices and profits than price competition in the one-sided (demand side) market (see, e.g., Singh and Vives 1984) to a two-sided market. In particular, Proposition 3 demonstrates that the precommitment to the matching quantity is more effective than the precommitment to either wage or price. The intuition behind this is as follows. In a two-stage sequential competition, when the commitment is made indirectly through either price or wage, there is still some competition on either the demand or supply side, and thus such a commitment is not as effective as a direct commitment to the matching quantity.

Kreps and Scheinkman (1983) study a model in which the firms play a two-stage capacity (quantity) precommitment game, where they set a capacity (quantity) in the first stage and then compete on price in the second stage, subject to the capacity constraint chosen earlier. The authors establish that the equilibrium of such a two-stage game has the same outcome as that of the Cournot model. Next, we examine whether an analogous equivalency holds in a two-sided market, which could help justify how to reach the Pareto-dominating equilibrium under the quantity competition compared with other competition modes.

We first define the quantity precommitment game in a two-sided market. In the first stage, any platform i simultaneously sets a quantity level q_i before the realization of the market size, representing the maximum amount each platform can match between supply and demand. For instance, the maximum amount could be controlled by the number of drivers recruited by a platform or the number of reminders or coupons sent to a driver pool for a given time slot. Denote by $\boldsymbol{q}$ the matching quantity vector. In the subsequent stage, the platforms compete in the spot market by setting both price $\boldsymbol{p}$ and wage $\boldsymbol{w}$ simultaneously contingent on the market size realization. The profit function of platform i as the outcome of matching can therefore be written as

$$\pi_i(\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{q}) = (p_i - w_i)\min\{d_i(\boldsymbol{p}), s_i(\boldsymbol{w}), q_i\}.$$

Proposition 4 (Extended Kreps-Scheinkman Equivalency to Two-Sided Market). *Suppose there is no demand variance. The equilibrium outcome of the two-stage quantity precommitment competition, in terms of price, wage, matching quantity, and profit for any firm, is the same as that of the single-stage quantity competition.*

Proposition 4 confirms that the Kreps and Scheinkman equivalency still holds in a two-sided market without demand uncertainty [7]. Combined with Proposition 3, Proposition 4 implies that by adopting capacity constraints, the platforms will achieve a more desirable situation than through all previously considered commitment devices. Such constraints can be achieved through, e.g., limiting the size of the labor pool, which ironically seems to be the regulatory objective of some governmental agencies.

5. Comparisons with a Relatively High Demand Uncertainty

The previous section assumes a sufficiently low demand uncertainty to examine the performance of various precommitment devices. In this section, we explore how different precommitment devices would fare in a two-sided market competition with a relatively high demand uncertainty.

The objective of this section is two-fold. First, we show that the impact of a relatively high demand uncertainty on the wage/price precommitment has opposite effects. On the one hand, the precommitted wage/price limits the firm's flexibility to react to the relatively high demand uncertainty. On the other hand, when demand/supply side is rather competitive, the wage/price precommitment alleviates the competition on the other side compared to no precommitment. Second, we show that the comparison between the spot-market price/wage competition and the commission-rate precommitment competition stays the same as that with a sufficiently low demand uncertainty.

To avoid tedious pairwise comparisons, we just present three sets of comparisons in the main body of the paper: modes P vs. C, modes wp vs. P, and modes pw vs. P, and relegate all the other comparisons to Online Appendix A. Moreover, to help the readers navigate our results, we summarize in Table 1 the comparisons of various modes for the setting with a sufficiently low demand uncertainty as well as the setting with a relatively high demand uncertainty.

5.1. Comparison of Modes P and C.

We observe from expressions (4) and (10) that the firm's expected profit under both modes P and C is proportional to the second moment of the potential market size $E[\Omega^2]$. As both competition

[7] When the demand variance is sufficiently small, we find that the equilibrium of the two-stage quantity precommitment competition is converging to that of the single-stage quantity competition.

Model	Sufficiently small Var(Ω)	Relatively large Var(Ω) (Analytic)	Relatively large Var(Ω) (Numerical)
P vs. wp vs. pw	$wp \succeq P \succeq pw$ if $\gamma > \beta$; $wp \preceq P \preceq pw$ if $\gamma \leq \beta$.	Suppose $\beta = 0$ (resp., $\gamma = 0$), $wp \preceq P$ if γ (resp., β) is small; $wp \succeq P$ if γ (resp., β) is large. $P \succeq pw$ if $\beta = 0$ or $\gamma = 0$ and β is small; $P \preceq pw$ if $\gamma = 0$ and β is large.	For medium or large Var(Ω), $wp \preceq P$ if β and γ are small; $wp \succeq P$ if β and γ are large; $P, wp \succeq pw$ in most region.
C vs. wp vs. pw	$wp \succeq C \succeq pw$ if $\gamma > \beta$; $wp \preceq C \preceq pw$ if $\gamma \leq \beta$.	For medium Var(Ω), $wp \preceq C$ if $\beta = 0$ and γ is small, or $\gamma = 0$ and β is small.	For medium or large Var(Ω), $wp \preceq C$ if β and γ are small; $wp \succeq C$ if β and γ are large; $C \succeq pw$ in most of the region.
C vs. P	$C \succeq P$ unless γ is slightly lower than β.	$C \succeq P$ unless γ is slightly lower than β.	$C \succeq P$ unless γ is slightly lower than β.
Q vs. Others	$Q \succeq \max\{P, wp, pw, C\}$	NA	NA

Table 1 Summary of the Comparisons of Various Modes

modes of C and P retain flexibility in response to market changes, their comparison is independent of the demand uncertainty, which we summarize as follows.

COROLLARY 1. *The comparison between modes C and P in the presence of a relatively high demand uncertainty stays the same as that with a sufficiently low demand uncertainty.*

5.2. Comparison of Modes wp and P.

PROPOSITION 5. *In the presence of a relatively high demand uncertainty, compare modes wp and P.*

(a) Suppose $\beta = 0$.

(i) If γ is sufficiently small and the variance of Ω is not sufficiently large, $E[\pi^*_{wp}] \leq E[\pi^*_P]$.

(ii) If γ is sufficiently large, $E[\pi^*_{wp}] > E[\pi^*_P]$.

(b) Suppose $\gamma = 0$.

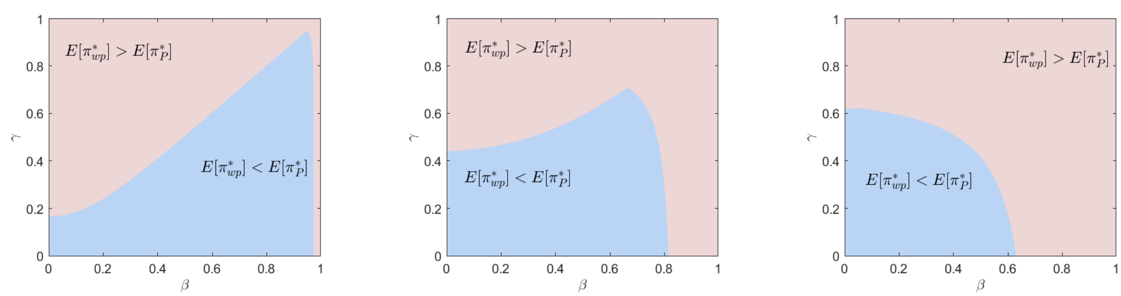
(i) If β is sufficiently small and the variance of Ω is not sufficiently large, $E[\pi^*_{wp}] \leq E[\pi^*_P]$.

(ii) If β is sufficiently large, $E[\pi^*_{wp}] > E[\pi^*_P]$.

Proposition 5(a-i) says that when there is no competition on the supply side, for which the precommitment on the supply side would have value with a sufficiently low market size uncertainty (see Proposition 1(a)), the wage precommitment, in the presence of a relatively large market size uncertainty, could do harm to the firms' profit. In wage precommitment competition, when the variance of the market size is not sufficiently large, intuitively, it is still optimal to set the wage ex ante such that the firms adopt a supply-depletion price for any market size. In this case, the equilibrium wage takes the value of

$$w_{wp}^* = \frac{1+\gamma}{4-3\beta+\gamma-2\gamma^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2}E[\Omega].$$

We observe that the equilibrium wage is decided before the realization of the market size as if there is no uncertainty and the market size equals its expectation, which would lead to excess supply for a low realized market size and insufficient supply for a high realized market size, resulting in a lower expected profit. Moreover, this effect is reinforced as the demand variance increases. Therefore, compared with simultaneous price and wage competition (with no precommitment), wage rigidity could hurt the platforms. That is, in the presence of a relatively large market size uncertainty, the precommitted wage could limit the platforms' flexibility to react to market conditions, resulting in a lower profit than contingently setting price and wage. Figure 2 displays the comparison between $E[\pi_{wp}^*]$ and $E[\pi_P^*]$ as the market size Ω takes a two-point distribution and changes with an increasing variance but a fixed mean.[8] As the market size uncertainty increases, Figure 2 shows that even when $\beta = 0$, the wage precommitment competition can be dominated by simultaneous price and wage competition for small values of γ.



(a) $\Omega_H = 4.1, \Omega_L = 3.9, q = 0.5$ (b) $\Omega_H = 4.5, \Omega_L = 3.5, q = 0.5$ (c) $\Omega_H = 5, \Omega_L = 3, q = 0.5$

Figure 2 Comparison between $E[\pi_{wp}^*]$ and $E[\pi_P^*]$ under market size uncertainty

To understand part (a-ii), note that when $\beta = 0$, simultaneous price and wage competition boils down to price competition only (as there is no competition on the supply side) and the wage precommitment competition turns to a two-stage problem where the wage is decided without competition in the first stage. According to Kreps-Scheinkman equivalency, the wage precommitment

[8] Figure 2 assumes a two-point distribution, which is the worst distribution for the platforms' profit levels of all distributions that share the same mean and variance and allows us to study the effect of demand uncertainty on precommitment devices in the worst case.

competition as a two-stage game is equivalent to Cournot competition, leading to a less intense competition and thus a higher profit than the price competition. That is, when $\beta = 0$, with a sufficiently low or without market size uncertainty, the wage precommitment competition is more profitable for firms. In the presence of a relatively large market size uncertainty, such a benefit of the precommitment still persists especially when there is intense competition on the demand side (i.e., γ is sufficiently large), which is confirmed by Figure 2 with $\beta = 0$.

As for part (b-i), note that with a sufficiently low market size uncertainty, when the supply side is more competitive than the demand side, the wage precommitment leads to a more intense market outcome than simultaneous price and wage competition (see Proposition 1(b)). In the presence of a relatively large market size uncertainty, this effect still exists, which is confirmed by Figure 2 with $\gamma = 0$.

Although the inflexibility due to the wage precommitment may hurt the firm's profit, part (b-ii) shows that such a precommitment can improve the platform's profit in some cases because the supply cap committed through wage can alleviate the intense competition on the supply side. To see this, consider that when there is no competition on the demand side (i.e., $\gamma = 0$), simultaneous price and wage competition boils down to a wage-only competition ex post, while the wage precommitment competition reduces to a wage-only competition ex ante. Suppose there is fierce competition on the supply side (i.e., β is sufficiently large). Compared to contingent simultaneous price and wage competition, the wage precommitment can prevent the platforms from competing too aggressively by setting wages too high in the spot market when the demand turns out to be high.

In summary, other than the impacts of the wage precommitment revealed in the case with a sufficiently low demand uncertainty, a relatively high demand uncertainty may undermine or enhance the value of the wage precommitment. On the one hand, the wage precommitment can tie the hands of platforms whn it comes to responding to market changes. On the other hand, the wage precommitment may restrain cutthroat wage competition when the potential market size turns out to be large. With a sufficiently low uncertainty, Figure 2(a) shows that the comparison depends on the competitiveness of the two sides; see Proposition 1. As the demand variability increases (still in a relatively small range), the region where competition mode P is preferred shifts to the left and expands slightly in Figures 2(b)-(c), indicating that the market size uncertainty undermines the value of the wage precommitment when the demand-side competition is slightly more intense than the supply-side competition (i.e., γ is slightly larger than β). As the variability further increases, the region where the competition model wp is preferred becomes enlarged in

Figures 2(d)-(f), indicating that high-enough market size uncertainty can strengthen the value of the wage precommitment, in particular, when the supply-side competition is sufficiently intense (i.e., β is sufficiently large). Figures 2(d)-(f) also show that when the competitions on both sides are less intense (i.e., both β and γ are small), the negative effect of a high demand uncertainty on mode *wp* is more prominent and thus mode P performs better; when the competitions on both sides are more intense (i.e., both β and γ are large), the reinforcement effect is more prominent, rendering mode *wp* better than mode P. We formally prove this result when $\beta = \gamma$ with the restriction of a two-point distribution; see Proposition D.1 in Online Supplement D.

5.3. Comparison of Modes *pw* and P.

Proposition 6. *Suppose the variance of demand uncertainty is not sufficiently large.*[9]

(a) Suppose $\beta = 0$*, then* $E[\pi^*_{pw}] \leq E[\pi^*_P]$.

(b) Suppose $\gamma = 0$.

(i) If β *is sufficiently small, then* $E[\pi^*_{pw}] \leq E[\pi^*_P]$.

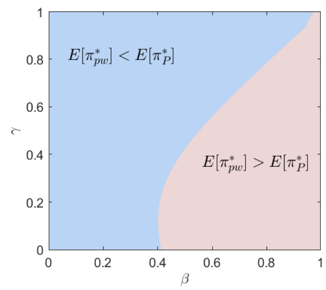
(ii) If β *is sufficiently large, then* $E[\pi^*_{pw}] > E[\pi^*_P]$.

Recall that with a sufficiently low demand uncertainty, when the demand side is more competitive than the supply side, price precommitment competition leads to a more intense outcome than simultaneous price and wage competition; see Proposition 1(a). Proposition 6(a) shows that in the presence of a high demand uncertainty, this effect still exists when there is no competition on the supply side; see also Figure 3 when $\beta = 0$.

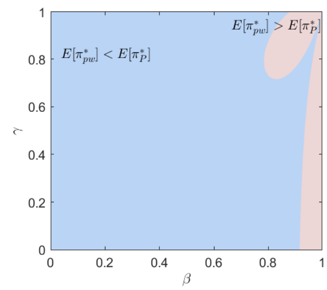
Proposition 6(b) shows that when there is no competition on the demand side, for which the precommitment on the demand side would have value with a sufficiently low demand uncertainty (see Proposition 1(b)), the price precommitment, in the presence of a moderate demand uncertainty, can harm the firm's profit. Like the wage precommitment competition, the impact of market size uncertainty on the price precommitment competition has two opposite effects. On the one hand, the precommitted price limits the firm's flexibility to react to market size uncertainty. On the other hand, when the supply side is rather competitive, the price precommitment alleviates the competition on the supply side compared with no precommitment. However, unlike the wage precommitment which commits the supply quantities (as there is no supply uncertainty), the price precommitment does not directly determine demand, and thus the alleviation of competition on the supply side is somewhat handicapped. To summarize, with a relatively large market size

[9] Online Supplement D provides Proposition D.2, an extended version of Proposition 6, which includes the specific conditions for the magnitude of the variance of demand uncertainty.

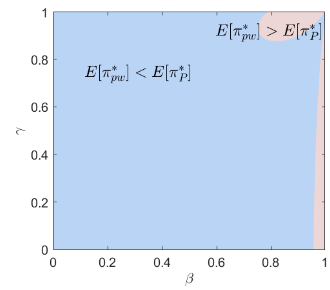
uncertainty, the price precommitment does not work as effectively as the wage precommitment. As the variability increases, Figures 3(a)-(c) show that the region where pw is better than P shrinks significantly, indicating that market size uncertainty undermines the value of the price precommitment. Compared with Figure 2, the region where pw is better than P is much smaller than where wp is better than P.



(a) $\Omega_H = 4.1, \Omega_L = 3.9, q = 0.5$



(b) $\Omega_H = 4.5, \Omega_L = 3.5, q = 0.5$



(c) $\Omega_H = 5, \Omega_L = 3, q = 0.5$

Figure 3 Comparison between $E[\pi^*_{pw}]$ and $E[\pi^*_P]$ under market size uncertainty

6. Conclusion

Our model is motivated by the ride-hailing markets. But to keep it parsimonious, we assume away some salient features in those markets, such as that demand and supply may be sensitive to the chance of being matched or the time it takes for them to be matched. We focus on studying various sequential movements in two-sided market competition–wage precommitment competition (followed by price competition), price precommitment competition (followed by wage competition), commission-rate precommitment competition (followed by price competition), and quantity/capacity precommitment (followed by capacitated price competition)–and comparing them with simultaneous price and wage competition (with no precommitment).

For the setting with a sufficiently low demand uncertainty, we obtain a set of sharp results that deepen our understanding of the nature of competitive markets beyond the celebrated Kreps-Scheinkman equivalency in one-sided market competition. First, the precommitment on the less competitive side alleviates the competition. Second, the commission precommitment is more profitable than no commitment when the competition intensity on one side is sufficiently higher than the other, but it can also perform worse than no commitment. Third, the quantity precommitment, sustained through a two-stage game with the capacity (quantity) precommitment, leads to the most profitable outcome of all modes. In the Online Appendix, we also extend the model to account for asymmetric platforms and matching friction, and find that our main insights tend to be robust.

For the setting with a relatively high demand uncertainty, we find that demand uncertainty has two opposite effects on wage/price precommitment. On the one hand, a precommitted wage/price limits the firm's flexibility to react to demand uncertainty. On the other hand, when demand/supply side is rather competitive, the wage/price precommitment alleviates the competition on the other side compared with no precommitment. Moreover, the comparison between the spot-market price and wage competition and the commission-rate precommitment competition stays the same as that with a sufficiently low demand uncertainty.

Our results have the following managerial implications. First, our results caution platforms that the precommitment to price or wage on the more competitive side can be worse than no commitment at all and suggest platforms to precommit on the less competitive side. Second, because the precommitment on the less competitive side benefits platforms, the regulation that gig workers should be classified as employees so to sustain the mode of the wage precommitment may alleviate competition between platforms and hurt drivers, if the labor market is less competitive than the consumer market. Third, governmental regulation that restricts the for-hire vehicle licenses and is often viewed as opposing the market expansion efforts by the ride-hailing platforms can lead to more rides, more traffic, and less sustainable commuting. However, it can also help restrain fierce price competition between platforms, benefiting their profitability and hurting social welfare.

Our paper has some limitations. First, for tractability, our main model assumes linear demand and supply systems, under which we obtain a set of sharp and clean results. It is worthwhile to verify the robustness of our results under alternative demand and supply systems such as MultiNomial Logit models, though we expect that many of our results would still hold qualitatively. Second, our model ignores spatial pricing. Due to the spatial feature of the ride-hailing market, a platform may be able to charge a local monopoly price because its competitor may not have available cars close to a customer. Third, we adopt numerical experiments to derive results in the model extension with matching friction. Specifically, we only verify the existence of equilibrium numerically and then compare the equilibrium outcomes in this setting. Given that it is very challenging to analyze such a model extension, some approximation methods may be adopted to obtain analytical results.

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Online Appendix to
"Precommitments in Two-sided Market Competition"

The online appendix consists of four sections.

- Section A includes the comparisons of modes with a relatively high demand uncertainty that are not presented in the main body.
- Section B extends the analysis from symmetric platforms to asymmetric platforms.
- Section C incorporates the friction in the process of matching supply with demand.
- Section D includes proofs of lemmas and propositions in the main text.

A. Comparisons of Modes

Comparison of Modes wp and C. The following corollary is an immediate result of Proposition 5 and Corollary 1.

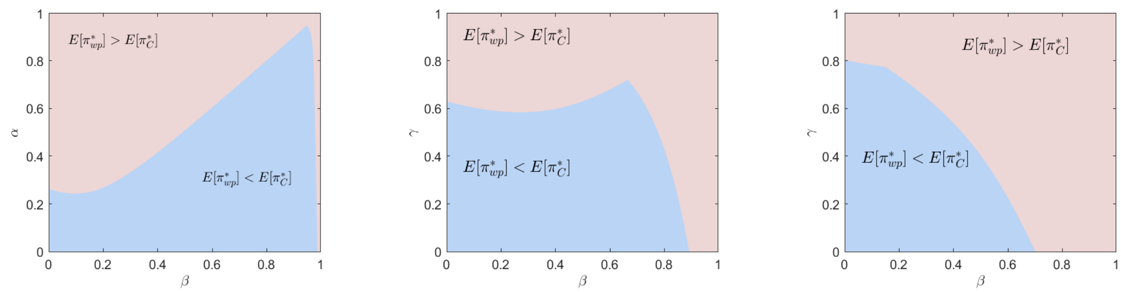
COROLLARY 2. *Suppose the variance of Ω is not sufficiently large. Then $E[\pi^*_{wp}] \leq E[\pi^*_C]$ if either of the following conditions holds:*

(a) $\beta = 0$ and γ is sufficiently small;

(b) $\gamma = 0$ and β is sufficiently small.

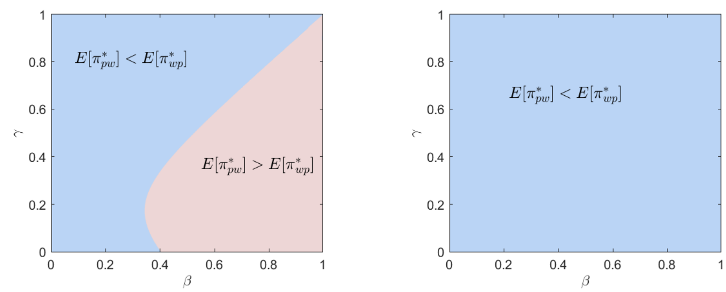
Proposition 5(a-i) and (b-i) specify the conditions under which mode wp performs worse than mode P, while Corollary 1 shows that mode P is worse than C either when $\beta = 0$ or when $\gamma = 0$. Therefore, Corollary 2 holds immediately. Figure A.1 indicates that the comparison between modes wp and C is very similar to that between wp and P, although the region where C is better is larger than that where P is better in Figure 2. Recall that demand variability benefits the firms under modes P and C on the same scale, so the comparison between P and C is fixed as the variability increases, which explains the similarity between Figures 2 and A.1. Moreover, Figure 1 shows that C is better than P in most of the parameter space with a sufficiently low uncertainty. Hence, it is not surprising that the region where C is better than wp is larger than that where P is better than wp.

Comparison of Modes wp and pw. Since the price precommitment fails to do well in alleviating the competition on the demand side, it is no surprise that the region where wp does better keeps expanding as the variability increases (see Figure A.2). When the variability is large enough (e.g., $\Omega_H = 4.5$ and $\Omega_L = 3.5$), mode pw is completely dominated by mode wp.



(a) $\Omega_H = 4.1, \Omega_L = 3.9, q = 0.5$ (b) $\Omega_H = 4.5, \Omega_L = 3.5, q = 0.5$ (c) $\Omega_H = 5, \Omega_L = 3, q = 0.5$

Figure A.1 Comparison between $E[\pi^*_{wp}]$ and $E[\pi^*_C]$ under market size uncertainty



(a) $\Omega_H = 4.1, \Omega_L = 3.9, q = 0.5$ (b) $\Omega_H = 4.5, \Omega_L = 3.5, q = 0.5$

Figure A.2 Comparison between $E[\pi^*_{wp}]$ and $E[\pi^*_{pw}]$ under market size uncertainty

B. Asymmetric Platforms

To facilitate navigating the results in the extensions, we provide the following Table 2 to summarize the comparison results among different competition modes for the model extensions with asymmetric platforms and matching friction. To simplify the analysis, we assume that the market size is deterministic.

B.1. Asymmetry in market sizes

We first assume that the two platforms face different potential market sizes with everything else the same as in the deterministic model. That is, the demand functions become $d_i(\boldsymbol{p}) = d_i(p_i, p_j) = [\Omega_i - p_i + \gamma p_j]^+,\ j \neq i, i = 1, 2,$ where $\Omega_1 \neq \Omega_2$. Without loss of generality, we assume $\Omega_1 \geq \Omega_2$. Following the same procedure as in the deterministic model, we can characterize the equilibrium for each competition mode and then compare those equilibria whenever they exist.

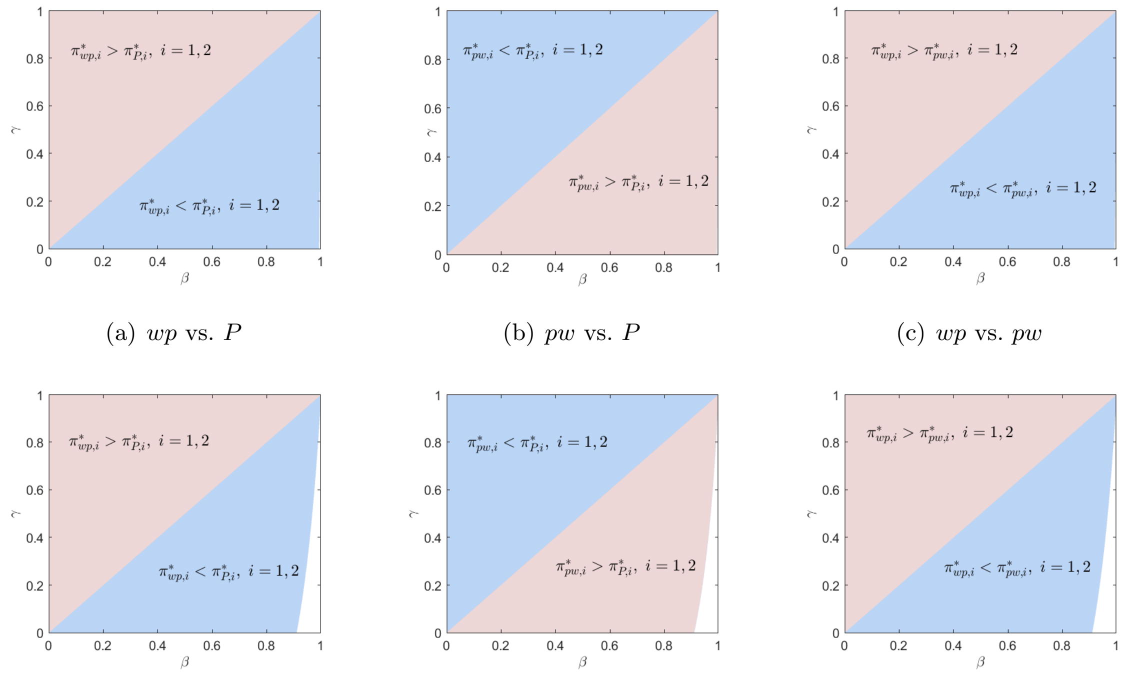
Observation 1 (Comparison of Modes P, wp, pw Under Asymmetric Market Sizes). Suppose asymmetric potential market sizes and the equilibria exist. If the demand side is more competitive than the supply side (i.e., $\gamma \geq \beta$), the preference ranking by the platform is: $wp \succeq P \succeq pw$. Otherwise, the ranking is reversed.

Model	Asymmetric Ω: $\Omega_1 > \Omega_2$	Asymmetric γ and β	Matching Friction
P vs. wp vs. pw	$wp \succeq P \succeq pw$ if $\gamma > \beta$; $wp \preceq P \preceq pw$ if $\gamma \leq \beta$.	if $\min\{\gamma_1,\gamma_2\} > \max\{\beta_1,\beta_2\}$, then $wp \succeq P \succeq pw$; if $\max\{\gamma_1,\gamma_2\} \leq \min\{\beta_1,\beta_2\}$, then $wp \preceq P \preceq pw$.	$wp \succeq P \succeq pw$ if $\gamma > \beta$; $wp \preceq P \preceq pw$ if $\gamma \leq \beta$.
C vs. wp vs. pw	$wp \succeq C \succeq pw$ if $\gamma > \beta$; $wp \preceq C \preceq pw$ if $\gamma \leq \beta$.	if $\min\{\gamma_1,\gamma_2\} > \max\{\beta_1,\beta_2\}$, then $wp \succeq C \succeq pw$; if $\max\{\gamma_1,\gamma_2\} \leq \min\{\beta_1,\beta_2\}$, then $wp \preceq C \preceq pw$.	$wp \succeq C \succeq pw$ if $\gamma > \beta$; $wp \preceq C \preceq pw$ if $\gamma \leq \beta$.
C vs. P	$C \succeq P$ unless γ is slightly lower than β.	$C \succeq P$ if $\gamma_i > \beta_i$, $i=1,2$; $C \preceq P$ if γ_i is slightly lower than β_i, $i=1,2$.	$C \succeq P$ unless γ is slightly lower than β.
Q vs. Others	$Q \succeq \max\{P, wp, C\}$; If $\Omega_1 \gg \Omega_2$, $\beta \to 1$, $\gamma \to 0$, then $Q \preceq pw$ for firm 1, and $Q \succeq pw$ for firm 2.	$Q \succeq \max\{P, wp, pw, C\}$.	$Q \succeq \max\{P, wp, pw, C\}$.

Table 2 Result Summary of the Comparisons for Asymmetric Platforms and Matching Friction

Observation 1 extends Proposition 1 (without demand uncertainty) from symmetric market sizes to asymmetric market sizes. Figure B.1 numerically displays the comparison among wage precommitment competition, simultaneous price and wage competition, and price precommitment competition. There is a region where the equilibrium does not exist when β is sufficiently large, and as the asymmetry increases (i.e., Ω_1 is increasingly greater than Ω_2), this region grows. This is because, facing a larger market size, platform 1 has an incentive to increase its supply. When β is sufficiently large, the competition on the supply side is so intense that both platforms need to offer very high wages to achieve their targeted level of supply, making it possible that the wage is higher than its price for platform 2 who faces a smaller market size. Consequently, platform 2 may be driven out of the market and the competition does not exist anymore. The larger the asymmetry, the larger the region. But as long as those equilibria exist, our result from the deterministic model that the comparison among wp, P, and pw depends on the comparison of the competition intensities of the two sides continues to hold.

Here, we comment on how to establish Observation 1. We first analytically derive the equilibrium profit for each mode (the profit expression for each mode is given in Online Supplement E). Then, we compare the equilibrium profits between any two modes. As one may notice, each profit expression includes four parameters, β, γ, Ω_1, and Ω_2. This makes it very challenging to analytically compare equilibrium profits as functions of four parameters. We normalize Ω_2 to 1 (without loss of generality) and vary Ω_1 by increasing it from 1 with a step size of 0.001. Then, for each fixed Ω_1, we enumerate β and γ both with a step size of 0.001, and check the ranking between the profit expressions. Finally, we do not observe any exceptions that Observation 1 fails to hold. The following Observation 2 is obtained similarly.



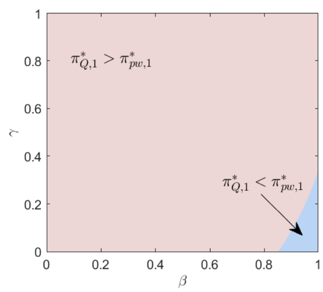
(a) wp vs. P (b) pw vs. P (c) wp vs. pw

(d) wp vs. P (e) pw vs. P (f) wp vs. pw

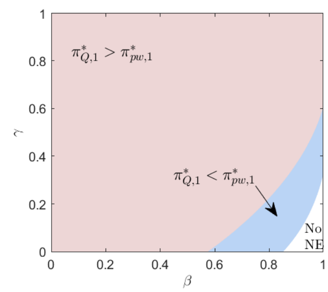
Figure B.1 **Comparison among wp, P, and pw with asymmetric market sizes. Figures 2(a)-(c) assume $(\Omega_1, \Omega_2) = (4.1, 3.9)$, while Figures 2(d)-(f) assume $(\Omega_1, \Omega_2) = (5, 3)$.**

Observation 2 (Comparison of Mode Q with Others Under Asymmetric Market Sizes). Suppose asymmetric potential market sizes and the equilibria exist. (a) The platforms prefer Q over wp and P. (b) When the difference between Ω_1 and Ω_2 is sufficiently large, if β is sufficiently large and γ is sufficiently small, platform 1 prefers pw over Q, while platform 2 always prefers Q over pw.

Observation 2 shows that the insight from the deterministic model that quantity competition is the most profitable largely holds. Surprisingly, with asymmetric market sizes, when the demand side competition is not very intense and the supply side competition is sufficiently intense, price precommitment competition can enable the platform with a larger market size to obtain a higher profit than in quantity competition. Figure B.2 displays the comparison between price precommitment competition and quantity competition with asymmetric market sizes. In the lower-right corner, price precommitment competition earns platform 1 a higher profit, and as the asymmetry becomes more prominent, this region is growing.



(a) $\Omega_1 = 5, \Omega_2 = 3$



(b) $\Omega_1 = 6, \Omega_2 = 2$

Figure B.2 Comparison between pw and Q with asymmetric market sizes.

Finally, we numerically compare mode C with other modes. We provide a representative example to show the robustness of Proposition 2. Table 3(a) sets $(\Omega_1, \Omega_2, \gamma, \beta) = (5, 3, 0.7, 0.1)$ where the demand side is more competitive than the supply side and shows the equilibrium for each mode. We observe that $\pi^*_{wp,i} \geq \pi^*_{C,i} \geq \pi^*_{pw,i}$ for each $i = 1, 2$, implying that similar to simultaneous price and wage competition, the precommitment to the commission is less profitable than the direct precommitment on the less competitive side, but more profitable than the direct precommitment on the more competitive side, which is consistent with Proposition 2(a). Second, $\pi^*_{P,i} \leq \pi^*_{C,i}$ for each $i = 1, 2$, implying that the commission precommitment performs better than no commitment when one side is sufficiently more competitive than the other, which is consistent with Proposition 2 (b-i). Finally, $\pi^*_{C,i} \leq \pi^*_{Q,i}$ for each $i = 1, 2$, confirming that the precommitment to the matching quantity is more profitable than the commission precommitment.

Table 3(b) sets $(\Omega_1, \Omega_2, \gamma, \beta) = (5, 3, 0.45, 0.5)$ and compares mode C with mode P. It shows $\pi^*_{P,i} \geq \pi^*_{C,i}$ for each $i = 1, 2$, implying that the commission precommitment can lead to a more intense market outcome than no commitment when the two-sided intensities are close enough, which is consistent with Proposition 2(b-ii).

B.2. Asymmetry in β

This subsection assumes $\gamma_1 = \gamma_2$ in the demand system $d_i(\boldsymbol{p}) = [\Omega - p_i + \gamma_i p_j]^+, j \neq i, i = 1, 2$, but $\beta_1 \neq \beta_2$ in the supply system $s_i(\boldsymbol{w}) = [w_i - \beta_i w_j]^+, j \neq i, i = 1, 2$. That is, platforms are symmetric on the demand side but asymmetric on the supply side. Note that a large value of β_i means that platform i's supply is easily attracted away by the other platform, so $\beta_i > \beta_j$ implies that platform i is less attractive than platform j in the supply market.

Figure B.3 compares the three modes P, wp, and pw by fixing $\gamma_1 = \gamma_2 = 0.5$ and varying the values of β_1 and β_2 from 0.15 to 0.85 with a step size of 0.05. Although Figure B.3 shows the profit comparison for platform 2, the comparison for platform 1 can be obtained by reversing the

Mode	P	wp	pw	C	Q
p_1	6.9261	7.7264	6.8589	7.3984	7.8505
p_2	5.9475	6.7101	5.8838	6.4185	6.8289
w_1	2.4518	2.1621	2.4763	2.2935	2.1175
w_2	2.1459	1.9146	2.1651	1.9897	1.8782
z_1	2.2372	1.9707	2.2598	2.0945	1.9297
z_2	1.9008	1.6984	1.9174	1.7604	1.6664
π_1	10.0098	10.9653	9.9036	10.6924	11.0631
π_2	7.2259	8.1446	7.1304	7.7963	8.2501

(a) $(\gamma, \beta) = (0.7, 0.1)$

Mode	P	C
p_1	5.5479	5.5455
p_2	4.4521	4.4486
w_1	2.637	2.6397
w_2	2.363	2.3667
z_1	1.4555	1.4563
z_2	1.0445	1.0468
π_1	4.2368	4.2319
π_2	2.182	2.1795

(b) $(\gamma, \beta) = (0.45, 0.5)$

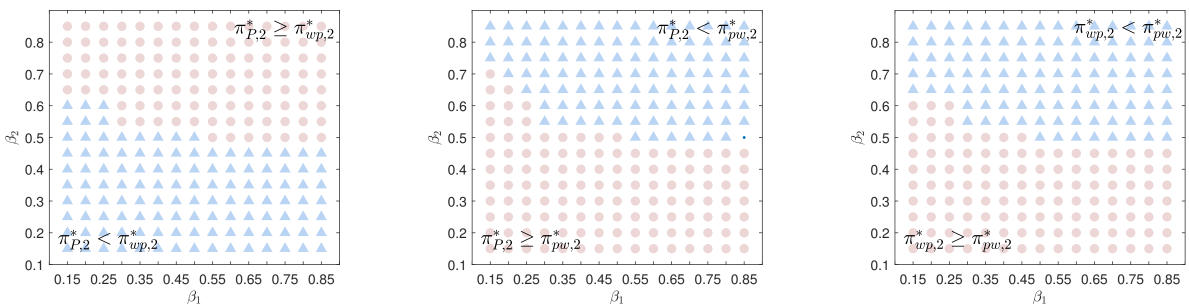
Table 3 Asymmetry in Market Sizes with $(\Omega_1, \Omega_2) = (5, 3)$

values of β_1 and β_2. For example, platform 1's profit comparison when $(\beta_1, \beta_2) = (0.15, 0.85)$ is the same as platform 2's profit comparison when $(\beta_1, \beta_2) = (0.85, 0.15)$. We focus on the comparison between modes wp and pw while their comparison with mode P is similar. In the upper-right region of Figure B.3(c), both β_1 and β_2 are greater than γ, indicating that the supply side is more competitive than the demand side. As expected, the price precommitment alleviates the competition and brings a higher profit for both platforms than the wage precommitment. Similarly, in the lower-left region where the demand side is more competitive than the supply side, the wage precommitment brings a higher profit for both platforms than the price precommitment. This is consistent with Proposition 1.

Moreover, in the lower-right region of Figure B.3(c) where $\beta_1 > \gamma$ and $\beta_2 < \gamma$, although it is not clear which of the two sides is more competitive, platform 1 is less attractive than platform 2 on the supply side. We find that the wage precommitment brings a higher profit to platform 2 than the price precommitment. This is because platform 2 has an advantage over platform 1 on the supply side, so the precommitment on the supply side enables platform 2 to take advantage of platform 1, thus increasing platform 2's profit. The upper-left region of of Figure B.3(c) can be interpreted similarly. In summary, a precommitment on the platform's advantageous (disadvantageous) side can bring a higher (lower) profit to that platform than a precommitment on the other side.

B.3. Asymmetry in both γ and β

This subsection analyzes the case in which the two platforms are asymmetric in the substitution factors on both the demand and supply sides. That is, the demand and supply systems take a general form: $d_i(\boldsymbol{p}) = [\Omega - p_i + \gamma_i p_j]^+, j \neq i, i = 1, 2,$ $s_i(\boldsymbol{w}) = [w_i - \beta_i w_j]^+, j \neq i, i = 1, 2$. Note that a large value of γ_i means that platform i has a strong capability to attract demand from the other platform, that is, platform i is more attractive on the demand side. Similarly, a large value of β_i means platform i is less attractive on the supply side. Due to the complicated nature of this



(a) P vs. wp (b) P vs. pw (c) wp vs. pw

Figure B.3 Comparisons among P, wp, and pw for asymmetry in β.

general case, we provide several representative examples of the parameter set to investigate the comparisons among different modes.

In Tables 4 and 5, $\max\{\gamma_1, \gamma_2\} < \min\{\beta_1, \beta_2\}$, which means that the demand side is less competitive than the supply side. We find $\pi^*_{wp,i} \leq \pi^*_{P,i} \leq \pi^*_{pw,i}$ for each $i = 1, 2$, indicating that a precommitment on the demand side (the less competitive side) alleviates the competition and generates a higher profit, which is consistent with Proposition 1. Second, $\pi^*_{wp,i} \leq \pi^*_{C,i} \leq \pi^*_{pw,i}$ for each $i = 1, 2$, implying that similar to simultaneous price and wage competition, the commission precommitment is less profitable than the direct precommitment on the less competitive side, but more profitable than the direct precommitment on the more competitive side, which is consistent with Proposition 2(a). Third, $\pi^*_{Q,i}$ for each $i = 1, 2$ is the highest among different modes, implying that the precommitment to the matching quantity is the most profitable compared to the precommitment to the wage, price, or commission, which is consistent with Proposition 3. In Tables 6 and 7, $\min\{\gamma_1, \gamma_2\} > \max\{\beta_1, \beta_2\}$, which means that the demand side is more competitive than the supply side. Again, we obtain results consistent with those from the deterministic model. Finally, in Tables 4-7, $\pi^*_{P,i} < \pi^*_{C,i}$ for each $i = 1, 2$, implying that the commission precommitment performs better than no commitment when one side is sufficiently more competitive than the other. Table 8 shows $\pi^*_{P,i} > \pi^*_{C,i}$ for each $i = 1, 2$, implying that the commission precommitment can lead to a more intense market outcome than no commitment when the two-sided intensities are close enough. These results are consistent with Proposition 2(b).

In Tables 4 and 6, $\gamma_1 < \gamma_2$, indicating that platform 1 is less attractive than platform 2 on the demand side, and $\beta_1 > \beta_2$, indicating that platform 1 is less attractive than platform 2 on the supply side. That is, platform 1 is less advantageous than platform 2 on both demand and supply sides. Not surprisingly, platform 1's profit is substantially lower than that of platform 2. In Tables 5

Mode	P	wp	pw	C	Q
p_1	0.98	0.9899	0.99	0.99	1.01
p_2	1.0925	1.083	1.12	1.1	1.1335
w_1	0.62	0.63	0.59	0.613	0.5532
w_2	0.5915	0.61	0.55	0.583	0.5243
z_1	0.1764	0.1725	0.1775	0.1758	0.16
z_2	0.2505	0.2635	0.2255	0.2459	0.22
π_1	0.0635	0.0621	0.071	0.0663	0.0731
π_2	0.1249	0.1246	0.1285	0.1271	0.134

Table 4 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.15, 0.35, 0.75, 0.55)$

Mode	P	wp	pw	C	Q
p_1	0.964	0.9678	0.98	0.9719	0.9921
p_2	1.1079	1.0987	1.13	1.1136	1.1472
w_1	0.559	0.56	0.522	0.5443	0.4936
w_2	0.649	0.66	0.604	0.6348	0.57
z_1	0.20205	0.197	0.1898	0.1951	0.1801
z_2	0.22975	0.24	0.2125	0.2266	0.1998
π_1	0.0818	0.0803	0.08693	0.0835	0.0898
π_2	0.1054	0.1053	0.1118	0.1085	0.1153

Table 5 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.15, 0.35, 0.55, 0.75)$

Mode	P	wp	pw	C	Q
p_1	1.46	1.554	1.42	1.503	1.5966
p_2	1.5945	1.681	1.57	1.6428	1.7574
w_1	0.62	0.57	0.65	0.6014	0.553
w_2	0.5935	0.57	0.592	0.575	0.523
z_1	0.4123	0.3705	0.4428	0.4001	0.37
z_2	0.5005	0.4845	0.4945	0.4848	0.44
π_1	0.3463	0.3646	0.341	0.3608	0.3861
π_2	0.501	0.5383	0.4836	0.5177	0.5432

Table 6 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.55, 0.75, 0.35, 0.15)$

Mode	P	wp	pw	C	Q
p_1	1.43	1.522	1.41	1.49	1.5813
p_2	1.6016	1.71	1.57	1.6668	1.766
w_1	0.54	0.51	0.5558	0.5216	0.4781
w_2	0.6599	0.61	0.682	0.6334	0.5873
z_1	0.441	0.4185	0.4535	0.4266	0.39
z_2	0.4709	0.4315	0.4875	0.4508	0.42
π_1	0.3925	0.4235	0.3874	0.4131	0.4302
π_2	0.4434	0.4747	0.4329	0.4659	0.495

Table 7 **Asymmetry in both γ and β with** $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.55, 0.75, 0.15, 0.35)$

Mode	P	C
p_1	1.4	1.3956
p_2	1.44	1.4321
w_1	0.75	0.7536
w_2	0.78	0.7877
z_1	0.321	0.3204
z_2	0.33	0.3355
π_1	0.2087	0.2057
π_2	0.2178	0.2162

Table 8 Asymmetry in both γ and β with $(\gamma_1, \gamma_2, \beta_1, \beta_2) = (0.5, 0.55, 0.55, 0.6)$

and 7, $\gamma_1 < \gamma_2$, indicating that platform 1 is still less attractive than platform 2 on the demand side, but $\beta_1 < \beta_2$, indicating that platform 1 is more attractive than platform 2 on the supply side. Surprisingly, platform 1's profit is still fairly lower than that of platform 2, implying that the demand-side advantage can be more critical to the platform than the supply side.

C. Matching Friction

Now we extend our deterministic model to incorporate the friction in the process of matching supply with demand. In view of Bernstein et al. (2021), we capture customer's wait time in the steady state of the matching process before getting matched as follows: $W_i = \begin{cases} (\frac{d_i}{s_i})^2 & \text{if } d_i < s_i, \\ \infty & \text{otherwise.} \end{cases}$ If $s_i \leq d_i$, the process becomes unstable, in which case customer's wait time is defined as ∞. Only if $d_i < s_i$, the system can serve all the demand and reach a steady state, and the more supply, the less wait time for customers. Thus the demand system can be enriched as follows: $d_i(\boldsymbol{p}) = d_i(p_i, p_j) = [\Omega - (p_i + hW_i) + \gamma(p_j + hW_j)]^+, j \neq i, i = 1, 2.$

Since the demand is always smaller than supply in equilibrium, not all drivers will be utilized all the time. Hence, the effective wage expected by each driver is $w_i \frac{d_i}{s_i}$, where $\frac{d_i}{s_i}$ is the utilization rate of drivers. Thus the supply system becomes $s_i(\boldsymbol{w}) = s_i(w_i, w_j) = \left[w_i \frac{d_i}{s_i} - \beta w_j \frac{d_j}{s_j}\right]^+, j \neq i, \, i = 1, 2.$

We note that unlike mode Q of the deterministic model in which supply is equal to demand and equal to the committed matching quantity, mode Q here assumes that the platforms commit to both demand and supply quantities, because in this extension demand is always smaller than supply; otherwise, the system would not be stable.

Due to the intractable nature of this extension, we resort to a numerical study to check the robustness of our findings. We provide several representative examples of the parameter set to investigate the comparisons among different competition modes.

In Tables 9-12,[10] we allow $h \in \{0, 0.1, 0.2, 0.3\}$, $\gamma \in \{0.6, 0.7\}$, and $\beta \in \{0.1, 0.2\}$, where the demand side is more competitive than the supply side. In every single table of the 16 combinations

[10] The value of w in mode C is calculated by multiplying the equilibrium price and equilibrium commission rate.

of the above parameter set, first, we find that $\pi^*_{wp} \geq \pi^*_P \geq \pi^*_{pw}$, implying that a precommitment on the supply side (the less competitive side) alleviates the competition and generates a higher profit than no precommitment, which, in turn, has a higher profit than the precommitment on the demand side (the more competitive side). This result is consistent with Proposition 1(a), indicating that taking consumers' wait time into consideration seems not to alter the effects of precommitments in various devices. Here we examine the underlying rationale in detail. Unlike the deterministic model, the wage precommitment cannot determine the supply quantity in the presence of matching friction; however, compared with mode P, a relatively lower wage still leads to a relatively lower supply quantity, even though the supply quantity is also affected by the utilization rate (but the effect of the utilization rate on the supply quantity is less prominent than that of the wage). This observation is confirmed by every single table in Tables 9-12. Suppose for a contradiction that a relatively lower wage leads to a relatively higher supply quantity, which means the utilization rate should be unreasonably high, which in turn implies a higher demand quantity. Given that the high utilization rate plays a less significant role in the demand system, a higher demand quantity requires a lower price, and thus a lower profit margin, which is not optimal. In summary, the incorporation of customers' wait time makes the system of matching demand and supply more complicated, but its effect on the demand or supply is secondary compared with that of the price or wage. Therefore, although the wage precommitment cannot determine the supply quantity directly, the reduced power of the precommitment can still alleviate the competition intensity in the second stage, leading to a less intense outcome than mode P. The comparison between modes P and pw can be interpreted similarly.

Second, $\pi^*_{wp} \geq \pi^*_C \geq \pi^*_{pw}$, implying that the commission precommitment is less profitable than the precommitment on the less competitive side, but more profitable than the precommitment on the more competitive side, which is consistent with Proposition 2(a). Again, although the presence of customers' wait time weakens the power of the precommitments, the commission precommitment is still not as direct and effective as the wage precommitment. Moreover, $\pi^*_C \geq \pi^*_P$, implying that when one side is more competitive than the other, the commission precommitment can be better than no precommitment at all. On the other hand, as shown in Table 13, the commission precommitment can lead to a more intense market outcome than no commitment when the two-sided intensities are close enough. These results are consistent with Proposition 2(b).

Finally, π^*_Q is the highest among various modes, implying that the precommitment to the demand and supply quantities is still the most effective compared to the precommitment to wage, price, or commission, which is consistent with Proposition 3, because, compared with other precommitments, the precommitment to both supply and demand quantities is more direct and suffers the least impact from the matching friction. With the ordering of the two-sided intensities reversed, Tables 14-17 with $\gamma < \beta$ also display consistent results.

Mode	P	wp	pw	C	Q
p	1.4	1.72	1.4	1.54	1.49
w	0.52	0.41	0.53	0.4928	0.4511
d	0.44	0.312	0.44	0.384	0.404
s	0.454	0.3395	0.4579	0.4125	0.405
π	0.3872	0.4087	0.3828	0.4021	0.4197

(a) $h = 0$

Mode	P	wp	pw	C	Q
p	1.37	1.54	1.35	1.43	1.4305
w	0.5	0.46	0.51	0.4719	0.4333
d	0.4151	0.3501	0.4231	0.3911	0.388
s	0.432	0.3809	0.4406	0.4077	0.389
π	0.3612	0.3782	0.3554	0.3747	0.3869

(b) $h = 0.1$

Mode	P	wp	pw	C	Q
p	1.32	1.47	1.3	1.42	1.37
w	0.48	0.44	0.49	0.4544	0.4156
d	0.3983	0.3428	0.4063	0.3613	0.372
s	0.4147	0.3683	0.4234	0.3844	0.373
π	0.3345	0.3531	0.3291	0.3489	0.3555

(c) $h = 0.2$

Mode	P	wp	pw	C	Q
p	1.29	1.4	1.24	1.41	1.3142
w	0.45	0.42	0.45	0.4521	0.3967
d	0.3734	0.334	0.3888	0.3366	0.355
s	0.3888	0.3553	0.3969	0.3695	0.356
π	0.3137	0.3273	0.3071	0.3227	0.3257

(d) $h = 0.3$

Table 9 **Matching Friction with** $(\Omega, \gamma, \beta) = (1, 0.6, 0.1)$

Mode	P	wp	pw	C	Q
p	1.51	1.74	1.45	1.67	1.5175
w	0.56	0.43	0.57	0.501	0.4938
d	0.396	0.304	0.42	0.332	0.393
s	0.4211	0.3234	0.4378	0.3647	0.394
π	0.3762	0.3982	0.3696	0.3881	0.4023

(a) $h = 0$

Mode	P	wp	pw	C	Q
p	1.48	1.67	1.39	1.61	1.458
w	0.54	0.44	0.54	0.4991	0.4738
d	0.3734	0.2981	0.4064	0.3236	0.377
s	0.4018	0.3238	0.419	0.3594	0.378
π	0.351	0.3667	0.3454	0.3595	0.3711

(b) $h = 0.1$

Mode	P	wp	pw	C	Q
p	1.37	1.56	1.32	1.55	1.3961
w	0.54	0.48	0.54	0.5425	0.455
d	0.3813	0.3112	0.3983	0.3208	0.362
s	0.4061	0.3456	0.4147	0.3732	0.363
π	0.3165	0.3361	0.3107	0.3232	0.3407

(c) $h = 0.2$

Mode	P	wp	pw	C	Q
p	1.29	1.45	1.247	1.41	1.4094
w	0.48	0.47	0.49	0.4935	0.3988
d	0.3688	0.3184	0.3768	0.3344	0.317
s	0.3763	0.3459	0.3842	0.3632	0.318
π	0.2987	0.3121	0.2939	0.3065	0.3204

(d) $h = 0.3$

Table 10 **Matching Friction with** $(\Omega, \gamma, \beta) = (1, 0.6, 0.2)$

Note that Tables 9-12 allow $h = 0$, which is an extension of the deterministic model's supply system to account for the drivers' utilization while keeping the demand system unchanged as being linear. We find that the main results in Propositions 1-3 still hold in this extension. Compared

Mode	P	wp	pw	C	Q
p^*	1.64	1.87	1.62	1.83	1.8367
w^*	0.6	0.54	0.62	0.5673	0.5011
d^*	0.508	0.439	0.514	0.451	0.449
s^*	0.5238	0.4617	0.5357	0.4799	0.45
π^*	0.5283	0.5839	0.514	0.5695	0.5997

(a) $h=0$

Mode	P	wp	pw	C	Q
p^*	1.56	1.84	1.53	1.79	1.7805
w^*	0.57	0.53	0.58	0.537	0.4867
d^*	0.5026	0.4215	0.5116	0.4359	0.436
s^*	0.5079	0.4484	0.5168	0.4591	0.437
π^*	0.4976	0.5522	0.486	0.5462	0.5641

(b) $h=0.1$

Mode	P	wp	pw	C	Q
p	1.52	1.81	1.49	1.74	1.7276
w	0.55	0.52	0.56	0.522	0.4711
d	0.4852	0.4051	0.4942	0.4239	0.422
s	0.49	0.4352	0.499	0.4463	0.423
π	0.4706	0.5226	0.4596	0.5162	0.5302

(c) $h=0.2$

Mode	P	wp	pw	C	Q
p	1.51	1.71	1.47	1.66	1.6715
w	0.56	0.5	0.56	0.5312	0.4567
d	0.4641	0.4058	0.4743	0.4225	0.409
s	0.4838	0.4275	0.4889	0.4494	0.41
π	0.4409	0.491	0.4316	0.4769	0.4969

(d) $h=0.3$

Table 11 Matching Friction with $(\Omega,\gamma,\beta)=(1,0.7,0.1)$

Mode	P	wp	pw	C	Q
p^*	1.73	1.98	1.61	1.85	1.87
w^*	0.68	0.6	0.66	0.629	0.5513
d^*	0.481	0.406	0.517	0.445	0.439
s^*	0.5114	0.4416	0.5227	0.473	0.44
π^*	0.5051	0.5603	0.4912	0.5433	0.579

(a) $h=0$

Mode	P	wp	pw	C	Q
p^*	1.7	1.97	1.61	1.82	1.8138
w^*	0.67	0.58	0.65	0.6188	0.535
d^*	0.4641	0.3842	0.4888	0.4281	0.426
s^*	0.4985	0.4222	0.5044	0.4604	0.427
π^*	0.478	0.5339	0.4692	0.5142	0.5448

(b) $h=0.1$

Mode	P	wp	pw	C	Q
p^*	1.6	1.94	1.59	1.79	1.7576
w^*	0.63	0.57	0.65	0.6086	0.5188
d^*	0.4647	0.3694	0.4689	0.4122	0.413
s^*	0.4838	0.4104	0.494	0.4479	0.414
π^*	0.4508	0.5061	0.4407	0.4869	0.5117

(c) $h=0.2$

Mode	P	wp	pw	C	Q
p	1.51	1.77	1.47	1.78	1.7048
w	0.6	0.58	0.6	0.623	0.5013
d	0.4606	0.3928	0.4708	0.3947	0.399
s	0.4704	0.4269	0.4752	0.4436	0.4
π	0.4191	0.4675	0.4096	0.4566	0.4802

(d) $h=0.3$

Table 12 Matching Friction with $(\Omega,\gamma,\beta)=(1,0.7,0.2)$

Mode	P	C
p^*	1.38	1.32
w^*	0.57	0.6204
d^*	0.241	0.274
s^*	0.2622	0.2916
π^*	0.1952	0.1917

(a) $h=0$

Mode	P	C
p^*	1.26	1.19
w^*	0.57	0.595
d^*	0.2574	0.2915
s^*	0.2708	0.2945
π^*	0.1776	0.1735

(b) $h=0.1$

Table 13 Matching Friction with $(\Omega,\gamma,\beta)=(1,0.45,0.5)$

with the deterministic model, the equilibrium wage increases because of the consideration of the utilization rate. Moreover, since the demand needs to be smaller than the supply, a higher price is charged to regulate the demand. Hence, due to a smaller demand and a higher wage, the platforms

Mode	P	wp	pw	C	Q
p^*	0.85	0.87	0.83	0.84	0.8312
w^*	0.44	0.43	0.42	0.4368	0.4125
d^*	0.1555	0.1425	0.1678	0.1611	0.163
s^*	0.1654	0.1565	0.1663	0.1677	0.164
π^*	0.0637	0.0627	0.0676	0.0649	0.0682

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.78	0.86	0.76	0.81	0.7505
w^*	0.43	0.39	0.39	0.405	0.3725
d^*	0.1456	0.105	0.1466	0.1284	0.147
s^*	0.1582	0.128	0.1513	0.1442	0.148
π^*	0.051	0.0493	0.0543	0.052	0.0556

(b) $h = 0.2$

Table 14 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.1, 0.6)$

Mode	P	wp	pw	C	Q
p^*	1.01	1.03	1	0.96	0.9261
w^*	0.38	0.32	0.31	0.4704	0.455
d^*	0.1258	0.1083	0.1216	0.1628	0.18
s^*	0.1383	0.1178	0.1228	0.175	0.181
π^*	0.0792	0.0769	0.0839	0.0797	0.0848

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.95	0.98	0.92	0.96	0.8462
w^*	0.43	0.33	0.38	0.384	0.4175
d^*	0.1244	0.0977	0.1286	0.1137	0.165
s^*	0.1462	0.1135	0.1398	0.1321	0.166
π^*	0.0647	0.0635	0.0694	0.0655	0.0707

(b) $h = 0.2$

Table 15 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.2, 0.6)$

Mode	P	wp	pw	C	Q
p^*	0.9	0.93	0.89	0.88	0.8626
w^*	0.5	0.47	0.42	0.5016	0.4567
d^*	0.1187	0.0995	0.1161	0.1302	0.135
s^*	0.1335	0.1184	0.121	0.1399	0.136
π^*	0.0475	0.0458	0.0545	0.0492	0.0548

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.84	0.89	0.82	0.79	0.7788
w^*	0.5	0.49	0.41	0.4898	0.4134
d^*	0.1109	0.0895	0.1063	0.123	0.122
s^*	0.129	0.1147	0.1144	0.138	0.123
π^*	0.0377	0.0358	0.0436	0.039	0.0446

(b) $h = 0.2$

Table 16 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.1, 0.7)$

Mode	P	wp	pw	C	Q
p^*	1.01	1.07	1.01	1.06	0.9626
w^*	0.56	0.43	0.49	0.4028	0.51
d^*	0.13	0.0889	0.1243	0.0914	0.151
s^*	0.1478	0.1071	0.1352	0.1051	0.152
π^*	0.0585	0.0569	0.0646	0.0601	0.0683

(a) $h = 0.1$

Mode	P	wp	pw	C	Q
p^*	0.97	1.04	0.97	0.96	0.88
w^*	0.55	0.32	0.33	0.5376	0.4667
d^*	0.1338	0.063	0.0856	0.1164	0.138
s^*	0.137	0.0778	0.0921	0.1371	0.139
π^*	0.0478	0.0454	0.0548	0.0492	0.0571

(b) $h = 0.2$

Table 17 Matching Friction with $(\Omega, \gamma, \beta) = (1, 0.2, 0.7)$

admit a lower profit than in the deterministic model. Lastly, when h increases from 0 to 0.3, the profit decreases, because the wait time reduces consumers' utility. Meanwhile, the price also decreases, but the demand does not necessarily increase, because the wait time also hurts the demand.

In addition to the robustness check, we would like to point out several differences from the deterministic model. First, in the deterministic model, a precommitment to either wage or price can determine the supply or demand quantity, constraining the subsequent competition on the other side. However, due to the existence of consumers' waiting, the precommitment to either wage or price cannot determine the supply or demand quantity, thereby limiting the power of the precommitment to alleviate the competition on the other side. But such a reduced power of the precommitment still plays a critical role in the competitions, and thus our main results remain robust after considering the wait time. Second, in the deterministic model, a higher equilibrium price corresponds to a higher equilibrium profit and a lower equilibrium wage and matching quantity (thus lower consumer surplus and service provider surplus). This may not be true in the presence of consumers' waiting. (i) Since the equilibrium demand is not equal to the supply, a higher price is not necessarily coupled with a lower wage; i.e., see modes wp and Q in Tables 9(b)-(d). (ii) Wait time also makes the relationship between price and demand more involved. On the one hand, a higher price may drive some consumers out of the market. On the other hand, a smaller amount of potential consumers leads to a lower wait time, which can boost the demand to some extent. Although the overall relationship between price and demand does not change, the wait time acts in the opposite direction to what the price does. Similarly, although a higher wage leads to more service providers, the utilization rate of each service provider is lower. That is, although there are more service providers, each service provider earns less, and the surplus of each individual service provider is reduced.

D. Proof of Lemmas and Propositions in the Main Body

Proof of Lemma 1. Suppose the realized market size is x. For any fixed (p_2, w_2), platform 1's best choice is to render the demand equal to the supply quantity. Otherwise, the platform could always improve its profit by either raising the price or lowering the wage. Let z_1 denote the matching quantity, then

$$x - p_1^* + \gamma p_2 = z_1 = w_1^* - \beta w_2.$$

Hence,

$$p_1^* = x + \gamma p_2 - z_1, \quad w_1^* = z_1 + \beta w_2. \tag{D.1}$$

Platform 1 faces such an optimization problem:

$$\max_{z_1} \quad (x + \gamma p_2 - 2z_1 - \beta w_2)z_1$$
$$s.t. \quad z_1 \leq \frac{1}{2}(x + \gamma p_2 - \beta w_2).$$

Note that the above constraint is derived by $p_1^* \geq w_1^*$. Solving the problem yields $z_1^* = \frac{x+\gamma p_2 - \beta w_2}{4}$. Plugging back into (D.1) gives the best response of platform 1 as follows,

$$p_1^*(p_2, w_2) = \frac{3}{4}(x + \gamma p_2) + \frac{1}{4}\beta w_2, \quad w_1^*(p_2, w_2) = \frac{1}{4}(x + \gamma p_2) + \frac{3}{4}\beta w_2.$$

By the same token, we obtain

$$p_2^*(p_1, w_1) = \frac{3}{4}(x + \gamma p_1) + \frac{1}{4}\beta w_1, \quad w_2^*(p_1, w_1) = \frac{1}{4}(x + \gamma p_1) + \frac{3}{4}\beta w_1.$$

Solving the set of the above equations yields the resulting equilibrium price p_P^*, wage w_P^*, matching quantity z_P^*, and profit π_P^*. Taking expectation with respect to Ω yields the expected equilibrium profit $E[\pi_P^*]$. □

Proof of Lemma 2. Online Supplement B.1 derives the equilibrium of wage precommitment competition under demand uncertainty. The general idea is as follows. We first analyze the subgame equilibrium conditional on a fixed wage in the first stage and the realization of market size (shown in Lemma B.1), and show that for any realized market size x, the equilibrium price in the second stage takes either the supply-depletion price or profit-maximizing price. Based on the optimal decision in the second stage, we then come back to the first stage and discuss different cases regarding to the magnitude of the wage set in the first stage. There are three cases:

1. The wage is too low such that for any realized market size x, both firms adopt the supply-depletion price in the second stage;
2. The wage is in a medium range such that there exists a threshold of the market size, beyond which both firms adopt the supply-depletion price and adopt the profit-maximizing price otherwise;
3. The wage is too high such that for any realized market size x, both firms adopt the profit-maximizing price in the second stage.

When there is no demand uncertainty, we show that the resulting equilibrium wages and prices are such that the supply is equal to the demand (shown in Lemma B.2), which means the equilibrium price in the second stage must be the supply-depletion price. When the demand variance is sufficiently small, it is still optimal to set the wage ex ante such that both firms adopt a supply-depletion price for any market size. That is, Case 1 occurs. According to Case 1 in Online Supplement B.2, since $\text{Var}(\Omega)$ is sufficiently small, one can check

$$\frac{1+\gamma}{4 - 3\beta + \gamma - 2\gamma^2 - 2\beta\gamma + \beta^2\gamma + \beta\gamma^2} < \frac{1}{3 - 2\beta - 2\gamma + \beta\gamma},$$

which implies $w_{i,c1}^* < w_{ub}^1$. Therefore,

$$w_{wp}^* = \frac{1+\gamma}{4 - 3\beta + \gamma - 2\gamma^2 - 2\beta\gamma + \beta^2\gamma + \beta\gamma^2} E[\Omega].$$

Putting w^*_{wp} back to (B.18) and (B.19) gives the equilibrium price p^*_{wp} and the equilibrium expected profit $E[\pi^*_{wp}]$. □

Proof of Lemma 3. Online Supplement C derives the equilibrium of price precommitment competition under demand uncertainty. The general idea is as follows. We first analyze the subgame equilibrium conditional on a fixed price in the first stage and the realization of market size (shown in Lemma C.1), and show that for any realized market size x, the equilibrium wage in the second stage takes either the demand-depletion wage or profit-maximizing wage. Based on the optimal decision in the second stage, we then come back to the first stage and discuss different cases regarding to the magnitude of the price decided in the first stage. There are three cases:

1. The price is too high such that for any realized market size x, both firms adopt the demand-depletion wage in the second stage;

2. The price is in a medium range such that there exists a threshold of the market size, beyond which both firms adopt the profit-maximizing wage and adopt the demand-depletion wage otherwise;

3. The price is too low such that for any realized market size x, both firms adopt the profit-maximizing wage in the second stage.

When there is no demand uncertainty, we show that the resulting equilibrium wages and prices are such that the supply is equal to the demand (shown in Lemma C.2), which means the equilibrium wage in the second stage must be the demand-depletion wage. When the demand variance is sufficiently small, it is still optimal to set the price ex ante such that both firms adopt a demand-depletion wage for any market size. That is, Case 1 occurs. According to Case 1 in Online Supplement C.2, since Var(Ω) is sufficiently small, one can check

$$\frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2} > \frac{2+\beta-\beta^2}{3-2\gamma+\beta-\beta\gamma-2\beta^2+\beta^2\gamma},$$

which implies $p^*_{i,c1} > p^1_{lb}$. Therefore,

$$p^*_{pw} = \frac{3+\beta-\beta^2-\beta\gamma}{4-3\gamma+\beta-2\beta^2-2\beta\gamma+\beta^2\gamma+\beta\gamma^2}E[\Omega].$$

Putting p^*_{pw} back to (C.8) and (C.9) gives the equilibrium wage w^*_{pw} and the equilibrium expected profit $E[\pi^*_{pw}]$. □

Due to the space limit of the Online Appendix, the remaining proofs of lemmas and propositions in the main body are relegated to part D of an Online Supplement, which can be found on the authors' websites.