

# CFD-DEM modeling of seepage erosion around shield tunnels

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**Abstract:** When tunnels are built in saturated silty sand, the tunnel leakage can carry fine particles into tunnels and generate seepage erosion process. During this process sand particles are subjected to high confining and hydraulic pressures and then are eroded through the seams of segmental joints. This paper investigates the mechanism of seepage erosion process using Computational Fluid Dynamics and Discrete Element Method (CFD-DEM) coupling simulations. The seepage erosion processes are simulated for loose, medium dense and dense silty sand, respectively. The evolution of the fine particles loss and the volumetric strain are investigated. Results show that the fine particles are eroded in two patterns. The first pattern is induced by axial pressure extruding fine particles through seams without hydraulic pressure. The second pattern is induced by fluid drag force dragging fine particles under hydraulic pressure. Correspondingly, the erosion process is divided into two stages as initial extruding stage and the following eroding stage. Result shows that dense sand is more prone to particle erosion in the first pattern while loose sand are gradually more prone to particle erosion in the second pattern. The quantitative relationship between the fine particles loss, the volumetric strain and the four influencing factors (i.e. time, hydraulic pressure, consolidated stress ratio and void ratio) are investigated using regression analysis based on 81 numerical simulations, respectively. The flow paths of the eroded fine particles are also investigated during the erosion process, which demonstrates that flow paths change alternatively between the blocked state and the opening state and then more flow paths in the model will open as the erosion process carries on.

**Keywords:** Seepage erosion; silty sand; fluid dynamics; discrete element; tunnel

## 34 **1. Introduction**

35 For shield driven tunnels assembled with segmental lining built in saturated soils, the  
36 tunnel convergence and differential longitudinal settlements may induce the opening and  
37 dislocation of segmental joints. Therefore, tunnel leakage frequently occurs through  
38 circumferential and longitudinal segmental joints. In low permeability soils, the tunnel  
39 leakage provides a new drainage boundary which will cause the decrease of pore pressure  
40 around the tunnel, and thus results in ground and tunnel settlements. The leakage-induced  
41 ground and tunnel responses can be significant in saturated fine soils and have been of great  
42 concern in tunnel engineering (O'Reilly et al., 1991; Yi et al., 1993; Mair & Taylor, 1997;  
43 Cooper et al., 2002; Shin et al., 2002; Asakura & Kojima, 2003; Wongsaroj et al., 2007; Mair,  
44 2008; Zhang et al., 2012; Shen et al., 2014; Wu et al., 2015a, 2015b, 2016; Zhang et al., 2015;  
45 Xu et al., 2016 ). The common understanding on the effect of tunnel leakage on high  
46 permeable soil, for example sand and silty sand, is that it would not significantly reduce the  
47 pore pressure and would not induce ground or tunnel settlements. However, fine particles  
48 could be easily eroded by water seepage into the tunnel through the segmental joints because  
49 of the low cohesive force between the particles in sand or silty sand (see Fig. 1). This erosion  
50 of fine particles may result in the ground loss around the tunnel and increase the void ratio of  
51 the soil. The excessive ground loss will induce longitudinal differential settlements, which  
52 will reduce the stability of shield tunnels and then deteriorate the serviceability of tunnels.  
53 The erosion-induced increase of void ratio will seriously reduce the strength and stiffness of  
54 the soils around tunnel (Yin et al., 2014, 2016a), which will further induce settlements under  
55 external loads (e.g. traffic loading of subway). However, up to now the seepage erosion  
56 induced hazards have been mainly studied for earth dams (Wan & Fell, 2004; Fox, 2006;  
57 Richards, 2007; Chang & Yin, 2011; Midgley, 2013; Yin et al., 2016b). More attention should  
58 be paid to the tunnel engineering.

59 Previous studies of erosion mainly focused on dam engineering. The grain size  
60 distribution (GSD), the confining pressure and the hydraulic pressure, as well as the size of  
61 the soils constriction affect the erosion process. For example, [Kenny \(1986\)](#) suggested an H-F  
62 geometrical curve to distinguish the stable grading from the unstable grading for particle  
63 erosion; [Tomlinson \(2000\)](#) presented that high confining pressure can also produce significant  
64 particle erosion because high confining pressure can break the arching bridges formed by the  
65 fine particles across the inter-particle voids and result in more erodible fine particles; the  
66 constriction size, defined as the size of narrow voids along the flow path which is the key  
67 obstacle for fine particles to travel successfully through the flow path, is another important  
68 factor affecting the particle erosion. The constriction size distribution was found to be closely  
69 related to the GSD, material relative density and the cumulative amount of eroded fine  
70 particles ([Indraratna, 2007](#); [Reboul, 2010](#)).

71 Compared with earth dams, the characteristic of particle erosion around tunnels are as  
72 follows: (1) the soil particles are subjected to various confining and axial pressures depending  
73 on the tunnel embedded depth; (2) the erosion boundary is specific because particle erosion  
74 happens only through the seams of tunnel segmental joints. For silty sand foundation, the  
75 various confining and axial pressures are always associated with various void ratios along the  
76 depth. Therefore, the void ratio is used to characterize the stress states in this paper. Besides,  
77 under different hydraulic pressure and consolidated stress ratio, silty sand can have different  
78 performance during the erosion process. Furthermore, time is another important factor to  
79 define the erosion process. All these factors deeply influence the seepage erosion process  
80 around shield tunnel and these processes need to be clarified. Among many ways, the  
81 coupling CFD-DEM should be an effective method ~~with fundamentals of physics~~ which can  
82 **provide insights of fundamental physics**.

83 DEM appears to be promising for investigating the seepage erosion process coupled with

84 CFD, particularly the migration and erosion of fine particles (Zhao & Shan, 2013; Sibille et al.,  
85 2015). DEM treats soils as an assembly of discrete particles. It starts with the basic  
86 constitutive laws at inter-particle contacts at microscopic scale and develops into the  
87 responses of the particle assembly under different loading conditions at the macroscopic scale.  
88 The advantage of DEM lies in that it can physically capture the behavior of particulate  
89 materials, and as a discontinuous analysis method it can simulate the large deformation and  
90 discontinuous process of discrete particle assembly under quasi-static and dynamic condition  
91 (see Jiang & Yin, 2012, 2014; Jiang et al., 2016). Therefore, it can potentially overcome the  
92 shortcomings of the finite element method and is a powerful numerical tool for computing the  
93 motion of large number of particles in the large deformation and discontinuous analyses of  
94 particle eroding. However, few investigations by CFD-DEM simulating the seepage erosion  
95 process in granular materials around shield tunnels have been conducted up to now.

96 The main objectives of this paper are: (1) to investigate the seepage erosion mechanism  
97 of soils around tunnels under various stress states and hydraulic pressures, and (2) to obtain  
98 the quantitative relationship between the fine particles loss, the volumetric strain and the four  
99 influencing factors: time, hydraulic pressure, consolidated stress ratio and void ratio. For this,  
100 the extensive CFD-DEM modelling of erosion is performed under various influencing factors.  
101 The confining and axial pressures applied on the DEM model are used to simulate the stress  
102 states of the soils around shield tunnels. In this paper, three stress states are applied on the  
103 DEM model to generate three void ratios of soil models representing loose, medium dense and  
104 dense silty sand, respectively. The specific erosion boundary of longitudinal and  
105 circumferential segmental joints is simulated using the perpendicular erosion seams at the  
106 bottom plate. The seepage erosion mechanism under various stress states and the specific  
107 erosion boundary is interpreted.

## 108 2. Numerical simulations

### 109 2.1 Description of CFD-DEM

110 The CFD option for PFC 3D enables the combination of the DEM calculation with the  
111 computational fluid dynamics model (Itasca Consultant Group, 2003). In this paper, the CFD  
112 solves the incompressible Navier-Stokes equation on a 3D discretized fluid cell consisting of  
113 mapped hexahedra grid as shown in Fig.4(c). In each fluid cell, the fluid force acting on each  
114 particle is calculated and assigned based on the fluid conditions in the fluid cell that the  
115 particle occupies. The presence of particles in each fluid cell is accounted by the porosity  
116 terms in the fluid equation including the Navier-Stokes equation and the continuity equation.

117 The Navier-Stokes equation can be modified to include the effect of a particulate solid  
118 phase into the fluid. Since the average effects over many particles are focused on (as opposed  
119 to attempting to model the details of fluid flow between particles), they can be characterized  
120 in terms of porosity  $n$  and a coupling force  $f_b$ , as show in the Eq. (1).

$$121 \quad \rho_f \frac{\partial v}{\partial t} + \rho_f \frac{v}{n} \cdot \nabla(v) = -n \nabla p + \mu \nabla^2(v) + f_b \quad (1)$$

122 where  $\rho_f$  is the density of the fluid,  $p$  is the fluid pressure,  $\mu$  is the dynamic viscosity of the  
123 fluid,  $v$  is the Darcy fluid velocity.

124 The Darcy fluid velocity can be calculated by the Darcy's law using Eq.(2).

$$125 \quad v = ki = k \frac{\Delta P}{\gamma_w l} \quad (2)$$

126 where  $i$  is the hydraulic gradient,  $P$  is the hydraulic pressure,  $\gamma_w$  is the unit weight of water,  $l$  is  
127 the length of the flow path which is the height of soil model in this study,  $k$  is the permeability  
128 (or hydraulic conductivity coefficient) of soil with units of length over time.

129 The permeability can be calculated according to the Kozeny-Carman relation using  
130 Eq.(3).

131 
$$k = \frac{\gamma_w}{\mu_f} B \frac{n^3}{(1-n)^2} d^2 \quad (3)$$

132 where  $\mu_f$  is the dynamic fluid viscosity in Pa·s,  $B$  is a geometric factor taken as 1/180,  $d$  is the  
 133 grain diameter. According to Eq.(3), the value of  $k$  of models in this study ranges from  
 134  $1.317 \times 10^{-6} \text{m/s}$  to  $2.880 \times 10^{-5} \text{m/s}$  for different void ratios.

135 The drag force per unit volume applied by the particles to the fluid in each fluid element  
 136 is defined as

137 
$$f_b = \beta(u - v) \quad (4)$$

138 where  $\beta$  is an empirical parameter. According to Tsuji (1993), the parameter  $\beta$  can be  
 139 calculated in one of two ways, depending on the porosity of the fluid element. For high values  
 140 of porosity ( $n \geq 0.8$ ),  $\beta$  is derived from the corrected nonlinear drag force exerted on a spherical  
 141 particle by a fluid relating to the Reynolds number. For low values of porosity ( $n < 0.8$ ),  $\beta$  is  
 142 derived from the Ergun relations (see Ergun 1952) as shown in Eq.(5). In this study, the  
 143 porosity is lower than 0.8.

144 
$$\beta = \frac{(1-n)}{d^2 n^2} (150(1-n)\mu_f + 1.75\rho_f d(u-v)) \quad (5)$$

145 where  $d$  is the average diameter of particles in the fluid element.

146 The drag force  $f_{drag}$  is the body force experienced by the fluid as a result of moving  
 147 particles. A force equal and opposite is distributed to the particles in each fluid element, which  
 148 is proportional to the volume of each particle, defined as fluid drag force,  $f_{drag}$ .

149 
$$f_{drag} = \frac{4}{3} \pi r^3 \frac{f_b}{1-n} = \frac{4}{3} \pi r^3 \beta (v-u)(1+e) \quad (6)$$

150 where  $r$  is the particle radius,  $e$  is the void ratio.

151 The coupling between the CFD and DEM in each time step is volume-averaged and two-  
 152 way, which means that the force acting on the particles in one fluid cell is also applied to the  
 153 fluid as an average over the same fluid cell. The total fluid drag force is determined in PFC

154 and divided by the volume of the fluid cell in CFD. The CFD determines the fluid velocity  
155 and fluid pressure gradient in each fluid cell. So in each time step when the coupling  
156 information are exchanged, PFC sends CFD the current porosity and the total fluid drag force.  
157 At the same time CFD sends PFC the current fluid velocity and fluid pressure gradient in each  
158 fluid cell. After that, the PFC program runs forward one time step for given interval seconds  
159 when the fluid drag force and porosity in each fluid cell are recalculated. Then the PFC  
160 program moves to the next time step.

161 The DEM code of PFC 3D was used to build the soil models. The inter-particle contact  
162 and the particle-wall contact in normal and tangential directions were modelled by the linear  
163 contact model proposed by Cundall (1979) as shown in Fig. 2. The contacts in normal and  
164 tangential directions contain a spring to resist the inter-particle force and a dashpot that allows  
165 energy dissipation and quasi-static deformation. A divider is set in the normal and tangential  
166 directions so that the contact force can be reset to zero if the two particles are separated. A  
167 slider is set in the tangential direction to provide shear force controlled by the Coulomb  
168 friction between two contact particles.

169 The instantaneous translational and angular accelerations can be calculated based on the  
170 Newton's 2<sup>nd</sup> Law using the contact force and moment applied on each particle. The position  
171 and the force state of each soil particle can be recorded at each time step. By generating tens  
172 of thousands of soil particles, the erosion process can be simulated with recording its  
173 deformation.

## 174 **2.2 Simulation process**

### 175 *2.1.1 Grain size distribution*

176 In order to have an axisymmetric condition which can reduce the particle number  
177 compared to cubic DEM model, the cylindrical DEM model was designed with 0.7 mm in  
178 diameter and 0.7 mm in height within the CFD domain. It contains about twenty thousand  
179 particles with six different diameters as shown in Fig. 3. The grain size distribution (GSD)

180 follows the gap-graded pattern proposed by Wan and Robin (2004) which was widely used to  
181 study the erosion related issues. In Fig. 3, the dashed line represents the on-site GSD of  
182 Shanghai silty sand around tunnels, and the solid line represents the gap-graded GSD in the  
183 model. The simplified gap-graded GSD represents the main characteristic of the GSD of on-  
184 site soil. The width of the erosion seam is 0.1 mm. So the gap-graded GSD is simplified as  
185 two categories of coarse particles and fine particles. For coarse particles, the diameters are  
186 between 0.25 mm to 0.1 mm, and for the fine particles, the diameters are between 0.025 mm  
187 to 0.0125 mm. The transitional particles with diameters ranging from 0.1 mm to 0.025 mm  
188 were ignored because they can hardly pass the constrictions formed by the coarse particles  
189 with the maximum diameter of 0.25 mm. The particles with diameters smaller than 0.0125  
190 mm were also ignored, since minor effects could be induced by these too fine particles with  
191 computational demand.

192 The coarse particles form the skeleton force chain in the model and they can not be  
193 eroded through the erosion seam. The fine particles are erodible and the erosion of fine  
194 particles will induce re-arrangement of particles and their skeleton force chain, thus result in  
195 volumetric strain of the model. So the simplified GSD represents the main characteristic of the  
196 GSD of on-site soil and ignores the minor one to save the computational time.

### 197 2.1.2 Model preparation

198 The model preparation followed four steps. First, the homogeneous model was generated  
199 using the Multi-layer Method according to Jiang & Yin (2012, 2014), as shown in Fig. 4(a).  
200 The model was generated by three layers in this step. The first layer was in the bottom and  
201 particles were randomly generated in this layer. The axial pressure was then applied on the top  
202 wall to compress the first layer, while the confining pressure was applied on the lateral wall to  
203 keep the lateral walls still. The axial and confining pressures were gradually applied to the  
204 target value as presented in Table 1. The target pressures can be determined corresponding to



205 the on-site pressures around the tunnel. This process was repeated for the second and third  
206 layers until the model was fully generated. Along this way, three types of DEM models  
207 (Model-1, Model-2 and Model-3) were generated by applying three stress states of confining  
208 and axial pressures and the corresponding void ratios were 0.14, 0.31 and 0.43, representing  
209 dense, medium dense and loose sand respectively, as presented in Table 1.

210 Then, the model was consolidated under target confining and axial pressures as presented  
211 in Table 1. In the deposition process, an efficient servo-control algorithm coded based on the  
212 manual from the [Itasca Consulting Group \(2003\)](#) was used to keep the confining and axial  
213 pressures constant and to save the computational time.

214 Afterwards, the perpendicular seams with the width of 0.1 mm were generated at the  
215 bottom of the model as shown in Fig. 4(b), through which the fine particles would be  
216 gradually eroded by seepage. The perpendicular seams were used to simulate the  
217 perpendicular longitudinal and confederal joints of segmental tunnel lining.

218 Finally, the hydraulic pressure was applied using the fluid cell ([Shimizu, 2004](#);  
219 [Ravichandran, 2010](#)). The  $3 \times 3 \times 3$  grids of fluid cell were used and shown in Fig. 4(c). If the  
220 size of the grid is too small, singularity in the governing equations of the fluid may happen  
221 when one coarse particle occupies one fluid cell and reduces the porosity of the fluid cell to  
222 nearly zero. Since the diameter of the coarse particle is larger than 0.2 mm and the height of  
223 the soil model is 0.7 mm, the  $3 \times 3 \times 3$  grids of fluid cell were used to avoid the computational  
224 singularity.

### 225 *2.1.3 Seepage erosion progress*

226 After the completion of DEM model, the seepage erosion progress was started and  
227 particles migration was triggered. Both the axial pressure and hydraulic pressure caused the  
228 fine particles erosion. Particles near the seams at the bottom were firstly pushed out of the  
229 model by the axial pressure. And soon, particle arch formed above the seams and stopped

230 further erosion. Afterwards, further erosion occurred due to the fluid drag force induced by  
231 the differential hydraulic pressure. A single particle will lose the balance and start migrating  
232 following certain erosion paths when subjected to the fluid drag force. Correspondingly, in  
233 shield tunnel engineering, when the opening and dislocation of the segmental joints develop  
234 large enough for particles passing through, the fine particles will flow into the tunnel from the  
235 segmental joints. This process is repeating up to a stable state.

236 Different values of hydraulic pressure were used as shown in Table 1 to study its  
237 influence on the seepage erosion.

### 238 **2.3 Model Parameters**

239 The parameters for soil particle, wall cell and fluid cell are presented in Table 2. For the  
240 soil particle, a granular density of  $2650 \text{ kg/m}^3$  and a friction coefficient of 0.3 were used based  
241 on previous studies (Jiang & Yin, 2012, 2014). The normal and shear stiffnesses were  
242 calibrated from the macro-parameters of the Shanghai silty sand by trial and error tests  
243 according to Luo (2007). Note that the unit of stiffness for spherical particles is “pa” instead  
244 of “N/m” in two-dimensional discs, and in this study the particle stiffness is constant and  
245 independent of the particle radius (Itasca Consulting Group, 2003). For the wall cell, the  
246 normal and shear stiffnesses were set to be 10 times bigger than the particle stiffness to  
247 prevent the particles from passing through the walls. The parameters of the fluid cell were  
248 derived based on the behaviour of pure water under the pressure of 100 kPa and the  
249 temperature of  $20^\circ\text{C}$ .

## 250 **3. Progression of seepage erosion**

### 251 **3.1 Seepage erosion-induced fine particles loss**

252 The fine particles loss was defined as the mass ratio of the eroded fine particles to all  
253 particles in the model, which can be distinguished by the axial pressure extruding induced

254 fines loss and the hydraulic pressure induced fines erosion. For instance, when the head  
255 pressure is 0, the particle loss is only generated by the axial compression extrusion, which can  
256 be defined as the Case 1 (pure extrusion); when the head pressure is increased to 100 kPa  
257 keeping other conditions the same, the particle loss is caused by the axial compression as well  
258 as the drag force due to seepage, which can be defined as Case 2. Then, the amount of particle  
259 loss caused only by the seepage drag force is equal to the Case 2 minus Case 1.

260 The progression of fine particles loss with time for all three models under the same  
261 hydraulic pressure of 100kPa was presented in Fig. 5(a) with results under no applied  
262 hydraulic pressure (0 kPa) shown in Fig. 5(b) as a reference. For the Model-1 with dense sand,  
263 the progression of erosion-induced fine particles loss can be divided into two stages so-called  
264 as the extruding-eroding coupled stage and the eroding-dominated stage. In the former, the  
265 fine particles loss increases rapidly with time during which an important part of fines loss (up  
266 to around  $t = 1s$  when the increasing rate becomes small and stable shown in Fig. 5(b)) is ~~and~~  
267 ~~it is mainly~~ induced by axial pressure extruding particles through the erosion seam on the  
268 bottom. In the latter, the increase of fine particles loss with time slows down and the fine  
269 particles loss gradually reaches stable, during which fine particles loss is mainly induced by  
270 the fluid drag force dragging particles to the bottom of the model. The fine particles loss for  
271 medium dense sand by Model-2 and loose sand by Model-3 keeps increasing in the eroding-  
272 dominated stage at different paces. The fine particles loss of the Model-3 is most significant  
273 during the progression of seepage erosion.

274 The fine particles loss at a reference time of seepage erosion was defined as the reference  
275 fine particles loss. Note that the reference time of seepage erosion is set at  $t=6s$  which is  
276 enough for the comparison with saving computational time. The reference fine particles loss  
277 of the three models under different hydraulic pressures was presented in Fig. 5(b,c). The  
278 exponential relation was found between the reference fine particles loss and the hydraulic

279 pressure. The interception on the  $w$  axis is the reference fine particles loss corresponding to  
280 the null hydraulic pressure, which is caused by the axial pressure squeezing the fine particles  
281 passing through the bottom seams. As interpreted in the former section, the axial pressure-  
282 induced fine particles loss is the first pattern of erosion. The axial pressure-induced reference  
283 fine particles loss for Model-1, Model-2 and Model-3 are 1.1%, 0.69% and 0.49%  
284 respectively. Further erosion is prevented by the particle-arch formed above the seam. It is  
285 larger for the dense sand Model-1 than for the loose sand Model-3 because the axial pressure  
286 applied on the Model-1 is larger than that on the on Model-3. When the hydraulic pressure  
287 was applied, the reference fine particles loss increases with the hydraulic pressure because the  
288 fluid drag force is applied on each particle. However, the increasing rate for the dense sand  
289 Model-1 is much slower than the loose sand Model-3. For instance, when the hydraulic  
290 pressure increased from 0 to 100 kPa, the reference fine particles loss for the loose sand  
291 Model-3 increased from 0.49% to 5.8% while the reference fine particles loss for the dense  
292 sand Model-1 only increased from 1.1% to 1.67%. This implies that for dense sand, the  
293 reference fine particles loss is mainly induced by the axial pressure. Whilst, for loose sand the  
294 reference fine particles loss is mainly induced by the hydraulic pressure. Generally, the axial  
295 pressure results in more fine particles loss for dense sand than for loose sand, while the  
296 hydraulic pressure results in more fine particles loss for loose sand than for dense sand. The  
297 combined reference fine particle loss caused by axial pressure and hydraulic pressure is more  
298 important for loose sand and less important for dense sand.

299 The coupling effect of void ratio and hydraulic pressure on the reference fine particles  
300 loss is illustrated in Fig. 6, which indicates that the coupling of high void ratio and high  
301 hydraulic pressure results in the very significant reference fine particles loss of 14%.

### 302 **3.2 Seepage erosion-induced volumetric strain**

303 In the models, force chains are formed in the granular materials to resist axial and

304 confining pressures. The force chains are composed of coarse particles and part of fine  
305 particles. Particles erosion results in particles re-arrangement in the force chains, and thus  
306 leads to volumetric strain of the model.

307 Fig. 7(a) shows the evolution of the volumetric strain with time under the same hydraulic  
308 pressure of 100 kPa for three models. The evolution of volumetric strain can also be divided  
309 into two stages: extruding stage and eroding stage. During the extruding stage, the volumetric  
310 strain increases rapidly. Then, the increasing of volumetric strain slows down with time and  
311 the volumetric strain gradually reaches a stable state at the end of the eroding stage. However,  
312 for the medium dense sand Model-2, the volumetric strain in the eroding stage increases  
313 slowly. This will be explained in the section 4 of this paper in terms of the flow path in  
314 medium dense sand Model-2 later on.

315 The volumetric strain at the reference time of seepage erosion was defined as the  
316 reference volumetric strain. Fig. 7(b) shows the reference volumetric strain of the three  
317 models under different hydraulic pressures. The linear correlation was found between the  
318 reference volumetric strain and the hydraulic pressure. The interception on the  $\varepsilon_v$  axis is the  
319 reference volumetric strain with null hydraulic pressure and is induced by the axial pressure.  
320 The axial pressure-induced reference volumetric strain for Model-1, Model-2 and Model-3 are  
321 1.27%, 1.33% and 0.93% respectively. It is larger for the dense sand Model-1 than for the  
322 loose sand Model-3, which implies that the axial pressure introduces more volumetric strain  
323 for dense sand than for loose sand. But the differences between the three models are not  
324 obvious. Under the hydraulic pressure, the volumetric strain of the loose sand Model-3  
325 increases faster than that of the dense sand Model-1. When the hydraulic pressure increased  
326 from 0 to 100 kPa, the reference fine particles loss for the Model-3 increased from 0.93% to  
327 1.2% (0.27% increased), while the reference fine particles loss for the Model-1 increased from  
328 1.27% to 1.4% (0.13% increased). It implies that the hydraulic pressure produces more

329 volumetric strain for loose sand than for dense sand. This is in consistency with the  
330 progression of reference fine particles loss described in the previous section. However, the  
331 hydraulic pressure-induced increasement of volumetric strain is not obvious compared to axial  
332 pressure-induced volumetric strain. This implies that the axial pressure plays the major role  
333 for volumetric strain in both dense and loose sand. Besides, under the same hydraulic pressure,  
334 the reference fine particles loss is more important for loose sand Model-3 as shown in Fig.  
335 5(b,c), while the reference volumetric strain is less important for loose sand Model-3 as shown  
336 in Fig. 7(b). This is because that the fine particles loss is defined as the mass ratio of the  
337 eroded fine particles to all particles of the model, and the total mass of Model-1 is much  
338 bigger than Model-3 which gives a smaller ratio, even though the mass of eroded fine  
339 particles in Model-1 surpasses that of Model-3.

340 Comparing Fig. 5(a) with Fig. 7(a), for loose sand Model-3, about 85% of the reference  
341 volumetric strain with only 32% of the reference fine particles loss happened in the extruding  
342 stage. In the eroding stage, the volumetric strain reaches stability while the fine particle loss  
343 keeps increasing. It means that further particle loss in this stage does not increase the  
344 volumetric strain any more. These particles can be defined as the floating particles as shown  
345 in Fig. 8. The erosion of skeleton particles will directly lead to the increase of volumetric  
346 strain. Whilst, the erosion of floating particles will only increase the voids between the  
347 skeleton force chains and isolate the skeleton particles from each other. The isolated skeleton  
348 force chains in loose sand Model-3 are able to balance the static forces but with less resistance  
349 to the dynamic traffic loading. Once the force chains collapse under the dynamic traffic  
350 loading, further volumetric strain will develop. So the erosion of floating particles in the  
351 eroding stage increases the risk of further volumetric strains under cyclic traffic loading in  
352 shield tunnels.

353 For dense sand Model-1, about 90% of the reference volumetric strain with 72% of the

354 reference fine particles loss happened in the extruding stage. In the eroding stage, both the  
355 volumetric strain and fine particles loss reach a stable state. There are two reasons leading to  
356 this phenomenon. First, the mass ratio of floating particles in dense sand model is smaller than  
357 that in loose sand model. Secondly, particles in the dense sand Model-1 are less likely to be  
358 hollowed out by fluid drag force compared with loose sand Model-3.

359 The coupling effect of void ratio and hydraulic pressure on the volumetric strain is  
360 illustrated in Fig. 9. It shows that high hydraulic pressure coupled with high void ratio results  
361 in a larger volumetric strain. The maximum volumetric strain in Fig. 9 can be as large as 2.5%,  
362 which could cause additional damage to the tunnel lining.

#### 363 **4. Flow path analysis**

364 The flow path was recorded to investigate the particle movements under the hydraulic  
365 pressure. In the model, the erodible fine particles are dragged by the fluid drag force following  
366 many possible flow paths among the skeleton force chains. Each flow path has a constriction  
367 with the minimum diameter which may stuck the fine particles. Tracing the fine particles can  
368 record the flow path in 3D coordinate system.

369 The positions of the particles were monitored every 750 steps and recorded by the  
370 coordinates. The flow path can be then obtained by connecting the monitored positions. Dense  
371 points on the flow path imply that the particle is blocked for a relatively long time, whereas  
372 loose points on the flow path imply that the particle is unobstructed.

373 Fig. 10 shows the flow path of two eroded particles. The origin of coordinate is at the  
374 bottom center of the model. The coordinate for the top center of the model is (0,0,0.7). The  
375 fine particle with the ID of 28968 is blocked at the height of 0.3 mm temporarily. The fine  
376 particle with the ID of 26237 is blocked at the height of 0.5 mm temporarily. These two flow  
377 paths closed during the period that the fine particles were blocked and then re-opened when  
378 the constriction hole gets enlarged due to the movement of the force chain. And the fine

379 particles continued moving until they met the next blocking condition or got out of the model  
380 from the perpendicular seams at the bottom. Fig. 11 shows the flow paths of two fine particles  
381 blocked at the height of 0.1 mm and 0.4 mm respectively. These two flow paths were not re-  
382 opened until the end of the simulation.

383 During the erosion process, the erodible fine particles always find the least energy  
384 consuming path among many other paths. The erosion process can be described using three  
385 modes as shown in Fig. 12. Mode 1 is shown in Fig. 12(a), where the fine particle is smaller  
386 than the constriction hole so that the particle can go through without consuming extra energy.  
387 During this process, the volumetric strain did not develop. Mode 2 is shown in Fig. 12(b),  
388 where a fine particle was blocked at the constriction hole and the path was closed temporarily.  
389 Fine particles blocked around the entrance of the path have to find other least energy  
390 consuming paths to go through. Mode 3 is shown in Fig. 12(c), where the skeleton force chain  
391 formed by the coarse particles was removed by the unbalanced force due to particle re-  
392 arrangement. The constriction hole was enlarged and the previous blocked fine particles could  
393 go through again. This is why fine particles with IDs of 28968 and 26237 continue migrating  
394 after temporary obstruction. The three modes of erosion can appear separately or alternatively.

395 The modes of flow path described in Fig. 12 are only for an individual one. The real flow  
396 paths in the model consist of thousands of such individual flow paths, which form a complex  
397 flow net. Since it is very difficult to describe the flow net directly, the distribution of flow  
398 paths was described by analysing the variation of numbers of fine particles in different layer  
399 of the studied model.

400 Each model was equally divided into three layers, denoted as top, middle and bottom  
401 layer along its height. Fig. 13(a) shows the fine particle distribution with time for different  
402 layers of Model-3 under the hydraulic pressure of 100 kPa. For Model-3, the amount of fine  
403 particles in the top layer decreases with time, whereas the number of fine particles in the



404 middle and bottom layers is unchanged with time. It implies that the eroded fine particles flow  
405 through the top, middle and bottom layers successively. So the particles flow process follows  
406 the mode 1 of erosion as shown in Fig. 12(a). This can explain that the loose sand Model-3  
407 experienced most significant fine particle loss but results in least significant volumetric strain  
408 among the three models.

409 For Model-2 in Fig. 13(b), the number of fine particles in the top and bottom layers  
410 decreases with time during the extruding stage and the eroding stage, whereas the number of  
411 fine particles in the middle layer increases with time during the extruding stage and reaches a  
412 stable state during the eroding stage. It indicates that during the extruding stage, the flow  
413 paths from middle layer to bottom layer closed so that the fine particles from the top layer  
414 were blocked in the middle layer. Whereas in the eroding stage, the paths are re-opened so  
415 that the number of fine particles in the middle layer tends to stabilize. For the medium dense  
416 sand Model-2, the particle flow in each layer follows different flow mode. The re-opening of  
417 the flow path in the eroding stage results in the increase of the volumetric strain as shown in  
418 Fig. 7(b).

419 For Model-1 in Fig. 13(c), the number of fine particles in bottom layer decreases with  
420 time, whereas the number of fine particles in the top and middle layers remains unchanged. It  
421 indicates that most of the flow paths from the top layer to the bottom layer were blocked. Fine  
422 particles near the perpendicular seams at the bottom layer were eroded until the particle arch  
423 is formed and the further erosion is prevented. This can explain the evolution of volumetric  
424 strain with hydraulic pressure as shown in Fig. 7 (b). Most of the volumetric strain was caused  
425 by the axial pressure rather than hydraulic pressure.

426 From the above analysis, the development of the flow paths can be described in different  
427 layers without portraying the complex and unpredictable flow network in the model.

## 428 **5. Regression analysis**

429 In order to obtain the quantitative relationship between the reference fine particles loss,  
430 the volumetric strain and the influencing variables, a regression analysis was conducted based  
431 on the numerical results. The influencing variables were considered in this study are: 1) time  $t$ ;  
432 2) hydraulic pressure  $P_u$ ; 3) consolidation stress ratio  $K_0$ ; and 4) void ratio  $e$ . To eliminate the  
433 influence of dimensions in the regression analysis, the time  $t$  and hydraulic pressure  $P_u$  were  
434 normalized by two reference parameters  $t_0$  and  $P_0$ , where  $t_0$  is 1 second and  $P_0$  is the  
435 confining pressure listed in Table 1. The upper and lower bounds for the influencing variables  
436 are listed in Table 3. The specific value for each variable was given between the upper and  
437 lower bounds. For  $t/t_0$ , it has been given 9 values of 0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5. Five values  
438 were given from 0 to 1 because the fine particles loss and the volumetric strain increase  
439 significantly during this period of extruding stage of seepage erosion. For the other three  
440 variables, the values are averagely set between the upper and lower bounds.

441 The amount of DEM simulation can be  $9 \times 5 \times 6 \times 5 = 1350$  if fully combining the four  
442 variables. Therefore, the principle of orthogonal experiment was used for the four variables to  
443 optimize the simulation, based on which 81 simulations were carried out after optimization  
444 using the orthogonal combination (Zhang et al. 2015; Kung et al. 2007).

445 The variations of averaged fine particles loss  $w$  and volumetric strain  $\varepsilon_v$  with the  
446 influencing variables are presented from Fig. 14 to Fig. 17, in which the averaged values of  $w$   
447 and  $\varepsilon_v$  were obtained by taking the arithmetic average of  $w$  and  $\varepsilon_v$  at a certain value of the  
448 influencing variable. For example, using 18 simulations with  $K_0 = 0.75$  presented in Table 4,  
449 the averaged values of  $w$  and  $\varepsilon_v$  were obtained by taking the arithmetic average of  $w$  and  $\varepsilon_v$   
450 values. Along this way, the averaged values of  $w$  and  $\varepsilon_v$  corresponding to various influencing  
451 variables can be obtained using the same method.

452 The averaged  $w$  with the four variables is presented in Fig. 14. The averaged  $w$  varies

453 nonlinearly with each normalized variable. These nonlinear trends can be empirically fitted by  
 454 an exponential or a power function with a coefficient of determination ( $R^2$ ) higher than 0.75.  
 455 Based on the four best fitted curves in Fig. 14, a function can be established to interpret the  
 456 systematic variation of  $w$  with the four normalized variables as

$$457 \quad w = e^{a_0} \times e^{a_1 \ln(t/t_0)} \times e^{a_2/K_0} \times e^{a_3 \ln(P_u/P_0)} \times e^{a_4/e} \quad (7)$$

458 where  $w$  is the fine particle loss.  $a_i$  are the function coefficients. Taking the logarithm scale  
 459 for both sides of Eq. (7), a multi-linear function can be obtained as

$$460 \quad \ln w = a_0 + a_1 \ln\left(\frac{t}{t_0}\right) + \frac{a_2}{K_0} + a_3 \ln\left(\frac{P_u}{P_0}\right) + \frac{a_4}{e} \quad (8)$$

461 Then, the multiple linear regression analysis can be carried out using Statistical Product  
 462 and Service Solutions (SPSS). Eq. (8) was used to fit all data from 81 simulations. The best-  
 463 fitted function coefficients are listed in Table 5. Eq. (8) associated with these coefficients has  
 464 the  $R^2$  of 0.87.

465 Similarly, the averaged  $\varepsilon_v$  with four variables is presented in Fig. 15. The nonlinear trend  
 466 of averaged  $\varepsilon_v$  with each variable can be empirically fitted by an exponential or a power  
 467 function. The function of  $\varepsilon_v$  is established as

$$468 \quad \varepsilon_v = e^{b_0} \times e^{b_1 \ln(t/t_0)} \times e^{b_2/K_0} \times e^{b_3 \ln(P_u/P_0)} \times e^{b_4 e^2 + b_5 e} \quad (9)$$

$$469 \quad \ln \varepsilon_v = b_0 + b_1 \ln\left(\frac{t}{t_0}\right) + \frac{b_2}{K_0} + b_3 \ln\left(\frac{P_u}{P_0}\right) + b_4 e^2 + b_5 e \quad (10)$$

470 where,  $\varepsilon_v$  is volumetric strain,  $b_i$  are function coefficients.

471 The function coefficients in Eq. (10) can also be obtained by conducting multiple linear  
 472 regression analysis. The function coefficients are listed in Table 6. Eq. (10) associated with  
 473 these coefficients has the  $R^2$  of 0.64.

474 Thus, the quantitative function between the averaged  $w$  and  $\varepsilon_v$  and the four influencing  
 475 variables were obtained. It is worth noting that Eq. (8) and Eq. (10) should be limited within

476 the range of values discussed in this study.

477 Comparing Fig. 14 and Fig. 15, the evolution tendency for fine particles loss and  
478 volumetric strain with the first three variables is similar. Both fine particles loss and  
479 volumetric strain increase with time  $t/t_0$  and hydraulic pressure  $P_u/P_0$  and decrease with  
480 consolidation stress ratio  $K_0$ . But the tendency for the last variable (void ratio  $e$ ) is opposite.  
481 This is because the variation of fine particles loss and volumetric strain is caused by axial  
482 pressure and hydraulic pressure, as explained in the former section. From Fig. 14(d), the  
483 contribution of these two factors can be separated into Fig. 16(a) adding Fig. 16(b), which  
484 shows the evolution of averaged fine particles loss with void ratio by axial pressure and  
485 hydraulic pressure respectively. And similarly, Fig. 15(d) can be separated into Fig. 17(a) and  
486 Fig. 17(b) which shows the evolution of averaged volumetric strain with void ratio by axial  
487 pressure and hydraulic pressure respectively.

488 The separated evolutions for fine particles loss and volumetric strain are similar in Fig.  
489 16 and Fig. 17. This similar evolution tendency shows that the dense sand is more prone to  
490 particle erosion caused by axial pressure, and loose sand is more prone to particle erosion  
491 caused by hydraulic pressure. But the variation rate of fine particles loss is always faster than  
492 volumetric strain. This conclusion is in consistency with the conclusion in the former section.

## 493 **6. Conclusions**

494 The mechanism of seepage erosion around shield tunnels was investigated for dense,  
495 medium dense and loose silty sand using CFD-DEM coupling simulations. The gap-graded  
496 GSD of soil particles was used in all simulations. Different confining pressure, axial pressure  
497 and hydraulic pressure were applied on the soil model to simulate different stress state of the  
498 soils around shield tunnel in saturated silty sand foundation. The opening and dislocation of  
499 tunnel segmental joints were modelled by the perpendicular erosion seam in the bottom of the  
500 model. The erosion process was studied in terms of the evolutions of fine particles loss and

501 volumetric strain for dense, medium dense and loose sand, respectively. The findings are  
502 summarized as follows:

503 (1) The erosion of fine particles around tunnels is caused by axial and hydraulic  
504 pressures. Particles near the seams at the bottom are firstly extruded through the seams by the  
505 axial pressure. Further erosion is caused by fluid drag force due to hydraulic pressure.

506 (2) For loose sand, fine particles loss is mainly induced by fluid drag force. While for  
507 dense sand, fine particles loss is mainly induced by axial pressure. The combined fine particle  
508 loss caused by axial pressure and hydraulic pressure is most for loose sand and least for dense  
509 sand.

510 (3) For the volumetric strain of both dense and loose sand, the axial pressure plays the  
511 major role with the fluid drag force due to hydraulic pressure playing the minor role. Under  
512 the hydraulic pressure, the volumetric strain of loose sand increases relatively faster than  
513 dense sand. Thus under low hydraulic pressure, the volumetric strain for dense sand is more  
514 pronounced than loose sand. But the volumetric strain of loose sand can surpass that of dense  
515 sand under high hydraulic pressure.

516 (4) The major role of fluid drag force is to migrate the fine particles between the skeleton  
517 force chains and isolate the skeleton particles from each other, thus reducing the resistance to  
518 the dynamic traffic loading. The minor role of fluid drag force is that when particle migration  
519 changes the skeleton force chain and enlarges the flow path, it will also lead to the increase of  
520 volumetric strain.

521 (5) The erodible fine particles always find the least energy consuming path among many  
522 other paths during migration. Loose sand has more unblocked flow paths than dense sand.  
523 More flow paths will open during the erosion process and the change of flow path is always  
524 accompanied with the increase of volumetric strain.

525 (6) Two quantitative relations between the two objective variables of  $w$ ,  $\varepsilon_v$  and the four

526 influencing variables of  $t/t_0$ ,  $K_0$ ,  $P_u/P_0$ ,  $e$  were obtained by regression analysis. They can be  
527 used to predict the variation tendency of  $w$  and  $\varepsilon_v$  under different conditions.

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641 **Tables**

642 **Table 1 Pressures applied on the soil models**

Model No.	Model-1				Model-2				Model-3			
Confining pressure(kPa)	3000				250				130			
Axial pressure(kPa)	4000				360				210			
Hydraulic pressure(kPa)	500	250	100	50	180	100	50	22.5	100	50	25	10
Void ratio	0.14				0.31				0.43			

643

644

645 **Table 2 Parameters for soil particles and fluid**

	Granular density $\rho(\text{kg/m}^3)$	Normal stiffness $K_n$ (Pa)	Shear stiffness $K_s$ (Pa)	Friction coefficient $f_c$	Fluid density (kg/m <sup>3</sup> )	Dynamic viscosity (Pa·s)
Soil particle	2650	5e6	5e6	0.3	-	-
Wall cell	-	5e7	5e7	0.3	-	-
Fluid cell	-	-	-	-	998.23	1×10 <sup>-3</sup>

646

647

648 **Table 3 Upper and lower bounds of influencing variables and magnitude of specific**  
 649 **value for each variable**

Influencing variables	Upper and lower limits	Number of specific values
$t/t_0$	0-5	9
$K_0$	0.6-1.0	5
$P_u/P_0$	0.0-1.2	6
$e$	0.14-0.43	5

**Table 4 Value of the variables corresponding to  $K_0=0.75$** 

No.	$K_0$	$t/t_0$	$P_u/P_0$	$e$	$\varepsilon_v$ (%)	$w$ (%)
1	0.75	0.4	0.2	0.37	0.81	0.63
2	0.75	0.6	0.4	0.43	0.83	0.95
3	0.75	0.2	0.4	0.22	0.85	0.65
4	0.75	0.2	0.2	0.22	0.86	0.54
5	0.75	0.6	0.6	0.37	0.88	1.12
6	0.75	2	0.6	0.43	0.97	1.75
7	0.75	0.8	0.4	0.22	1.05	1.17
8	0.75	0.8	0.8	0.16	1.13	1.50
9	0.75	0.4	0.6	0.31	1.15	0.84
10	0.75	3	0.2	0.22	1.20	1.67
11	0.75	1	0.2	0.31	1.23	0.95
12	0.75	5	0.4	0.37	1.24	2.62
13	0.75	1	1.2	0.31	1.29	1.66
14	0.75	4	0.6	0.37	1.37	2.64
15	0.75	2	1	0.16	1.44	1.75
16	0.75	3	1	0.16	1.46	1.81
17	0.75	5	1.2	0.16	1.55	1.97
18	0.75	4	0.8	0.31	1.74	2.81
averaged value					1.17	1.50

**Table 5 Coefficients in the regression equation for  $w$** 

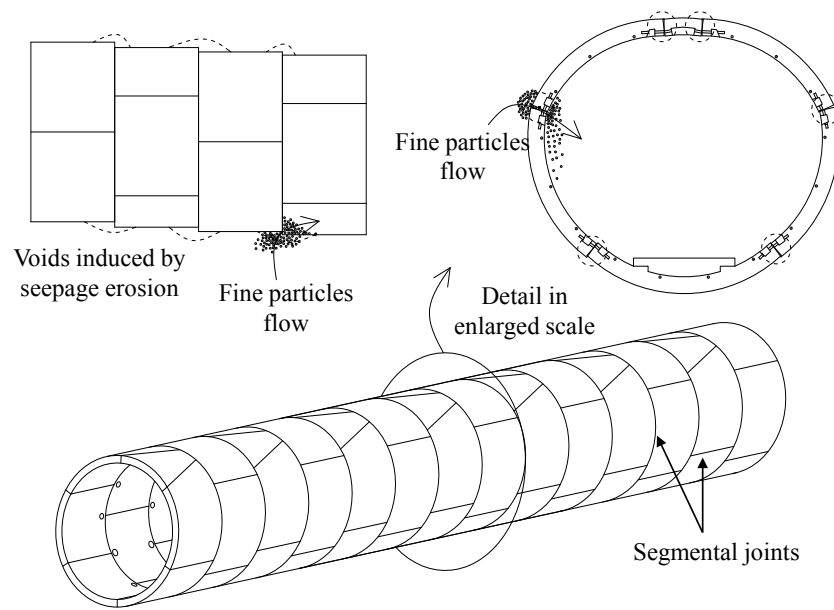
Coefficients	Values
$a_0$	-0.148
$a_1$	0.391
$a_2$	0.426
$a_3$	0.214
$a_4$	0.009
$R^2$	0.87

**Table 6 Coefficients in the regression equation for  $\varepsilon_v$** 

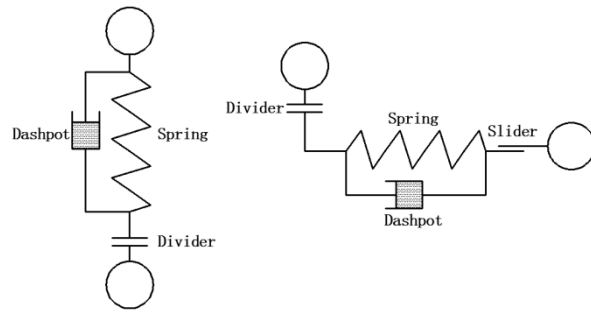
Coefficients	Values
$b_0$	-0.222
$b_1$	0.137
$b_2$	0.129
$b_3$	0.12
$b_4$	-7.397
$b_5$	3.197
$R^2$	0.64

657 **Figure list:**

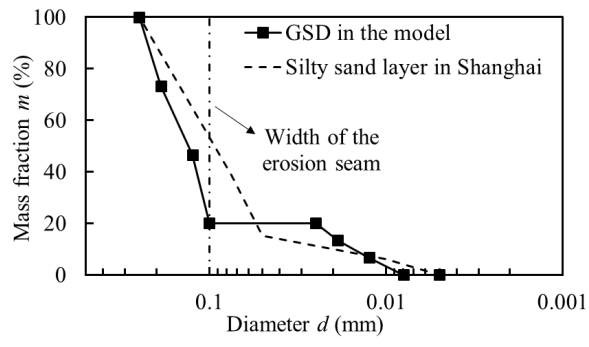
- 658 Fig. 1 Schematic of seepage erosion around tunnels  
659 Fig. 2 Contact model of two disks in DEM  
660 Fig. 3 Grain size distribution (GSD) in the numerical simulation (gap-gradation)  
661 Fig. 4 Simulation process  
662 Fig. 5 Progression of fine particles loss for three types of models (A: Extruding stage; B:  
663 Eroding stage)  
664 Fig. 6 Coupling effect of hydraulic pressure and void ratio on the reference fine particles loss  
665 Fig. 7 Progression of volumetric strain for three types of models (A: Extruding stage; B:  
666 Eroding stage)  
667 Fig. 8 Floating particles and skeleton particles in the soil model  
668 Fig. 9 Coupling effect of hydraulic pressure and void ratio on the reference volumetric strain  
669 Fig. 10 Flow path of the eroded particles  
670 Fig. 11 Flow path of the blocked particles  
671 Fig. 12 Three modes of particle erosion process  
672 Fig. 13 Fine particle distribution in three layers for three models under the hydraulic pressure  
673 of 100kPa (A: Extruding stage; B: Eroding stage)  
674 Fig. 14 Evolution of averaged fine particles loss with influencing variables: (a) time ( $t/t_0$ ); (b)  
675 consolidation stress ratio ( $K_0$ ); (c) hydraulic pressure-consolidation pressure ( $P_u/P_0$ ); (d) void  
676 ratio  
677 Fig. 15 Evolution of averaged volumetric strain with influencing variables: (a) time ( $t/t_0$ ); (b)  
678 consolidation stress ratio ( $K_0$ ); (c) hydraulic pressure-consolidation pressure ( $P_u/P_0$ ); (d) void  
679 ratio ( $e$ )  
680 Fig. 16 Evolution of averaged fine particles loss with void ratio: (a) caused by axial pressure  
681 (b) caused by hydraulic pressure  
682 Fig. 17 Evolution of averaged volumetric strain with void ratio: (a) caused by axial pressure  
683 (b) caused by hydraulic pressure.



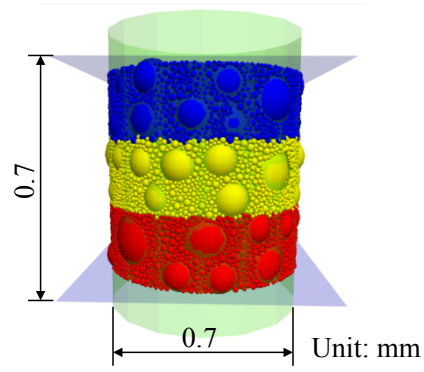
**Fig. 1 Schematic of seepage erosion around tunnels**



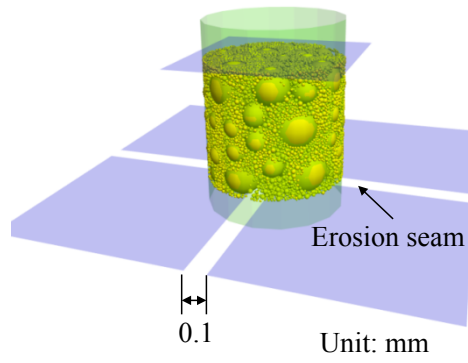
**Fig. 2 Contact model of two disks in DEM**



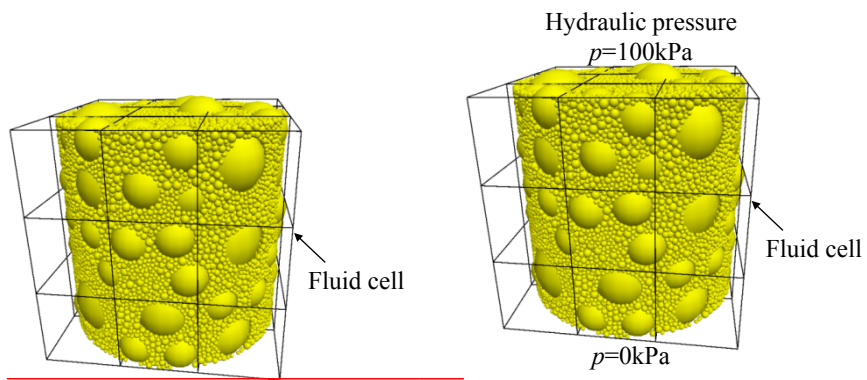
**Fig. 3 Grain size distribution (GSD) in the numerical simulation (gap-gradation)**



(a) Model generation



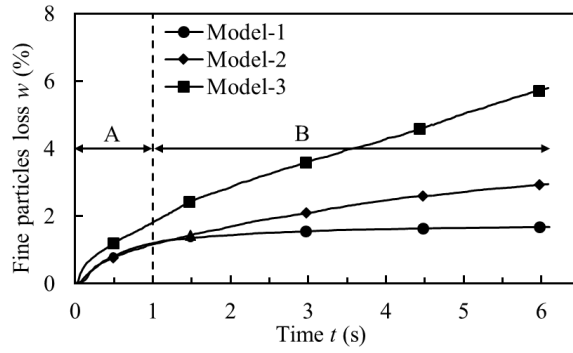
(b) Seam



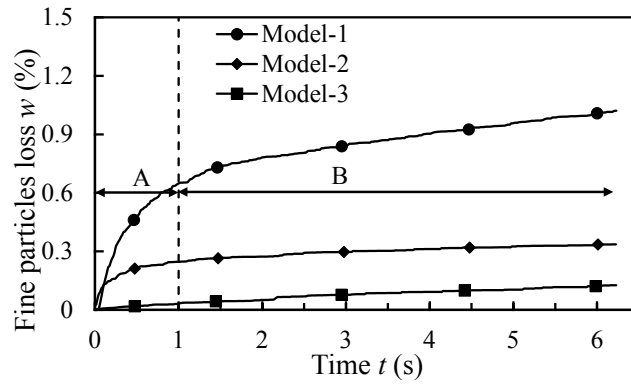
(c) Fluid cell under the hydraulic pressure of 100kPa

**Fig. 4 Simulation process**

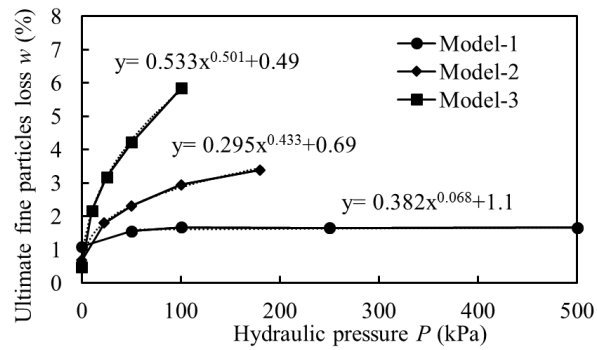




(a) Fine particle loss under the hydraulic pressure of 100 kPa

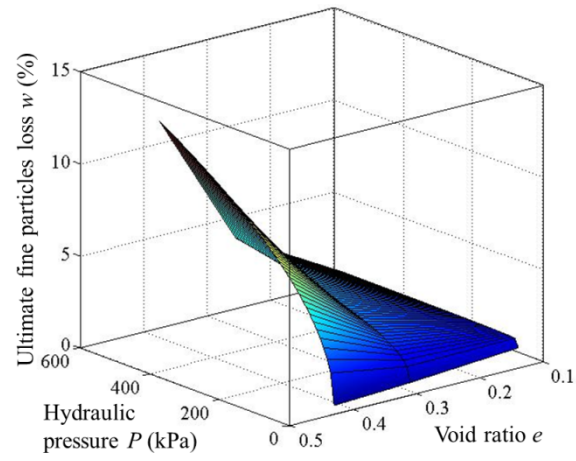


(b) Fine particle loss without hydraulic pressure (0 kPa)

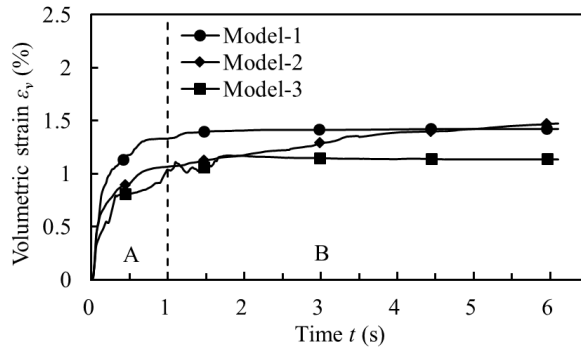


(c) reference fine particles loss under various hydraulic pressures

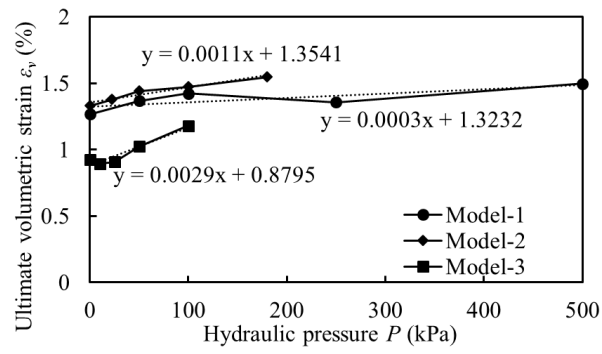
**Fig. 5 Progression of fine particles loss for three types of models (A: Extruding-Eroding coupled stage; B: Eroding-dominated stage)**



**Fig. 6** Coupling effect of hydraulic pressure and void ratio on the reference fine particles loss

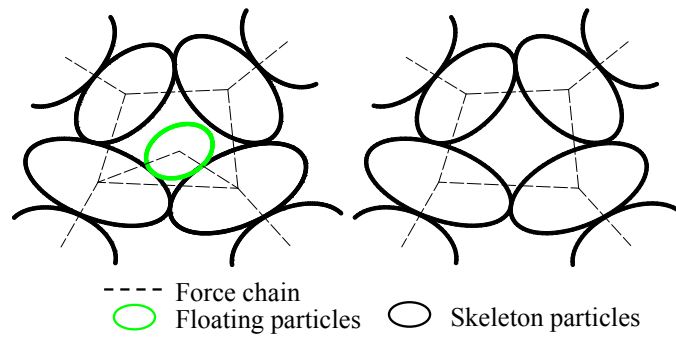


(a) Volumetric strain under the hydraulic pressure of 100 kPa

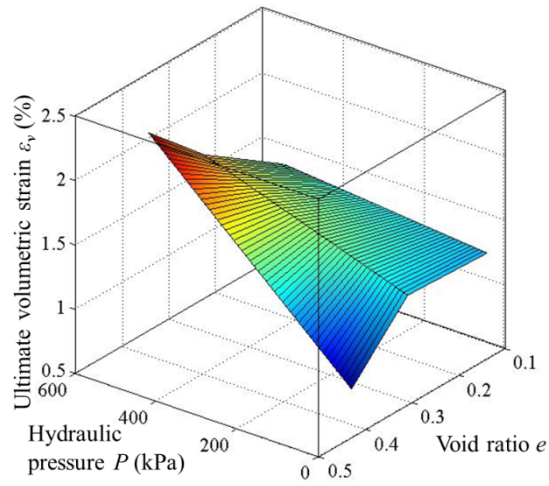


(b) reference volumetric strain under various hydraulic pressures

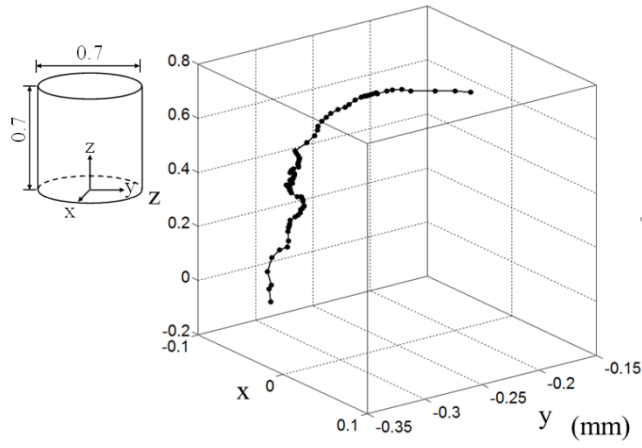
**Fig. 7 Progression of volumetric strain for three types of models (A: Extruding stage; B: Eroding stage)**



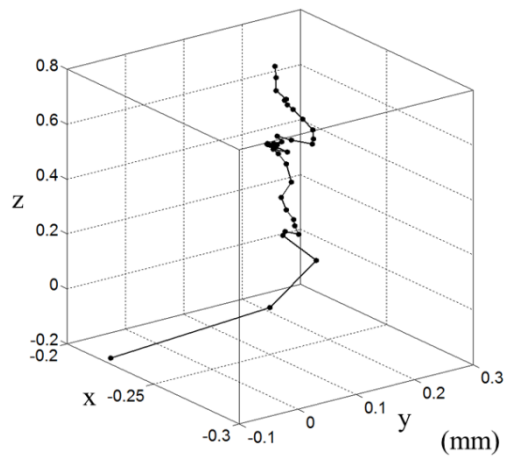
**Fig. 8 Floating particles and skeleton particles in the soil model**



**Fig. 9** Coupling effect of hydraulic pressure and void ratio on the reference volumetric strain

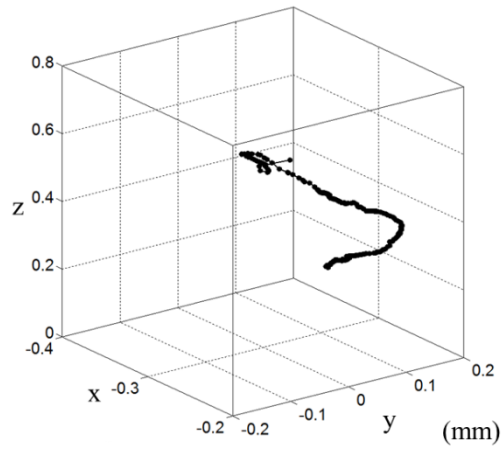


(a) Erosion path (particle id 28968)

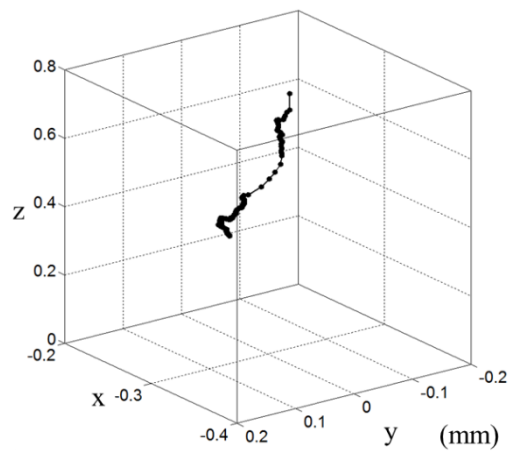


(b) Erosion path (particle id 26237)

**Fig. 10** Flow path of the eroded particles

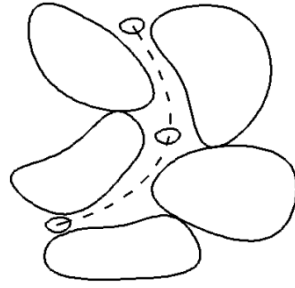


(a) Erosion path (particle id 26809)

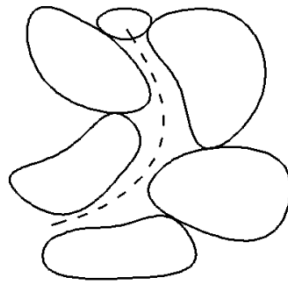


(b) Erosion path (particle id 26664)

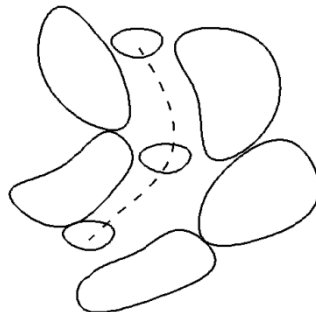
**Fig. 11 Flow path of the blocked particles**



(a) Mode 1



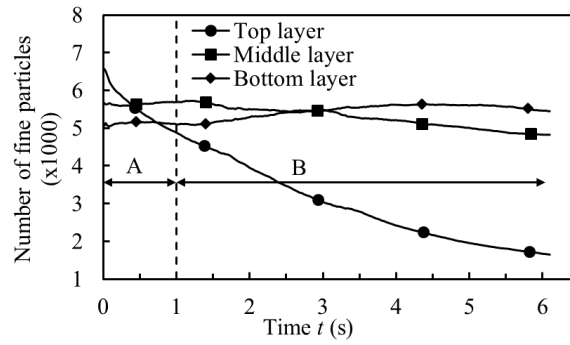
(b) Mode 2



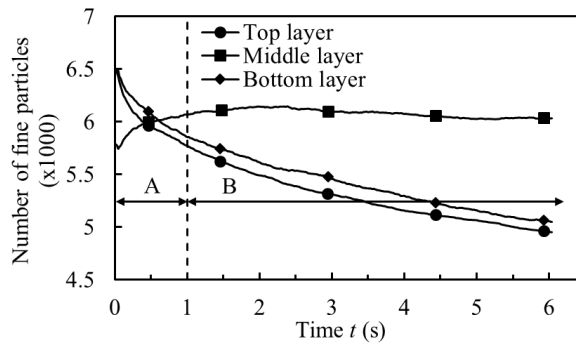
(c) Mode 3

**Fig. 12 Three modes of particle erosion process**

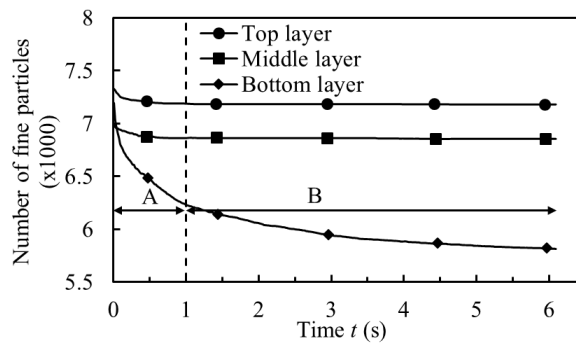




(a) Fine particles in each layer of Model-3



(b) Fine particles in each layer of Model-2

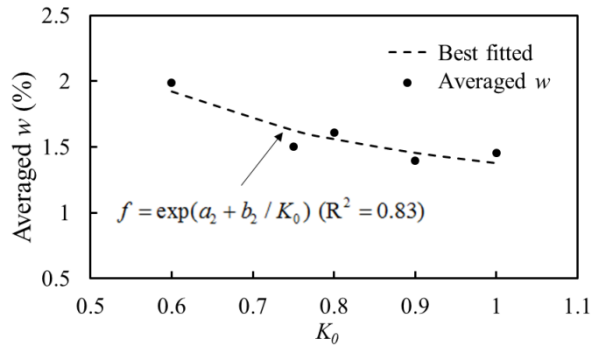
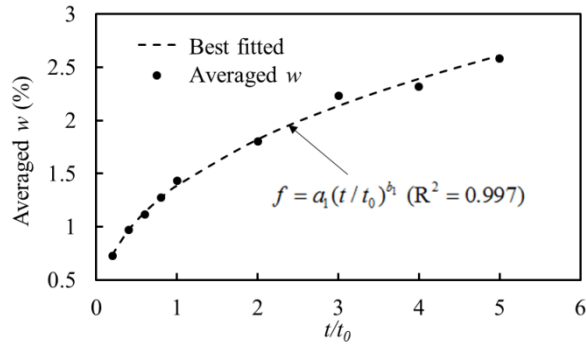


(c) Fine particles in each layer of Model-1

**Fig. 13 Fine particle distribution in three layers for three models under the hydraulic pressure of**

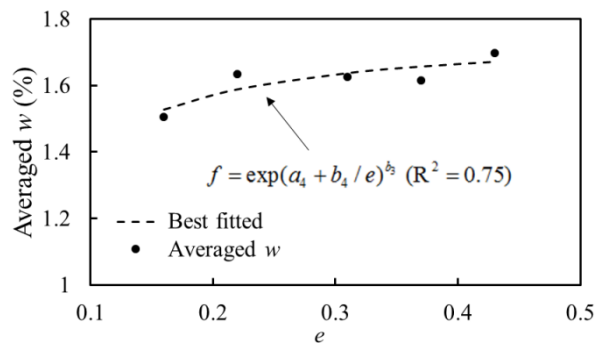
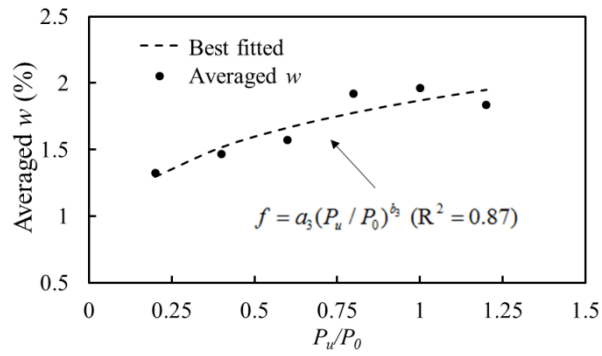
**100kPa**

**(A: Extruding stage; B: Eroding stage)**



(a)

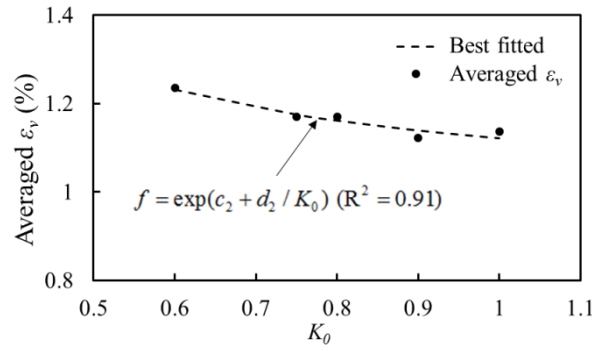
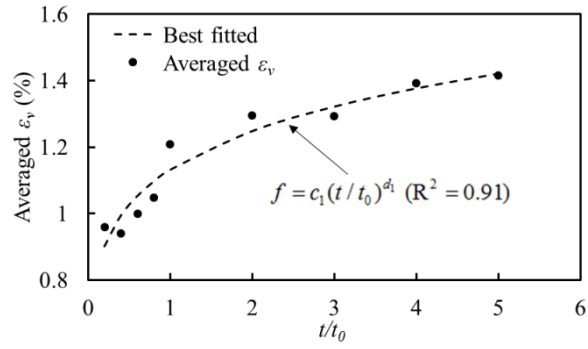
(b)



(c)

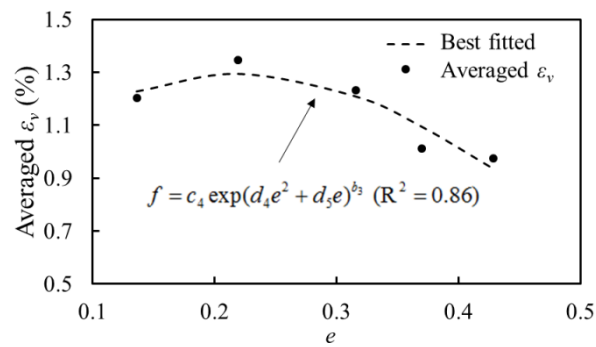
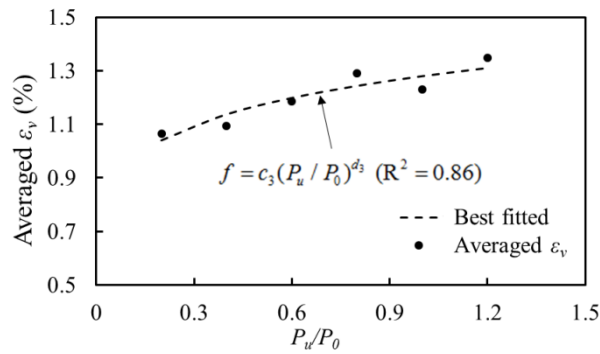
(d)

**Fig. 14 Evolution of averaged fine particles loss with influencing variables: (a) time ( $t/t_0$ ); (b) consolidation stress ratio ( $K_0$ ); (c) hydraulic pressure-consolidation pressure ( $P_u/P_0$ ); (d) void ratio ( $e$ )**



(a)

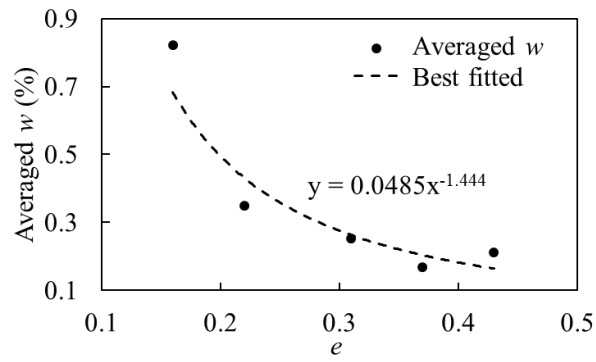
(b)



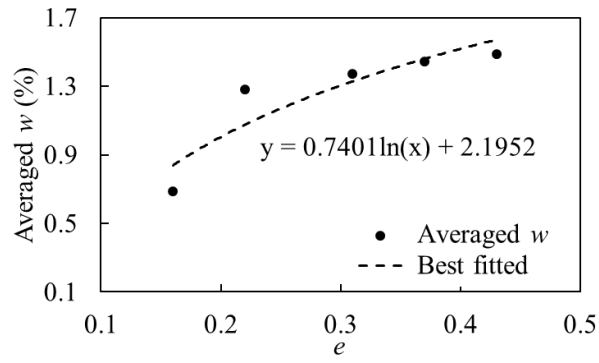
(c)

(d)

**Fig. 15 Evolution of averaged volumetric strain with influencing variables: (a) time ( $t/t_0$ ); (b) consolidation stress ratio ( $K_0$ ); (c) hydraulic pressure-consolidation pressure ( $P_u/P_0$ ); (d) void ratio ( $e$ )**

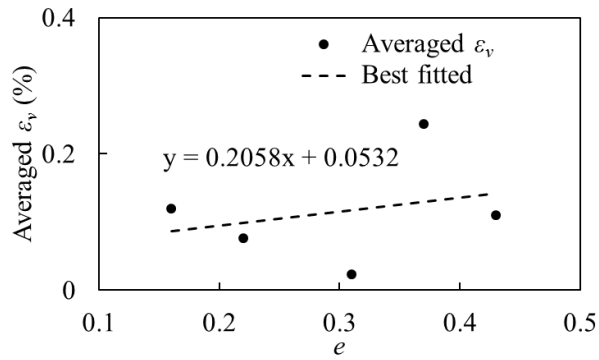
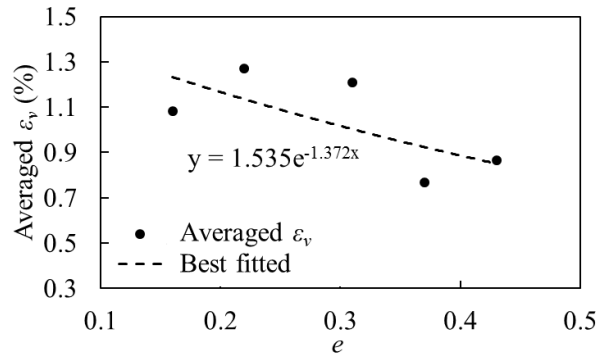


(a)



(b)

**Fig. 16 Evolution of averaged fine particles loss with void ratio: (a) caused by axial pressure (b) caused by hydraulic pressure**



(a)

(b)

**Fig. 17 Evolution of averaged volumetric strain with void ratio: (a) caused by axial pressure (b) caused by hydraulic pressure**