Fast Integration Algorithms for the Evaluation of the Time-dependent Failure Probability

Abstract: Structural safety during the entire service life is essentially a time-dependent reliability problem because structural resistance and load effect are generally functions of time. To evaluate the time-dependent failure probability of a structure, various time-dependent reliability methods have been developed, among which the conditional probability theory-based method is relatively realistic and has a good prospect for predicting the time-dependent reliability of structures in most cases. However, when the performance function is complicated or involves multiple random variables, the conditional probability theory-based method involves a multi-dimensional integral over time space and random-variate space, and the computation of the multi-dimensional integral by direct integration is almost impossible. In this study, a fast integration algorithm for evaluating the time-dependent failure probability is proposed. In the proposed method, the integral with respect to time is firstly estimated by Gauss-Legendre quadrature and the integral corresponding to random-variate space is then obtained by the point-estimate method based on bivariate dimension-reduction integration. The efficiency and accuracy of the proposed method are illustrated by several numerical examples. It can be concluded that the proposed method provides a useful tool for evaluating the time-dependent failure probability especially when performance functions are complicated or involve multiple random variables.

Keywords: Time-dependent failure probability; Fast integration algorithms; Gauss-Legendre quadrature; Point-estimate method; Bivariate dimension-reduction integration;

Introduction

In classical structural reliability theory, the probability of structural failure is expressed as

$$P_f = \Pr ob\left(R \le S\right) = \int_{r \le s} f\left(r, s\right) drds \quad (1)$$

where R is structural resistance; S is load effect; $f(r, s)$ is the joint probability density function (PDF) of R and S; r and s are respectively the sample of R and S; and $r \le s$ is the domain of integration, which denotes the failure region of the structure.

The structural resistance and load effect are treated as time-invariant random variables in Eq. (1). However, in practical engineering, structural resistance may deteriorate due to ageing brought on by the aggressive service environment (Wang et al. 2016), and the occurrence and intensity of load may change significantly over time due to changing service and environmental demands. Therefore, structural safety during the entire service life is essentially a time-dependent reliability problem (Mori and Ellingwood 1993; Enright and Frangopol 1998; Stewart and Rosowsky 1998; Ciampoli 1998; Melchers 1999; Vu and Stewart 2000; Hong 2000; Li et al. 2005; Duprat 2007; Li et al. 2015; Biondini and Frangopol 2016; Yang et al. 2017). And the time-dependent failure probability during a given period [0, T] can be formulated as

$$P_f\left(T\right) = \Pr ob\left\{R\left(t\right) \le S\left(t\right), \forall t \in \left[0, T\right]\right\} = \int_{r(t) \le s(t)} f\left[r(t), s(t)\right] dr(t) ds(t) \quad (2)$$

where $R(t)$ and $S(t)$ are respectively resistance and load effect which are varying with time t. Evaluating the time-dependent failure probability through Eq. (2) is generally a formidable work.

To determine the time-dependent probability of failure, various approximated methods have been proposed based on different assumption. Under the hypothesis that the load is described by a non-stochastic discrete approach, Duprat (2007) developed a time-dependent reliability method, in which the time-dependent probability of failure is expresses as a cumulative failure probability of a series system. In this method, the given period [0, T] is uniformly divided into Z independent discrete sub-periods, and the length of each independent sub-period is k years. Then the time-dependent probability of failure is expressed as (Duprat 2007)

$$P_f(T) = 1 - \prod_{i=1}^{Z}[1 - p_{fk}(i)] \tag{3}$$

where $p_{fk}(i)$ is the elementary k year probability of failure for any ith period, which can be obtained by general reliability methods such as first-order reliability method (Hasofer and Lind 1974; Rackwitz and Fiessler 1978; Shinozuka 1983). The computation of this method is relatively simple and efficient. However, the time-dependent probability of failure has significant difference for the different length of sub-period. Moreover, the number and the time of occurrence of live load events during the lifetime are deterministic, which is inconsistent with the practical engineering.

Based on the assumption of the occurrence in time of live load events described by a Poisson point process and conditional probability theory, Mori and Ellingwood (1993) proposed a time-dependent reliability method. When a structure is subjected to two statistically independent load processes with intensities S_1 and S_2, respectively, but only S_1 varies with time, the time-dependent failure probability is given as (Mori and Ellingwood 1993)

$$P_f(T) = 1 - \int_0^\infty \int_0^\infty \exp\left\{-\lambda_{S_1}\left\{T - \int_0^T F_{S_1}\left[r \cdot g(t) - s_2\right]dt\right\}\right\} f_{S_2}(s_2) f_{R_0}(r) ds_2 dr \tag{4}$$

where λ_{S_1} is the mean occurrence rate of S_1; $g(t)$ is the degradation function of structural resistance; $F_{S_1}(\cdot)$ is the cumulative distribution function (CDF) of S_1; $f_{S_2}(\cdot)$ is the PDF of S_2; and $f_{R_0}(\cdot)$ is the PDF of the initial resistance R_0.

According to Eq. (4), the determination of time-dependent probability of failure involves a complex multi-dimensional integration over time space and random-variate space. To simplify the computation of Eq. (4), the degradation function $g(t)$ can be substituted by a resistance reduction factor g^*. Then, Eq. (4) can be rewritten as (Mori and Nonaka 2001)

$$P_f(T) = 1 - \int_0^\infty \exp\left\{-\lambda_{S_1} T\left[1 - F_{S_1}\left(r \cdot g^* - \mu_{s_2}\right)\right]\right\} f_{R_0}(r)dr \tag{5a}$$

$$g^* = \frac{F_{S_1}^{-1}\left\{\dfrac{1}{T}\int_0^T F_{S_1}\left[\mu_{R_0} g(\tau) - \mu_{S_2}\right]d\tau\right\} + \mu_{S_2}}{\mu_{R_0}} \tag{5b}$$

where μ_{S_2} and μ_{R_0} are the mean of S_2 and R_0, respectively; and $F_{S_1}^{-1}(\cdot)$ is the inverse function of $F_{S_1}(\cdot)$. To obtain the resistance reduction factor g^* expressed in Eq. (5*b*), the integration with respect to time and the inverse CDF of S_1 are required. Hence the computational efficiency of time-dependent probability of failure in Eq. (5*a*) does not be improved remarkably compared with Eq. (4). Moreover, Eq. (5*a*) and (5*b*) are only suitable for the cases that the occurrence rate of live load is time independent and the variability of S_2 is relatively small.

If the live load is described as a non-stationary process with a time-variant occurrence rate $\lambda_{S_1}(t)$ and time-variant CDF of intensity, Eq. (4) can be rewritten as (Li et al. 2015)

$$P_f(T) = 1 - \int_0^\infty \int_0^\infty \exp\left\{-\int_0^T \lambda_{S_1}(t)\left(1 - F_{S_1}\left[r \cdot g(t) - s_2, t\right]\right)dt\right\} f_{S_2}(s_2) f_{R_0}(r) ds_2 dr \tag{6}$$

From Eq. (4), Eq. (5*a*) and (5*b*) or Eq. (6), it can be observed that the computation of time-dependent probability of failure by direct integration is almost impossible especially when performance function is complicated or involves multiple random variables.

The theoretically most rigorous method for the time-dependent reliability problem is proposed by Melchers (1999), which is based on the assumption that both the resistance and the load effect are continuous stochastic processes. The time-dependent failure probability of the structure is estimated directly from the first-passage probability. When the probability of no up-crossing in [0, *T*] is approximated using Poisson distribution with zero events occurred, the time-dependent failure probability can be formulated as (Melchers 1999; Li and Melchers

1993)

$$P_f(T) = 1 - \left[1 - p_f(0)\right] \cdot \exp\left[-\int_0^T \nu(t)dt\right] \tag{7a}$$

$$\nu(t) = \nu_a^+(t) = \int_{\dot{a}}^{\infty} (\dot{x} - \dot{a}) f_{X\dot{X}}(a, \dot{x}) d\dot{x} \tag{7b}$$

where $p_f(0)$ is the probability of failure at $t = 0$; ν_a^+ is the up-crossing rate of the process $X(t)$ relative to the barrier level to be up-crossed a; $\dot{a}$ is the slope of a with respect to time; $\dot{X}$ is the time-derivative process of stochastic process $X(t)$; and $f_{X\dot{X}}(\cdot)$ is the joint PDF for X and $\dot{X}$. Closed-form solutions of this method are not tractable for the complicated situations such as highly nonlinear limit state functions, non-Gaussian random variables, and time-varying load and/or resistance quantities (Stewart and Rosowsky 1998). Therefore, analytical solutions for the first-passage probability only exist for few specific processes (Yang et al. 2017).

Of interesting here is to facilitate application of the conditional probability theory-based method [i.e., Eq. (4), Eq. (5*a*) or Eq. (6)] for complicated or multiple-dimensional performance functions. In the present paper, a fast integration algorithm for evaluating the time-dependent failure probability is proposed. The main procedure of the proposed method includes two steps: (1) the integral with respect to time t is obtained by Gauss-Legendre quadrature; and (2) the integral corresponding to random-variate space is estimated by point estimate method based on bivariate dimension-reduction integration. Then, the efficiency and accuracy of the proposed method for evaluating the time-dependent failure probability for performance function with multiple random variables is demonstrated through several numerical examples. Finally, findings of the present paper are summarized.

Fast integration algorithms for the time-dependent failure probability based on Gauss-Legendre quadrature and point estimate method

When the performance function involves n independent random variables, the time-dependent probability of failure in Eq. (6) can be rewritten as

$$P_f(T) = 1 - \int_{\Omega} \exp\left\{-\int_0^T \lambda_Q(t)\left(1 - F_Q\left[G^*(\mathbf{x}, t), t\right]\right)dt\right\} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{8}$$

where $\mathbf{X}$ is the (n-1)-dimensional independent random vector excluding the live load Q; $F_Q(\ )$ is the CDF of Q; $f_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of $\mathbf{X}$; Ω is the domain region of $\mathbf{X}$; and $G^*(\ )$ denotes a function determined from time-dependent limit-state function $G(\mathbf{X}, Q, t)=0$ [i.e., $Q=G^*(\mathbf{X}, t)$].

Assume $t = T\cdot\tau/2+T/2$, the integral interval of time t, [0, T], in Eq. (8) can be transformed into [-1, 1], leads to

$$\int_0^T \lambda_Q(t)\left(1 - F_Q\left[G^*(\mathbf{x}, t), t\right]\right)dt = \frac{T}{2}\int_{-1}^{1} \lambda_Q\left(\frac{T}{2}\tau + \frac{T}{2}\right)\left\{1 - F_Q\left[G^*\left(\mathbf{x}, \frac{T}{2}\tau + \frac{T}{2}\right), \frac{T}{2}\tau + \frac{T}{2}\right]\right\}d\tau \tag{9}$$

Based on the Gauss-Legendre quadrature, Eq. (9) can be rewritten as follows:

$$\int_0^T \lambda_Q(t)\left(1 - F_Q\left[G^*(\mathbf{x}, t), t\right]\right)dt = \frac{T}{2}\sum_{k=1}^{N} \lambda_Q\left(\frac{T}{2}\tau_k + \frac{T}{2}\right)\left\{1 - F_Q\left[G^*\left(\mathbf{x}, \frac{T}{2}\tau_k + \frac{T}{2}\right), \frac{T}{2}\tau_k + \frac{T}{2}\right]\right\}W_k \tag{10}$$

where τ_k and W_k (k = 1, 2, …, N) are the abscissas and weights of Gauss-Legendre quadrature, respectively. If N = 4, they are expressed as

$$\tau_{1+} = -\tau_{1-} = 0.3399810,\ \ W_{1+} = W_{1-} = 0.6521452 \tag{11a}$$

$$\tau_{2+} = -\tau_{2-} = 0.8611363,\ \ W_{2+} = W_{2-} = 0.3478548 \tag{11b}$$

Substituting Eq. (10) into Eq. (8), leads to

$$P_f(T) = 1 - \int_{\Omega} H(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = 1 - E\left[H(\mathbf{X})\right] \tag{12a}$$

$$H(\mathbf{X}) = \exp\left\{-\frac{T}{2}\sum_{k=1}^{N} \lambda_Q\left(\frac{T}{2}\tau_k + \frac{T}{2}\right)\left\{1 - F_Q\left[G^*\left(\mathbf{X}, \frac{T}{2}\tau_k + \frac{T}{2}\right), \frac{T}{2}\tau_k + \frac{T}{2}\right]\right\}W_k\right\} \tag{12b}$$

where $E(\ )$ denotes the expectation.

From Eq. (12a), it can be observed that the key issue of evaluating the time-dependent failure probability is essentially to determine the mean of $H(\mathbf{X})$, which can be obtained by the point-estimate method based on bivariate dimension-reduction integration.

Based on the inverse Rosenblatt transformation (Hohenbichler and Rackwitz 1981), the function $H(\mathbf{X})$ can be rewritten as

$$H(\mathbf{X}) = \exp\left\{-\frac{T}{2}\sum_{k=1}^{N}\lambda_Q\left(\frac{T}{2}\tau_k+\frac{T}{2}\right)\left\{1-F_Q\left[G^*\left(T^{-1}(\mathbf{U}),\frac{T}{2}\tau_k+\frac{T}{2}\right),\frac{T}{2}\tau_k+\frac{T}{2}\right]\right\}W_k\right\} = h(\mathbf{U}) \quad (13)$$

where $T^{-1}(\mathbf{U})$ denotes the inverse Rosenblatt transformation; and $\mathbf{U}$ is a (n-1)-dimensional independent standard normal random vector.

According to bivariate dimension-reduction method (Xu and Rahman 2004), the function h(U) can be approximated by

$$h(\mathbf{U}) \approx h_2 - (n-3)h_1 + \frac{(n-2)(n-3)}{2}h_0 \quad (14)$$

where h_2 is a summation of $(n-1)(n-2)/2$ two-dimensional functions; h_1 is a summation of (n-1) one-dimensional functions; and h_0 is a constant. h_2, h_1, and h_0 are respectively expressed as

$$h_2 = \sum_{i<j} h(0,\cdots,U_i,\cdots,U_j,\cdots,0) = \sum_{i<j} h_{ij}(U_i,U_j) \quad (15a)$$

$$h_1 = \sum_{i=1}^{n-1} h(0,\cdots,U_i,\cdots,0) = \sum_{i=1}^{n-1} h_i(U_i) \quad (15b)$$

$$h_0 = h(0,\cdots,0,\cdots,0) \quad (15c)$$

where h_{ij} is a two-dimensional function, $i, j = 1, 2, \ldots, n$ and $i < j$; and h_i is a one-dimensional function.

With the aid of Eq. (14) and the definition of expectation, the mean of $H(\mathbf{X})$ can be obtained as

$$E\left[H(\mathbf{X})\right] = E\left[h(\mathbf{U})\right] = \sum_{i<j}\mu_{ij} - (n-2)\sum_{i=1}^{n-1}\mu_i + \frac{(n-1)(n-2)}{2}h_0(0,\cdots,0) \quad (16)$$

where μ_{ij} and μ_i are respectively the mean of h_{ij} and h_i, which are respectively expressed as

$$\mu_{ij} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_{ij}(u_i, u_j)\phi(u_i)\phi(u_j)du_i du_j \tag{17a}$$

$$\mu_i = \int_{-\infty}^{+\infty} h_i(u_i)\phi(u_i)du_i \tag{17b}$$

where $\phi(u_i)$ and $\phi(u_j)$ are the PDF of U_i and U_j, respectively.

According to point-estimate method (Zhao and Ono 2000) in standard normal space, μ_{ij} and μ_i can be estimated as

$$\mu_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{m} P_i \cdot P_j \cdot h_{ij}(u_i, u_j) \tag{18a}$$

$$\mu_i = \sum_{i=1}^{m} P_i \cdot h_i(u_i) \tag{18b}$$

where u_r and P_r ($r = i, j$) are estimating points and the corresponding weights, respectively; and m is the number of estimating points in one dimensional case. The estimating points and their corresponding weights can be readily obtained as

$$u_r = \sqrt{2}x_r, \quad P_r = \omega_r / \pi \tag{19}$$

where x_r and ω_r are the abscissas and weights for Hermite integration with weight function $\exp(-x^2)$ (Abramowitz and Stegun 1972).

For a seven point estimate, the estimating points and their corresponding weights are readily obtained as (Zhao and Ono 2000)

$$u_0 = 0, \quad P_0 = 16/35 \tag{20a}$$

$$u_{1+} = -u_{1-} = 1.1544054, \quad P_{1+} = P_{1-} = 0.24012318 \tag{20b}$$

$$u_{2+} = -u_{2-} = 2.3667594, \quad P_{2+} = P_{2-} = 3.0757124 \times 10^{-2} \tag{20c}$$

$$u_{3+} = -u_{3-} = 3.7504397, \quad P_{3+} = P_{3-} = 5.4826886 \times 10^{-4} \tag{20d}$$

In summary, the computational procedure of the developed method for evaluating the time-dependent failure probability is described as follows, with the flowchart illustrated in Fig. 1.

1. Determinate the abscissas τ_k and the corresponding weights W_k of Gauss-Legendre quadrature in Eq. (12*b*).
2. Transform the function $H(\mathbf{X})$ into the function $h(\mathbf{U})$ using the inverse Rosenblatt transformation [i.e., Eq. (13)];
3. Approximate the function $h(\mathbf{U})$ by bivariate dimension-reduction method [i.e., Eq. (14)];
4. Evaluate all of the mean of h_{ij} and h_i (i.e., μ_{ij} and μ_i) by the point-estimate method with the aid of Eqs. (18*a*)-(18*b*);
5. Obtain the mean of the function $h(\mathbf{U})$, i.e., $E[h(\mathbf{U})]$, by substituting μ_{ij} and μ_i into Eq. (16);
6. Compute the time-dependent failure probability $P_f(T)$ using Eq. (12*a*).

Numerical Examples and Investigations

In order to investigate the simplicity, efficiency and accuracy of the proposed method for structural time-dependent reliability analysis, the bending moment safety of a simply supported rectangular reinforced concrete beam is investigated in this example, which has been analyzed by Val (2007). The beam is subjected to uniformly distributed loads (dead load and live load) and chloride attack shown in Fig. 2. The flexural performance function is expressed as

$$G(\mathbf{X}) = \alpha \cdot \min\left\{ A_s(t) f_y \left[d - \frac{A_s(t) f_y}{1.7 f_c b} \right],\ \frac{1}{3} f_c b d^2 \right\} - \frac{(S+Q)L^2}{8} \tag{21}$$

where α is the resistance model uncertainty; f_y is yield strength of reinforcing steel; f_c is compressive strength of concrete; b is the cross-sectional width (b = 0.35m); d is the effective depth of cross-section; L is the span of beam (L= 10m); S is the dead load; Q is live load; and $A_s(t)$ is the total cross-sectional area of longitudinal bars at time t, which is expressed as

$$A_s(t) = n_0 \frac{\pi D_0^2}{4} - \sum_{i=1}^{n_0} A_{p,i}(t) \tag{22}$$

where n_0 is the number of reinforcing bars (n_0 = 9); D_0 is the initial diameter of reinforcing bar (D_0 = 25.4 mm); and $A_{p,i}(t)$ is the cross-sectional area of pitting corrosion in the ith reinforcing bar in time t, which is determined by the hemispherical model (Val 2007) shown in Fig. 3, expressed as

$$A_{p,i}(t) = \begin{cases} A_1 + A_2 & p(t) \le \frac{D_0}{\sqrt{2}} \\ \frac{\pi D_0^2}{4} - A_1 + A_2 & \frac{D_0}{\sqrt{2}} < p(t) \le D_0 \\ \frac{\pi D_0^2}{4} & p(t) > D_0 \end{cases} \tag{23}$$

in which

$$A_1 = \frac{1}{2}\left[\theta_1\left(\frac{D_0}{2}\right)^2 - \zeta\left|\frac{D_0}{2} - \frac{p(t)^2}{D_0}\right|\right];\quad A_2 = \frac{1}{2}\left[\theta_2 p(t)^2 - \zeta\frac{p(t)^2}{D_0}\right] \tag{24a}$$

$$\zeta = 2p(t)\sqrt{1-\left[\frac{p(t)}{D_0}\right]^2};\quad \theta_1 = 2\arcsin\frac{\zeta}{D_0};\quad \theta_2 = 2\arcsin\frac{\zeta}{2p(t)} \tag{24b}$$

where $p(t)$ is the depth of a pit after t years since corrosion initiation, which can be evaluated as

$$p(t) = 0.0116 i_{\text{corr}} t R \tag{25}$$

where i_{corr} denotes the corrosion current density; and R is the ratio between the maximum pit depth and the average corrosion depth.

It is assumed that the corrosion of reinforcements is initiated at time t = 0, and the corrosion current density is deterministic and time independent (i_{corr} = 1μA/cm^2). The occurrence rate of the live load is considered to be constant ($\lambda(t)$ = 1/year). In this example, α, R, f_y, f_c, d, S, and Q are mutually independent random variables, and their statistical properties are listed in Table 1.

The time-dependent failure probabilities obtained from the proposed method, are shown in Figure 4(a), in which the semi-logarithmic coordinates is used, together with the results obtained by Monte Carlo simulation (MCS) (see Appendix) with 1,000,000 samples (the coefficient of variation of MCS is less than 5.00%). And the corresponding time-dependent reliability indices are shown in Figure 4(b). It can be seen that: (1) the time-dependent failure probability increases with time, and increases more rapidly in the late stage of the reference period. (the reliability index decreases with time); (2) the results obtained from the proposed method are in close agreement with those from MCS.

Concluding Remarks

The present paper proposes a fast integration algorithm for evaluating the time-dependent failure probability of a structure. In the proposed method, the integral with respect to time is firstly estimated by Gauss-Legendre quadrature and the integral corresponding to random-variate space is then obtained from the point-estimate method based on bivariate dimension-reduction integration. From the investigation of this paper, it can be concluded that the time-dependent failure probabilities obtained by the proposed method are almost the same with those obtained by MCS. The proposed method provides a useful tool for evaluating the time-dependent failure probability of a structure especially for performance function with multiple random variables.

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Appendix

The exact values of the time-dependent failure probabilities of a structure during the given period [0, T] are obtained by MCS. When the performance function involves n independent random variables, including live load Q and other variables x_j (j = 1, 2, …, n-1), the procedure of MCS is described as following:

1. Generate a sample vector **x** of the random vector **X**;
2. Obtain the number of occurrence of live load event, N_T, during the time interval [0, T] that is a sample of Poisson distribution with parameter $\lambda(t)$;
3. Determine the times of occurrence of live load t_i (i=1, 2, …, N_T). When the occurrence rate of live load $\lambda(t)$ is constant, t_i (i = 1, 2, …, N_T) are samples of uniform distribution on the interval (0, T]. If $\lambda(t)$ is a function of time, t_i (i = 1, 2, …, N_T) are generated from their CDFs, which are expressed as

$$F_{t_i}\left(t\right)=\frac{\int_0^t \lambda\left(\tau\right)d\tau}{\int_0^T \lambda\left(\tau\right)d\tau} \tag{26}$$

4. Generate samples q_i (i = 1, 2, …, N_T) of the live load Q at t_i;
5. Calculate the performance functions $G(t_i$, **x**) (i=1, 2, …, N_T). If all results are greater than zero, the structure is safe in time interval [0, T]; otherwise, the structure is failure;
6. Reiterate steps 1-5, and count the number of structural failure. If the number of reiteration is sufficiently large, the time-dependent failure probabilities are approximated by the ratio of the number of structural failure and the number of reiteration.

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List of Table Captions:

Table 1. Statistical properties of random variables for Example 1

Parameter	Mean	Coefficient of variation	Distribution
Model uncertainty α	1.1	0.12	Normal
Parameter of pitting corrosion R	11.1	0.12	Gumbel
Steel strength f_y (MPa)	490	0.10	Lognormal
Concrete compressive strength f_c (MPa)	26.2	0.18	Lognormal
Effective depth of cross section d (mm)	710	0.02	Normal
Dead load S (kN/m)	21	0.10	Normal
Live load Q (kN/m)	17.5	0.44	Gamma

List of Figure Captions:

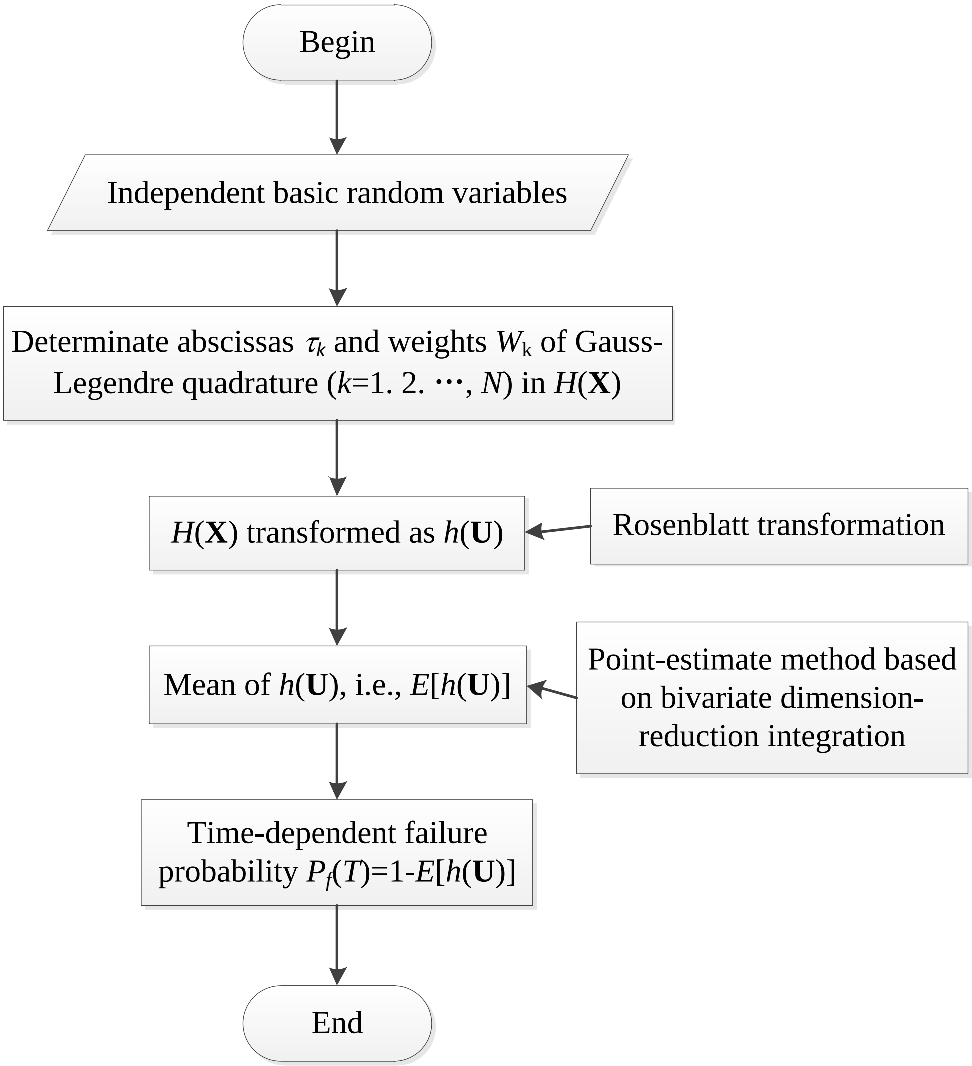


Fig. 1. Flowchart of procedure for time-dependent failure probability

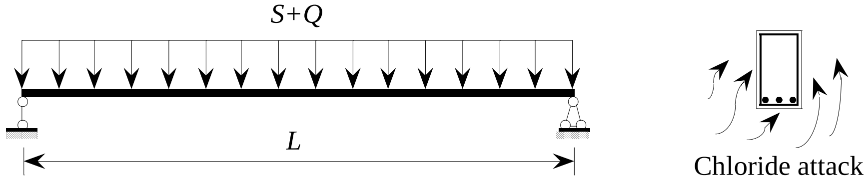
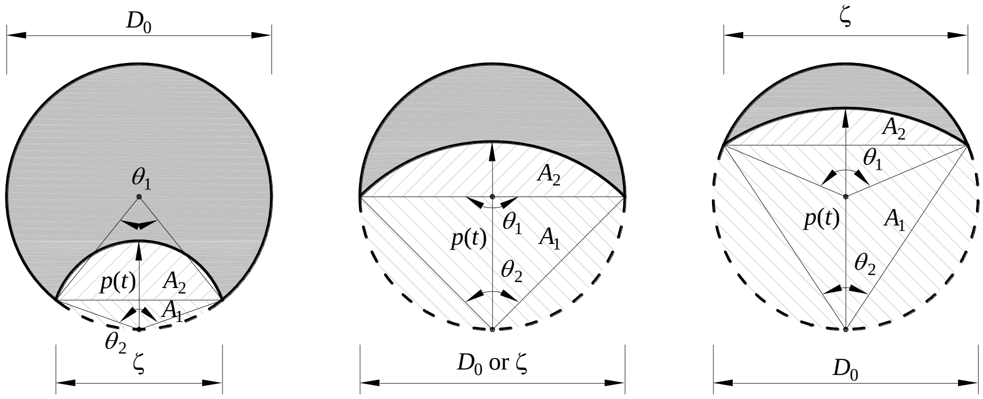
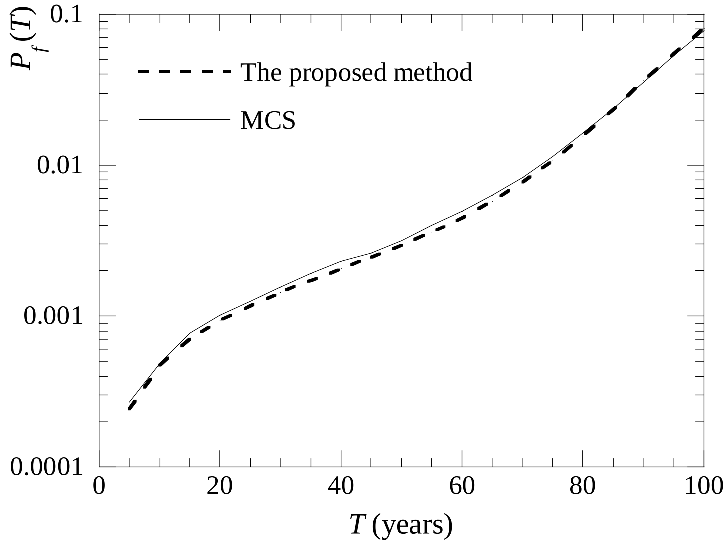


Fig. 2. Beam subjected to uniform loads and chloride attack for Example

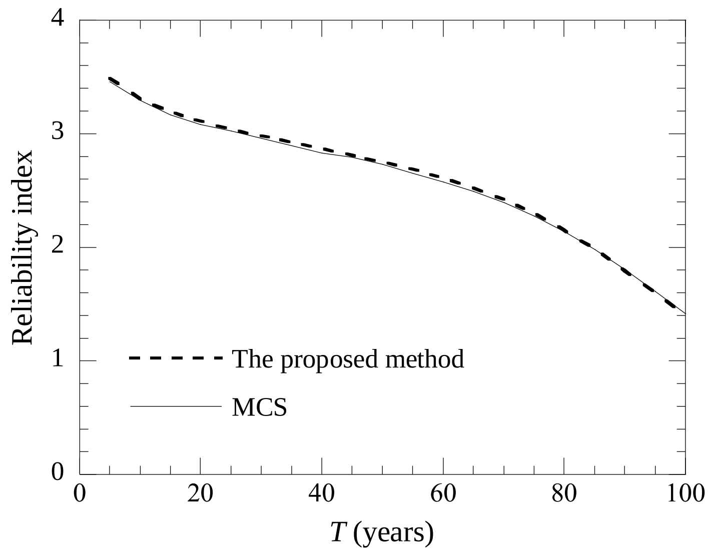


(a) $0 < p(t) < D_0/\sqrt{2}$ (b) $p(t) = D_0/\sqrt{2}$ (c) $D_0/\sqrt{2} < p(t) < D_0$

Fig. 3. Pitting corrosion with hemispherical shape for Example



(a) Time-dependent failure probability



(b) Time-dependent reliability index

Fig. 4. Comparison of different methods for Example