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Enhancing the effectiveness of urban drainage system design with an improved ACO-based method

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Abstract: In the context of climate change and urbanization, urban floods have been one of the major issues around the world, causing significant impacts on the society and environment. To effectively handle these floods, an appropriate design of the urban drainage system (UDS) is highly important as its function can significantly influence the flooding severity and distribution. In recent years, evolutionary algorithms (EAs) have been increasingly used to design UDS due to their great ability in identifying optimal solutions. However, low computational efficiency and low solution practicality (i.e. the final solutions do not satisfy the design criteria) are major challenges for the majority of EA-based methods. To this end, this paper proposes an improved ant colony optimization (ACO, a typical type of EAs) based method to enhance the UDS design effectiveness, where the optimization efficiency is enhanced by initializing the ACO using an approximate design solution identified by the engineering design method, and the solution practicality is improved by explicitly accounting for the design criteria within the optimization using a proposed sampling method. The utility of the proposed method is demonstrated using two real-world UDSs with different system complexities. Results show that the proposed method can identify design solutions with significantly improved efficiency and solution practicality compared to the traditional design approach, with advantages being more prominent for larger UDS design problems. The proposed method can be used by researchers/ practitioners to explore and develop better understanding of the UDS design alternatives under various challenges of climate change and rapid urbanization.

Keywords: urban drainage system (UDS); ant colony optimization (ACO); design criteria; optimization efficiency; solution practicality

1. Introduction

With the influences of global climate change and rapid urbanization, extreme rainstorm events are increasing in intensity and frequency worldwide, resulting in occurrences of floods that can cause significant economic losses as well as social and environmental problems (IPCC, 2014; Pumo et al., 2017). For instance, a severe flooding event in Beijing on July 21, 2012 affected over 1.6 million residents and killed 79 people (Wang, 2012). Within the flooding events, it has been widely recognized that the urban drainage systems (UDSs) are often unable to deliver the runoffs resulted from these complex hydrological and hydraulic situations in an effective manner (Duan et al., 2016a, 2016b). This is, at least partly, because the UDSs are designed based on the traditional treatment way where the assumed water depths/pipe flows are used in the UDS design within the local catchments (Pan et al., 2017). Consequently, the resultant design solutions are vulnerable to extreme rainfall events as their truly underlying process curves are highly complex and variable in a changing climate (Berg et al., 2013; Wasko and Sharma, 2015; Tung, 2018). This motivates many studies to optimally design the UDSs with the true rainfall process curves explicitly considered over the past decade, in order to maximize their effectiveness in dealing with floods (Fu and Bulter, 2014).

To explicitly consider the rainfall process curves over the catchment, a physical drainage simulation model has to be developed where the hydrological and hydraulic properties (e.g., runoff in the urban catchment, water depth in the pipes, or overflows at the manholes) can be simulated as a result of the specified rainfall curves. This, however, brings challenges for the UDS design optimization as the majority of the traditionally deterministic optimization techniques (e.g., linear programing or nonlinear programming) are difficult to incorporate UDS simulation models in their implementations (Li et al., 2015; Eckart et al. 2018). To solve this issue, evolutionary algorithms (EAs) have been employed to enable the UDS design due to their great flexibility in linking drainage simulation models that can explicitly consider rainfall process curves (Fu and Bulter, 2014), as well as their great effectiveness in handling the optimization problems with highly nonlinear and multi-dimensional complexities (Nicklow et al., 2010; Maier et al., 2014).

Among the various EAs, the ant colony optimization (ACO) has gained a great popularity in handling urban water resource optimization problems over the past two decades (Peng et al., 2013; Afshar et al., 2015). Previous studies have demonstrated that the ACO-based techniques are particularly suited to the optimization problems represented by multi-graph structures in the form of nodes and links (Zecchin et al., 2005). Typical examples include the water distribution systems (WDSs) and UDS design problems (Maier et al., 2003; Peng et al., 2013), where nodes and links are used to represent the underlying hydraulic properties of the systems. In terms of WDS design problems, Maier et al. (2003) reported that the ACOs exhibited better performance than genetic algorithms (GAs) in providing optimal solutions for their studied WDS cases. Subsequently, Zecchin et al. (2005) derived a set of equations to identify the optimal parameter values for the ACOs applied to WDS design problems. In more recent years, Zheng et al. (2017) developed an innovative ACO variant based on controlling the convergence trajectory in decision space to follow the pre-specified path, aimed at finding the best possible solution within a given and limited computational budget.

Within the area of the UDS design, Afshar (2006) has proposed an adaptive refinement procedure for the application of ACOs, aimed to improving the optimization efficiency. In subsequent papers of the same author (Afshar 2007; Afshar 2010), a number of constrained ACO algorithms were formulated and successfully applied to UDS design problems. It was claimed in these studies that the ACOs were able to effectively locate the near optimal solutions and were efficient in convergence characteristics. In more recent years, Moeini and Afshar (2013) developed a hybrid optimization technique, where an ACO was combined with a tree growing algorithm (TGA) to simultaneously solve the layout and size optimization problems for the UDSs. This hybrid method utilized the TGA to construct feasible tree layouts, followed by the determination of the optimal pipe diameters with the aid of ACOs.

While many different ACO variants have been successfully used to optimally design UDSs, critical challenges and issues still exist in their practical implementations, which have limited their wider applications to large and real-world UDSs. The first issue is the low computational efficiency associated with the ACO-based optimization techniques, and this issue has been clearly pointed out in a recent review paper (Afshar et al., 2015). This is because the ACO is a population-based searching method and hence it requires a large number of objective function evaluations to ensure the optimal solutions to be identified (Liu et al., 2016). Additionally, the objective function evaluation often involves a simulation model of the UDS, which has been widely evidenced to be very time-demanding (Haghighi and Bakhshipour, 2014). For example, for a UDS with 102 pipes as examined later in the present study, if the storm water management model (SWMM, Rossman, 2010) is used for hydraulic simulation, each evaluation of such model within the ACO based optimization takes approximately 3 seconds on a Dell PC with 2.9GHz (Inter R). As a result, the total optimization time by the standard ACO would take over 800 hours (around 34 days) with about 1,000,000 evaluations (Wang et al., 2015). Such computational overheads can significantly go beyond the computational resources that are typically available for industries and consultants (Beh et al., 2017). While a few ACO variants have been developed to improve the optimization efficiency (Afshar, 2006), their performance when dealing with large and real-world UDSs are still unsatisfactory (Afshar, 2015).

In addition to low computational efficiency, the other critical issue for the current EA-based methods is the difficulty in directly implementing the identified optimal solutions for the UDS under investigation. This is mainly because the constraints considered in these EAs are often specified in the solution space (e.g., no overflows), and hence the resultant design solutions cannot guarantee the feasibility in the decision space. For instance, a common regulation for the UDS design requires that the size of a drainage pipe in the upstream is usually not larger than the pipe in its immediately downstream, otherwise such a design solution is considered to be impractical for implementation (Walski et al., 2001). However, such design criterion has not been explicitly included in the optimization framework of the currently available EA-based optimization methods. As a result, the identified optimal solutions may have many pipes that do not obey this regulation and hence they cannot be adopted as final design schemes for practical applications. This is actually one of the important reasons that practitioners are reluctant to use EAs in their design work.

To address these two issues, this paper proposes an improved ACO-based method to enhance the optimization effectiveness (efficiency and solution practicality) of the UDS design by ccombining an engineering design method (EDM) and the existing algorithm of the Rank-Based Ant System (AS_{rank}, one type of ACO variants). The AS_{rank} is selected as it has been demonstrated to be a more effective ACO variant than its counterparts (Zecchin et al. 2005; Zheng et al., 2017). In the proposed ACO-based method, the EDM is first implemented to identify the approximate design solution for a UDS being considered, followed by the employment of the AS_{rank} initialized by this approximate solution to enable the further optimization, thereby enhancing the optimization efficiency. A constraint in the decision space with regard to a local design criterion of the UDS is proposed to improve the solution practicality. Two real-world UDSs with different complexities are used to examine the effectiveness of the proposed ACO-based optimization method.

2. Methodology

Figure 1 presents the overall methodological framework of the proposed optimization method. As shown in this figure, a single objective optimization function is proposed, followed by the identification of the approximate solution using an engineering design method (EDM). An improved ACO variant is introduced, within which a probability density function is proposed to generate initial solutions based on the approximate design solution (so as to enhance optimization efficiency), and a sampling rule is proposed to account for the design criterion in the decision space (in order to improve solution practicality). Then, two real-word UDSs are used as the case studies to demonstrate the utility of the proposed method. Finally, the effectiveness of the proposed method in terms of the optimization efficiency and solution practicality is discussed.

2.1 Formulation of the single objective optimization for the UDS design problems

In the proposed optimization method, the decision variables considered are the sizes of the pipes for the given layout, that is, $\mathbf{D} = [D_1, D_2, ..., D_n]^T$ where D_i is the diameter of pipe i=1, 2,…,n (n is the total number of pipes of the UDS being considered). The objective considered is the minimization of the design cost under a given rainfall return period. The detailed formulation is given as

Minimize the cost: $$C = \sum_{i=1}^{n} C(D_i)L_i \quad (1)$$

Constraints

(i) no overflow at each manhole: $$OF_j = 0, \forall j = 1, \ldots, m \quad (2)$$

(ii) Velocity range at pipe peak flows : $$v_{min} \le v_i \le v_{\max} \quad (3)$$

(iii) Diameter choices: $$D_i \in \mathbf{S}, \forall i = 1, \ldots, n \quad (4)$$

where C is the total cost of pipes, which is calculated based on the cost per unit length of the pipe multiplied by its length; L_i is the length of pipe i; $C(D_i)$ is the cost per unit length of pipe i for diameter D_i, which is comprised of pipe material cost and construction cost; OF_j is the amount of overflow in the manhole j=1,2,…,m (m is the total number of manhole, which is represented by the nodes in the UDS model). OF_j=0 is one of the typical constraints considered within the UDS design process (Afshar, 2015), indicating that no overflows are allowed at the UDS manholes for the rainfall return period considered. As indicated in Equation (3), the minimum and maximum velocities at peak flow in each pipe are often limited to avoid sediment deposition (v_{min}) and flushing (v_{max}), which are 0.75m/s and 10m/s respectively (Beijing General Municipal Engineering Design & Research Institute, 2017). Another constraint in the decision space (Equation 3) is that the diameters that can be selected for pipes are commercially discrete, indicated by the set of **S**. It is noted that the constraint (2) is often handled using a penalty function method within the optimization process (Zecchin et al., 2005). The values of OF_j as well as the pipe peak velocity v_i are computed with the aid of a hydraulic simulation model in this paper, which is the Storm Water Management Model (SWMM, Rossman, 2010).

It may be possible to optimally design the UDS (especially the large UDSs) in a sequential manner, where sizes of the upstream pipes can be optimized first, followed by the downstream pipe sizing optimization with the fixed configuration of upstream pipes. This is because the physical structure of the UDS is often tree-based, making the sequential optimization method possible. If the UDS design problem is defined in such a sequential manner, the size of the optimization problem and consequently the computational time would be reduced. However, despite the possible variation in problem definition, the applicability of the proposed method would be not affected as it also can be applied to the sub-problem straightforward.

2.2 The engineering design method

The engineering design method (EDM), such as the rational method (Mccuen, 1998), has been commonly used to design the UDSs in many countries, such as China (CDOWE, 2014). It is noted that while different engineering design approaches can be available, the rational method is used in this study due to its wide applications (Mccuen, 1998). Within the EDM (rational method), the flows of each pipe are first calculated using (Gupta, 2016)

$$Q_i = \varphi_i q F_i \tag{5}$$

$$q = \frac{a(1+\log(P))c}{(t_i^1 + t_i^2 + b)^d} \tag{6}$$

where φ_i is the runoff coefficient of the sub-catchment i associated with pipe i with contributing area of F_i (m^2); q (litre/second/m^2) is rainfall intensity over the entire catchment, with value semi-empirically determined, where a, b, c and d are parameters identified based on the fit of rainfall observations; P is the rainfall return period; t_i^1 is the concentration time (Westra et al., 2014) of the sub-catchment associated with pipe i and t_i^2 is total travelling time of the runoff from the upstream pipes of pipe i ($t_i^2 = 0$ if pipe i is the first upstream pipe). t_i^1 is computed as the time a drop of rainwater spends to arrive to the basin outlet section starting from the most hydraulically distant point of the catchment (Grimaldi et al., 2012). In engineering practice, t_i^1 is often estimated based on engineering experience, ranging from 5 to 15 minutes as stated in Beijing General Municipal Engineering Design & Research Institute (2017).

Once the Q_i has been identified for each pipe, the pipe diameter can be accordingly computed using the Manning Equation (Gupta, 2016)

$$Q_i = \frac{1}{nc_i} A_i (R_i)^{2/3} (I_i)^{1/2} \tag{7}$$

where nc_i is the manning coefficient of pipe i; A_i is the cross sectional area of flow; R_i is the hydraulic radius with $R_i = A_i / W_i$ (W_i is the wetted perimeter), and I_i is the slope of the hydraulic grade line. In the EDM, the flows of the pipes are often considered to be full for simplicity, as so $R_i = D/4$. Consequently, the pipe diameter can be determined using

$$D_i = (\frac{4^{5/3} nc_i Q_i}{\pi I_i^{1/2}})^{3/8} \tag{8}$$

The diameters identified using Equation (8) are continuous and hence they have to be rounded to their nearest large discrete values from the commercially available options **S** in Equation (3). As shown in Equations from (5) to (8), the EDM can be computationally efficient in determining the pipe diameters for a given rainfall return period *P*. However, this method has ignored the underlying rainfall process curve that can be quite different with that in Equation (5-6), which is especially the case in the context of a changing climate (Wasko and Sharma, 2015).

2.3 The Rank-Based Ant System (AS_{rank})

The details of the existing Rank-Based Ant System (AS_{rank}) algorithm used in this study are presented in this section. For dealing with combinatorial problems (e.g., the UDS design problems considered in the present work), each ant generates a solution by probabilistically constructing a permissible path through a directed graph *G*. At each decision point, edges are probabilistically selected based on two factors (Dorigo et al., 1996), namely an edge's pheromone value and the visibility value. Within iteration *e*, at decision point *i*, the edge *j*, denoted as edge (*i, j*), is selected with the following probability $P_{ij}(e)$ (Zecchin et al., 2005):

$$P_{ij}(e) = \frac{\tau_{ij}^{\alpha}(e)\eta_{ij}^{\beta}}{\sum_{g=1}^{Ni}\tau_{ig}^{\alpha}(e)\eta_{ig}^{\beta}} \tag{9}$$

where $\tau_{ij}(e)$ is the pheromone value on edge (*i, j*) at iteration *e*; η_{ij} is the visibility value for edge (*i, j*), N_i is the number of decision options for decision point *i*; α and β are the weighting exponents for the pheromone and visibility values. The updating process of the pheromone rule depends on two factors: pheromone decay and pheromone reinforcement, which can be expressed as (Zecchin et al., 2005)

$$\tau_{ij}(e+1) = \rho\tau_{ij}(e) + \Delta\tau_{ij}(e) \tag{10}$$

$$\Delta\tau_{ij}(e) = \sigma\Delta\tau(l_{ij}, \theta^{best}(e)) + \sum_{g=1}^{\sigma-1}(\sigma - g)\Delta\tau(l_{ij}, \theta^{(g)}(e)) \tag{11}$$

where ρ is the pheromone persistence factor (0< ρ <1); $\Delta\tau_{ij}(e)$ is the pheromone addition for edge (*i,j*). The incorporation of pheromone decay allows for a greater emphasis to be placed on more recent information, as edges that are not regularly updated will experience continual decay. σ is the number of elitist ants; $\theta^{best}(e)$ is the best solution found at iteration *e*; l_{ij} is the *j*th edge for the decision point *I* (i.e., the *j*th diameter option for pipe *i* of the UDS design problem), $\theta^{(g)}(e)$ is the k^{th} best solution found at iteration e. Only the top $\sigma - 1$ ranked solutions receive a pheromone addition, rather than the solutions from the entire colony. The pheromone additions from the ranked solutions are also scaled up by a factor ranging from 1 to $\sigma - 1$, depending on their rank. To this end, a pheromone update equation is introduced as:

$$\Delta\tau(l,\theta) = \begin{cases} \dfrac{R}{C(\theta) + p(\theta)}, & \textbf{if } \ l \in \theta \\ 0, & \textbf{otherwise} \end{cases} \tag{12}$$

where R is the pheromone reward factor; $C(\theta)$ is the cost of the solution θ determined using Equation (1) and $p(\theta)$ is the penalty cost for the infeasible solutions that violate the constraint (Equation 2). The advantage of the AS_{rank} scheme is that it guides the optimization search towards promising regions of the search space, encouraging a degree of exploration through the reinforcement from the ranked ants.

2.4 Proposed probability density function to generate initial solutions

A probability density function is needed to generate initial solutions for AS_{rank} based on the approximate solution from the EDM to enable further optimization with the rainfall process curve considered, which is

$$f(d_{k,j}) = \frac{1}{1 + A\left|j - s_k\right|}, \forall j \in [1, M] \tag{13}$$

where S_k is the index number of the diameter for pipe k in the approximate design solution (the total options in **S** are ranged from the smallest to the largest); M is the total number of the options; $f(d_{k,j})$ is the probability density of pipe k in choosing the diameter with index j from the total options; A is a scale parameter to adjust the characteristics of the probability density function. It is seen from Equation (13) that the diameter with the index of S_k (the diameter selected by the approximate design solution for pipe k) can be selected with the highest probability for pipe k within the initialization process. In addition, the diameter with larger distances to S_k will be assigned a lower selection probability. The rationale behind the proposed probability function is that the diameters selected by the EDM can be considered as the approximate optimal solutions and hence initializing the AS_{rank}'s searching around this approximate solution is more likely to identify optimal solutions in an efficient manner. A larger value of A in Equation (13) indicates a steeper distribution of the density function, representing a more biased initialization toward the approximate solution identified by the EDM. The proposed probability density distribution can be easily normalized to facilitate the practical application, which can be

$$P_{k,j} = \frac{f(d_{k,j})}{\sum_{j=1}^{M} f(d_{k,j})} \tag{14}$$

where $P_{k,j}$ is the probability of pipe k in selecting the diameter with index j from the total options. It is highlighted that Equation (14) is used to replace Equation (9) for the generation of initial solutions in the proposed method, but Equation (9) is again used to enable the generation of the offspring solutions afterwards.

2.5 Proposed sampling rule to account for the design criterion

In engineering practice, the pipe sizes for the upstream drainage sections/regions should not be larger than those of the downstream areas. For example, a three-pipe connection is shown as in Figure 2, with pipe flow directions described, that is: Pipe 1 and Pipe 2 are upstream pipes connecting to the downstream Pipe 3 via Node 2 (N_2). As a result, according to the engineering design practice, the diameter size of Pipe 3 should not be smaller than any one of Pipe 1 and Pipe 2. Generally, this engineering criterion for the UDS design can be expressed as:

$$Min(\Omega_j^d) \geq Max(\Omega_j^u) \tag{15}$$

where Ω_j^d represents the set of downstream pipes connected node *j* (i.e., the manhole *j*), and Ω_j^u represents the set of upstream pipes connected node *j*. This design criterion can be explicitly considered within the proposed optimization method, thereby improving the likelihood of the final optimal solutions to be practically implemented.

To account for the design criterion (Equation 15), a sampling rule is proposed as part of developed optimization method. To implement this sampling rule, all the decision variables (i.e., pipes) are indexed, with a lower index value representing a pipe at a further upstream location (e.g., the index 1 indicates the first upstream pipe). Such an indexing can be easily achieved with the aid of the topology of the UDS. More specifically, when applying Equation (14) or (9) to generate the initial solutions or the offspring solutions for the AS_{rank}, the selected diameter of the pipe is checked against its upstream pipes if available using Equation (15) immediately. If Equation (15) is not satisfied (i.e., the pipe diameter is lower than its upstream pipes), Equation (14) or (9) is used to generate the diameter for this pipe again until Equation (15) is satisfied. Such a process is undertaken sequentially for all pipes based on the assigned index (i.e., upstream pipes are considered before downstream pipes).

2.6 Different ACO methods considered and the performance evaluation

In this paper, three different ACO methods are considered and compared to evaluate the effectiveness of the proposed approach for the design of UDSs. These are ACO_1: the standard AS_{rank} method initialized by purely random solutions without the consideration of the design criterion (Equation 15), ACO_2: the proposed probability density function is used to generate initial solutions for AS_{rank} but the design criterion is not considered, and ACO_3 (the proposed optimization method) Equation (14) is used to generate initial solutions and Equation (15) is used to explicitly account for the design criterion. These three ACO schemes are applied to two real-word UDSs to compare their performances in terms of efficiency solution practicality.

As shown in Fig. 1, the result analysis is conducted in the solution space to demonstrate the efficiency of the proposed method, and in the decision space to demonstrate the utility of the proposed method in ensuring the engineering practicality of the design solutions. A metric of practicality level (*PL*) is used to facilitate the result analysis, which is defined as

$$B_i = \begin{cases} 1, & D_i \geq Max\{\Omega(D_i)\} \\ 0, & otherwise \end{cases} \tag{16}$$

$$PL = \frac{1}{n}\sum_{i=1}^{n} B_i \times 100\% \tag{17}$$

with $\Omega(D_i)$ being the set of diameters of all upstream pipes for pipe i with a diameter of D_i. A larger value of PL indicates that a larger number of pipes of the design solution conform to the design criterion (i.e. the downstream pipes are no lower than their corresponding upstream pipes).

3 Cases Study

3.1 Descriptions of two real-world UDSs

Two real-world UDSs from Hangzhou city of China (denoted as UDS1 and UDS2) are used to demonstrate the effectiveness of the proposed method. The UDS1 has a drainage area of 0.081 km^2 with 19 sub-catchments. This system contains 19 pipes with a total length of 1.3 km, 19 manholes, and one outlet, with slopes of pipes ranging from 3‰ to 10‰. Fig. 3(a) shows the catchment land uses of the UDS1, while Fig. 3(b) presents the schematic of network used for the hydraulic simulation. The UDS2 has a drainage area of 3.2 km^2 consisting of 102 sub-catchments. This system has 102 pipes with a total length of 39,984 m, 102 manholes and one outlet. The different types of land uses for the UDS2 are outlined in Figure 3(c), and Figure 3(d) shows the typology of this drainage system.

The hyetograph of a three-hour rainfall event plotted in Fig. 4 was considered for the design the two UDSs, and this event was extracted from historic rainfall data with a five year return period. This implies that the design solutions of the UDS1 and UDS2 should be able to deliver the runoff caused by this rainfall event without any overflows. It should be noted that the selected three-hour rainfall event may less critical than the Chicago curve derived from the local rainfall intensity-duration-frequency relation in producing peak manhole water depths/pipe flows. However, the selection of the most representative rainfall event was not the focus of this study (which is the proposal of an effective UDS design method), and hence the rainfall event selection would not affect the application of the proposed design method. Table 1 gives the unit cost of the discrete diameters that can be used for the two case studies. For the UDS1, the diameters ranging from 200 to 1000 mm were used, while for the larger UDS2, all these 11 diameters were considered. The SWMM was used to conduct the hydrology and hydraulic simulations for the two case studies, in which the kinematic-wave method and the Horton equation were employed to simulate the hydraulics and the infiltrations respectively, with details given in Rossman (2010). It should be noted that the optimization results could be affected by the parameterizations/ methods selected for the SWMM. For example, the Green-Ampt method could be used as an alternative to simulate the infiltration process for the catchments. However, since all the ACO methods used the same SWMM model parameterizations/ methods, the performance comparison (efficiency and practicality) was meaningful.

In the proposed method, the rainfall intensity estimated using Equation (6) was only employed to design pipe diameters in the EDM method, where the concentration time (t_i^1 in Equation 6) for each sub-catchment was assumed to be 10 minutes based on the engineering experience (Beijing

General Municipal Engineering Design & Research Institute, 2017). This was because the sub-catchments within the selected two case studies had a large proportion of the impermeable area. However, when using the ACO methods to further optimize pipe sizes based on the EDM initial solutions, the concept of the concentration time was not used as the rainfall-runoff process was physically simulated using the SWMM. In addition, it was observed that the design rainfall used in the EDM method significantly differed to the observed rainfall event in Figure 4, highlighting the importance to account for the true rainfall curve within the UDS design process.

3.2 Settings for the ACOs

For the two cases considered herein, the runoff coefficient was 0.8 (φ =0.8) according to the land uses offered by the local water utility, and the Manning coefficient nc=0.013 for all pipes. For the rainfall intensity equation (Equation 5), a=57.694, b=31.546, c=0.93 and d=1.008, which were provided by the local water utility. For the proposed density function in Equation (13), a value of A=1 was used to generate the initial solutions based on the approximate design solution, which was determined based on a preliminary analysis.

For AS_{rank} applied to the two case studies, all the parameters used including $\tau_{ij}(e=0)$, η_{ij} α, β ρ σ and R were taken from the calibration method recommended by Zecchin et al. (2005). It should be noted that all the three ACO methods used the identical parameter values for a fair performance comparison. The numbers of ants used for AS_{rank} were 100 for the UDS1 and 500 for the UDS2. Since the UDS1 was a relatively small system, the optimization was conducted 20 trials with different random number seeds, and each trial was run 500 generations, in order to obtain a statistically meaningful characterization of algorithm performance. For the large UDS2 case study, five trials were undertaken and each trial allowed 400 generations. Given that one model evaluation of the UDS2 took about 3 seconds on a Dell PC with 2.9GHz (Inter R), and the total number of hydraulic simulations was 1×10^6, the total computational time was about 35 days to finalize the optimization trials of the UDS2.

4 Results and Discussion

4.1. Computation efficiency analysis

Figures 5 and 6 plots the results of solution costs versus the number of generations for the three ACO optimization methods applied to the UDS1 and UDS2 respectively. The average cost across multiple trials of the ACO_1, ACO_2 and ACO_3 are represented by $ACO_{1\text{-ave}}$, $ACO_{2\text{-ave}}$ and $ACO_{3\text{-ave}}$ respectively, indicating their overall performance. The best solution of the multiple trials of the ACO_1, ACO_2 and ACO_3 are represented by $ACO_{1\text{-best}}$, $ACO_{2\text{-best}}$ and $ACO_{3\text{-best}}$ respectively for illustration. As shown Figure 5, initialized by the approximate design solution with the aid of the proposed probability density function, $ACO_{2\text{-ave}}$ exhibited significantly better performance than the standard ACO ($ACO_{1\text{-ave}}$ in Figure 5). While an additional constraint in the decision space (the design criterion) was considered in the proposed ACO_3, $ACO_{3\text{-ave}}$ still significantly outperformed $ACO_{1\text{-ave}}$ with 20 trials as shown in Figure 5.

In terms of the best solutions, $ACO_{2\text{-best}}$ was able to produce significantly lower cost solution than the standard ACO ($ACO_{1\text{-best}}$). However, it was found that the best solution identified by $ACO_{3\text{-best}}$ was worse than that provided by $ACO_{1\text{-best}}$. Similarity, the performance of $ACO_{3\text{-ave}}$ was worse than $ACO_{2\text{-ave}}$ for this small UDS design problem. This is because $ACO_{3\text{-best}}$ and $ACO_{3\text{-ave}}$ explicitly account for the pipe size design criterion (Equation 15) as an additional constraint relative to other ACO methods, and hence the costs of the final optimal solutions identified by ACO_3 can be higher than those from ACO_1 and ACO_2. It is noted that all final solutions from ACO_3 can satisfy the pipe size constraint (Equation 15) for that the sizes of the downstream pipes are no lower than their corresponding upstream pipes, but this cannot be guaranteed for ACO_1 and ACO_2, with the practicality level results given in section 4.2.

For the relatively large UDS2 problem, ACO_2 and ACO_3 clearly dominated the performance of ACO_1 as shown in Figure 6 in terms of both the average cost and best cost of the final optimal solutions. The approximate solution identified by the engineering design method (EDM) was consistently significantly better than the optimal solution provided by ACO_1 after 400 generations (approximate 35 days). This highlights great challenges and difficulties of the standard ACO in identifying the optimal solutions for large and real-world UDS design problems, as well as great effectiveness of the EDM in generating approximate solutions to enable the UDS design optimization. The latter was proved by the fact that the $ACO_{2\text{-ave}}$ and $ACO_{3\text{-ave}}$ was substantially lower than the $ACO_{1\text{-ave}}$. As shown in Figure 6, the best solution found by $ACO_{3\text{-best}}$ was approximately 0.28 million US dollars, which was about 28.5% lower than the best solution identified by $ACO_{1\text{-ave}}$. As the same for the UDS1, the performance of ACO_3 ($ACO_{3\text{-ave}}$ and $ACO_{3\text{-best}}$) was slightly worse than that of ACO_2 ($ACO_{2\text{-ave}}$ and $ACO_{2\text{-best}}$) for the UDS2 in terms of the solution cost (Figure 6) due to that ACO_3 has considered an additional constraint on pipe size in the decision space.

Figure 7 outlines the solution costs versus the number of generations for all different 20 trials of ACO_1 and ACO_3 applied to the UDS1. In addition to the relative overall better performance in identifying optimal solutions, the performance variation of ACO_3 caused by different random number seeds was significantly lower than that of ACO_1. Similar results were observed for the UDS2 problem. This demonstrates that the proposed optimization method with initial design solutions from the EDM, was robust to identify optimal solutions with performance not heavily relied on the random number seeds. This benefit is especially attractive for practical applications, as typically a very limited number of trials (i.e., one or two trials) are performed for the optimization techniques when dealing with large and real-world UDS design problems.

Figure 8 shows the percent of different pipe sizes between the EDM design solution and the final optimal solutions identified by the proposed method for the UDS1 (20 trials) and the UDS2 (five trails). It was observed that the approximate design solutions produced by the EDM had around 45% and 35% pipes with different sizes compared to the final optimal solutions found by the proposed method for the UDS1 and UDS2 respectively. This means that the optimal sizes of a large proportion of UDS pipes have already identified by the EDM before the implementations of the ACO algorithm. Therefore, the proposed method (combine the EDM and the ACO) was capable of finding good quality optimal solutions with a significantly improved efficiency compared to the standard ACO.

The relative poor performance of the standard ACO (ACO_1) compared to the proposed ACO (ACO_2 and ACO_3) can be caused by (i) ACO_1 was initialized by randomly generated solutions, and hence it was more difficult to identify good quality optimal solutions with a relatively limited time framework compared to the proposed ACO with initial solutions provided by the EDM, which was especially the case when dealing with large UDSs, and (ii) the most appropriate ACO parameterization can be varied as a function of the searching stages as demonstrated in Zheng et al. (2017), and hence the searching ability of the standard ACO can be deteriorated during the later stages (see Figures 5-7). However, it was noted that the adaptive parameterization was not the focus of this study.

Results in Section 4.1 clearly show that the proposed method is able to significantly improve the optimization efficiency compared the standard ACO. In other words, the proposed method can identify substantially better solutions than ACO_1 with the same time framework. This implies that the approximate design solution provided by the EDM is effective in guiding the search towards the promising regions with optimal solutions, as well as that the proposed probability distribution is effective to sample the initial solutions in the neighborhood region of this approximate design solution. The solution practicality is discussed in the next section.

4.2 Solution practicality analysis and discussion

Figures 9 and 10 present the solution practicality levels defined in Equation 16 and 17 as a function of the number of generations for the three ACO methods applied to the UDS1 and UDS2 respectively. Results of practicality levels in these two figures were averaged values of the best solutions across multiple trials for a statistically meaningful discussion. Interestingly, many pipes of the final optimal design solutions identified by the standard ACO method could not satisfy the pipe size design criterion (Equation 15) and hence these solutions are infeasible for practical implementations. For example, for the UDS1 and UDS2, the practicality levels of the ACO_1 were about 60% at the beginning of searching. While this number gradually increased as the searching continued, its final value was about 90% for the UDS1 and about 78% for the UDS2 on average as shown in Figures 9 and 10.

As expected, the practicality levels of the ACO with the constraint in the decision space explicitly considered (ACO_3) were consistently to be 100% across various trials for both case studies. Interestingly, while the design criterion was not considered for ACO_2, its practicality levels were significantly higher than those of the standard ACO (ACO_1). This is because the initial solutions of ACO_2 were generated based on the approximate design solution with pipe size design criterion satisfied, and hence the optimal solutions can possess relatively high values of practicality level. It is noted that the practicality level of the best solution at the first generation was consistently 100% for ACO_2 as shown in Figures 9 and 10. This is because the best solutions at the first generation were the approximate design solutions form the EDM, which have satisfied the pipe size design criterion (Equation 15).

Figure 11 shows the final design solution of a typical optimization trial of ACO_1 applied to the UDS2, with a cost of 0.36 million US dollars. The dotted lines represent the pipes that do not satisfy the pipe size design criterion in Equation (15). As observed in this figure, a total of 20 pipes violated this constraint in the decision space, and hence this optimal solution could not be adopted for practical implementation. Similar observations were found for other optimal solutions identified by ACO_1 applied to the UDS1 and UDS2. This highlights the great necessity and importance to explicitly account for the engineering design criteria within the optimization process; otherwise the final optimal solutions are practically not accepted.

5 Summary and conclusions

The urban drainage system (UDS) design is of great importance to the society and environment as it can significantly affect the severity and spatial distribution of urban floods, especially for flashing flooding events that have often occurred in developing countries such as China (Li et al., 2015). This motivates many studies to optimally design the UDSs, in order to maximize their effectiveness in runoff delivery under a given rainfall return period. While the conventional engineering design method (EDM) is efficient in identifying design solutions for a given UDS, the performance of the resultant designs are often unsatisfactory as the EDM is often based on assumed water depths/flows. Such assumed water depths/flows can be significantly different to those produced by the rainfall process curves (the critical storm event), especially in a changing climate (Wasko and Sharma, 2015). To solve this problem, evolutionary algorithms (EAs) are introduced to explicitly account for the rainfall process curves with the aid of physically based simulation models of the UDSs. However, high computational overheads and low solution practicality have significantly hampered the practical applications of EA-based design methods.

To address the issues stated above, this paper proposed a new optimization method to improve the optimization effectiveness, where the optimization efficiency was enhanced through combining the conventional EDM and the Rank-Based Ant System (AS_{rank}, one type of ACO variants), and the solution practicality was improved by explicitly considering a pipe size design criterion (Equation 15) within the optimization process. More specifically, the EDM was used to generate the approximate solution for the given UDS being considered, and this approximate solution was then used to generate initial solutions for AS_{rank} using the proposed probability density function. Two UDSs with different complexities were used to examine the effectiveness of the proposed optimization method. Observations from this study and their implications are outlined below:

(i) The proposed method, initialized by the approximate solution with the aid of the proposed probability density function, exhibited significantly improved optimization efficiency compared to the standard ACO initialized by purely random solutions. The efficiency improvement became more pronounced when dealing with larger UDS design problems.

(ii) The proposed method was able to ensure that the final optimal solutions entirely satisfy the engineering design criterion, thereby greatly improving the likelihood of the final design solutions to be adopted for practical implementations (i.e. improving the solution practicality).

(iii) It was noted that the average cost solutions were only used to enable the performance comparison between different ACO variants in the solution space. In engineering practice, the best solution of the multiple algorithm trials was often used for practical implementation. The appropriate number of trials can be dependent on the robustness level of the algorithms, as well as the time budgets allowed. Based on the experience of the present study, about five different trials of the proposed ACO were overall sufficient in identify near-optimal solutions.

It is noted that the optimization results of the two UDSs were conditioned on the selected parameterizations/methods for the SWMM. While the utility of the proposed method in both computational efficiency and solution practicality has been demonstrated, the practical implementations of these final solutions need to account for the potential impacts of different model parameterizations/methods. In closing, the proposed ACO-based optimization method can be a useful and necessary tool for researchers to explore and develop better understanding of the UDS design alternatives. An important future focus is to account for other hydraulic facilities (e.g., pumps and ponds) within the proposed optimization method.

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Table 1 Unit cost of the pipe diameters

Diameters (mm)	Cost (US $/m)
200	10.5
300	16.0
400	20.6
500	27.5
600	33.5
800	51.3
1000	69.4
1200	111.5
1500	163.3
1800	172.8
2000	206.5

Figure 1

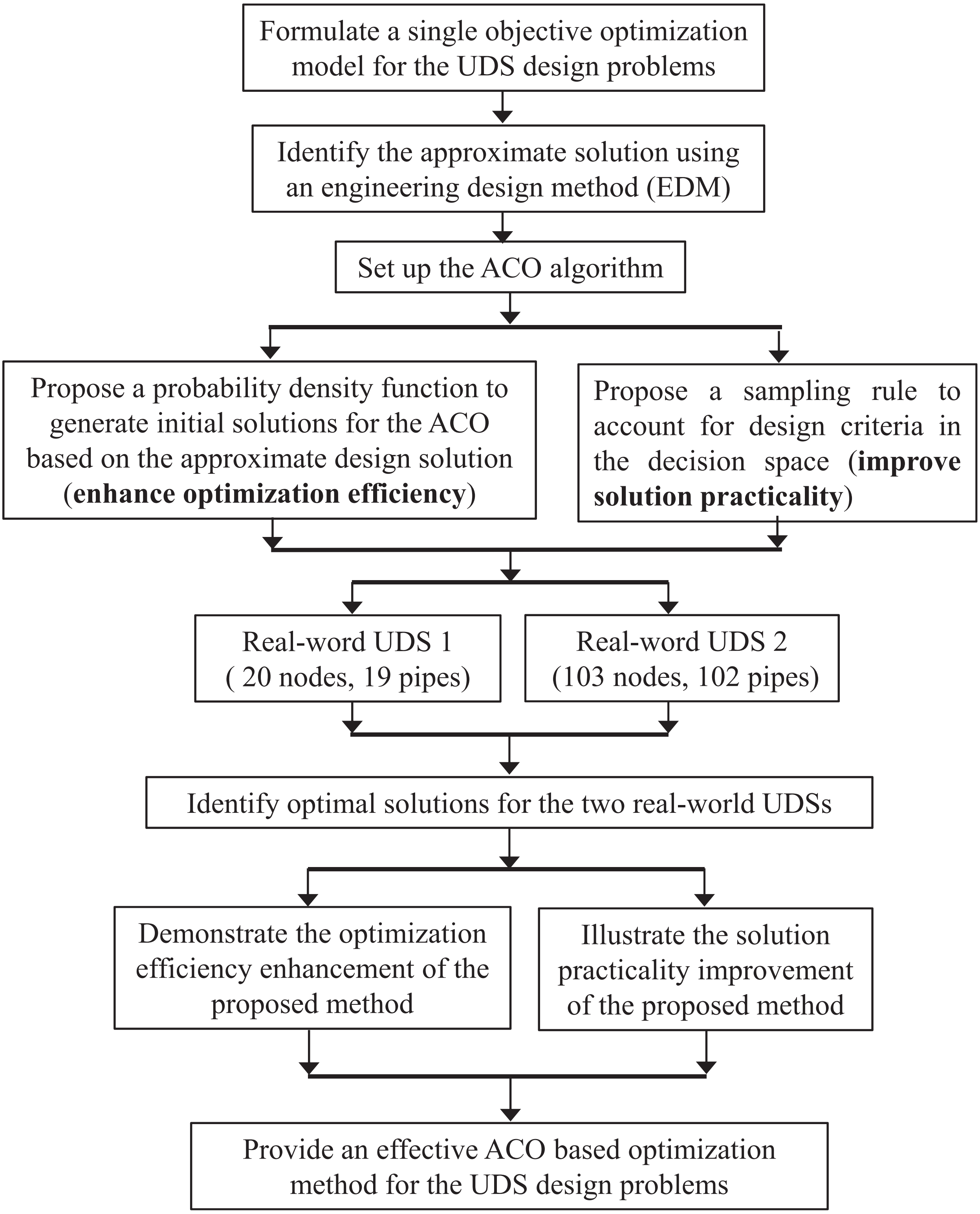

Formulate a single objective optimization model for the UDS design problems
Identify the approximate solution using an engineering design method (EDM)
Set up the ACO algorithm
Propose a probability density function to generate initial solutions for the ACO based on the approximate design solution (enhance optimization efficiency)
Propose a sampling rule to account for design criteria in the decision space (improve solution practicality)
Real-word UDS 1 (20 nodes, 19 pipes)
Real-word UDS 2 (103 nodes, 102 pipes)
Identify optimal solutions for the two real-world UDSs
Demonstrate the optimization efficiency enhancement of the proposed method
Illustrate the solution practicality improvement of the proposed method
Provide an effective ACO based optimization method for the UDS design problems

Figure 2

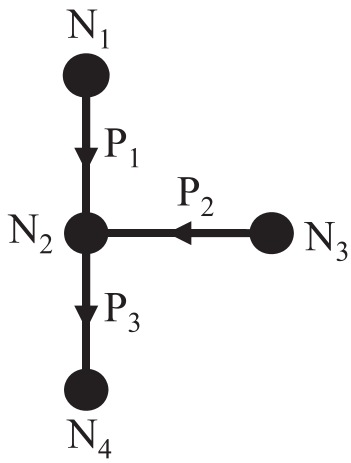


Figure 3

Outlet
Manhole
Pipeline
Green space
Building zon
Roadway
Catchment
(a)
(b)
Manhole
Outlet
Pipeline
Residential
Municipal
Schools
Commercial
Hospital
Industrial
Green space
Water area
Others
Catchment
（c）
（d）

Figure 4

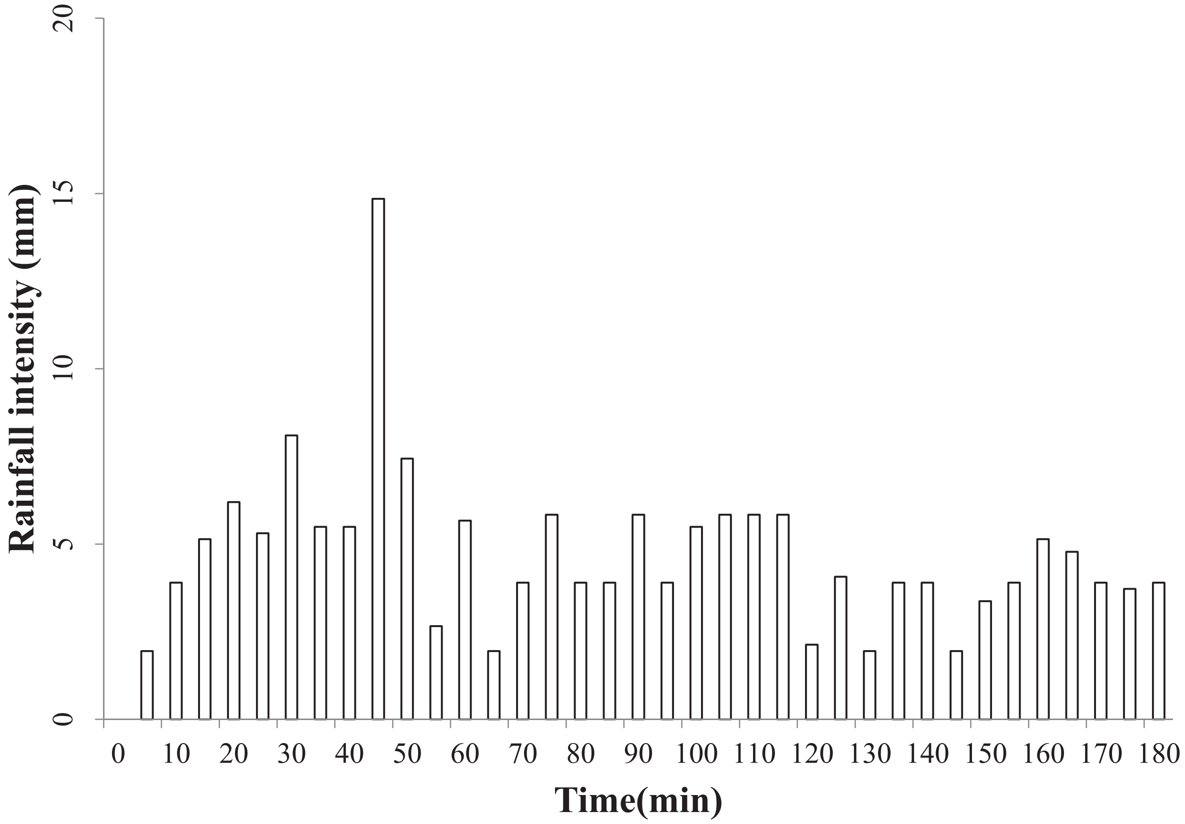
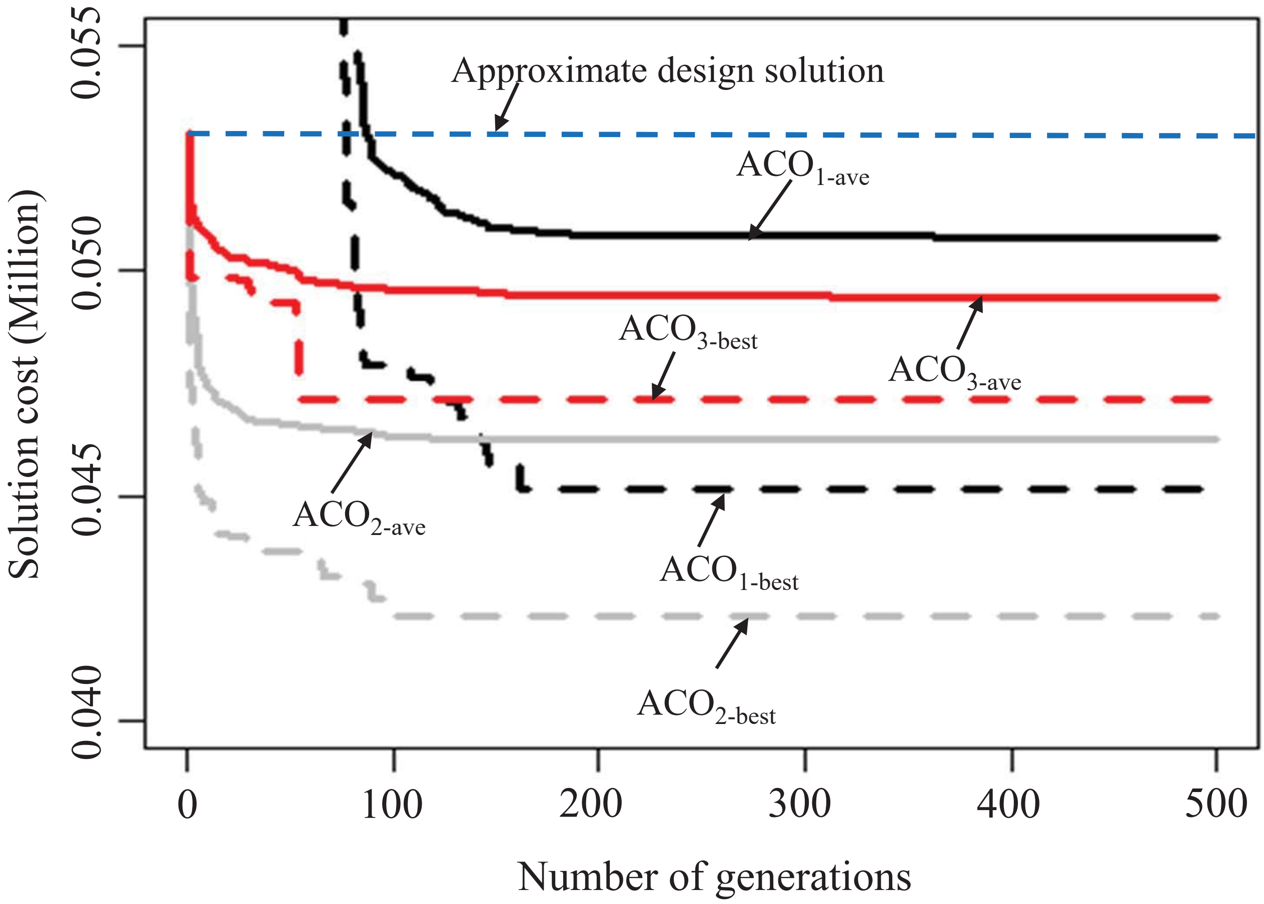
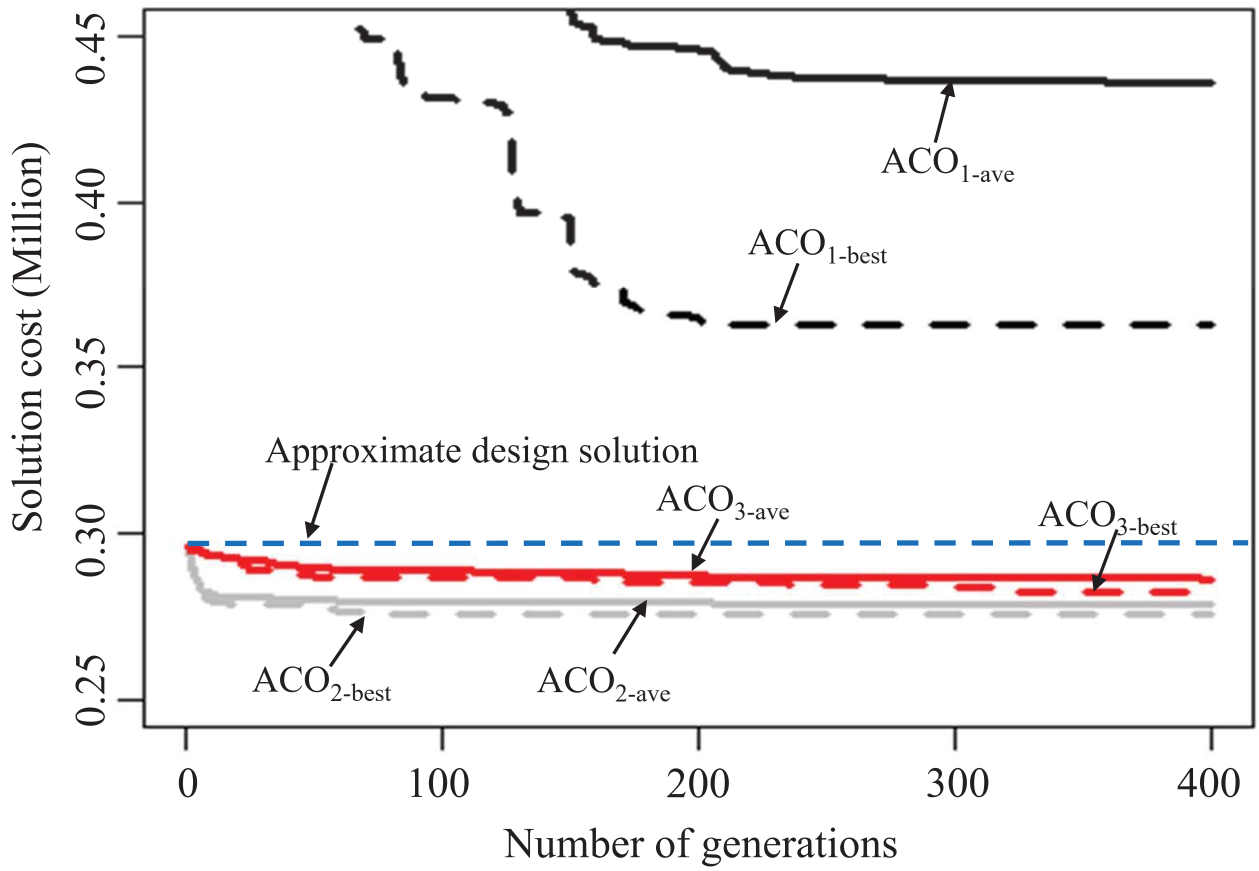

Rainfall intensity (mm)
0
5
10
15
20
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
Time(min)

Figure 5



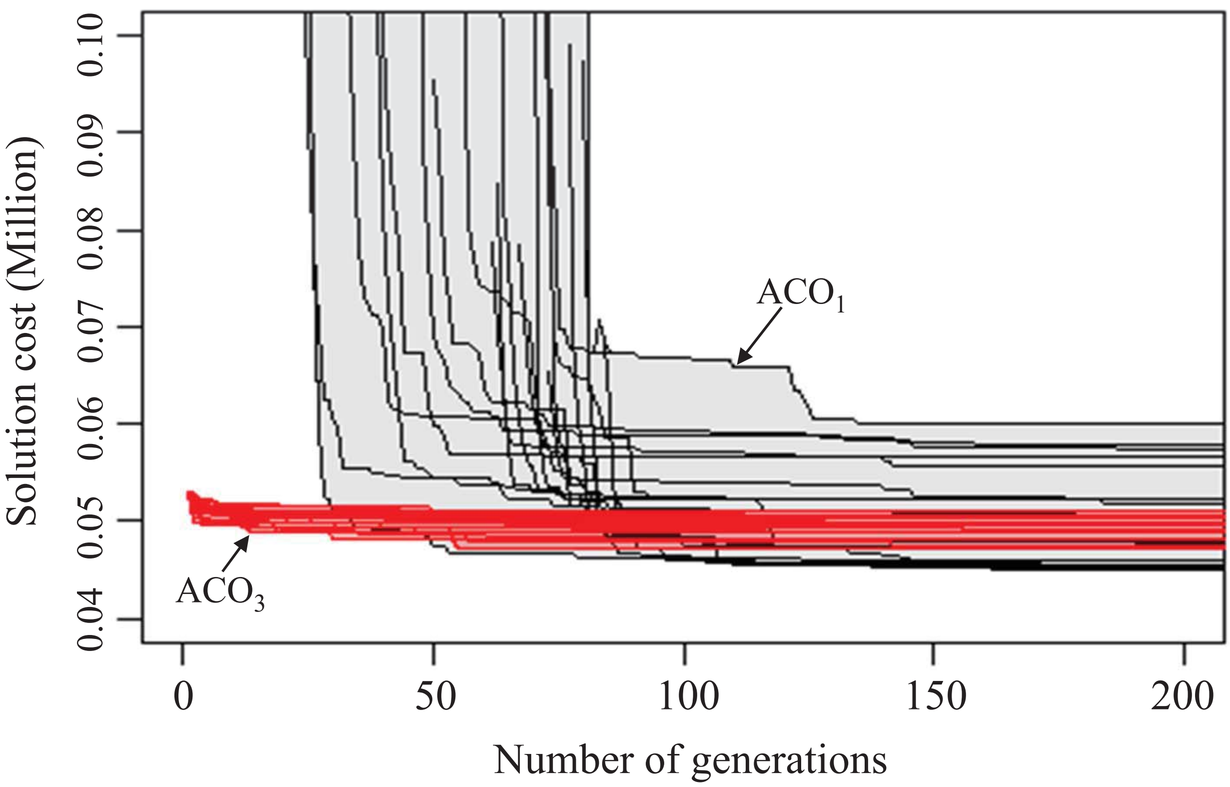
Approximate design solution
ACO1-ave
ACO3-best
ACO3-ave
ACO2-ave
ACO1-best
ACO2-best
Solution cost (Million)
0.040
0.045
0.050
0.055
0
100
200
300
400
500
Number of generations

Figure 6



Solution cost (Million)
0.45
0.40
0.35
0.30
0.25
ACO1-ave
ACO1-best
Approximate design solution
ACO3-ave
ACO3-best
ACO2-best
ACO2-ave
0
100
200
300
400
Number of generations

Figure 7



Solution cost (Million)
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0
50
100
150
200
Number of generations
ACO1
ACO3

Figure 8

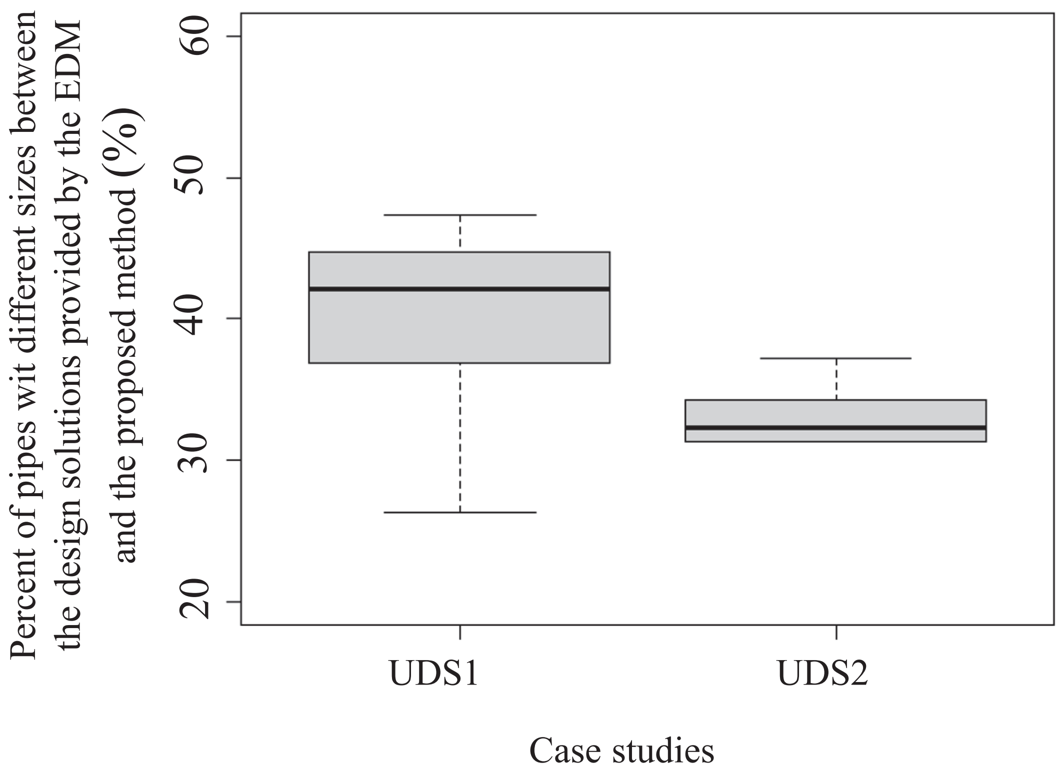
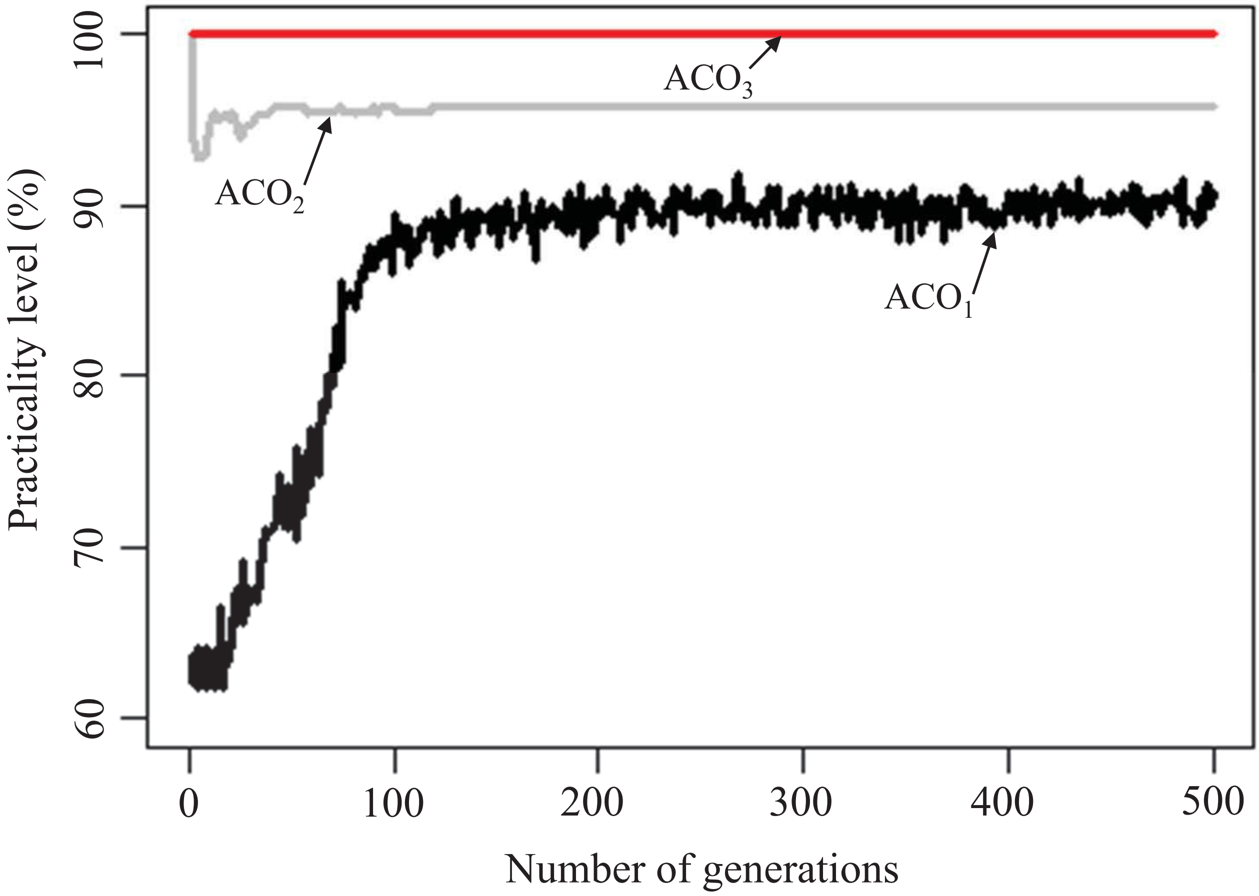
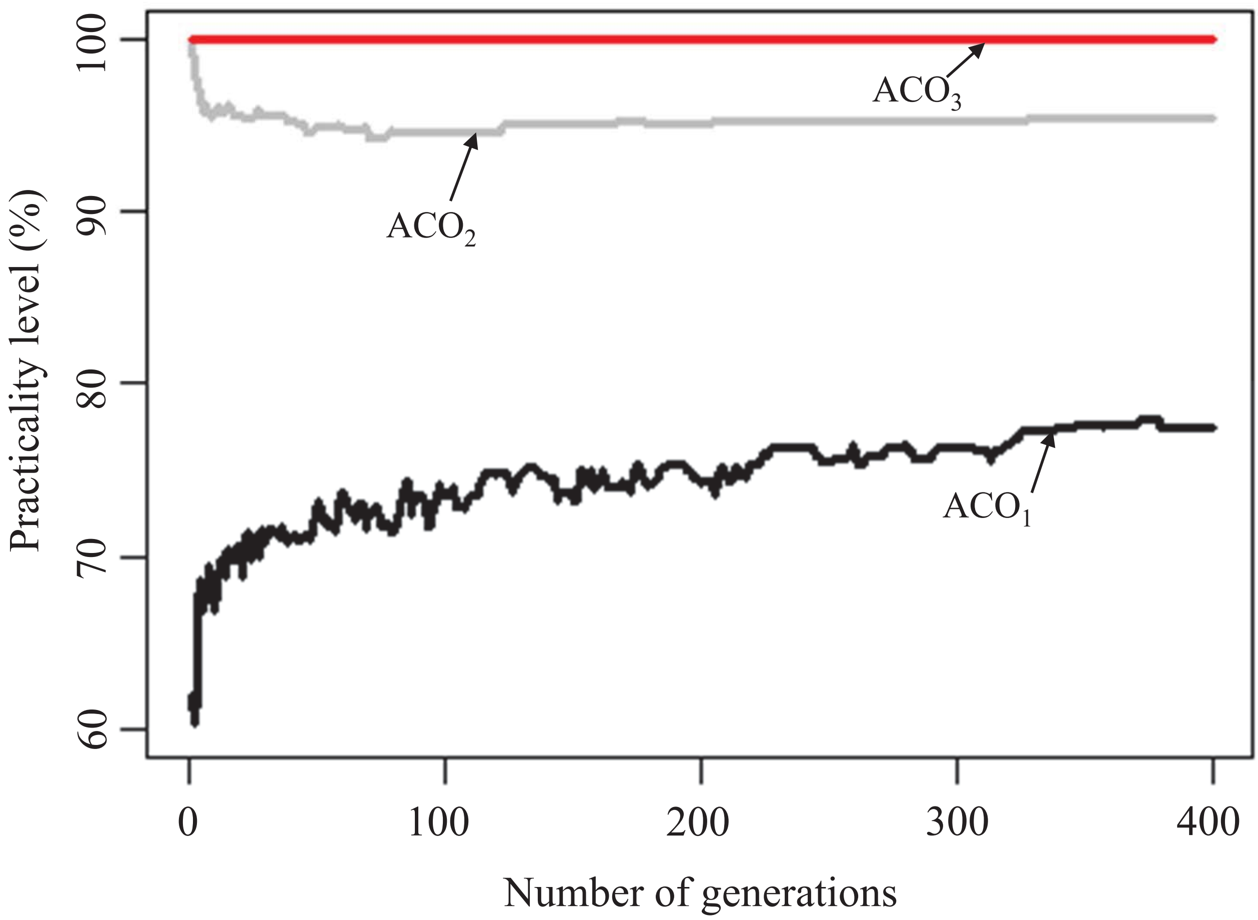

Percent of pipes wit different sizes between the design solutions provided by the EDM and the proposed method (%)
60
50
40
30
20
UDS1
UDS2
Case studies

Figure 9



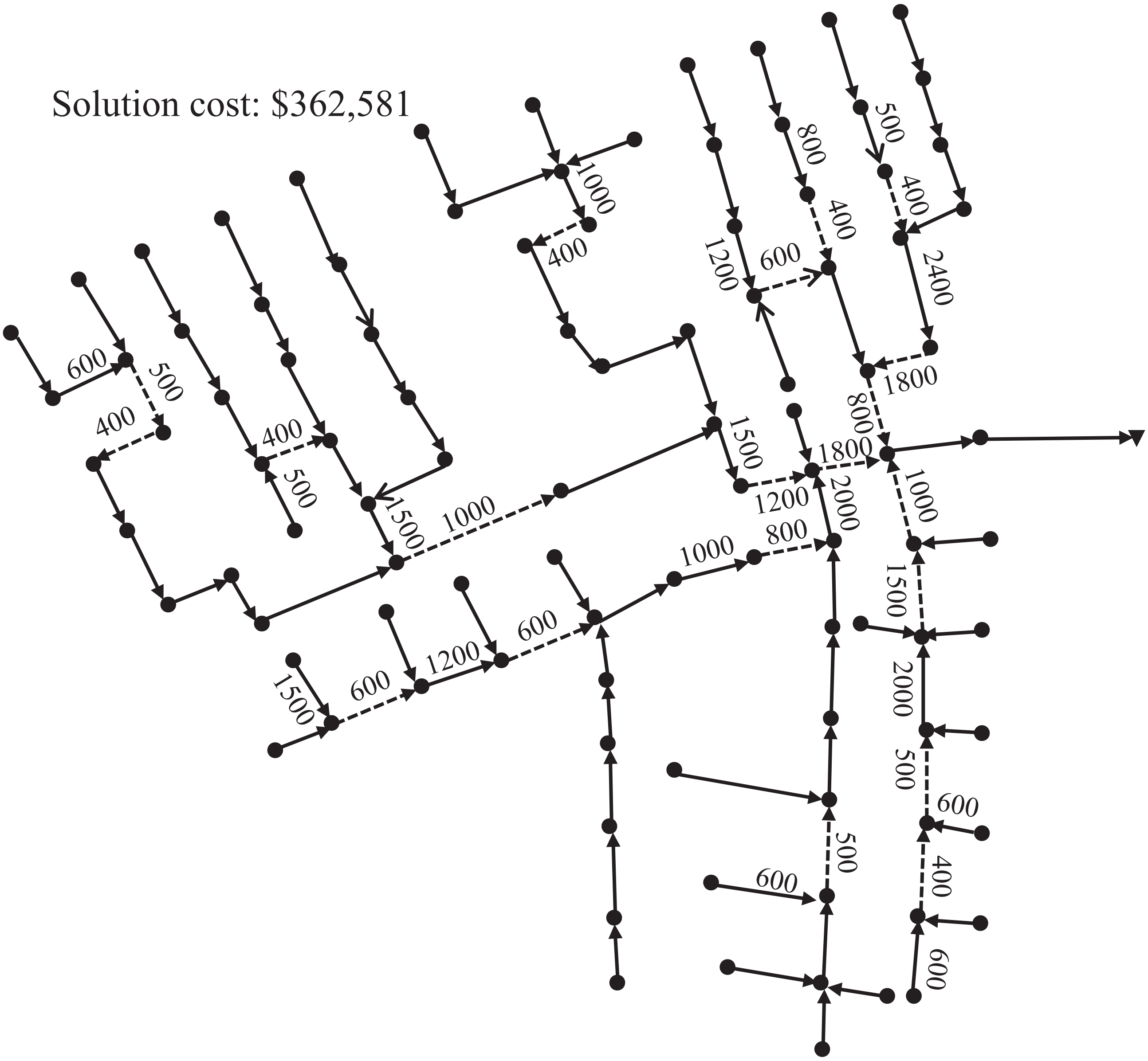
100
90
80
70
60
Practicality level (%)
ACO3
ACO2
ACO1
0
100
200
300
400
500
Number of generations

Figure 10



Practicality level (%)
60
70
80
90
100
0
100
200
300
400
Number of generations
ACO_1
ACO_2
ACO_3

Figure 11



Solution cost: $362,581
600
500
400
400
500
1500
1000
1000
400
1200
600
800
400
500
400
2400
1800
800
1500
1800
1200
2000
1000
1000
800
1500
1500
600
1200
600
2000
500
600
500
600
400
600

Figure Captions

Figure 1. The overall methodology of the proposed method for UDS design problems

Figure 2 Illustration of the engineering design criterion

Figure 3 Land uses of the UDS1 (a) and UDS2 (c), and the network topology of the UDS1 (b) and UDS2 (d)

Figure 4 The hyetograph considered for the UDS design

Figure 5. Solution cost versus number of generations for the three ACO methods applied to the UDS1 (the average cost and best cost are indicated by solid and dotted lines respectively)

Figure 6. Solution cost versus number of generations for the three ACO methods applied to the UDS2 (the average cost and best cost are indicated by solid and dotted lines respectively)

Figure 7. Solution costs versus the number of generations for all 20 runs of ACO_1 and ACO_3 applied to the UDS1 (Each line represents the best solution for each optimization trial)

Figure 8 Percent of pipes with different sizes between the EDM design solution and the final optimal solutions identified by the proposed method

Figure 9. Practicality level (average of the best solution across 20 trials) versus the number of generations of the three ACO methods applied to UDS1

Figure 10. Practicality levels (average of the best solution across multiple trials) versus the number of generations of the three ACO methods applied to UDS2

Figure 11. A Typical optimal solution of ACO_1 applied to the UDS2, where the dotted lines represent pipes do not satisfy the design criterion in Equation (14)