The following publication Qiao, J., & Chen, W. (2019). Continuous precise ephemerides for Beidou-2 maneuvered satellites based on a new thrust force model. Advances in Space Research, 64(3), 696-706 is available at https://doi.org/10.1016/j.asr.2019.05.017.

Continuous Precise Ephemerides for Beidou Maneuvered Satellites Based on a New Thrust Force Model

Jing Qiao · Wu Chen*

Department of Land Surveying and Geo-Informatics, Hong Kong Polytechnic University, Hong Kong 999077, China

* Correspondence: wu.chen@polyu.edu.hk

Abstract Beidou geostationary earth orbit (GEO) and inclined geo-synchronous orbit (IGSO) satellites are frequently maneuvered to keep them in the designed orbits by thrust forces. However, the thrust forces are generally unknown and satellite orbits are difficult to be determined during maneuver periods. At present, precise ephemerides are not available or not complete for the satellite on the day of maneuver, and sometimes even the day after. Our aim is to provide continuous precise orbit for Beidou satellite during the maneuver period. In this study, we present a thrust model based on the detected thrust behavior and carry out precise orbit determination (POD) of the maneuvered satellite. Apart from the parameters of satellite initial state and solar radiation pressure (SRP), the thrust parameters are also estimated. Using observations from the Multi-GNSS Experiment (MGEX) network in Sep and Oct 2017, we have detected Beidou maneuvers and conducted POD for the maneuvered satellites. The accuracy of our recovered orbit has been evaluated by comparing to a concatenation of the dynamic orbit before/after the maneuver and kinematic orbit during the maneuver. The orbit differences before/after maneuvers are about 0.30, 2.34 and 0.42 m in the RAC directions for GEOs, and 0.33, 0.62 and 0.26 m for IGSOs. A similar level of accuracy has been achieved during maneuvers as shown by the orbit differences and the stable POD residuals during the

whole maneuver day. This recovered orbit accuracy is comparable to that of the normal Beidou precise ephemerides from IGS analysis centers (WHU/GFZ).

Keywords Beidou; Maneuver detection; Thrust force; Precise ephemeris

Introduction

Satellite maneuver is a procedure taken by the master control to reposition the satellite when it deflects from its assigned orbit slot due to various perturbations. It is realized by thrust forces generated by the onboard propulsion system. Compared to other navigation satellites, e.g., medium earth orbit (MEO), Beidou GEO and IGSO satellites are more frequently maneuvered, typically about 9 to 15 times per year for each GEO and twice per year for each IGSO (Qiao and Chen 2018), mainly due to the ellipticity of earth's equator and the 1:1 resonance between the satellite and the earth rotation periods (Hugentobler et al. 1999). In the case of a normal Beidou GEO/IGSO maneuver, the satellite velocity change is on the level of dm/s due to the thrust-induced-acceleration of $10^{-4} m/s^2$; whereas the velocity can increase or decrease by 28 m/s with the $10^{-3} m/s^2$ thrust-induced-acceleration in a GEO out-of-plane maneuver which happens biennially (Qiao and Chen 2018; Xie et al. 2012). The frequent satellite maneuvers and generally confidential thrust forces of Beidou satellites are intractable for their POD. Currently, three analysis centers of the International GNSS Service (IGS) which provides open access and high-quality GNSS data, products and service to the society (Villiger and Dach 2018), i.e., Wuhan University (WHU), Deutsches GeoForschungsZentrum (GFZ) and Center for Orbit Determination in Europe (CODE) provide final orbit product for Beidou (Dach et al. 2009; Guo et al. 2016; Z. Deng et al. 2014). CODE only contributes to the orbits of Beidou IGSO and MEO satellites. GFZ excludes the maneuvered satellite from the POD solution once it is detected by the broadcast ephemerides and data preprocessing. Similarly, WHU does not conduct POD for the satellite when the appearance of a maneuver is indicated by orbit fitting and navigation message health status (Montenbruck et al. 2017). CODE also removes the maneuvered Beidou IGSO satellite from their final product or provides its orbit before the epoch of maneuver and set the positions to zero during the rest of the day. As a result, no continuous precise ephemerides are available for Beidou maneuvered satellites.

Though various ways have been investigated to successfully detect satellite maneuver (Gienger and Pereira 2012; Huang et al. 2017; Kelecy et al. 2007; Patera 2008; Qiao and Chen 2018), research on the POD of the maneuvered satellite is insufficient, in particular for the navigation satellites.

A dedicated procedure has been adopted by CODE for maneuver detection and calibration for the IGSO satellites of Beidou and QZSS. When a repositioning event is assumed to have happened, the satellite orbit arcs before and after the event are extrapolated to the day of the maneuver and compared. The closest approach epoch of these two overlapping arcs is supposed to be the maneuver time. The velocity difference at this epoch is assumed as the instantaneous velocity change due to maneuver (Prange et al. 2016). Accurate maneuver-free orbits can be obtained by a POD of the arcs separately before and after the maneuver, excluding a certain time interval around the detected epoch since the thrust forces usually last for a certain amount of time (Dach et al. 2009; Qiao and Chen 2015). However, satellite orbits during the main maneuver period cannot be calibrated, and as mentioned before, the orbits of Beidou IGSO satellites after maneuver are not provided by CODE either.

Some Chinese institutes accessible to the satellite control information, such as the maneuver start and end time, maximum of the thrust in radial, along track and cross track (RAC) directions, mass of the satellite, pulse width and period, force efficiency, etc., also made some efforts for maneuvered satellite orbit recovery. The thrust amplitudes indicated by the satellite control information are not accurate enough and thrust parameters, such as thrust forces or force corrections in the RAC directions, have to be estimated while using the maneuver force model (Cao et al. 2014; Huang et al. 2009). Research on the long-arc orbit determination of the maneuvered GEO satellites using Chinese Area Positioning System (CAPS) ranging data was formerly conducted, two orbit determination strategies that using the telemetry data or estimating constant accelerations in RAC directions have achieved better than 20 m position consistency (Huang et al. 2009). Aiming at a fast orbit recovery, China Beijing Satellite Navigation Center carried out dynamic orbit determination of Beidou GEO satellites by estimating the thrust force errors contained in the telemetry parameters. The 3-dimentional (3D) and radial orbit accuracy were around 30 and 3 m two hours after maneuver, 20 and 2.5 m four hours after (Li et al. 2015). Using C-band pseudo-range observations one day before and two hours after maneuver and the pulse information, continuous orbit determination of the GEOs achieved decimeter to 1 m accuracy in the radial direction, 9-18 m in the horizontal direction, i.e., a combination of the along- and cross-track directions, and a URE of better than 2 m not accounting for satellite clock errors (Cao et al. 2014). Despite all the efforts made for maneuvered satellite orbit recovery, no institute has been able to obtain continuous orbits for Beidou GEO/IGSO maneuvered satellites with comparable accuracy to current precise ephemerides.

In this study, we try to obtain continuous precise orbits for Beidou-2 maneuvered satellites based on a proposed thrust model in the absence of the telemetry data. A description of the thrust force model and processing strategy adopted for maneuver detection and POD is given in the section below. Afterward, the maneuver detection result in the studied period is presented, followed by the orbit validation of the POD result. Finally the conclusions are drawn in the last section.

Maneuvered satellite POD method

Thrust force model

According to the previous maneuver detection results (Qiao and Chen 2018), in normal maneuver cases, the thrust forces increase steadily at the beginning of maneuver, then stay nearly constant for around 20 min, and come back to zero abruptly at the end of maneuver. Based on this changing tendency, we propose a general thrust force model: using a piecewise linear function to model thrust forces in each of the RAC directions. The thrust forces are estimated in three segments: the initial stage of maneuver, the main period for maneuver and the final phase of maneuver. As is shown in Figure 1, t_0, t_1, t_2 and t_3 are the turning points of the thrust model. To compensate for the discrepancies between the assumed and real thrust model and thrusting epochs, we suppose that the forces at t_0 and t_3 are not zero. Therefore, four constant force vectors $\mathbf{F}_0, \mathbf{F}_1, \mathbf{F}_2$ and $\mathbf{F}_3$ in RAC directions at these turning points are estimated. As thrust is merely executed in the radial direction, to reduce the number of estimated parameters and enhance the robustness of the observation equation, we assume a linear force in the radial direction and only two constant force parameters at the start and end of maneuver are estimated. Thus, totally ten thrust parameters, i.e., $\mathrm{f_{ai}}, \mathrm{f_{ci}}$ $(\mathrm{i} = 0,1,2,3)$ and $\mathrm{f_{ri}}$ $(\mathrm{i} = 0,3)$, where $\mathrm{f_{ri}}, \mathrm{f_{ai}}, \mathrm{f_{ci}}$ stand for the force components in the RAC directions, will be estimated by the proposed model. The thrust acceleration $\mathbf{a}_{thrust}$ at any time within the maneuver period can be obtained by linear interpolation:

$$\boldsymbol{a}_{\mathrm{thrust}} = \begin{cases} \boldsymbol{F}_0 + (\boldsymbol{F}_1 - \boldsymbol{F}_0)\dfrac{t - t_0}{t_1 - t_0}, t_0 \le t < t_1 \\ \boldsymbol{F}_1 + (\boldsymbol{F}_2 - \boldsymbol{F}_1)\dfrac{t - t_1}{t_2 - t_1}, t_1 \le t < t_2 \\ \boldsymbol{F}_2 + (\boldsymbol{F}_3 - \boldsymbol{F}_2)\dfrac{t - t_2}{t_3 - t_2}, t_2 \le t \le t_3 \end{cases} \quad (1)$$

The modeled thrust shall be integrated into satellite equations of motion as follows:

$$\begin{aligned}\ddot{\boldsymbol{r}} &= -GM\cdot\frac{\boldsymbol{r}}{r^3}+\boldsymbol{a}^*(\mathrm{t},\boldsymbol{r},\dot{\boldsymbol{r}},\mathrm{p}_1,\mathrm{p}_2,...)+\boldsymbol{a}_{\mathrm{thrust}}(\mathrm{t}) \\ &= -GM\cdot\frac{\boldsymbol{r}}{r^3}+\boldsymbol{a}(\mathrm{t},\boldsymbol{r},\dot{\boldsymbol{r}},\mathrm{p}_1,\mathrm{p}_2,...,\mathrm{f}_{\mathrm{r}0},\mathrm{f}_{\mathrm{a}0},\mathrm{f}_{\mathrm{c}0},\mathrm{f}_{\mathrm{a}1},\mathrm{f}_{\mathrm{c}1},...)\end{aligned} \tag{2}$$

where:

GM is the product gravitational constant times mass of the earth

$\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}$ are the geocentric position vector of the satellite at time t and its first and second order time derivatives, respectively

$\mathbf{a}^*$ is the perturbing acceleration without the thrust

$\mathbf{a}$ is the perturbing acceleration with the thrust

p_k, k=1,2,… are parameters of other dynamic forces, such as solar radiation pressure (SRP)

The equations of motion shall be linearized with the approximate values of those to be estimated parameters: satellite state vector at the initial epoch $(\mathbf{r_0},\dot{\mathbf{r}}_0)$ or $(a,e,i,\Omega,\omega,\mathrm{u}_0)$ can be obtained from the broadcast or ultra-rapid ephemeris; $\mathrm{p}_1,\mathrm{p}_2,...$ may be taken from the estimated ones of previous processing or assumed as zero; $\mathrm{f}_{\mathrm{r}0},\mathrm{f}_{\mathrm{a}0},\mathrm{f}_{\mathrm{c}0},\mathrm{f}_{\mathrm{a}1},\mathrm{f}_{\mathrm{c}1},...$ are set as zero. The thrust parameters are estimated together with other orbital parameters.

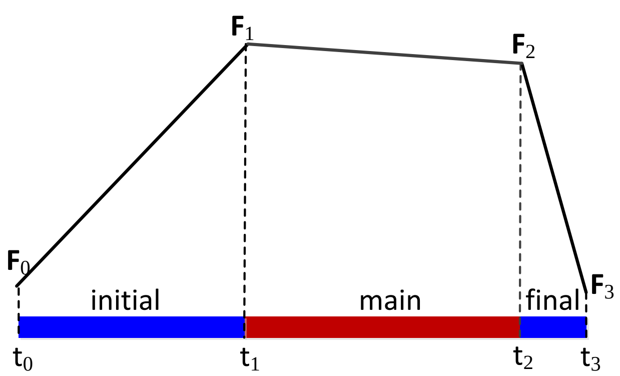

Fig. 1 Three stages of satellite maneuver

POD strategy

Accurate estimation of the turning points of different maneuver stages is crucial for satellite POD. They can be determined by the change of satellite velocity or acceleration due to the thrust force. Satellite velocity or acceleration change can be derived by comparing the kinematic orbit with the integrated one: the kinematic orbit is determined with ground tracking observations and represents the real state of the satellite, while the orbit integration is conducted considering all the possible perturbations but the thrust; the difference between these two orbits can show the effect of satellite maneuver to some extent(Qiao and Chen 2018). The kinematic orbit in this study also serves the purpose of orbit validation during maneuver period in the along- and cross-track directions, which requires a better accuracy than the previous study of maneuver detection.

Kinematic orbit determination

The kinematic orbit is determined in a process taking no account of the satellite dynamic information and using the geometric observations only. The strategy adopted for kinematic orbit determination of the maneuvered satellite is similar to that used for maneuver detection (Qiao and Chen 2018). The only difference is that the initial orbit of the maneuvered satellite is obtained from the precise orbit before maneuver rather than the broadcast ephemeris. As the accuracy of the short-arc orbit determination just in the period before maneuver on the day of maneuver is quite poor, the POD is carried out in a period that starts from the first epoch of the day before maneuver to the epoch before maneuver on the maneuver day. Detailed POD strategy of the satellite before maneuvering is similar to the dynamic orbit determination of the satellite on the whole maneuver day, which is illustrated in the next subsection. Here, in the case of POD during maneuver-free period, the thrust model is not used and the integration step is not narrowed down as introduced later.

With the accurate initial orbit from POD before maneuver, the ambiguities can be fairly fixed for the observations without cycle slips in the kinematic orbit determination. Therefore, the kinematic orbit during the maneuver period can achieve accuracy similar to that of the precise ephemeris in the along- and cross-track directions, benefited from the superior geometry strength in the horizontal direction. However, with the Vertical Dilution Of Precision (VDOP) 10-20 times larger than the horizontal DOP (HDOP), orbit in the radial direction can jump to several meters due to the observation noise (Qiao and Chen 2018) .

Similarly, the precise orbit used to provide initial position of the maneuvered satellite in kinematic orbit determination will also serve as the reference orbit. By extrapolation of the precise orbit, the reference orbit free from thrust during the maneuver period can be obtained. Satellite velocity and acceleration change due to the thrust can be extracted by difference between the reference orbit and the kinematic orbit. Due to the observation noise, the kinematic orbit is unsmooth and so do the extracted velocity and acceleration change. Furthermore, the proposed thrust model is just an approximation and the actual thrust behavior can be more complex. Therefore, the thrust turning points cannot be directly identified from the acceleration change.

Figure 2 demonstrates an example of the thrust-induced velocity and acceleration change of a maneuvering case. A thrust turning points determination procedure has been proposed: Though the thrust-induced-acceleration varies around zero at the beginning of maneuver, its effect can be observed from the velocity change which shifts towards a certain direction. A 2-min sliding window is used here. The start of maneuver is identified if the velocity changes in a certain direction are all below or above zero in the window.

Considering the observation noise, the first point t_0 is set at 2 min before the first epoch of the first window satisfying this condition. As shown in the amplified period in upper row of Figure 2, velocity changes are negative in the window. During the main maneuver period, the acceleration change is obvious even jumps are still exist. The second point t_1 is determined as the first epoch of the first sliding window during which all the thrust-induced accelerations are negative or positive; similarly, the third point t_2 is assumed to be the last epoch of the last window meeting the same condition. Based on our experience, the last point t_3 is set at the epoch 3 min after t_2.

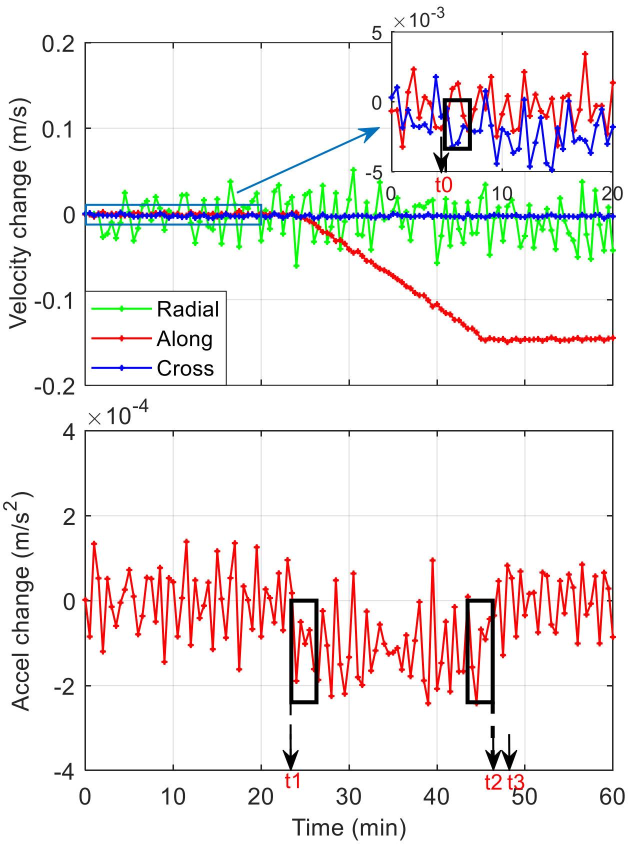

Fig. 2 Thrust-induced velocity and acceleration change of C03 during 09:05:30-10:05:30 on Oct 23, 2017

Dynamic orbit determination

An initial orbit on the whole maneuver day has to be provided to conduct dynamic orbit determination. A gap of 5-7 hours exists in the broadcast ephemeris for normal Beidou maneuvered satellite, so the broadcast ephemeris is not a good choice to derive the initial orbit. We obtain the initial orbit by concatenating the kinematic orbit in the maneuver period and extrapolated orbits before and after maneuver based on the precise ephemerides. The extrapolated orbits are acquired following two steps: orbit fitting of the precise orbits on the two days before/after maneuver to estimate satellite initial state vector and SRP parameters; with these initial elements, carrying out forward/backward orbit integration and satellite orbit before/after maneuver on the maneuver day can be obtained. The orbit models and reference frames used in orbit fitting are the same as those used for orbit determination, shown in Table 1, and the thrust model is not implemented for these maneuver-free periods. Figure 3 plots the differences between the forward three-day integrated orbits and precise orbits of one IGSO and one GEO satellite without maneuver. As the data in the first two days are used for orbit fitting, orbit differences on the former two days indicate orbit fitting accuracy and the RMS errors in RAC directions are 0.08, 0.07, 0.18 m and 0.26, 0.25, 0.07 m for the IGSO and GEO satellite respectively; orbits on the last day are predicted and the prediction accuracies of the IGSO and GEO satellite are 0.20, 0.31, 0.46 m and 0.33, 0.85, 0.07 in each of the RAC directions respectively. The predicted orbit accuracy is much better than that of the broadcast ephemerides and close to that of the precise ones.

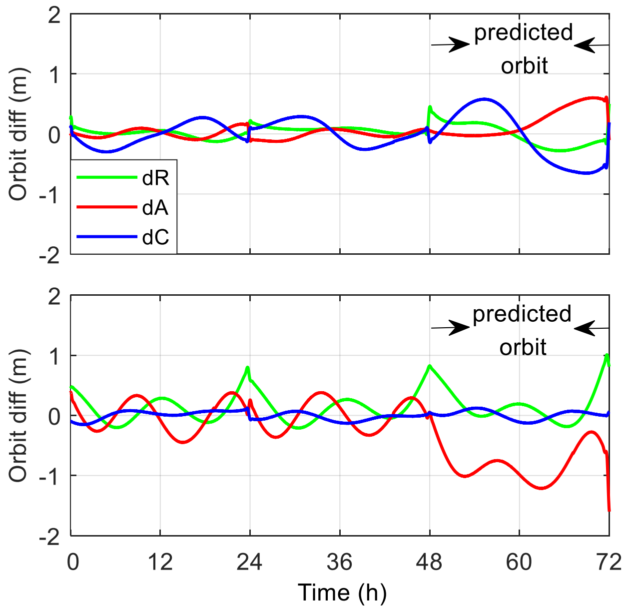

Fig. 3 Coordinate differences between the integrated and precise orbits. Satellite C09 on Jan 5-7, 2015 (*top*), satellite C01 on Jan 6-8, 2015 (*bottom*)

With the a priori information of the turning points and the initial orbit, POD of the maneuvered satellite using the high accuracy dynamic orbit determination method can be conducted. One point of crucial importance is the step-size of numerical integration. Since the maneuver period is generally very short and the thrust force is not as smooth as the natural perturbations, mismatch of the turning points and the integral nodes will severely deteriorate the orbit accuracy. A detailed analysis of this issue has been given by Ju et al. ((Ju et al. 2017), and they proposed a modified Adams–Cowell integrator that combines the multi-step and single-step method around the maneuver. It was successfully used for the LEO with known burning timing information with an uncertainty of 1-2 s and duration that can be shorter than 1 min. Although Beidou thrust force is not continuous and usually characterized as pulses (Cao et al. 2014), it can last for a period of 20 min and averaged continuous force is generally assumed. In the case of our study, the maneuver timing uncertainty is 30 s.

Therefore, we just shorten the integration step-size, with interval of 30 s for the motion equation and 1 min for the variation equations.

POD is carried out by modified Bernese software (Dach et al. 2015) and the reference frames and dynamic models are listed in Table 1. The ionosphere-free linear combination of B1 and B2 phase and code observations with sampling rate of 30 s are used in the double difference POD process. Only data files with observable of the maneuvered satellite are used and the orbits of other satellites are not estimated but fixed to the precise ephemeris of WHU. Fixing the orbits of other satellites on the one hand improves the computation efficiency, and on the other hand, avoids the possible deterioration of other satellite orbits by the maneuvered satellite. The station coordinates are tightly constrained to the PPP results, earth rotation parameters are constrained to the IERS C04 product, and zenith tropospheric delays are estimated with the globe mapping function (GMF) with a parameter spacing of 2 h and the DRY GMF is used as the a priori troposphere model. Phase center offsets (PCOs) of some Beidou satellites are obtained from the estimated values (Guo 2014) and nominal value of (0.6, 0.0, 1.1 m) is adopted for others; satellite phase center variation (PCV) correction is not applied. The ground antenna PCO and PCV of Beidou satellites are assumed the same as those of GPS from igs14.atx. The ambiguities are not fixed but eliminated before normal equation saving (Dach et al. 2015). POD of the maneuvered satellite is conducted on the maneuver day as well as the day preceding and following maneuver, then combine the three daily normal equations using the ADDNEQ2 program of Bernese to estimate the final orbit. The estimated orbit parameters are the six Keplerian elements $a, e, i, \Omega, \omega, \mathrm{u}_0$, five SRP parameters of the reduced ECOM and a constant acceleration in the along-track direction, as well as ten thrust force parameters of the proposed model.

Table 1 Reference frames and dynamic orbit models in POD

Reference frames	
Time system	GPS Time
Inertial frame	ICRF at J2000.0
Terrestrial frame	ITRF2014
Precession model	IAU 2000 Precession theory
Nutation model	IAU 2000R06 Nutation theory (Petit and Luzum 2010)
EOP parameters	Polar motion & UT1 from IERS C04
Orbit models	
Geo-potential (static)	EGM2008 model up to degree and order 12 (+C_{21}+S_{21})
Tidal variations in geo-potential	Solid earth tides: IERS 2010 Ocean tides: FES2004 model Solid earth and oceanic pole tide: IERS 2010 (Petit and Luzum 2010)
Third-body	Sun, Moon, Jupiter, Venus, Mars as point masses
Ephemeris used	JPL DE421(Folkner et al. 2009)
SRP model	Reduced ECOM and a constant along track acceleration (Zhao et al. 2013)
Relativistic effects	IERS 2010 (Petit and Luzum 2010)
Thrust forces	Proposed piecewise thrust force model
Attitude model	Orbit normal for GEO, yaw steering and orbit normal for IGSO and MEO (Montenbruck et al. 2015)

Results and orbit validation

We studied orbital maneuvers of all Beidou satellites for two months, i.e., Sep and Oct, 2017, using daily observations from the MGEX network. More than 80 tracking stations located worldwide can be used for POD of each IGSO satellite, and around 50 stations near the longitude of GEO satellites can be adopted in their POD. Examples of the tracking network for the IGSO C07 and GEO C01 are shown in Figure 4. In this section, we first present the detected maneuvers of Beidou satellites in the studied months; then the POD results of the maneuvered satellites are validated by the kinematic orbit concatenated with precise orbit and orbit from the IGS data analysis centers.

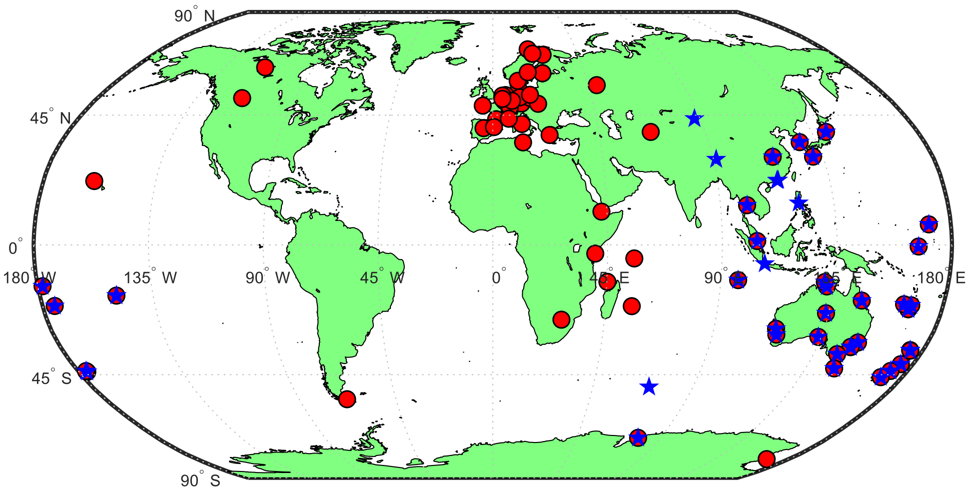

Fig. 4 Observation networks in POD of satellite C07 on Oct 16 (*red circle*) and C01 on Oct 31 (*blue star*)

Maneuvers of Beidou satellites

Fourteen maneuvers were detected in the two-month period following the previous maneuver detection method (Qiao and Chen 2015; Qiao and Chen 2018) and the availability of broadcast ephemeris and precise ephemeris from WHU and GFZ are listed in Table 2. The last three columns in the table indicate whether precise ephemerides are available on the day proceeding maneuver, the day of maneuver and the day following, respectively. No precise ephemerides are given by WHU on all the maneuver days, but ephemeris is available for the maneuvered satellite C05 on Oct 10 in GFZ final orbit product. It can also be noticed that after two successive days of maneuver for C05 on Oct 09-10, precise ephemeris on the day after maneuvers, i.e., Oct 11, is not available either.

Table 2 The availability of Beidou broadcast and precise ephemeris (EPH) in Sep and Oct, 2017

Satellite-Day	Unhealthy Broadcast EPH	Precise EPH (WHU/ GFZ)

C01-Sep 30	09:00:00-14:00:00	6 h	√/ √	×/ ×	√/ √
C02-Sep 11	09:00:00-14:00:00	6 h	√/ √	×/ ×	√/ √
C03-Sep 01	09:00:00-14:00:00	6 h	√/ √	×/ ×	√/ √
C03-Sep 27	09:00:00-14:00:00	6 h	√/ √	×/ ×	√/ √
C04-Sep 04	10:00:00-14:00:00	5 h	√/ √	×/ ×	√/ √
C05-Sep 18	00:00:00-05:00:00	6 h	√/ √	×/ ×	√/ √
C01-Oct 31	09:00:00-14:00:00	6 h	√/ √	×/ ×	√/ √
C02-Oct 19	10:00:00-14:00:00	5 h	√/ √	×/ ×	√/ √
C03-Oct 23	09:00:00-14:00:00	6 h	√/√	×/ ×	√/ √
C05-Oct 09	22:00:00-04:00:00	7 h	√/√	×/ ×	×/ ×
C05-Oct 10	23:00:00-04:00:00	7 h	×/ ×	×/ √	×/ ×
C06-Sep 24	10:00:00-14:00:00	5 h	√/ √	×/ ×	√/ √
C09-Sep 21	10:00:00-16:00:00	7 h	√/ √	×/ ×	√/ √
C07-Oct 16	10:00:00-15:00:00	6 h	√/ √	×/ ×	√/ √

Apart from the three IGSO maneuvers which are listed at the bottom three rows and one special case of GEO out-of-plane maneuver − C05 on Oct 09, all the maneuvers are GEO in-plane ones in which mainly along track thrust forces are executed. We didn't perform POD for the rare case of GEO out-of-plane maneuver due to the complexity of its thrust in this study.

Using the method for turning points determination proposed in the last section, the turning points of the thrust model can be obtained. The four turning points and intervals between t_0 and t_1, t_1 and t_2 of the studied cases are listed in Table 3. The last point t_3 is 3 min after t_2 as defined before. From the table, it can be observed that the main maneuver period, t_2-t_1, last 6.0 to 34.0 min for these two types of satellites. The initial stage of maneuver generally ranges from 12.0 to 29.0 min for the GEO satellites with an exceptional case of C05 on Oct 10 which lasts only 2.0 min, similar to the normal cases of IGSO satellites with a range of 1.0-2.5 min. Moreover, similar thrusting epochs have been found for some of the satellites in these two months.

Table 3 Thrust turning points of the studied maneuvering cases

Satellite-Day	t_0	t_1	t_2	t_3	$t_1 - t_0$	$t_2 - t_1$
C01-Sep 30	9:11:00	9:36:00	9:53:30	9:56:30	25.0 min	17.5 min
C02-Sep 11	9:14:00	9:39:00	9:51:00	9:54:00	25.0 min	12.0 min
C03-Sep 01	9:09:00	9:31:00	9:50:30	9:53:30	22.0 min	19.5 min
C03-Sep 27	9:04:30	9:30:00	9:51:30	9:54:30	25.5 min	21.5 min
C04-Sep 04	9:18:30	9:47:30	9:53:30	9:56:30	29.0 min	6.0 min
C05-Sep 18	00:25:30	00:37:30	00:51:30	00:54:30	12.0 min	14.0 min
C01-Oct 31	9:10:00	9:36:00	9:54:30	9:57:30	26.0 min	18.5 min
C02-Oct 19	9:16:30	9:39:30	9:51:00	9:54:00	23.0 min	11.5 min
C03-Oct 23	9:09:00	9:29:30	9:50:30	9:53:30	20.5 min	21.0 min
C05-Oct 10	23:21:30	23:23:30	23:57:30	00:00:30	2.0 min	34.0 min
C06-Sep 24	9:22:30	9:23:30	9:37:30	9:40:00	1.0 min	14.0 min
C09-Sep 21	9:32:30	9:35:00	9:45:00	9:48:00	2.5 min	10.0 min
C07-Oct 16	10:19:30	10:21:00	10:39:30	10:42:30	1.5 min	18.5 min

With an initial file containing the thrust turning points, POD of the maneuvered satellites using observations before, during, and after the maneuver can be conducted. Precise orbits for these maneuvered satellites with sampling intervals of 30 s have been obtained. Below the validation methods and accuracy of our determined orbits are introduced.

Orbit validation

Several ways are generally used for orbit validation. One is using the satellite laser ranging (SLR), which is adopted for independent validation of GNSS satellite orbit mainly in the radial direction. However, the SLR requires the knowledge of satellite predicted orbit to point their instrument to the correct location and acquire returns from the retroreflector on the satellite. For the particular case of satellite maneuver, the thrust forces are unknown, and the satellite orbit cannot be well predicted; thus SLR data cannot be obtained for the maneuvered satellites. Another way for external orbit validation is by comparing with the orbit of another analysis center; however, it not possible either for the maneuvered satellite, as the precise orbit products from WHU and GFZ generally exclude the maneuvered satellite.

The accuracy of the satellite POD result on the day of a maneuver is mainly validated by comparison with a reference orbit, which is obtained by concatenating the kinematic orbit during the maneuver with precise orbits determined before and after maneuvering. The 'reference orbit' here should not be confused with the 'reference orbit' used for turning points determination in the last section. As shown in Figure 5, the red segment denotes the maneuver period, and Day 1, Day 2, and Day 3 are the day before, the day of, and the day after maneuvering, respectively. The precise orbits free from the maneuver (blue segment) are determined in an arc that is one day before/after the maneuver and the period before/after maneuver on the day of maneuver. Firstly, the accuracy of the reference orbit has to be verified in the aspects of the two parts: precise orbits before/after maneuvering (blue segment) and kinematic orbit during maneuver (red segment).

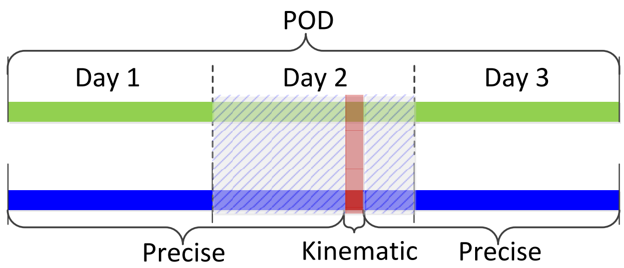

Fig. 5 Concatenated reference orbit for POD result validation

To evaluate the accuracy of the reference orbits before/after the maneuver, we compare the orbits on the days before and after maneuver that generated together with the reference orbits on the day of maneuver to WHU. As the orbits on the days preceding/following the maneuver and during the period before/after maneuvering on the maneuver day are determined in a long arc, the accuracy of the former ones should reflect that of the latter. Comparison statistics of our orbits preceding/following maneuvers with WHU final products are shown in Table 4, and our orbits are referred to as HK in the table. On the other hand, precise orbit difference between WHU and GFZ in Oct 2017 are also calculated to get a general picture of Beidou GEO/IGSO accuracy in the studied period; the results are also listed in Table 4 for comparison.

Table 4 RMS and the maximum of the orbit differences between the precise orbits of WHU and GFZ/ HK in Oct, 2017

	RMS (m) - WHU and GFZ/ HK			Maximum (m) - WHU and GFZ/ HK		
	Radial	Along	Cross	Radial	Along	Cross
C01	0.36/ 0.38	1.97/ 1.86	1.15/ 0.84	0.78/ 0.55	4.01/ 4.38	4.78/ 1.26
C02	0.46/ 0.28	3.06/ 3.85	1.16/ 1.04	0.94/ 0.46	6.33/ 8.51	3.96/ 1.62
C03	0.52/ 0.28	1.96/ 1.39	1.60/ 0.94	2.35/ 0.80	4.96/ 3.12	6.49/ 1.68
C04	0.51/ 0.40	2.71/ 3.59	1.42/ 0.85	3.90/ 0.85	5.44/ 7.26	3.74/ 1.87
C05	0.58/ 0.37	1.75/ 3.26	0.69/ 1.05	2.50/ 0.72	5.82/ 9.14	2.18/ 1.97
C06	0.43/ 0.11	0.12/ 0.22	0.15/ 0.18	5.18/ 0.41	1.10/ 0.70	1.00/ 0.59
C07	0.06/ 0.13	0.06/ 0.26	0.04/ 0.17	0.27/ 0.73	0.27/ 0.71	0.12/ 0.70
C08	0.05/ 0.17	0.06/ 0.22	0.08/ 0.22	0.27/ 0.59	0.21/ 0.54	0.27/ 0.58
C09	0.12/ 0.10	0.12/ 0.20	0.14/ 0.23	1.42/ 0.42	1.28/ 0.65	0.98/ 1.07
C10	0.06/ 0.11	0.07/ 0.30	0.05/ 0.21	0.31/ 0.56	0.22/ 0.65	0.17/ 0.60
C13	0.06/ 0.17	0.06/ 0.27	0.08/ 0.23	0.45/ 0.74	0.32/ 1.11	0.29/ 0.76

It can be seen that the RMSs of orbit differences between our results and WHU are no more than 0.4, 4.0 and 1.1 m in the RAC directions for GEOs; and considering the maximum differences, the consistency of our GEO results to WHU is more stable in the radial and cross-track components than GFZ. For the IGSOs, the RMSs of discrepancies between our results and WHU range from 0.1 to 0.3 m in each direction, comparatively larger than the normal cases of GFZ and WHU. Despite this, the orbits determined in an arc that is one day preceding/following the maneuver together with the period before/after maneuvering on the maneuver day have comparable accuracy with that of WHU and GFZ. Therefore, they can be safely used for orbit validation of the maneuvered satellite.

As introduced in the subsection of Kinematic orbit determination, the accuracy of kinematic orbit during the maneuver period is similar to that of the precise ephemeris in the along- and cross-track directions. The radial component of the kinematic orbit suffers from variations of several meters, and the examples during normal periods have shown RMS errors of 1.3 and 1.2 m for GEO and IGSO satellites, respectively (Qiao and Chen 2018). The kinematic orbit may not be an ideal choice for orbit validation due to its low accuracy in the radial direction; however, no other accurate orbit can be used. In this study, we use the kinematic orbit for orbit evaluation during the maneuver separately, and the POD residuals of the whole maneuver day are investigated. Below we give a detailed analysis of each maneuvered satellite POD result.

Validation with reference orbit

Figures 6 and 7 plot the difference between the reference orbits and the POD results of the maneuvered GEOs and IGSOs on the day of maneuver respectively, only the cases in Oct are plotted for simplicity. For C05 on Oct 10, as its maneuver period was during 23:21:30-00:00:30, the orbit from noon of the main maneuver day to the same time of the following day is evaluated. As can be seen from these two figures, fairly good accuracy is shown in the radial and cross-track components for both GEOs and IGSOs. The large jumps in the radial direction during the maneuver period may due to the fluctuations of the reference orbit. Fluctuations of orbit difference in the along-track direction during some of the maneuver periods may due to the not ideally modeled thrust. Jumps of the orbit differeces before and after maneuvering are induced by the inconsistency of the reference orbits before and after.

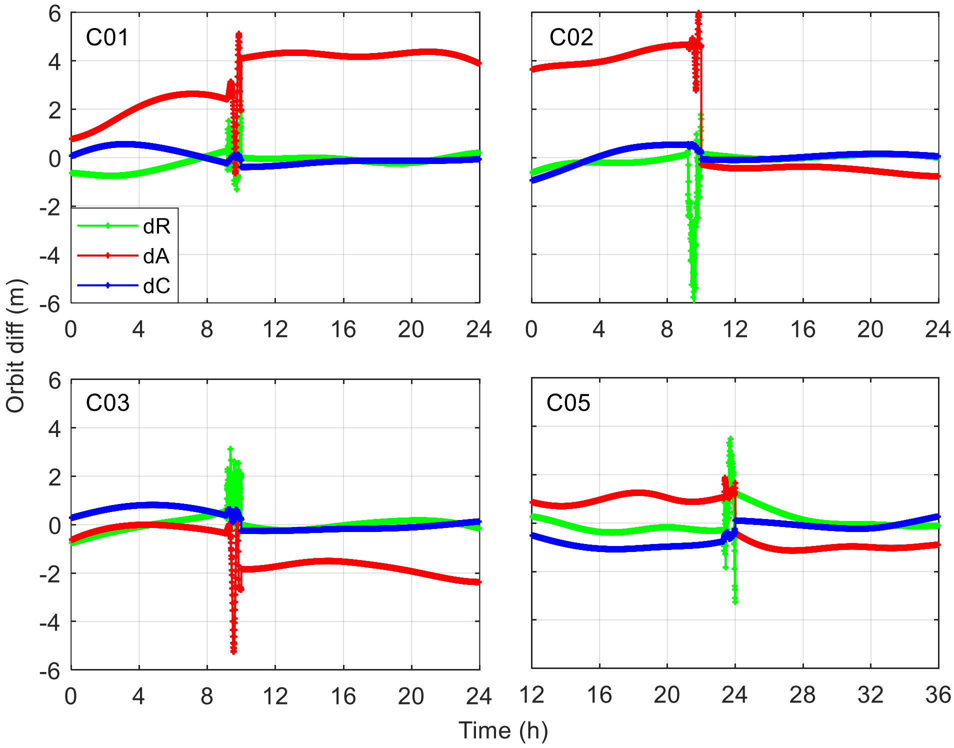

Fig. 6 Comparison of POD results of the GEO maneuvered satellites C01, C02, C03 and C05 with the reference orbits

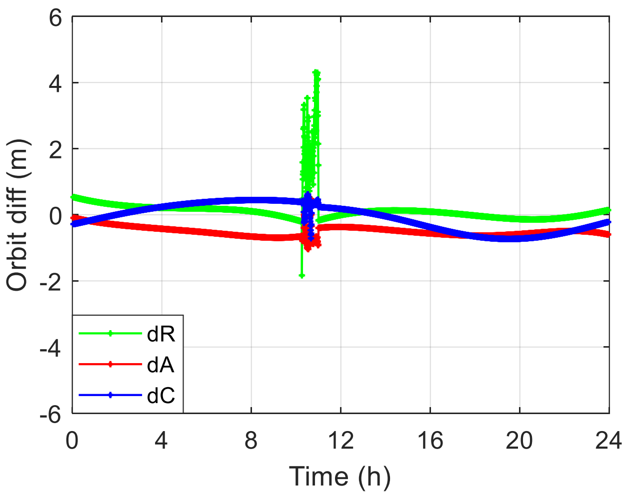

Fig. 7 Comparison of POD results of the IGSO maneuvered satellite C07 with the reference orbit

The RMSs of orbit difference between the POD results and the kinematic segment of the reference orbits during maneuvering and the precise sections of the reference orbits before and after maneuvering are listed separately in Table 5. Before and after the maneuvers, the GEO accuracy is generally better than 0.50 m in the radial direction, 0.98-5.65 m and 0.23-0.71 m in the along- and cross-track directions respectively; the IGSOs have decimeter level accuracy with mean values of 0.33, 0.62 and 0.26 m in the RAC directions respectively. During the maneuvers, similar level of accuracy to that before and after maneuvers has been achieved in both along- and cross-track directions; orbit difference RMSs in the radial direction have averages of 1.48 and 1.91 m for GEOs and IGSOs respectively, which may be due to the meter level noises of the referenced kinematic orbit (Qiao and Chen 2018).

Table 5 RMS of the differences between the POD results and reference orbits

	POD and Kinematic/ Precise		
	Radial (m)	Along (m)	Cross (m)
C01-Sep 30	0.98/ 0.13	1.54/ 5.65	0.24/ 0.51
C02-Sep 11	1.51/ 0.17	0.77/ 1.55	0.53/ 0.36
C03-Sep 01	1.10/ 0.42	2.30/ 1.44	0.75/ 0.71
C03-Sep 27	1.22/ 0.27	3.69/ 1.52	0.25/ 0.23
C04-Sep 04	1.63/ 0.57	1.09/ 2.27	1.31/ 0.34
C05-Sep 18	1.09/ 0.26	2.35/ 2.31	0.72/ 0.35
C01-Oct 31	0.70/ 0.34	2.97/ 3.56	0.12/ 0.28
C02-Oct 19	2.91/ 0.17	4.55/ 2.63	0.42/ 0.31
C03-Oct 23	1.49/ 0.24	2.43/ 1.43	0.44/ 0.43
C05-Oct 10	1.78/ 0.35	1.32/ 0.98	0.50/ 0.66
C06-Sep 24	1.38/ 0.46	0.39/ 0.85	0.18/ 0.22
C09-Sep 21	2.10 / 0.37	0.95/ 0.49	0.23/ 0.14

C07-Oct 16	2.26/ 0.17	0.70/ 0.51	0.37/ 0.41
GEO-Mean	1.48/ 0.30	2.28/ 2.34	0.51/ 0.42
IGSO-Mean	1.91/ 0.33	0.68/ 0.62	0.27/ 0.26

Comparison with available external orbits

Although the precise orbit of the maneuvered satellite is generally not available on the maneuver day, for the maneuver case of C03 on Oct 10, precise ephemeris is provided by GFZ. CODE sometimes issues the orbit of IGSO satellite before the maneuver epoch, and C07 on Oct 16 is of such a case in our study period with its orbit before 8 a.m. available from CODE. We compare POD results of these two satellites with the corresponding orbits from GFZ and CODE.

The differences of C07 orbits between our POD result and CODE final product in its available period are plotted in Figure 8. Relatively good consistency between our orbit and CODE final product is shown in the maneuver-free period, and the RMSs are 0.19, 0.15 and 0.11 m in the RAC directions. However, apparent orbit discrepancy is shown for C03 in Figure 9, especially during the first and last four hours of the day. Also, an abrupt jump appears in the along-track direction at the end of the day when the maneuver is detected.

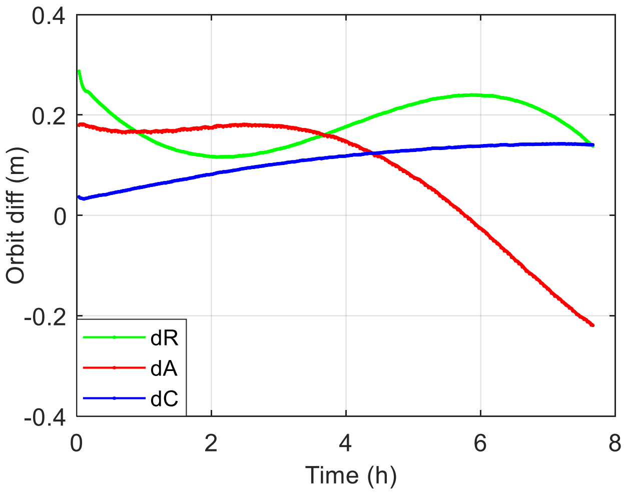

Fig. 8 Comparison of POD result of the satellite C07 on Oct 16 with precise orbit from CODE. Only the first nearly 8 h' orbits are compared

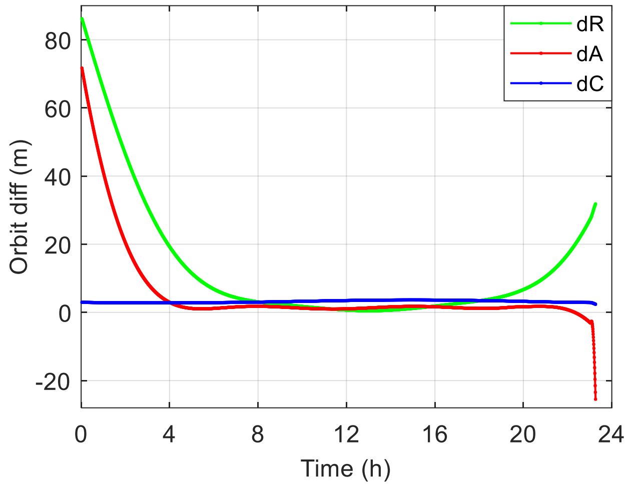

Fig. 9 Comparison of POD result of the satellite C03 on Oct 10 with precise orbit from GFZ

As the C03 orbits of our POD result and the GFZ product are not consistent, to check the accuracy of both results, we investigate the double difference phase observation residuals of the MGEX stations. The normal satellite orbits are from WHU, and the C03 orbits from our result and GFZ are used respectively. With the ionospheric effect reduced by ionosphere-free combination, the tropospheric delay and station coordinate estimated beforehand in a PPP process and the station/satellite clock errors eliminated by double difference, the double difference phase observations should only contain constant ambiguities. Using station DYNG and C03 as the reference station and satellite, examples of the double difference phase observation residuals are plotted in Figure 10, and the name of the other station and satellite for each observation are marked in the legend. The investigated period is from 21:00 to 23:57 which includes the maneuver period and the studied satellites are successively tracked by the stations. The ambiguities of each pair of observations are fixed as the double difference observations at 21:00. As can be seen from the figure, when the orbit of C03 from our POD result is used, the residuals remain almost zero in the nearly three hours; however, large residuals are shown while the GFZ product is used and a steep increase begins at around 23:21:30 when is the starting epoch of the maneuver. Therefore, we have the confidence to claim that our C03 POD result is correct and GFZ might not have successfully detected this maneuver and just carried out its POD as in normal cases.

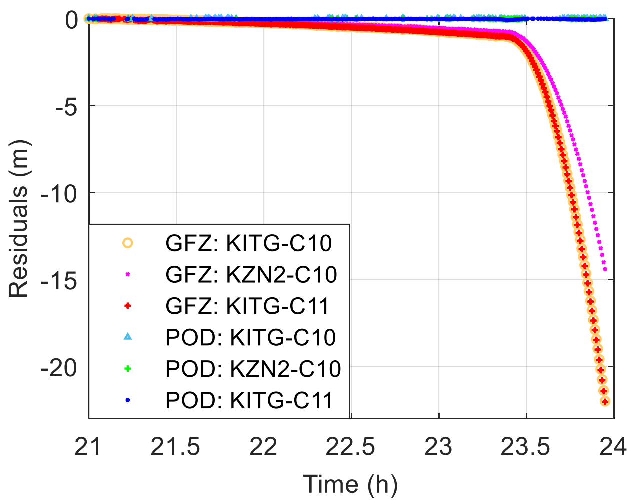

Fig. 10 Double difference phase observation residuals using orbits of C03 on Oct 10 from our POD result and GFZ

Residuals in POD

As reflected by Figures 6 and 7, the radial orbit differences have meter level jumps while comparing the POD results with the kinematic segment of reference orbits during maneuvers. We suppose it is due to the inaccuracy of the referenced kinematic orbits. To verify this, we investigated the post-fit double difference phase observation residuals in the POD. Figures 11 and 12 illustrate the residuals of one GEO and one IGSO maneuvered satellite respectively.

In each case, four double difference observations formed with the maneuvered satellite are randomly chosen for simplicity.

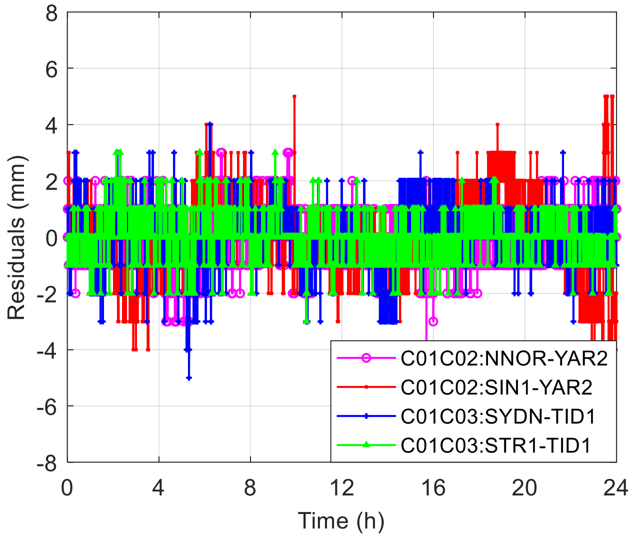

Fig. 11 Double difference phase observation residuals in POD of C01 on Oct 31

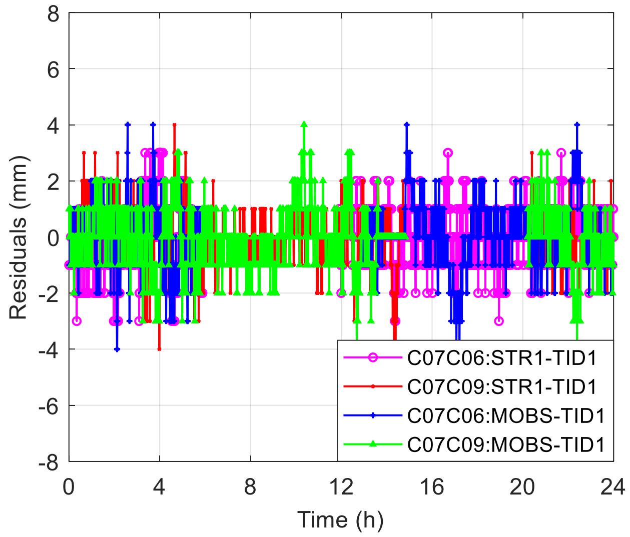

Fig. 12 Double difference phase observation residuals in POD of C07 on Oct 16

Figure 11 shows the observation residuals in the POD process of C01 on Oct 31, and it can be observed that there is no evident increase of residuals during the maneuver, 9:10:00-9:57:30. Similarly, the residuals in the POD process of C07 on Oct 16 also remain at a stable level on the whole day. The RMSs of the displayed residuals of C01 and C07 are 1.06 and 1.15 mm, respectively.

Conclusions

The Beidou satellites are maneuvered in such a high frequency that about once per month for each GEO, twice per year for each IGSO. Continuous precise ephemerides for these maneuvered satellites are generally not provided by the IGS analysis centers, or not necessarily correct when available, e.g., C03 orbit on Oct 10, 2017 from GFZ. Accurate and continuous orbit for the maneuvered satellite is needed to reduce the maneuver interruption on the users. Based on our previous study of the thrust forces, we proposed a thrust model for POD of the maneuvered satellites. ~~Precise ephemerides of the maneuvered satellites with comparable accuracy to those of the current normal Beidou satellites have been obtained.~~

We studied the maneuvers of Beidou satellites in Sep and Oct, 2017 using observations from the MGEX network. Fourteen maneuvers, including three IGSO out-of-plane maneuvers, ten GEO in-plane maneuvers and one special case of GEO out-of-plane one, have been detected. The GEO out-of-plane maneuver is comparatively tremendous and complex also has a rather low frequency, thus not taken into consideration for POD in this study.

The thrust forces in the primary components of GEO/IGSO maneuvers share the common changing tendency that increasing slowly at the initial stage of maneuver, steady-going during the main period and followed by a sharp turning back to zero. In the maneuvered satellite POD process, the thrust parameters relating to the three stages in all the RAC directions are estimated together with satellite initial state and SRP parameters.

The maneuvered satellite orbits have been validated by comparing to the concatenated orbits which have accuracy close to the precise ephemerides of WHU/GFZ, except the radial component during the maneuver. The RMSs of the differences between the maneuvered satellite POD results and reference orbits before and after maneuvering have average values of 0.30, 2.34 and 0.42 m in the RAC directions for GEOs, and 0.33, 0.62 and 0.26 m for the IGSOs. During maneuvers, similar statistics have been achieved in the along- and cross-track directions; whereas, the orbit radial differences are around 1-2 m which is similar to the accuracy of the referenced kinematic orbit. The post-fit observation residuals of the POD process present no increase or jumps during maneuvers; furthermore, as the orbit is determined in a long arc, the orbit after the maneuver cannot be that good if the thrust model during maneuvering is not adequately established. Hence, we refer that the 1-2 m orbit radial differences during maneuvers are due to the inaccuracy of the reference orbit rather than the POD results. Therefore, we can conclude that the maneuvered satellite POD results have comparable accuracy with the precise ephemeris and continuous ephemerides of the maneuvered satellites can be obtained by the proposed method.

As the proposed thrust model is just an approximation and the turning points determination accuracy is affected by the observation noise and sampling interval, the estimated thrusts may not always follow the pattern in Figure 1. A better thrust model may be developed in the future to mitigate the fluctuations in the primary thrusting directions. Furthermore, according to the different lengths of t_1-t_0 for GEOs and IGSOs in Table 4, different thrust models may be proposed for these two types of satellites respectively. In addition, we will try to resolve the problem of POD for GEO satellites undergoing complex out-of-plane maneuver.

Acknowledgement

The work was substantially supported by grants from the Hong Kong RGC Joint Research Scheme (E-PolyU501/16).

References

Cao F, Yang X, Li Z, Sun B, Kong Y (2014) Orbit determination and prediction of GEO satellite of BeiDou during repositioning maneuver. Adv Space Res 54(9):1828–1837. doi:10.1016/j.asr.2014.07.012

Dach R, Brockmann E, Schaer S, Beutler G, Meindl M, Prange L, Bock H, Jäggi A, Ostini L (2009) GNSS processing at CODE: status report. J Geodesy 83(3-4):353-365. doi:10.1007/s00190-008-0281-2

Dach R, Lutz S, Walser P, Fridez P (2015) Bernese GNSS Software Version 5.2. Astronomical Institute, University of Bern. doi:10.7892/boris.72297

Folkner WM, William JG, Boggs DH (eds) (2009) The planetary and lunar ephemeris DE 421. IPN Progress Report, 42-178, Jet Propulsion Laboratory

Gienger G, Pereira FL (2012) Towards Automated Determination of Orbit Maneuvers for GNSS Satellites. In: Conference on Dynamics and Control of Space Systems, Porto, Portugal, March. pp 131-150

Guo J (2014) The impacts of attitude, solar radiation and function model on precise orbit determination for GNSS satellites. Dissertation (in Chinese with English abstract). Dissertation, Wuhan University

Guo J, Xu X, Zhao Q, Liu J (2016) Precise orbit determination for quad-constellation satellites at Wuhan University: strategy, result validation, and comparison. J Geodesy 90(2):143-159. doi:10.1007/s00190-015-0862-9

Huang G, Qin Z, Zhang Q, Wang L, Yan X, Fan L, Wang X (2017) A Real-Time Robust Method to Detect BeiDou GEO/IGSO Orbital Maneuvers. Sensors (Basel) 17(12). doi:10.3390/s17122761

Huang Y, Hu X, Huang C, Yang Q, Jiao W (2009) Precise orbit determination of a maneuvered GEO satellite using CAPS ranging data. Science in China Series G: Physics, Mechanics and Astronomy 52(3):346-352. doi:10.1007/s11433-009-0052-y

Hugentobler U, Ploner M, Schildknecht T, Beutler G (1999) Determination of resonant geopotential terms using optical observations of geostationary satellites. Adv Space Res 23(4):767-770. doi:10.1016/S0273-1177(99)00153-2

Ju B, Gu D, Herring TA, Allende-Alba G, Montenbruck O, Wang Z (2017) Precise orbit and baseline determination for maneuvering low earth orbiters. GPS Solut 21(1):53-64. doi:10.1007/s10291-015-0505-x

Kelecy T, Hall D, Hamada K, Stocker MD (2007) Satellite Maneuver Detection Using Two-line Element (TLE) Data. In: Proceedings of the Advanced Maui Optical and Space Surveillance Technologies Conference, Wailea, Maui, Hawaii.

Li X, Huang Y, Guo R, Su R, Geng K (2015) A new dynamic orbit determination method for GEO satellite after orbit maneuver. In: Mechanics and Mechatronics (ICMM2015). pp 847-855. doi:10.1142/9789814699143_0105

Montenbruck O, Schmid R, Mercier F, Steigenberger P, Noll C, Fatkulin R, Kogure S, Ganeshan AS (2015) GNSS satellite geometry and attitude models. Adv Space Res 56(6):1015-1029. doi:10.1016/j.asr.2015.06.019

Montenbruck O et al. (2017) The Multi-GNSS Experiment (MGEX) of the International GNSS Service (IGS) – Achievements, prospects and challenges. Adv Space Res 59(7):1671-1697. doi:10.1016/j.asr.2017.01.011

Patera RP (2008) Space Event Detection Method. J Spacecraft Rockets 45(3):554-559. doi:10.2514/1.30348

Petit G, Luzum B (2010) IERS Conventions (2010). IERS Technical Note 36. Verlagdes Bundesamts für Kartographie und Geodäsie, Frankfurt am Main, Germany

Prange L, Orliac E, Dach R, Arnold D, Beutler G, Schaer S, Jäggi A (2016) CODE's five-system orbit and clock solution—the challenges of multi-GNSS data analysis. J Geodesy 91(4):345-360. doi:10.1007/s00190-016-0968-8

Qiao J, Chen W (2015) BeiDou satellites maneuvers detection for precise orbit determination. In: 26th IUGG General Assembly, Prague, Czech Republic, July 2015.p Presentation

Qiao J, Chen W (2018) Beidou satellite maneuver thrust force estimation for precise orbit determination. GPS Solut 22(2). doi:10.1007/s10291-018-0705-2

Villiger A, Dach R (eds) (2018) International GNSS Service Technical Report 2017 (IGS Annual Report). doi:10.7892/boris.116377

Xie J, Wang J, Mi H (2012) Analysis of Beidou Navigation Satellites In-orbit State. In: Sun J, Liu J, Yang Y, Fan S (eds) Proceedings of China Satellite Navigation Conference (CSNC) 2012. Springer, Berlin, Heidelberg, pp 111-122. doi:10.1007/978-3-642-29193-7_10

Z. Deng, M. Ge, M. Uhlemann, Zhao Q (2014) Precise orbit determination of Beidou Satellites at GFZ. In: Proceedings of IGS Workshop, Pasadena, USA, 23-27 June.

Zhao QL, Guo J, Li M, Qu LZ, Hu ZG, Shi C, Liu JN (2013) Initial results of precise orbit and clock determination for COMPASS navigation satellite system. J Geodesy 87(5):475-486. doi:10.1007/s00190-013-0622-7

Author Biographies

Jing Qiao received her Ph.D. degree from Hong Kong Polytechnic University in 2018. She received her B. S. degree in Geodesy from Wuhan University in 2013. Her research interests include satellite autonomous navigation, Beidou satellite maneuver detection, precise orbit determination and its application.

Wu Chen is Professor at Department of Land Surveying and Geo-Informatics, Hong Kong Polytechnic University. Prof Chen has been actively working on GNSS related research for more than 30 years. His main research interests are GNSS positioning quality evaluation, system integrity, various GNSS applications, seamless positioning and SLAM.