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Partial GNSS ambiguity resolution in coordinate domain

Abstract Traditionally, if full ambiguity resolution is not successful, partial ambiguity resolution will be tried. However, identifying which subset of ambiguities to fix is not easy and is still an open problem. Since the actual purpose of most applications is positioning, rather than fixing all or part of the ambiguities, in this research, we are trying to bypass the problem of identifying which subset of ambiguities to fix and provide a partial solution in the coordinate domain for the bias-free case. The basic idea is that with a user-defined failure rate, we can find a group of ambiguity candidates and each will provide one position. The partial solution is constructed based on these positions together with an indicator to show its maximum positioning error with user defined reliability. In order to meet various user requirements, different kinds of partial solutions in coordinate domain are proposed. Different from the traditional partial ambiguity resolution methods, the new method still works with all the ambiguities (i.e. the complete vector), but works with the different possible values that the complete ambiguity vector may take. The validness and applicability of the proposed partial solution are demonstrated based practical BeiDou triple-frequency observations. Numerical results show that some partial solutions can be more accurate, while others can meet higher reliability or integrity requirement.

Keywords: GNSS, RTK, partial ambiguity resolution, partial solution in coordinate domain, ambiguity validation

Introduction

Global navigation satellite systems (GNSS) are navigation systems that provide space-based positioning, navigation and timing (PNT) service in all weather conditions, anywhere on or near the Earth (Leick 2004). GNSS provide two common types of measurements: pseudo-range and carrier phase. These measurements enable the determination of the ranges between the receiver antenna and the satellites. The carrier phase based positioning results in more precise range than those from pseudo-range, if the carrier phase ambiguity can be reliably resolved (Han and Rizos 1999).

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Ambiguity resolution is an indispensable step for all Real-Time Kinematic (RTK) positioning related applications, and it is one of the most challenging problems in GNSS research. Traditionally, if the resolution of all ambiguities is not successful, partial ambiguity resolution (PAR) will be tried and if the latter also failed, float ambiguity solution will be used for positioning and navigation.

With launching of more and more GNSS systems, PAR will become more and more popular and important for RTK related applications, especially for long-distance ones. The reason is that with so many satellites, fixing only part of all ambiguities or ambiguities of part of all satellites is enough to meet the user required accuracy for most of time. Besides, not all carrier phase measurements have same data quality, as there are different GNSS systems, one GNSS system may have different kinds of satellites, such as BeiDou, same kind of satellites may have different observation environments, such as different elevation angle, and even one satellite generally has observations of different frequency bands, all these differences may lead to different data quality. Some ambiguities of carrier phase measurements with good data quality may be easy to fix, while others may be difficult to fix. If fixing only part of all ambiguities is enough to meet the user required accuracy, it is not necessary to wait more time to fix all. Hence, the research on PAR will become more and more important in order to improve the efficiency of precise navigation and positioning applications.

The first PAR method was introduced as early as in 1999 (Teunissen et al, 1999), which was proposed for the ionosphere-weighted geometry-free GPS model. Since then, a lot of research works have been done on PAR research. Some tried to fix only the (extra) wide-lane ambiguities in case two or more frequencies are being used (Cao et al, 2007; Li, 2010; Li J et al, 2015; Li B et al, 2015; Li et al, 2017). Some are to include only ambiguities with variance below a certain level (Teunissen et al, 1999; Gao and Shen, 2001; Takasu and Yasuda 2010), or ambiguities from satellites at a minimum elevation or with higher elevation (Li et al, 2015; Takasu and Yasuda 2010), with a minimum required signal-to-noise ratio, or which are visible for a certain time (Parkins, 2011). Some fix only (linear combinations of) ambiguities for which the best and second-best solutions are consistent (Lawrence, 2009). Others proposed a generalized version of the class of Integer Aperture estimators for PAR (Brack and Günther, 2014; Brack and Günther, 2015). In all the above mentioned research, bias-free case is assumed. In other words, when applying an ambiguity acceptance test, the underlying assumption is that the used mathematical model (including both functional and stochastic components) is correct. As this will not always be the case, so biasd cases of PAR are also studied in past research works (Henkel and Günther, 2010; Parkins, 2011; Verhagen et al, 2012; Wang and Feng, 2013; Li et al, 2014). Some proposed partial integer decorrelation in the presence of large biases (Henkel and Günther, 2010; Wang and Feng, 2013). Some proposed an iterative procedure in which many different subsets are evaluated (Parkins, 2011; Wang and Feng, 2013). However for both of bias-free and biased cases, the problem of identifying which subset of ambiguities to fix is not easy and it is still an open problem.

Practically, for most precise positioning applications, ambiguity resolution is not our final objective, and our true purpose is to provide user position as precise as possible. In this research, we are trying to bypass the PAR problem and provide a user position directly, which will be called a partial solution in the coordinate domain. Different from the traditional partial ambiguity resolution methods, the new method still works with all the ambiguities (i.e. the complete vector), but works with the different possible values that the complete ambiguity vector may take.

This research is targeting to the bias-free case. First, the basic idea of the proposed partial solution in the coordinate domain is described in details. And then, the numerical results are presented to demonstrate the validness of the proposed method and also compared to those of traditional PAR methods.

Partial solution in the coordinate domain

The following are the general form of linear observation equations of ambiguity resolution with the variance-covariance matrix $\boldsymbol{Q}$:

$$AX + BN = L \tag{1}$$

where, $\boldsymbol{L}$ is the observation vector; $\boldsymbol{N}$ is integer carrier phase ambiguity vector; $\boldsymbol{X}$ is the real-valued vector of the other unknown parameters; the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are the corresponding design matrices.

By simply disregarding the integer constraints on the ambiguity vector and performing a least-squares adjustment, the real-valued estimates of $\boldsymbol{X}$ and $\boldsymbol{N}$ can be derived as follows:

$$\begin{bmatrix} \hat{X} \\ \hat{N} \end{bmatrix} = \left([A \quad B]^T Q^{-1} [A \quad B] \right)^{-1} [A \quad B]^T Q^{-1} L \tag{2}$$

$\begin{bmatrix}\hat{X}\\ \hat{N}\end{bmatrix}$ is referred to as the float solution. With LAMBDA (Teunissen, 1995) search method, we can get the integer ambiguity resolution $\check{\boldsymbol{N}}$ with $\left\|\hat{N}-\check{N}\right\|^2_{Q_{\hat{N}\hat{N}}} = \left(\hat{N}-\check{N}\right)^T Q^{-1}_{\hat{N}\hat{N}}\left(\hat{N}-\check{N}\right) = min$. $\check{\boldsymbol{N}}$ should be validated and if it meets the user reliability requirement, it will be fixed. Otherwise, a partial solution will be considered.

Proposed partial solution in coordinate domain

In the ambiguity space, there are countless number of integer ambiguity candidates, denoted as $\check{\mathbf{N}}_{\mathbf{1}}$, $\check{\mathbf{N}}_{\mathbf{2}}$, …, $\check{\mathbf{N}}_{\mathbf{k}}$, … with probabilities $\mathbf{p_1}$, $\mathbf{p_2}$, …, $\mathbf{p_k}$, … from high to low. Define event A as that the correct ambiguity resolution $\tilde{\mathbf{N}}$ is one of $\check{\mathbf{N}}_{\mathbf{1}}$, $\check{\mathbf{N}}_{\mathbf{2}}$, …, $\check{\mathbf{N}}_{\mathbf{k}}$, and suppose $\sum_{\mathrm{i=k+1}}^{+\infty} \mathrm{p_i} < \mathrm{p_u}$ ($\mathbf{p_u}$ user-defined failure rate), we can say that the event A is true meets the user reliability requirement.

Denoting $\mathbf{x_1}$, $\mathbf{x_2}$, …, $\mathbf{x_k}$ as the derived user position corresponding to $\check{\mathbf{N}}_{\mathbf{1}}$, $\check{\mathbf{N}}_{\mathbf{2}}$, …, $\check{\mathbf{N}}_{\mathbf{k}}$, as $\tilde{\mathbf{N}}$ is one of $\check{\mathbf{N}}_{\mathbf{1}}$, $\check{\mathbf{N}}_{\mathbf{2}}$, …, $\check{\mathbf{N}}_{\mathbf{k}}$, so the correct position corresponding to $\tilde{\mathbf{N}}$ is also among $\mathbf{x_1}$, $\mathbf{x_2}$, …$\mathbf{x_k}$ with user required reliability. Taking Figure 1 as an example, there are four possible positions which meet the user reliability requirement. For bias-free case, the red one is $\mathbf{x_1}$ (best) which generally has much bigger probability than the others, for example larger than 90%. The blue ones are $\mathbf{x_2}$, $\mathbf{x_3}$, $\mathbf{x_4}$ which have generally much smaller probability than the red one. The distances between these positions can show the position sensitivity for different ambiguity candidates. If a solution is constructed based on these four positions, the maximum distance from the constructed solution to these four positions can reflect the possible positioning error (or bias) in some sense with the user reliability requirement. Of course this maximum distance is closely related to the user defined failure rate. The smaller the failure rate, the bigger the maximum distance and vice versa.

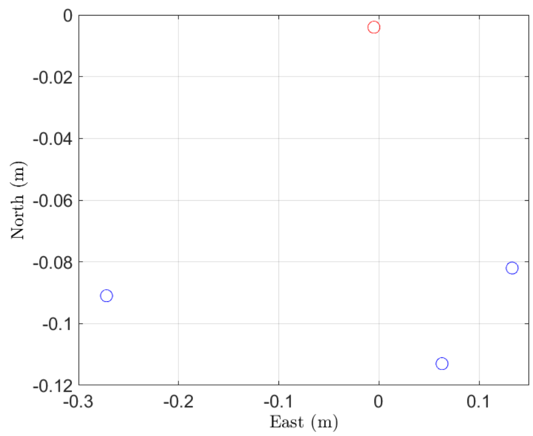

Fig 1 Positions corresponding to four ambiguity candidates meeting the user reliability requirement

Based on the positions $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, …, $\boldsymbol{x_k}$, the following partial solutions in coordinate domain are proposed:

1) Use $\boldsymbol{x_1}$ (Best) directly

As above mentioned, when the float ambiguity solution is unbiased, the position $\boldsymbol{x_1}$ generally has much bigger probability to be correct than the others, so in practical applications, if allowed, it is better to use $\boldsymbol{x_1}$ as the final solution as much as possible. In order to avoid big positioning error, the following is suggested to be used together:

$$\mathrm{Max\_D_1} = \mathrm{Max}(\|\mathrm{x_1} - \mathrm{x_i}\|)\ (\mathrm{i} = 2 \dots, \mathrm{k})$$

In figure 1, $\mathrm{Max\_D_1}$ is the maximum one among the three distances from $\mathrm{x_1}$ (on the top) to $\mathbf{x_2}$, $\mathbf{x_3}$, $\mathbf{x_4}$ (below). In some sense, $\mathrm{Max\_D_1}$ can be regarded as an indicator of the maximum possible positioning error (bias) of $\boldsymbol{x_1}$ with user defined reliability and it should meet the user requirement when using $\boldsymbol{x_1}$.

2) Use $\boldsymbol{x_{Mid}}$ (Middle) which fulfills the following conditions:

$\mathrm{Min}(\mathrm{Max}(\|x - x_i\|))$ ($i = 1,2 \dots, k$ and x is any point in user space)

$\boldsymbol{x_{Mid}}$ (the middle point among the below three ones shown in the Figure 2) is a position whose maximum distance to $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, …, $\boldsymbol{x_k}$ is the minimum. If $k = 2$, $\boldsymbol{x_{Mid}}$ will be the middle point of $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$.

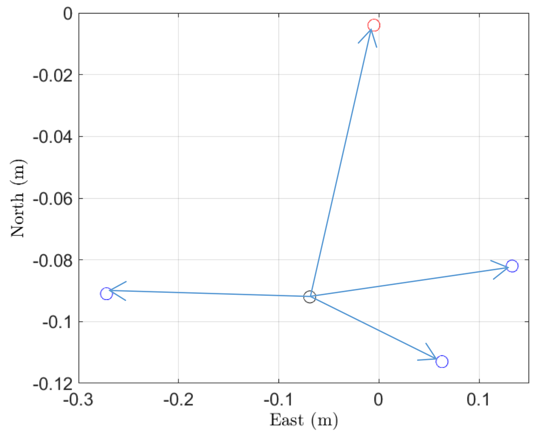

Fig. 2 x_{Mid} position (black point)

The following indicator is also suggested to be used together:

$$\text{Max\_D}_{Mid} = \text{Max}(\|x_{Mid} - x_i\|)\ (i = 1 \ldots, \text{k})$$

Max_D_{Mid} is the maximum distance from $\mathbf{x_{Mid}}$ to $\mathbf{x_1}$, x_2, …, $\mathbf{x_k}$ and it should also meet the user requirement. Different from $\mathbf{x_1}$ whose objective is to provide accurate solutions to the user as much as possible, the objective of $\mathbf{x_{Mid}}$ is to avoid big positioning bias as much as possible and provide better horizontal or 3-D protecting level of integrity in some sense.

3) Use the following weighted average solution (Average):

$$x_W = \frac{\sum_{i=1}^{k} p_i x_i}{\sum_{i=1}^{k} p_i}$$

It is a weighted average of the solutions $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, …, $\boldsymbol{x_k}$ based on the their probabilities, as shown in Figure 3.

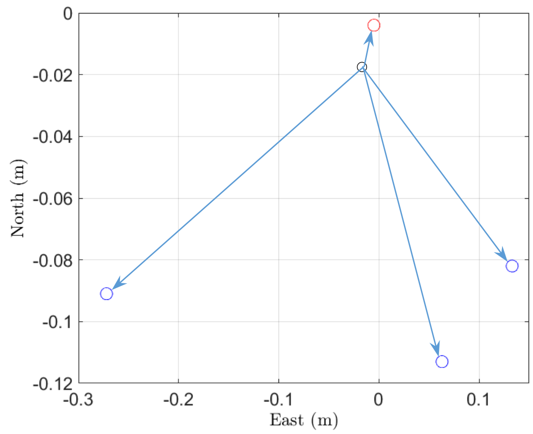

Fig. 3 x_W position (black point)

The following is also suggested to be used together:

$$\text{Max\_D}_W = \text{Max}(\|x_W - x_i\|)\ (i = 1, 2 \ldots, \text{k})$$

Max_D_W is the maximum distance from $\mathbf{x_W}$ to $\mathbf{x_1}$, $\mathbf{x_2}$, …, $\mathbf{x_k}$ and it should also meet the user requirement. Here, $\mathbf{x_W}$ is very similar to BIE-estimator (Teunissen 2003; Teunissen 2005c) which has the optimality property in both the ambiguity- and coordinate-domain of being the estimator with the smallest mean squared error (MSE) of all estimators and in some sense always outperforms the float ambiguity solution. The difference is that in the proposed $\mathbf{x_W}$, the number of used $\mathbf{x_1}$, $\mathbf{x_2}$, …, $\mathbf{x_k}$ are limited and confined only to those meeting user reliability requirement. So $\mathbf{x_W}$ can be regarded as an approximation to BIE-estimator.

4) Use the float solution $\hat{\boldsymbol{X}}$ directly

If none of the above three solutions meets the user requirement, the float solution $\hat{\boldsymbol{X}}$ is the final choice. In order to evaluate the maximum possible error of $\hat{\boldsymbol{X}}$, define $\text{Max\_D}_{float} = \text{Max}(\|\hat{X} - x_i\|)\ (i = 1 \ldots, \text{k})$ which is the maximum distance from the float coordinate solution $\hat{\boldsymbol{X}}$ to $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, …, $\boldsymbol{x_k}$. When using $\hat{\boldsymbol{X}}$, better make sure Max_D_{float} meets the user requirement. In fact, Max_D_{float} is always greater than Max_D_{Mid}, so according to this criteria, $\hat{\boldsymbol{X}}$ is not superior to $\boldsymbol{x_{Mid}}$.

Analysis and discussions

For some cases, especially short baseline, though the failure rate may not fulfill the user requirement, the success rate of the best ambiguity candidate can still be 90% or even 99%, much bigger than the others. In this case, if $\mathrm{Max\_D}_1$ fulfills the user requirement, the user may like to use $\boldsymbol{x_1}$ as the final solution as it may provide best accuracy with high probability.

Different from $\boldsymbol{x_1}$ (Best), the advantage of $\boldsymbol{x_{Mid}}$ (Middle) is to make the maximum possible positioning error as small as possible and it is more applicable to applications which have higher requirement on integrity. While $\boldsymbol{x_W}$ (Average) is something between $\boldsymbol{x_1}$ and $\boldsymbol{x_{Mid}}$. When the success rate of the best ambiguity candidate is much bigger than the others, $\boldsymbol{x_W}$ will be very close to $\boldsymbol{x_1}$. If all the ambiguity candidates have similar success rate, $\boldsymbol{x_W}$ will be close to $\boldsymbol{x_{Mid}}$.

For using each of these three solutions, it is important to know the corresponding maximum possible positioning errors as indicated by $\mathrm{Max\_D}_1$, $\mathrm{Max\_D}_{Mid}$ and $\mathrm{Max\_D}_{W}$. Though these three indicators are given in three dimensions (3D), they can also be constructed in 2D or even 1D form according to user’s practical needs.

Ambiguity validation method

In the above section, when we select the group of ambiguity candidates $\breve{\boldsymbol{N}}_{\boldsymbol{1}}$, $\breve{\boldsymbol{N}}_{\boldsymbol{2}}$, …, $\breve{\boldsymbol{N}}_{\boldsymbol{k}}$ and make sure that the correct solution is among this group with user defined reliability, it is in fact ambiguity validation, though it is not validation of one ambiguity candidate, it is validation of one group of ambiguity candidates.

In the past research, a lot of ambiguity validation methods have been proposed for full ambiguity resolution, such as Ratio tests (Euler and Schaffrin, 1991; Frei and Beuler, 1990; Han and Rizos, 1996), w-test (Wang et al, 1998), success rate (Teunissen, 1998; Teunissen, 1999;), Integer Aperture estimators (Teunissen, 2003; Teunissen, 2004a; Teunissen, 2004b; Teunissen, 2005a; Teunissen, 2005b; Verhagen, 2004), Ratio test combined with IA estimators (Ji et al, 2010; Li et al, 2014) etc. And theoretically, the proposed partial solution in this research can be used together with most of them. But for $\boldsymbol{x_W}$, none of them may be fit as they always provide a much bigger probability for the best ambiguity candidate than the others which make $\boldsymbol{x_W}$ always very close to $\boldsymbol{x_1}$ and meaningless. Thus, in this section, a new ambiguity validation method is proposed and used.

Take a two-dimensional case as example, in Figure 4, the point $\breve{\boldsymbol{N}}_{\boldsymbol{1}}$ is the correct integer ambiguity solution and the points $\breve{\boldsymbol{N}}_{\boldsymbol{2}}$, $\breve{\boldsymbol{N}}_{\boldsymbol{3}}$, … are other integer ones. As $\breve{\boldsymbol{N}}_{\boldsymbol{1}}$ is unknown, the best integer ambiguity candidate $\breve{\boldsymbol{N}}$ can be theoretically any point of $\breve{\boldsymbol{N}}_{\boldsymbol{1}}$, $\breve{\boldsymbol{N}}_{\boldsymbol{2}}$, $\breve{\boldsymbol{N}}_{\boldsymbol{3}}$, …. Correspondingly, the float ambiguity solution $\hat{\boldsymbol{N}}$ will be any point of $\hat{\boldsymbol{N}}_{\boldsymbol{1}}$, $\hat{\boldsymbol{N}}_{\boldsymbol{2}}$, $\hat{\boldsymbol{N}}_{\boldsymbol{3}}$, …, and $\hat{N}-\breve{N}=\hat{N}_1-\breve{N}_1=\hat{N}_2-\breve{N}_2=\cdots$. If we know the probabilities of $\hat{\boldsymbol{N}}$ falling on $\hat{N}_1$, $\hat{N}_2$, $\hat{N}_3$, … respectively, denoted as p_1, p_2, p_3, …, then the probability of $\hat{N}=\hat{N}_1$ will be $\frac{p_1}{\sum_{i=1}^{+\infty}p_i}$, which is also the probability of $\breve{N}=\breve{N}_1$. We can see that different from previous ones, it is a probability when we have already derived $\hat{\boldsymbol{N}}$, i.e., it is a conditional probability or posterior one.

In order to get the probability of $\hat{N}$ falling on the position $\hat{\boldsymbol{N}}_{\boldsymbol{1}}$, an area is constructed centered on $\hat{\boldsymbol{N}}_{\boldsymbol{1}}$, $\hat{\boldsymbol{N}}_{\boldsymbol{2}}$, $\hat{\boldsymbol{N}}_{\boldsymbol{3}}$, …respectively, with the same shape and size. The size of these areas is denoted as S which is small enough that the probability density of each point in the area can be regarded same and equal to that of the center. Define event A as $\hat{\boldsymbol{N}}$ falling in any of these areas, event B as $\hat{\boldsymbol{N}}$ falling in the area centered on $\hat{\boldsymbol{N}}_{\boldsymbol{1}}$ (correct one) and event C as $\hat{\boldsymbol{N}}$ falling in any of these areas except for the correct one. Then, the probability for the event A is $P(A)=\sum_{i=1}^{+\infty}f_iS$, for the event B, it is $P(B)=f_1S$ and for the event C, it is $P(C)=\sum_{i=2}^{+\infty}f_iS$. Here, f_i is the probability density of $\hat{N}_i$:

$$f_i=\frac{1}{\sqrt{det(Q_{\hat{N}\hat{N}})(2\pi)^{\frac{n}{2}}}}exp\{-\frac{1}{2}\left\|\hat{N}_i-\breve{N}_1\right\|_{Q_{\hat{N}\hat{N}}}^2\}=\frac{1}{\sqrt{det(Q_{\hat{N}\hat{N}})(2\pi)^{\frac{n}{2}}}}exp\{-\frac{1}{2}\left\|\hat{N}_1-\breve{N}_1+\breve{N}_i-\breve{N}_1\right\|_{Q_{\hat{N}\hat{N}}}^2\}=\frac{1}{\sqrt{det(Q_{\hat{N}\hat{N}})(2\pi)^{\frac{n}{2}}}}exp\{-\frac{1}{2}\left\|\Delta N+\breve{N}_i-\breve{N}_1\right\|_{Q_{\hat{N}\hat{N}}}^2\} \quad (5)$$

According to Bayes theorem, then the conditional success rate of ambiguity resolution is:

$$P(B|A)=\frac{f_1S}{\sum_{i=1}^{+\infty}f_iS}=\frac{f_1}{\sum_{i=1}^{+\infty}f_i} \quad (6)$$

and the conditional failure rate of ambiguity resolution is:

$$P(C|A)=\frac{\sum_{i=2}^{+\infty}f_iS}{\sum_{i=1}^{+\infty}f_iS}=\frac{\sum_{i=2}^{+\infty}f_i}{\sum_{i=1}^{+\infty}f_i} \quad (7)$$

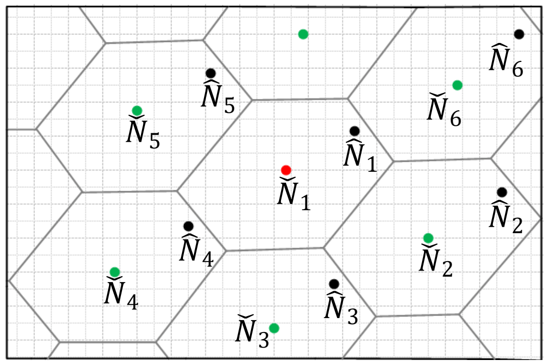

Fig. 4 2-D example of ambiguity resolution

As $\breve{\boldsymbol{N}}_{\boldsymbol{1}}$, $\breve{\boldsymbol{N}}_{\boldsymbol{2}}$, $\breve{\boldsymbol{N}}_{\boldsymbol{3}}$, …and $\hat{\boldsymbol{N}}_{\boldsymbol{1}}$, $\hat{\boldsymbol{N}}_{\boldsymbol{2}}$, $\hat{\boldsymbol{N}}_{\boldsymbol{3}}$, … are infinite, the summation in (6) and (7) cannot be made over all of them. The contribution of some of them obviously can be neglected. In this research, the following formula is used to determine the neglected ones:

$$\frac{f_i}{f_1} < \gamma \tag{8}$$

where γ is the critical value of the required failure rate of the practical applications.

For validation of a group of ambiguity candidates, define Event D as that the correct ambiguity resolution is one of $\boldsymbol{N}_{\boldsymbol{1}}$, $\boldsymbol{N}_{\boldsymbol{2}}$, …, $\boldsymbol{N}_{\boldsymbol{k}}$, then the probability of Event D is true will be:

$$P(D) = \frac{\sum_{i=1}^{\mathrm{k}} f_i}{\sum_{i=1}^{+\infty} f_i} \tag{9}$$

It can be used together with the partial solution proposed in this research.

We can see that unlike IA estimator, when two ambiguity candidates are almost equally close to the float ambiguity solution, the new method will derive two similar probabilities for these two candidates. Hence, it is more applicable to the partial solutions proposed in this research and also more applicable to practical applications.

Numerical results

As the proposed partial solution in the coordinate domain is only applicable to the bias-free case, in order to fully demonstrate its validness, a zero baseline is taken advantage of due to its easy data quality control. The zero baseline is formed by the two stations CUT0 and CUT2 in Curtin

University, the two receivers are all Trimble NetR9, and the shared antenna is TRM59800.00/SCIS. The observations are collected on February 08, 2017 at a sampling rate of 30 seconds and only triple-frequency observations of BeiDou are used in this research.

Data quality control and mathematical model

For zero baselines, the data quality of each type of observations can be easily controlled, since, after double differencing, the observations only contains observation noise, which is important to establish the correct stochastic model for ambiguity validation. For example, Figures 5 and 6 show the noise of the double differenced pseudorange and carrier phase observations between BD satellites PRN 01 and 02. From Figure 5, we can see that the observation noise of C3 (code type corresponding to B3) is obviously smaller than the other two, while for carrier phase observations, the observation noise levels are similar for all three frequency bands. The standard deviations for C1, C2 and C3 are 0.309 m, 0.279 m and 0.128 m respectively and for B1, B2 and B3, they are 2.8mm, 2.9mm and 2.8 mm respectively.

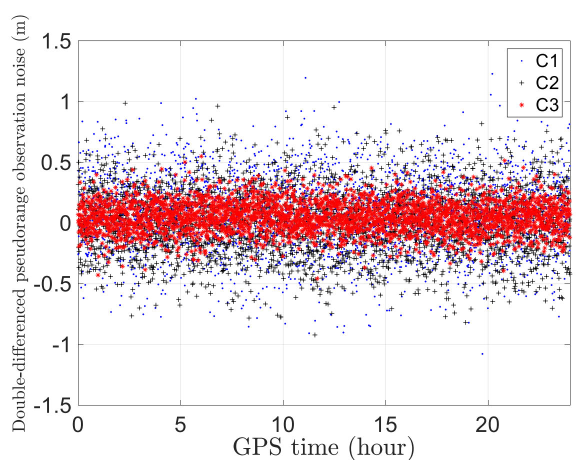

Fig. 5 Pseudorange observation noise (differenced between PRN 01 and 02)

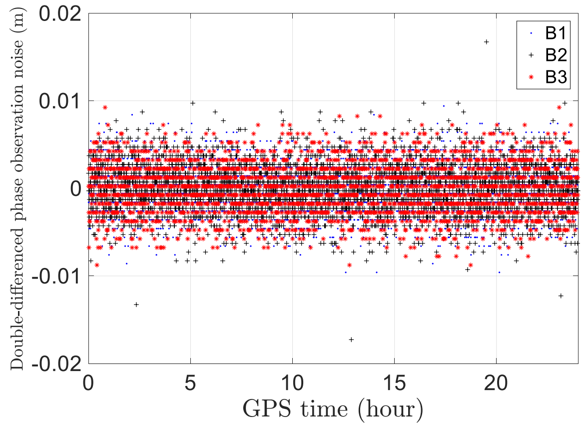

Fig. 6 Carrier phase observation noise (differenced between PRN 01 and 02)

As for short baseline, full ambiguity resolution is generally successful and PAR is generally not necessary, so in order to fully demonstrate the validness of the proposed partial solutions, the zero double difference baseline is regarded as a long-distance one. Otherwise, we can only get the full ambiguity resolutions. Taken one pair of satellites as an example, the double-differenced observation equations between satellites p and q are as follows:

$$\begin{cases} a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z + \Delta\nabla I + m^{pq}\Delta Z_{Tr} = \Delta\nabla L_{C1}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z + \frac{f_2^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} = \Delta\nabla L_{C2}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z + \frac{f_3^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} = \Delta\nabla L_{C3}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z - \Delta\nabla I + m^{pq}\Delta Z_{Tr} + \lambda_1\Delta\nabla N_1^{pq} = \Delta\nabla L_{B1}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z - \frac{f_2^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} + \lambda_2\Delta\nabla N_2^{pq} = \Delta\nabla L_{B2}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z - \frac{f_3^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} + \lambda_3\Delta\nabla N_3^{pq} = \Delta\nabla L_{B3}^{pq} \end{cases} \tag{10}$$

In which, Δx, Δy & Δz are the coordinate parameters and a^{pq}, b^{pq} & c^{pq} are the corresponding coefficients; $\Delta\nabla I$ is the double-differenced ionospheric delay parameter, f_1, f_2 & f_3 are frequencies corresponding to the frequency bands B_1, B_2 & B_3; ΔZ_{Tr} is the single-differenced zenith tropospheric delay between the two stations and m^{pq} is the single-differenced mapping function between satellites p & q, and in this research, Neill mapping function is used; $\Delta\nabla N_1^{pq}$, $\Delta\nabla N_2^{pq}$ & $\Delta\nabla N_3^{pq}$ are double-differenced ambiguity parameters of the frequency bands B_1, B_2 & B_3 and λ_1, λ_2 & λ_3 are corresponding wavelengths; $\Delta\nabla L_{C1}^{pq}$, $\Delta\nabla L_{C2}^{pq}$, $\Delta\nabla L_{C3}^{pq}$, $\Delta\nabla L_{B1}^{pq}$, $\Delta\nabla L_{B2}^{pq}$ &

$\Delta\nabla L_{B3}^{pq}$ are double-differenced observation vector corresponding to code types C1, C2 & C3 and carrier phase types B1, B2 & B3.

When forming double-differenced observation equations, the geostationary BeiDou satellite with PRN 03 is used as the reference satellite as it has the highest elevation angle among all observed BeiDou geostationary satellites. The ionospheric delay parameters are different every epoch and will be eliminated from the equations before using sequential least-squares adjustment to get the float coordinate and ambiguity solutions. Based on the float ambiguity solution, LAMBDA method will be used to search the group of possible ambiguity candidates which meet the failure rate 0.1% requirement.

In this research, the ambiguity resolution performance is investigated with only 5 minute observations (10 epochs), i.e., the 24-hour observations are processed every 5 minutes starting from the first epoch, second epoch … respectively. And the cutoff elevation angle is set to 15^{o} .

Analysis and discussion of numerical results

For about 70% cases, full ambiguity solution can be acquired, the number of partial solution cases is 789 and are indexed as 1, 2, 3… chronically. Figures 7 and 8 show the horizontal distance errors (Abbreviated as H error) and the maximum horizontal distance errors (Abbreviated as MaxH error) of the three partial solutions $\boldsymbol{x_1}$ (Best), $\boldsymbol{x_{Mid}}$ (Middle) and $\boldsymbol{x_W}$ (Average).

From Figures 7 and 8, we can see that the results of $\boldsymbol{x_1}$ (Best) and $\boldsymbol{x_W}$ (Average) are very similar except for a few cases, while they are very different from that of $\boldsymbol{x_{Mid}}$ (Middle). For H errors of $\boldsymbol{x_1}$ and $\boldsymbol{x_W}$, most of them are very small and only 1-2 cm, while for $\boldsymbol{x_{Mid}}$, most of them are bigger than 5 cm. But for MaxH errors, generally the results of $\boldsymbol{x_{Mid}}$ is smaller than that of $\boldsymbol{x_1}$ and $\boldsymbol{x_W}$.

Table 1 and 2 are the statistical results of the H and MaxH errors. From these two tables, we can see that $\boldsymbol{x_1}$ and $\boldsymbol{x_W}$ are only slightly different. For H errors less than 5 cm, the difference is less than 3%, and for that less than 2 dm, 5 dm, 1 m and 1.5 m, the difference is less than 1%. For MaxH errors, they are also very similar and the maximum difference is only 1.6% .

While when comparing $\boldsymbol{x_{Mid}}$ to $\boldsymbol{x_1}$ or $\boldsymbol{x_W}$, we can see that the difference is obvious. For H errors less than 5 cm, it is only 15.7% for $\boldsymbol{x_{Mid}}$, while for $\boldsymbol{x_1}$ or $\boldsymbol{x_W}$, it is more than 80%. For H errors less than 2 dm, it is only 76% for $\boldsymbol{x_{Mid}}$, while for $\boldsymbol{x_1}$ or $\boldsymbol{x_W}$, it is more than 90%. For MaxH errors less than 2 dm, it is 53.5% for $\boldsymbol{x_{Mid}}$, while for $\boldsymbol{x_1}$ or $\boldsymbol{x_W}$, it is less than 15%. For MaxH errors less than 5 dm, it is 95.6% for $\boldsymbol{x_{Mid}}$, while for $\boldsymbol{x_1}$ or $\boldsymbol{x_W}$, it is less than 90%.

From the comparison, we can say that $\boldsymbol{x_1}$ or $\boldsymbol{x_W}$ are more applicable to the users which have higher requirement on accuracy and they can benefit to precise positioning and navigation, but the prerequisite is that the maximum possible error $\mathrm{Max\_D_1}$ should also meet the user requirements. While $\boldsymbol{x_{Mid}}$ is more applicable to the users which have higher requirement on integrity.

What will happen if we combine $\mathbf{x_1}$ with $\mathbf{x_{Mid}}$? For example, if the maxH of $\mathbf{x_1}$ is less than user defined value, $\mathbf{x_1}$ is used, otherwise, $\mathrm{x_{Mid}}$ is used. The resulting horizontal distance errors are summarized and shown in Table 3. Compared to that of $\mathbf{x_{Mid}}$, we can see that the most affected is the positioning distance error falling within 5 cm. When the allowed maxH of $\mathbf{x_1}$ is set to 0.1 dm, the performance is same to that of $\mathbf{x_{Mid}}$, while when the allowed maxH increases to 0.2 dm, 0.3 dm and 0.4 dm, the percent of the positioning distance errors falling within 5 cm also increases to 24.1%, 56.2% and 68.1% dramatically, and the latter two values are much bigger than of $\mathbf{x_{Mid}}$. We can see that with the proposed partial solutions, we may on the one hand, control the maximum possible distance error, and on the other hand, improve the positioning accuracy as much as possible.

The proposed partial solutions in coordinate domain are also compared to the traditional partial ambiguity resolution. The subset of the ambiguities to be fixed is by selecting those shared by or consistent among $\breve{\mathbf{N}}_\mathbf{1}$, $\breve{\mathbf{N}}_\mathbf{2}$, …, $\breve{\mathbf{N}}_\mathbf{k}$. The method is similar to the methods proposed in the past research (Lawrence, 2009) and also a little similar to the selection method of the generalized integer aperture estimation (Brack and Günther, 2014) but with different validation test.

The used mathematical model is a little different from Equation (10) and the ambiguity parameters are those of the combinations listed in Table 4 and B2, as compared to the others, they have the biggest ratio of wavelength to noise and are independent. The relation between these ambiguity parameters and the original ones is, for satellites p and q , $\begin{bmatrix}\Delta\nabla N_{EW}^{pq}\\ \Delta\nabla N_{W}^{pq}\\ \Delta\nabla N_{2}^{pq}\end{bmatrix} = \begin{bmatrix}0 & 1 & -1\\ 1 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\nabla N_{1}^{pq}\\ \Delta\nabla N_{2}^{pq}\\ \Delta\nabla N_{3}^{pq}\end{bmatrix}$ and $\begin{bmatrix}\Delta\nabla N_{1}^{pq}\\ \Delta\nabla N_{2}^{pq}\\ \Delta\nabla N_{3}^{pq}\end{bmatrix} = \begin{bmatrix}-1 & 1 & 1\\ 0 & 0 & 1\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta\nabla N_{EW}^{pq}\\ \Delta\nabla N_{W}^{pq}\\ \Delta\nabla N_{2}^{pq}\end{bmatrix}$, so Equation (10) becomes:

$$\begin{cases} a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z + \Delta\nabla I + m^{pq}\Delta Z_{Tr} = \Delta\nabla L_{C1}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z + \frac{f_2^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} = \Delta\nabla L_{C2}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z + \frac{f_3^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} = \Delta\nabla L_{C3}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z - \Delta\nabla I + m^{pq}\Delta Z_{Tr} - \lambda_1\Delta\nabla N_{EW}^{pq} + \lambda_1\Delta\nabla N_{W}^{pq} + \lambda_1\Delta\nabla N_{2}^{pq} = \Delta\nabla L_{B1}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z - \frac{f_2^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} + \lambda_2\Delta\nabla N_{2}^{pq} = \Delta\nabla L_{B2}^{pq} \\ a^{pq}\Delta x + b^{pq}\Delta y + c^{pq}\Delta z - \frac{f_3^2}{f_1^2}\Delta\nabla I + m^{pq}\Delta Z_{Tr} - \lambda_3\Delta\nabla N_{EW}^{pq} + \lambda_3\Delta\nabla N_{2}^{pq} = \Delta\nabla L_{B3}^{pq} \end{cases} \tag{11}$$

In Figure 7, the blue part is the horizontal positioning error with the above mentioned traditional partial ambiguity resolution method and the last row of Table 1 is the statistics of the results. Compared to the results of the partial solutions in coordinate domain, we can find that the results of the traditional ambiguity resolution method are similar to or slightly better than that of $\mathbf{x}_{\mathbf{Mid}}$. And the positioning distance error falling within 5 cm is much smaller than that of the others.

Table 1 Statistics of horizontal positioning distance (H) errors (%)

	<5 cm	<2dm	<5dm	<1m	<1.5 m
Best x_1	87.6	90.2	95.3	99.5	100
Average x_W	84.9	90.7	95.7	99.8	100
Middle x_{Mid}	15.7	76.0	95.6	99.6	100
Partial ambiguity solution	23.8	86.9	96.3	99.7	100

Table 2 Statistics of maximum horizontal positioning distance (MaxH) errors (%)

	<5 cm	<2 dm	<5 dm	<1 m	<1.5 m
Best	1.1	13.4	86.2	99.4	100
Average	1.1	14.3	87.8	99.6	100
Middle	1.6	53.5	95.6	100	100

Table 3 Statistics of horizontal positioning distance (H) errors (%) with combined x_1 and x_{Mid}

MaxH (dm)	<5 cm	<2dm	<5dm	<1m	<1.5 m
1	15.7	76.1	96.6	99.6	100
2	24.1	76.1	96.6	99.6	100
3	56.2	76.1	96.6	99.6	100
4	68.1	76.8	96.5	99.6	100
5	78.6	84.4	96.6	99.8	100

Table 4 Combinations used in the mathematical model of the traditional PAR

	B1	B2	B3
extra-wide	0	1	-1
wide	1	0	-1

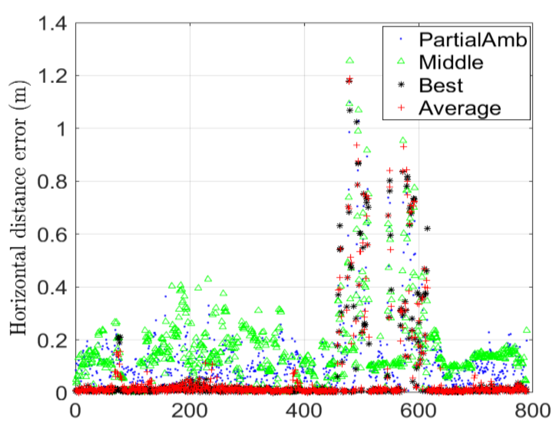

Fig. 7 Horizontal distance (H) errors of partial solutions in coordinate domain

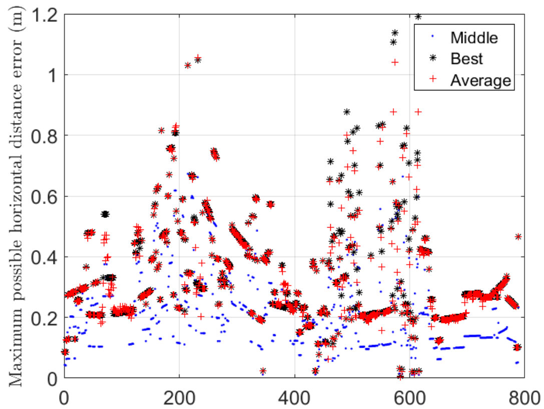

Fig. 8 Maximum possible horizontal (MaxH) errors of partial solutions in coordinate domain

Conclusions

In this research, we are trying to bypass the traditional PAR problem as identifying which subset of ambiguities to fix is not easy. Taking advantage of the position sensitivity of the possible ambiguity candidates, three partial solutions $\mathbf{x_1}$ (Best), $\mathbf{x_W}$ (Average) and $\mathbf{x_{Mid}}$ (Middle) in coordinate domains are proposed and each with an indicator of the maximum possible positioning error which is important for user to use these partial solutions and closely related to the user reliability requirement.

Each of these three partial solutions has its own characteristics. $\mathbf{x_1}$ and $\mathbf{x_W}$ are more applicable to the users which have higher requirement on accuracy, while $\mathbf{x_{Mid}}$ is more applicable to the users which have higher requirement on integrity. Hence, the proposed partial solutions can provide more choices to user and let them choose the suitable solution according to their practical needs. In addition, users can also construct the three indicators in 2D or even 1D form which is considered to be best suitable for their needs.

With these partial solutions and their indicators, we can on the one hand control the maximum possible positioning error and on the other hand, improve the positioning accuracy as much as possible. Compared to the traditional partial ambiguity resolution, numerical results with combined $\mathbf{x_1}$ and $\mathbf{x_{Mid}}$ show that the positioning performance in terms of accuracy is better than or at worst similar to that of the traditional partial ambiguity resolution.

The proposed partial solution in the coordinate domain is only applicable to the bias-free case. If available, bias should be detected and removed or properly weighted before deriving the proposed partial solutions. Though it is suggested that the proposed partial solutions had better be used with a new ambiguity validation method proposed in this research, in fact, they can be used with most previously proposed ambiguity validation methods, such as integer aperture estimators, but we should notice that $\boldsymbol{x}_{W}$ will become meaningless.

Though the proposed method is validated based on zero baseline observations, it is designed for non-zero baseline, especially for long baseline. However, for BeiDou observations of long baseline, multipath is an important and unavoidable error source which can affect the establishment of stochastic model and hence may affect the validation effect of the proposed method. But in future, validation of the proposed method based on observations of long baseline will be our further research work.

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