Postprocessing of the Linear Sampling Method in Inverse Electromagnetic Scattering Problem for Obstacles

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***Abstract*—The linear sampling method is known to be a simple and computationally efficient approach to retrieve the support of the scatterer using multistatic scattered field data. However, the recovered profile is always misleading, owing to the lack of robust edge detecting. This paper addresses this open issue. Using moving least square approximation, the upper and lower bounds of the profile of scatterers are pursued, and a sweeping process finds the optimal profile to match the scattered field data.**

***Index Terms*—Curvature, inverse scattering, linear sampling method, moving least square method.**

I. Introduction

The linear sampling method (LSM) could find the support of the scatterer without a priori information except its corresponding far field pattern of scattered electromagnetic wave. The LSM is based on the far field integral equation in the unknown $g(\varphi,\boldsymbol{r})$. Given the support of the scatterer, denoted as D, it has been proved that, $\|g(\varphi,\boldsymbol{r})\| \to \infty$ as $\boldsymbol{r} \to \partial D$ or $\boldsymbol{r} \in R^3\backslash D$ [1], where $\|\cdot\|$ is the norm operator and ∂D represents the profile of the domain D. Therefore, the singularities of $\|g(\varphi,\boldsymbol{r})\|$ are natural indicators of the profile of the scatterer, which will also impede the accurate numerical approximation of $g(\varphi,\boldsymbol{r})$ over the sampling points including D.

To overcome this difficulty, Tikhonov regularization is applied to solve the ill-posed linear system associated with the far field integral equation. The successful solving is at the cost of smoothing over $\|g(\varphi,\boldsymbol{r})\|$. This definitely eliminates the abrupt changes around the singularity as shown in Fig. 1. (*a*). It is believed that the contour line of $1/\| g(\varphi,\boldsymbol{r})\|$ still could extract the information of the profile of the scatterer. This gives rise to the problem of finding appropriate contour level to approximate ∂D. Many approaches were proposed, but they are either ad hoc or heuristic.

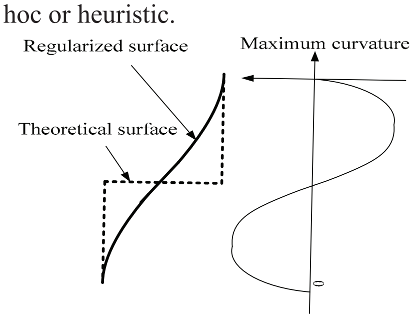

Fig. 1. The comparison between theoretical profile of $1/\|g(\varphi,\boldsymbol{r})\|$ and the regularized version (a) , the fault delineation given by maximum curvature (b).

II. The Proposed Postprocessing

This paper proposes a systematic approach to obtain the profile of the scatterer. Demonstrated in Fig.1 (*b*), the reconstructed profile of the indicator function $1/\|g(\varphi,\boldsymbol{r})\|$, thanks to the regularization, introduces fluctuations of the curvatures in the neighborhood of the singularity. It is clear that such artificial curvatures provide the upper and lower bounds of the location of singularities, i.e., the profile of the scatter. To capture the abrupt change of the surface associated with $1/\|g(\varphi,\boldsymbol{r})\|$, an adaptive moving least-square (AMLS) approximation is used [2], which could identify detailed features of the surface. On this basis, the average curvature along the contour line, K_a, could be computed as follows

$$K_a = \int_\Gamma K_m dl \Big/ \int_\Gamma dl \,, \tag{1}$$

where K_m is the maximal curvature at the points along the contour line Γ. Then the contour line with local maximum of K_a could be found, and they are the upper and lower bounds of the scatter. A sweeping process is thereby conducted: The contour lines between bounds are regarded as a potential profile of the scatter. The far field pattern of that profile can be computed via computational electromagnetics and compared with the available far field pattern. The one with minimal deviation is supposed to be the profile of the scatterer.

III. Numerical Examples

For the sake of simplicity, we consider two-dimensional scattering by a circular cylinder scalar electromagnetic scattering as shown in Fig. 2. The TM incident plane wave is presumed. As for more cases with arbitrary closed contour, they will be discussed in the full paper.

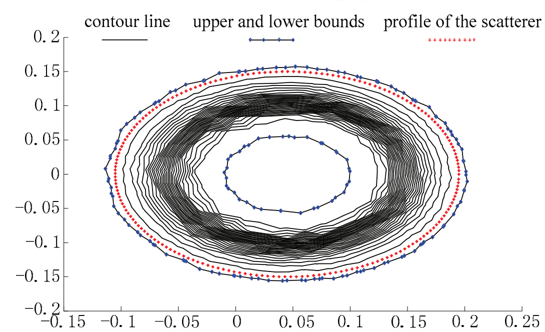

Fig. 2. The contour line plot obtained by using LSM and the contour levels achieving local maximum.

References

[1] F. Cakoni, D. Colton and P. Monk, *The Linear Sampling Method in Inverse Electromagnetic Scattering,* SIAM, 2011, pp. 41-43.

[2] Dey T K, Sun J., “An adaptive MLS surface for reconstruction with guarantees, ” *Symposium on Geometry processing*, 2005, pp. 43-52.