Fast Numerical Method for Computing Resonant Characteristics of Electromagnetic Devices based on Finite Element Method

Xiu Zhang[1,2], W. N. Fu[3] and S. X. Niu[3]

[1]College of Electronic and Communication Engineering, Tianjin Normal University, Tianjin, China
[2]Tianjin Key Laboratory of Wireless Mobile Communication and Wireless Power Transmission, Tianjin Normal University
[3]Department of Electrical Engineering, The Hong Kong Polytechnic University, Hong Kong, China.
zhang210@126.com eewnfu@polyu.edu.hk eesxniu@polyu.edu.hk

***Abstract*—The traditional method to obtain the response of electromagnetic devices over a wide frequency range is to use a frequency sweeping method. However, this method is very time consuming, especially in time-domain analysis. In order to save the heavy computational time, a novel fast algorithm based on finite-element method (FEM) to obtain the response under different frequencies is therefore proposed. With this proposed method, the FEM just needs to be solved once under a specific excitation function, and the frequency-domain responses at different frequencies are calculated using a simple integral formula. In this paper, the wireless power transfer system is used to showcase this method. The results indicate that this method can obtain the resonant frequency of the system precisely with significant savings in computation time.**

***Index Terms*—Finite element method, frequency sweeping, resonant frequency, time domain.**

I. Introduction

In the performance analysis of electromagnetic devices, the response needs to be calculated over a wide frequency range. For some electromagnetic devices such as micro-mechanical resonators and electric generation systems, it is very important to obtain their performance under different frequency excitations. To find the resonant devices' resonance characteristic, a traditional method is frequency sweeping [1]. In such method, the response of the electromagnetic devices will be computed under excitation with the frequency f using numerical methods. Then the response of devices will be computed repeatedly with the operating frequency f being changed. When the frequency changes N times, the devices' resonance characteristic can be found. The numerical method should be solved N times, which is very time consuming.

In this paper, a novel numerical method based on the time-domain analysis is proposed to obtain the devices' resonance characteristic. Compared with the frequency sweeping method, this method can save computational time significantly. In order to verify this method, the wireless power transfer (WPT) system is analyzed using FEM to obtain its resonant frequency.

II. Proposed Method

According to the time-domain convolution theory, the output response $y(t)$ ($t \geq 0$) of a linear, continuous time-invariant system with input $x(t)$ ($t \geq 0$) can be expressed as

$$y(t) = x(t) * y_\delta(t) = \int_0^\infty x(\tau) y_\delta(t-\tau) u(t-\tau) d\tau, \tag{1}$$

where, $y_\delta(t)$ is the impulse response of the system; $u(t - \tau)$ is the step function. If the input signals are sinusoidal function and cosinusoidal function, the outputs can be expressed as the following according to (1):

$$y_{\sin}(t) = \int_0^\infty \sin(\omega\tau) y_\delta(t-\tau) u(t-\tau) d\tau, \tag{2}$$

$$y_{\cos}(t) = \int_0^\infty \cos(\omega\tau) y_\delta(t-\tau) u(t-\tau) d\tau \cdot \tag{3}$$

If the input signal is

$$x_e(t) = u(t) - e^{-t}; \tag{4}$$

its output response is

$$y_e(t) + \frac{dy_e(t)}{dt} = y_u(t) + y_\delta(t) \cdot \tag{5}$$

If the input is $\sin(\omega t) + \cos(\omega t)/\omega$, its output is

$$\begin{aligned} y_{\sin}(t) + \frac{y_{\cos}(t)}{\omega} = \int_0^\infty \sin(\omega\tau)[y_e(t-\tau) + \frac{dy_e(t-\tau)}{dt}]u(t-\tau)d\tau \\ + \frac{\cos(\omega\tau)}{\omega} y_u(t-\tau)u(t-\tau)\Big|_{\tau=0}^{\tau=\infty} \end{aligned} \tag{6}$$

According to the operational rules of the trigonometric function, the above input can be deduced as

$$\sin(\omega t) + \frac{\cos(\omega t)}{\omega} = \sqrt{1 + \frac{1}{\omega^2}} \sin(\omega t + \varphi), \tag{7}$$

So, if the input signal is $\sin(\omega t + \varphi)$, its output response is

$$\begin{aligned} y(\omega t) = \frac{\omega}{\sqrt{\omega^2+1}} \int_0^\infty \sin(\omega\tau)[y_e(t-\tau) \\ + \frac{dy_e(t-\tau)}{dt}]u(t-\tau)d\tau + \frac{\cos(\omega\tau)}{\sqrt{\omega^2+1}} y_u(t-\tau)u(t-\tau)\Big|_{\tau=0}^{\tau=\infty} \end{aligned} \tag{8}$$

III. Initial Results

In order to verify the above proposed method, the WPT system is analyzed to obtain its performance. The simulation and experimental results are listed in Table I.

TABLE I Comperison Results of the WPT system

	Frequency Sweeping Method	Proposed Method	Experimental results	Error
Resonant frequency(MHz)	3.00	3.01	3.1	3%
Induced voltage in the receiver coil (V)	4.01	4.05	4.22	4%
Efficiency (%)	29.9	30.4	31.2	2.6%
Computational time	1*h*42*min*	5*min*32*s*	---	---

References

[1] S. E. Zhang, L. Mao, C. Ma. "Analysis and improvement of the frequency sweeping method for resonance characteristic measurement." *Antennas, Propagation & EM Theory (ISAPE), 2012 10th International Symposium on*, pp. 664-667, 2012.