

# A Two-degree of Freedom Spherical Motor

Y. Zou

K. W. Eric Cheng

Power Electronics Research Center, Department of Electrical Engineering,  
The Hong Kong Polytechnic University, Hong Kong.  
Email: y.zou@polyu.edu.hk eecheng@polyu.edu.hk

**Abstract**—In this paper, a direct-drive spherical motor to produce two-degree of freedom rotations has been proposed. The structure and prototype of the motor are given and the basic principles are described, including the operation principle and mathematic theories, etc. Importantly, the two air gaps between the mover and the rotor, the mover and the stator which mainly determine the performance of the motor are analyzed through the magnetic circuit method. Finite element analysis (FEA) is used to calculate some parameters of the motor and torque outputs from the two dimensions as well. The simulation results illuminate the effectiveness and feasibility of the motor proposed.

**Keywords**—Direct-drive, FEA, spherical motor, two-degree of freedom.

## I. INTRODUCTION

Motors are widely used by industrial applications and electrical devices such as electric vehicles. Likewise, they can also be employed by electric vessels propelled by two or three motors that can be controlled easily to realise fast mechanical responses [1]. However, traditional electricity-drive vessels usually employ several motors collaborated to adjust the directions by using complex immediate mechanical gears or their directions are manually controlled by a drone [2]. In this way, we find it difficult to automatically change the direction by employing one motor only for a vessel. Therefore, some motors and mechanical parts are used for vessels to realise an automatic drive and even an unmanned cruise. Since there are many mechanical devices employed by traditional electric vessels, we have to calibrate and maintain the vessel periodically. As a consequence, the whole cost of the driving system for the vessel will be increased. Also, the overall performances of these complicated mechanical motion systems are cumbersome and some mechanical components involved are vulnerable to hostile environmental in water, thus probably causing failures on the propulsion systems. Hence, employing one motor only to realise two dimensional rotations is of importance for the driving system of electric vessels, simplifying the whole structure and cutting costs. A new two-degree of freedom motor used for electric vessels is proposed in this paper.

Among electric actuators, direct-drive motors possess the characteristics of simple structure, robust control and easy regulation, etc. [3]. Due to the absence of intermediate mechanical devices, the structures of these motors are very simple and there is no need to maintain them periodically. The consequence is that the total costs on the direct drive systems will be declined. Plus, robust controls of the motors are quite suitable for hostile environments especially in water. Hence, applying direct-drive motors is a promising way for the construction of propulsion systems. Until now, several direct-drive motors have been investigated by researchers and can be employed by electric vessels as the propulsion system [4]. Brushless direct current (BLDC) motors are popularly used by drones because of its high speed and simple control approaches. Permanent magnets (PMs) synchronized motors are often employed in contemporary industrial productions

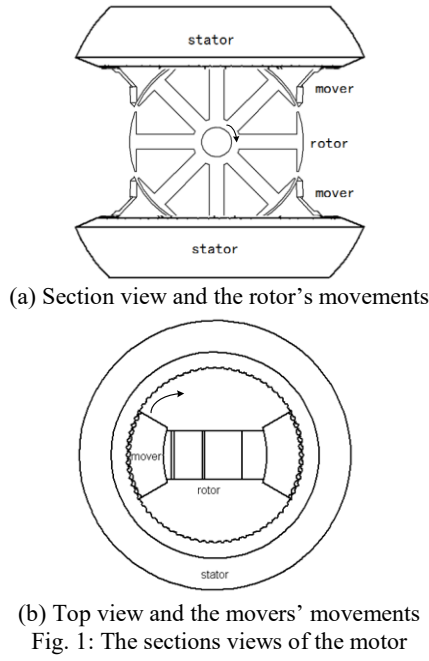
due to its stable speed and high precise positioning responses [5]. Some servo motion control systems employ stepping motors as their actuators because stepping motors are of low-cost and can be operated in an open looped way [6]. Recently, switched reluctance motors, owing simple structures and low cost due to the absence of rare earth materials, can be fixed on in-wheel driven electric vehicles [7]. However, all of the motors mentioned are only one degree of freedom, it is necessary for them to realise two-dimensional movements by using two motors together. Two or three degree of freedom actuators may appeal to some researchers and they are effective for contemporary applications in lots of fields.

There are a few direct drive two-degree of freedom motors that have been investigated by researchers or engineers nowadays [8]. The most popular is rotational and linear motor based on switched reluctance principles [9]. Their simple structures attract some researchers' attentions. But the coupled magnetic circuit structure fails to realise a high precise positioning control and is complicated by using decoupled control algorithms [10]. Apart from linear-rotary motors, some spherical motors have also been investigated recently. One of the most popular spherical motors is based on permanent magnets as synchronized motors [11]. Another one is that the motor employs a hybrid magnetic flux generation approach which consists of switched reluctance theory and permanent magnets principle [12]. However, these devices have disadvantages of low force output and short moving distance to each stroke, constraining their further applications.

In this paper, a direct drive two-degree of freedom spherical motor proposed can realize two-dimensional rotations. The two movements can be realized in a relative high speed and a low speed, respectively. To realize high efficiency in high-speed rotation and high torque outputs in low speed movements, this motor inherits the advantages of DC motors and variable stepping motors. An elaborated design method for this motor is given and discussed and, FEA is used to accurately calculate main parameters for the motor. A prototype is manufactured and some basic simulations have been carried out. The simulation results prove the effectiveness and stability of the motor and indicate that this motor is capable of being an effective drone for electric vessels.

## II. MECHANICAL STRUCTURE AND THE DESIGN OF THE MOTOR

This motor is designed as a spherical structure and it consists of four movers, a rotor and two stators as shown in figure 1. The structure is simple and Fig.1 is the section view of the motor. The mover and the stator combined can be treated as a variable reluctance stepping motor. The mover, supplying the main magnetic field with four poles for the rotor, can be designed as a DC motor working in high-speed rotation along the shaft shown in Fig.1(a). The mover can move along the curved stator as shown in Fig.1(b). The mover is fixed on the stator with curved guiders and the rotor is fixed on the rotor by using two bearings installed on the shaft.



The prototype of the motor is shown in Fig.2, including two stators, four movers and a rotor. The rotor is fixed on a shaft on which two bearings are installed. The rotor is embraced by two crusts that are fixed with the shaft by the two bearings. Four movers are screwed on the crusts evenly. The rotor and the four movers are sandwiched by the two crusts along the shaft. Four movers are divided in two groups and two stators are cascaded at the end of each group, respectively.

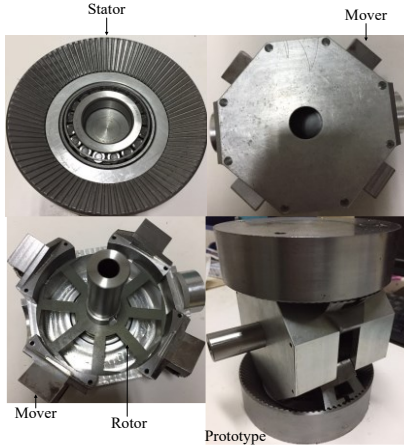


Fig. 2: The prototype of the motor

### III. PRINCIPLES OF THE MOTOR

This motor employs the rotor as the rotation part of a DC motor and the mover combining with the stator that construct a curved variable reluctance linear stepping motor. There are four movers that generate four magnetic poles. The poles are placed evenly with 45-degree interval in the space that is vertical to the shaft. The rotor is used to drive a turbine for the vessel and the mover is used to adjust the direction of the shaft. The stator has poles and slots with an equal arc length. There are four movers on which two coils are fixed. The movers can always be attracted by the teeth of the stator, when the coils of the four movers are excited in sequence. The rotor generates the rotation in another dimension. Four poles are divided operated in two groups, operated as a DC motor.

#### A. Basic operations

To reliaise a stable and robust control of the motor, PMs are not employed by the motor and it combines a variable reluctance stepping motor to adjust the direction of the shaft of the motor

and a DC motor which can realise high speed rotation to drive the vessel. The motor can be controlled in two modes that are variable reluctance mode and DC mode. Based on the principles, the stator, the movers and the rotor can be simplified as Fig.3. and the corresponding magnetic paths are given in Fig.4.

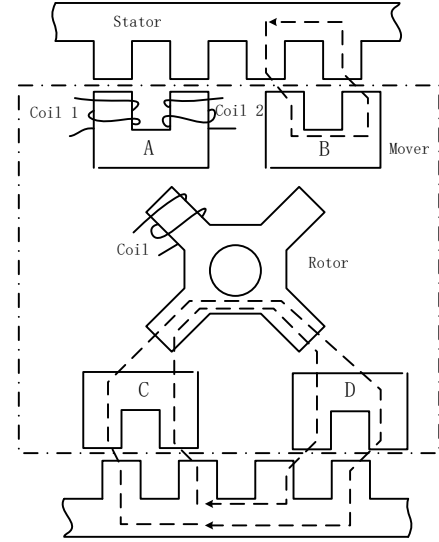
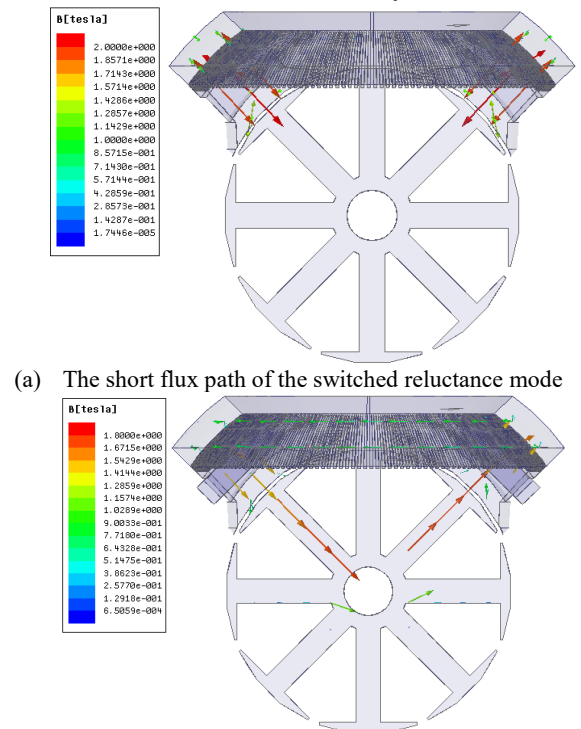


Fig. 3: A simplified structure of the motor

There are two stators and four movers (A, B, C and D) and each mover owns two coils (coil 1 and coil 2). When the two current-excited coils are connected in series, they will generate electromagnetic forces along the stator. The magnetic circuit would be very short according to the minimum reluctance principle and the flux path is shown in Fig.4 (a). The mover can move after its coils are excited in sequence. Under this situation, the motor works in variable reluctance mode. Another mode is under DC mode as shown in Fig.4 (b). The rotor of the motor is also a salient pole structure and each pole possesses a coil. Virtually, when the two coils of the mover are connected in parallel, the four movers can generate four magnetic poles and, put it another way, they form two pairs of magnetic poles for the rotor. After the coils of the rotor is excited in sequence, it can realise a rotation along the shaft. Hence, the motor can realise two dimensional movements simultaneously.



(b) The long flux path of the DC motor mode  
Fig. 4: The flux paths of the motor with two modes.

### B. Basic mathematic model

For this motor, there are two main principles followed. Since the air-gap between the movers and stators are very small, rudimentary magnetic field generated by the movers can be seen as a constant one. According to the law of  $F=BIL$ ,  $F$  is electromagnetic force.  $B$ ,  $I$  and  $L$  are the flux density, the phase current of the winding and its length, respectively. The function respecting to armature current and torque can be formulated as

$$T = K_t I_a \quad (1)$$

where  $I_a$  is the armature current and  $T$  is the torque output of the motor.  $K_t$  is a coefficient between the current and torque output. It can be seen that the torque is proportional to the current and the DC motor can be considered as a linear component under rated current if there is no local saturation in the motor. if the intensive magnetic field of the air gap between the mover and the rotor is  $B_p$ , the value of the coefficient of the motor can be calculated as

$$K_t = B_p l \quad (2)$$

And  $l$  is the stack length of the windings along the armature. Back magnetic motive force (MMF) of the armature of the DC motor can be expressed as

$$E_a = K_e \omega \quad (3)$$

$\omega$  is the angular speed of the rotor.  $K_e$  is a coefficient between the two items and it can be obtained by the following equation

$$K_e = \frac{nC \Phi_d}{2\pi m} \quad (4)$$

$\Phi_d$  is air gap flux between a pole of the rotor and the mover.  $C$  is total number of conductors in armature winding and  $m$  is the number of parallel paths of windings. Theoretically, the two coefficients of  $K_e$  and  $K_t$  are equal.

From above, it can be seen that the DC motor allows for a very simple linear relationship between armature current, torque, and velocity. The electrical expression for the DC motor can be formulated as

$$V = K_e \omega + \frac{L di}{dt} + RI \quad (5)$$

here  $V$  is the terminal voltage,  $L$  is the armature inductance, and  $R$  is the motor terminal resistance. The dynamic equation of the motor can be expressed as

$$T_r = \frac{J d\omega}{dt} + D\omega + T_f + T_L \quad (6)$$

where  $T_r$  is the generated torque,  $J$  is the sum of the moments of inertia for the motor and the load.  $D$  is the vicious damping coefficient.  $T_f$  is the internal friction torque, and  $T_L$  is the load torque.

Dynamic equation of each mover of the motor, according to the minimum reluctance principle, can be expressed as the following equation

$$T_m = J_m \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + T_l = \frac{\partial W_{co}(i, \theta)}{\partial \theta} \quad (7)$$

where torque  $T_m$  is electromagnetic force which is related to the phase current and the position of mover.  $T_l$ ,  $J_m$  and  $B$  are load force, mass of mover and damping coefficient respectively.  $W_{co}(i, \theta)$  is the co-energy of the motor and  $\theta$  is angle of the mover.

$$W_{co}(i, \theta) = \int_0^i \lambda(i, \theta) di = \int_0^i L(i, \theta) i di \quad (8)$$

$\lambda(i, \theta)$  is flux linkage of the motor and  $i$  is phase current. The value of inductance is a function with phase current and angle of mover. According to (7) and (8), we can obtain:

$$T_m = \frac{1}{2} i^2 \frac{\partial L(i, \theta)}{\partial \theta} \quad (9)$$

The inductance of the mover is determined by both the current and angle of the mover and, the torque out is nonlinear related with the current. The electrical express for the variable-reluctance motor is

$$u = R' i + \frac{d\lambda(i, \theta)}{dt} \quad (10)$$

where  $R'$ ,  $u$ , and  $i$  represent phase resistance, terminal voltage and current of phase, respectively.

### C. Analysis of magnetic circuit

Neglecting flux leakage, the half part of the whole magnetic circuit for the designed spherical motor can be simplified by Fig. 5 according to the magnetic circuit principle.

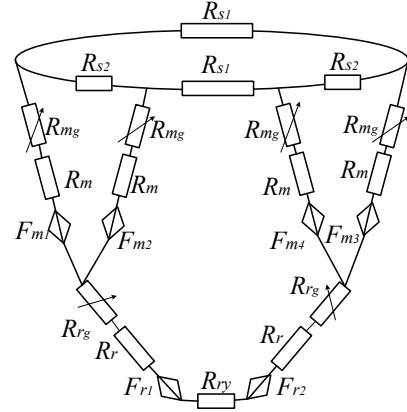


Fig. 5: Equivalent magnetic circuit of the half part of the motor

In Fig. 5,  $R_{s1}$  is the magnetic reluctance between two movers and  $R_{s2}$  is the magnetic reluctance between two poles of a mover.  $R_{mg}$  and  $R_{sg}$  denotes the magnetic reluctances of air gaps between the mover and the stator, and the rotor and mover, respectively.  $R_m$  and  $R_r$  signify the magnetic reluctances of poles of the mover and the rotor, respectively.  $R_{ry}$  is magnetic reluctance of yoke of the rotor. From  $F_{m1}$  to  $F_{m4}$  are magnetic potentials generated by four concentrated coils of the poles of two movers. Similarly,  $F_{r1}$  and  $F_{r2}$  are magnetic potentials produced by the coils of the rotor.

When the motor works in the first mode, the flux of each pole between the mover and the stator can be derived as

$$\Phi_m = \frac{F_{m1} + F_{m2}}{2(R_m + R_{mg}) + R_{s2}} \quad (11)$$

without magnetic saturation, the inductance of the two coils connecting in series can be calculated as

$$L = \frac{N\Phi_m}{i} \quad (12)$$

$N$  is conductor turns of each coil. Substituting equation (11) to (12), the inductance is

$$L = \frac{N^2}{R_m + R_{mg} + \frac{R_{s2}}{2}} \quad (13)$$

After obtaining the values of the inductance, the torque of the mover can be yielded.

Likewise, the fluxes in air gaps and inductance of the rotor working in the second mode can also be calculated

$$\Phi_s = \frac{F_{r1} + F_{r2} + F_{m2} + F_{m4}}{2(R_r + R_{rg} + R_m + R_{mg}) + R_{s1} + R_{ry}} \quad (14)$$

Then the inductance of the rotor is

$$L = \frac{N^2}{R_r + R_{rg} + R_m + R_{mg} + \frac{R_{s1} + R_{ry}}{2}} \quad (15)$$

Since the length of  $R_{rg}$  is much bigger than that of  $R_{mg}$ , the inductance of the rotor will be much less than that of the mover.

According to equation (13) and (15), the inductances of the mover and the rotor are mainly influenced by lengths of two air gaps. Therefore, designing the two air gaps plays an important role for the performance of the motor.

#### D. Air gaps determination

In order to analyse the mutual influences of the magnetic circuit for the two modes, Fig.5 can be simplified as two circuits shown in Fig. 6, with yoke magnetic reluctances from mover and stator neglected.

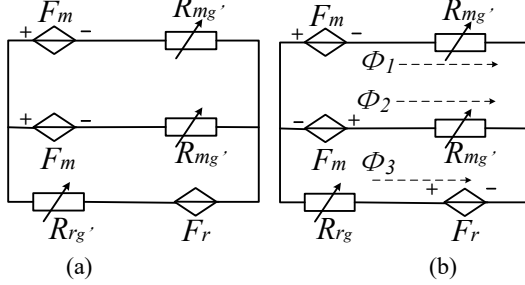


Fig. 6: Simplified model of the magnetic circuit

In the two figures, the value of  $F_m$  is half of that of  $F_{m1}$  which is equal to other corresponding potentials in Fig.5. Likewise,  $F_r$  is half of  $F_{r1}$  or  $F_{r2}$ , and  $R_{mg'}$  as well as  $R_{rg'}$  are half of the counterparts in Fig.5. we can come into a deduction for the two modes of the motor.

- When the motor works in long flux path mode, the flux generated from  $F_m$  would not go through the short flux path regardless of the lengths of those air gaps. The premise is that the air gaps in short flux path are identical and there is no saturation in the path.
- When the motor works in short flux path, the fluxes can be calculated as follows equations, assuming that the whole magnetic circuit is under a linear condition.

$$\begin{cases} \Phi_1 + \Phi_2 + \Phi_3 = 0 \\ 2F_m + \Phi_2 R_{mg'} - \Phi_1 R_{mg'} = 0 \\ F_m + \Phi_3 R_{rg'} - F_r - \Phi_1 R_{mg'} = 0 \\ F_r + \Phi_3 R_{rg'} + F_m + \Phi_2 R_{mg'} = 0 \end{cases} \quad (16)$$

Then the values of the fluxes can be obtained.

$$\begin{cases} \Phi_1 = \frac{F_m - F_r + \frac{2F_r R_{rg'}}{2R_{rg'} + R_{mg'}}}{R_{mg'}} \\ \Phi_2 = \frac{-F_m - F_r + \frac{2F_r R_{rg'}}{2R_{rg'} + R_{mg'}}}{R_{mg'}} \\ \Phi_3 = \frac{2F_r}{2R_{rg'} + R_{mg'}} \end{cases} \quad (17)$$

It can be seen that the fluxes passing through the rotor is only related by  $F_r$  and the value determined by  $F_r$  and the magnetic reluctances of the two air gaps. If  $F_r$  is zero, there is no fluxes will pass through the rotor. Therefore, the motor will be operated in the two modes only by switching the two coils of mover from parallel connection to series connection and the length of the two air gaps between the rotor and the mover, the mover and the stator can be designed easily.

#### IV. SIMULATIONS FOR THE MOTOR

The flux density of the motor is shown in Fig.7. It can be seen that the fluxes pass through the stator, the movers and the rotor. The inductance profile of the rotor is given in Fig.8. From this figure, the change of the inductance during a period is not significant because the length of the air gap between the rotor and the mover is bigger than that between the mover and the stator. Therefore, even though the rotor rotates along y

axis, the value of the inductance of the windings fixed on the rotor cannot be changed dramatically. It is suitable for the operation of the DC mode of the motor. Another feature to the inductance is that these values will decline with the increase of the current in windings of the rotor due to the physical characteristics of the soft magnetic material.

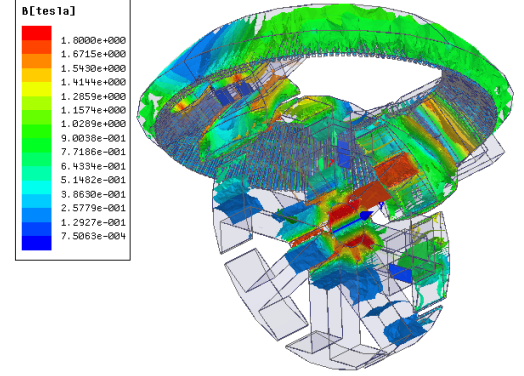


Fig. 7: The flux density of the motor

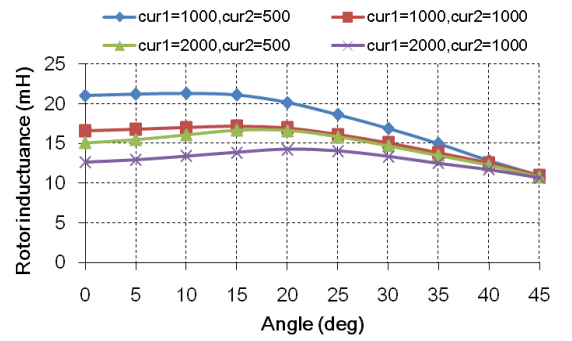


Fig. 8: Inductance profile of the rotor

The torque profile of the rotor is given in Fig.9. Torques are calculated with variety of ampere turns both in windings of movers and the rotor. Current 1 is the current in the windings of the movers and current 2 denotes the current value of windings of the rotor. When the current 1 is 1000 ampere turns, the torque of the rotor will increase with the increase of the current of the rotor windings. This trend is same to the condition that the excitation current for the mover is 2000 ampere turns. Meanwhile, it can be seen that the torque under the condition that current 1 and current 2 are 1000 ampere turns is bigger than the condition that current 1 is 2000 ampere turns and current 2 is 500 ampere turns. It suggests that the current of the windings of the rotor contributes more to the rotational torque. The maximum torque of the motor can exceed 6 N.m. Once there is no current for all windings, the motor cannot produce torques for both the mover and the rotor. If the windings of the mover are excited under rated current, the detent torque of the rotor will be generated. This detent torque of the rotor is given in Fig.10. The maximum value of the detent force is up to 1.7 N.m when the windings of the movers are excited only.

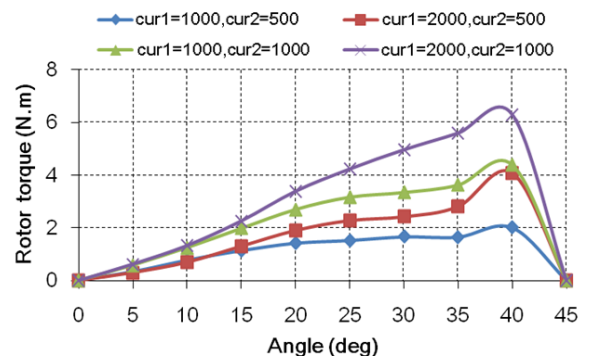


Fig. 9: Torque profile of the rotor under corresponding to angles

under different excitations

Fig.10 is the detent force of the rotor when the mover is excited under rated current

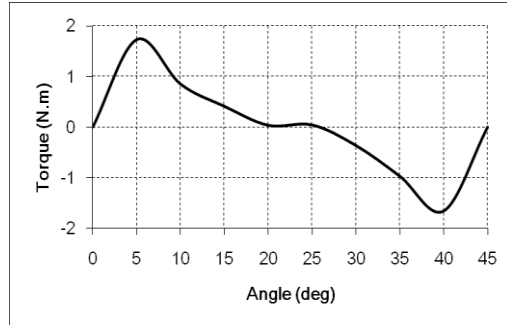


Fig. 10: The detent force of the rotor when the mover is excited under rated current

According to the 3 dimensional element finite calculations, the inductance of the mover is up to 260 mH when the motor is operated in the short flux path mode shown in Fig. 11. In the meantime, the torques of the mover under different excitations are obtained as shown in Fig.12. it can be seen that the torque of the mover to change the direction for the shaft of the mover can be up to 3 N.m.

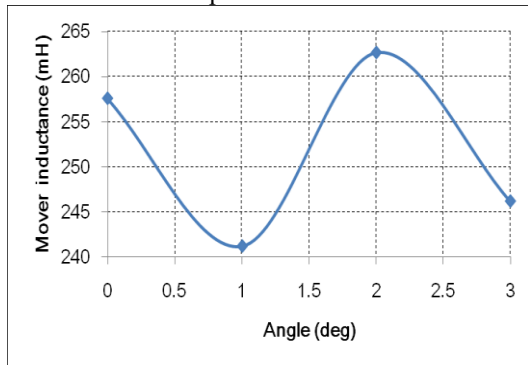


Fig. 11: Inductance of the mover versus angles

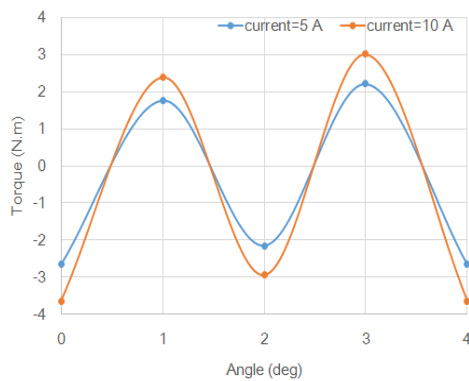


Fig. 12: The torque of the mover corresponding to different angles

#### IV. CONCLUSION

In this paper, a novel two-degree of freedom spherical motor is designed and analyzed. The structure and prototype of the motor are given to clarify the whole construction. The operation principle of the motor is introduced in the first part and its basic mathematic theories as well. More importantly, the magnetic circuit of the motor is built to elaborate the design of the motor, especially for how to determine the two air gaps that play vital roles for the entire performance and characteristics of the motor, including the self-inductances of the windings on the movers and the rotor and the torque outputs from the two dimensions. By FEA simulations, self-inductances and torque outputs from the rotor and the movers are obtained, which suggests that the motor is capable

of producing torque in two dimension simultaneously. We expect that the motor designed can be used in industrial instruments, electric vehicles and vessels, etc.

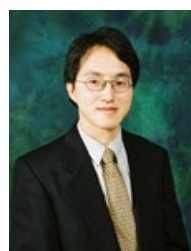
#### REFERENCES

1. K.W.E. Cheng, X. D. Xue and K. H. Chan, "Zero emission electric vessel development", International Conference on Power Electronics Systems and Applications (PESA), 2015, pp.1-5.
2. S. Y. Li and K.W.E. Cheng, "A new two-degree of freedom switched reluctance motor for electric vessel", International Conference on Power Electronics Systems and Applications (PESA), 2015, pp.1-6.
3. X. D. Xue, K. W. E. Cheng and S. L. Ho, "Optimization and evaluation of torque-sharing functions for torque ripple minimization in switched reluctance motor drives", IEEE Transactions on Power Electronics, Vol.24, no.9, 2009, pp.2076-2090.
4. X. D. Xue, K. W. E. Cheng, T. W. Ng and N. C. Cheung, "Multi-objective optimization design of In-wheel switched reluctance motors in electric vehicles", IEEE Transactions on Industrial Electronics, Vol.57, no.9, 2010, pp.2980-2987.
5. K. Matsuoka and S. Obata, "Automatic design method of brushless DC motors for VCRs", IEEE Transactions on Consumer Electronics, Vol.35, no.3, 1989, pp.642-648.
6. D. Howe and W. Low, "Design and dynamic calculations for miniature permanent magnet stepper motors", IEEE Transactions on Magnetics, Vol.20, no.5, 1984, pp.1768-1770.
7. X. D. Xue, K. W. E. Cheng and Y. J. Bao, "Control and integrated half bridge to winding circuit development for switched reluctance motors", Vol.10, no.1, 2014, pp. 109-116.
8. J. F. Pan, Yu Zou and Norbert C. Cheung, "Performance analysis and decoupling control of an integrated rotary-linear machine with coupled magnetic paths", IEEE Transactions on Magnetics, Vol.50, no.2, 2014, pp.7018804.
9. J. F. Pan, N. C. Cheung and Guangzhong Cao, "A rotary-linear switched reluctance motor", 2009 3rd International Conference on Power Electronics Systems and Applications (PESA), 2009, pp.1-5.
10. Jianfei Pan, Fanjie Meng and Guangzhong Cao, "Decoupled control for integrated rotary-linear switched reluctance motor", IET Electric Power Applications, Vol.8, no.5, 2014, pp.199-208.
11. Masahito Tsukano, Yo Sakaidani, Katsuhiro Hirata, Noboru Niguchi, Shuhei Maeda and Ariff Zaini, "Analysis of 2-degree of freedom outer rotor spherical actuator employing 3-D finite element method", IEEE Transactions on Magnetics, Vol.49, no.5, 2013, pp.2233-2236.
12. Shuhei Maeda, Katsuhiro Hirata and Noboru Niguchi, "Dynamic analysis of an independently controllable electromagnetic spherical actuator", IEEE Transactions on Magnetics, Vol.49, no.5, 2013, pp.2263-2266.

#### ACKNOWLEDGMENT

The author gratefully acknowledge of the financial support of the Research Office, The Hong Kong Polytechnic University under the project number G-YN27.

#### BIOGRAPHY



**K.W.E.Cheng** obtained his BSc and PhD degrees both from the University of Bath in 1987 and 1990 respectively. Before he joined the Hong Kong Polytechnic University in 1997, he was with Lucas Aerospace, United Kingdom as a Principal Engineer. He received the IEE Sebastian Z De Ferranti Premium Award (1995), outstanding consultancy award (2000), Faculty Merit award for best teaching (2003) from the University and Silver award of the 16<sup>th</sup> National Exhibition of Inventions. He has published over 200 papers and 7 books. He is now the professor and director of Power Electronics Research Centre.