This is the Pre-Published Version.

A Coordinated Dispatch Model for Distribution Network Considering PV Ramp

Jiayong Li, *Student Member*, *IEEE*, Zhao Xu, *Senior Member*, *IEEE*, Jian Zhao, *Student Member*, *IEEE*, Can Wan, *Member*, *IEEE*

Abstract—**The ramp events of photovoltaic (PV) generation will cause severe voltage variations in a distribution network. To address this issue, a two-stage robust optimization based intra-hour dispatch model is proposed for the coordination of the on-load tap changer and smart PV systems. Hence, the distribution network security can be ensured by adjusting substation voltage and controlling PV systems. Case study on 33-bus system verifies the effectiveness of the proposed model.**

Index Terms—**Coordination, PV ramp, distribution networks, voltage violations.**

I. Introduction

PV generation ramp event (PRE) occurs when solar irradiance suddenly changes due to cloud movement, which may lead to voltage violations in distribution networks (DNs), especially when the PV generation penetration is high [1]. These PREs are very difficult to be predicted due to the chaotic nature of the climate system. To address the PREs-induced voltage violation issues, the coordination between on-load tap changer (OLTC) and smart PV systems will be investigated.

This letter proposes a two-stage robust optimization based intra-hour dispatch model for the coordination of OLTC and admissibility-restricted PV systems. In the first stage, maximum admissible PV outputs (MAPO) and the OLTC are co-optimized to reinforce the coordination, where MAPO is proposed to quantify how much PV generation can be accommodated to ensure the DN security against PREs. In the second stage, the feasibility of the first-stage decision variables is evaluated for any realization of PREs. Finally, the column-and-constraint generation (CCG) algorithm is utilized to solve the model.

II. Robust Intra-hour Dispatch Model

A. An Enhanced PV Inverter Control Strategy

The PV inverters can support voltage regulations through curtailing renewable energy and absorbing/releasing reactive power [2]. An enhanced inverter control strategy is proposed herein with MAPO interpreted as a security bound. MAPO is regarded as an input of the PV inverter controller, where the excessive active power beyond MAPO is curtailed to prevent voltage violations. The set of PV inverter operating points is given by

$$\begin{aligned}\Phi_j = \{(p^s_{j,t}, q^s_{j,t}) : p^s_{j,t} = \min(P^{apo}_{j,t}, p^a_{j,t}) \ \text{(1a)}, \ \left|q^s_{j,t}\right| \le Q^{\max}_j \ \text{(1b)}, \\ -\tan\theta_j \cdot p^s_{j,t} \le q^s_{j,t} \le \tan\theta_j \cdot p^s_{j,t} \ \text{(1c)} \ \}, \text{where } Q^{\max}_j = \sqrt{S^2_j - (P^{\max}_j)^2}\end{aligned} \tag{1}$$

where $p^s_{j,t}$ and $q^s_{j,t}$ are active and reactive power set-points of j-th PV inverter at t-th time interval, S_j and $P^{\max}_j$ are the rated apparent power of inverter and the PV panel active power capacity. (1a) is to determine $p^s_{j,t}$ by selecting the smaller value between $P^{apo}_{j,t}$ and $p^a_{j,t}$, where $P^{apo}_{j,t}$ is maximum admissible PV output and $p^a_{j,t}$ is the maximum available active power, obtained from maximum power point tracking technique. (1b) is to keep the reactive power support below the upper bound $Q^{\max}_j$. (1c) describes the minimum power factor requirement according to the corresponding power factor angle θ_j.

B. Modeling PV Ramp Events

The uncertainty set of PRE is modeled as

$$\begin{aligned}U = \{p^a_{j,t} \mid z_{j,t} \in \{0,1\}, \ \ p^a_{j,t} = (1 - z_{j,t})P^f_{j,t} + z_{j,t}P^L_{j,t} \ \forall j, \ \forall t \ \text{(2a)}, \\ \sum_j z_{j,t} \le \Gamma_t \ \ \forall t \ \text{(2b)}, \sum_{t \in \mathrm{T}} z_{j,t} \le \Gamma_j \ \ \forall j \ \ \text{(2c)}\}\end{aligned} \tag{2}$$

where $z_{j,t}$ is a binary with 1 indicating an PRE, $P^f_{j,t}$ is forecasted PV output without considering PRE, $P^L_{j,t}$ is the lower bound of PRE. Γ_t and Γ_j are spatial and temporal uncertainty budgets. (2b) and (2c) describe the uncertainty budget limitation constraints.

C. Two-Stage Coordinated Intra-Hour Dispatch Model

A two-stage coordinated intra-hour dispatch model (CID) is formulated to maximize PV accommodation capability. The horizon and resolution are 1 hour and 5 minutes, respectively. The formulation of CID is given as follows,

$$\max_{P^a_{i,t}, V_0} \sum_{t=1}^{T} \sum_{j=1}^{n_s} (P^{apo}_{j,t}) \tag{3}$$

$$s.t. \ \ P^{apo}_{j,t} \le P^f_{j,t} \ \ \forall j, \ \forall t \tag{4}$$

$$V_0 = \sum_{k=1}^{n_r} V^0_k \cdot \chi_k \ , \ \chi_k \in \{0,1\} \ , \ \sum_{k=1}^{n_r} \chi_k = 1 \tag{5}$$

$$\forall \{p^a_{j,t}\} \in U, \ \exists \ \{p_{0,t}, q_{0,t}, p^s_{j,t}, q^s_{j,t}\}, \text{ such that}$$

$$(p^s_{j,t}, q^s_{j,t}) \in \Phi_j \ \ \ \forall j, \ \forall t \tag{6}$$

$$p_{0,t} = \sum_{j:0 \to j} P_{0j,t} \ \ \forall t, \ \ p^s_{\phi(i),t} - p^l_{i,t} = \sum_{j:i \to j} P_{ij,t} - \sum_{k:k \to i} P_{ki,t} \ \ \ \forall i \ /0 \ \forall t \tag{7}$$

$$q_{0,t} = \sum_{j:0 \to j} Q_{0j,t} \ \ \forall t, \ \ q^s_{\phi(i),t} - q^l_{i,t} = \sum_{j:i \to j} Q_{ij,t} - \sum_{k:k \to i} Q_{ki,t} \ \ \ \forall i \ /0 \ \forall t \tag{8}$$

$$V_{j,t} = V_{i,t} - (r_{ij}P_{ij,t} + x_{ij}Q_{ij,t}) / V_r \tag{9}$$

$$V_{0,t} = V_0 \ \ \forall t \ \text{(10a)}, \ V^{\min} \le V_{i,t} \le V^{\max} \ \ \ \forall i \setminus 0, \ \forall t \ \text{(10b)} \tag{10}$$

where the objective (3) is to maximize total MAPOs. (4) keeps MAPO smaller than the forecasted PV values. (5) represents the voltage regulation of OLTC, where V_0 denotes voltage at the substation bus; χ_i is a binary with 1 indicating the i-step of OLTC is selected and V^0_k is the corresponding voltage at step i. (6)-(10) describe the PV-embedded robust distribution network power flow for any realization of PRE, developed from linear DistFlow model [3]. Particularly, (6) restricts the operating points of PV inverters within the allowable regions. (7) and (8) describe the active and reactive power balance at the substation bus and other buses, respectively, where $p_{0,t}$ and $q_{0,t}$ are active

This work was partially supported by Hong Kong RGC Theme Based Research Scheme Grants No. T23-407/13N and T23-701/14N. The work of C. Wan was supported by the Hundred Talents Program of Zhejiang University.

J. Li, Z. Xu and J. Zhao are with the Department of Electrical Engineering, The Hong Kong Polytechnic University, Hung Hom, Hong Kong. (emails: j-y.li@connect.polyu.hk, eezhaoxu@polyu.edu.hk, zhaojianzju@gmail.com).

C. Wan is with the College of Electrical Engineering, Zhejiang University, Hangzhou 310027, China (email: canwan@zju.edu.cn).

The following publication J. Li, Z. Xu, J. Zhao and C. Wan, "A Coordinated Dispatch Model for Distribution Network Considering PV Ramp," in IEEE Transactions on Power Systems, vol. 33, no. 1, pp. 1107-1109, Jan. 2018 is available at https://doi.org/10.1109/TPWRS.2017.2735901.

and reactive power injection at the substation bus; $\phi(i)$ is to map bus index i to PV index j; $P_{ij,t}$ and $Q_{ij,t}$ are active and reactive power flow on branch (i, j); $p^l_{i,t}$ and $q^l_{i,t}$ are active and reactive load at bus i. (9) describes the voltage drops on branches, where $V_{i,t}$ and V_r denote the voltage magnitude at bus i and the reference value; r_{ij} and x_{ij} are resistance and reactance of branch (i, j). (10) denotes the voltage limit constraints at the substation bus and other buses, respectively, where $V^{\min}$ and $V^{\max}$ are the lower and upper bounds. To quantify the infeasibility, the second stage problem (6)-(10) is reformulated as (11) by introducing two positive slack variables $s^l_{i,t}$ and $s^u_{i,t}$.

$$\max_{p^a_{j,t}\in U(\boldsymbol{x})} \min_{y,s^l_{i,t}\geq 0,s^u_{i,t}\geq 0} \sum_{i,t}(s^l_{i,t}+s^u_{i,t})$$

$$s.t.\ V_{i,t}+s^l_{i,t}\geq V^{\min},\ V_{i,t}-s^u_{i,t}\leq V^{\max}\ \ \forall i\setminus 0,\forall t,\ (6)-(10a) \quad (11)$$

where $\boldsymbol{x}$ represents first stage variables, and $\boldsymbol{y}$ denotes the second stage variables. The positive optimal value of (11) implies the infeasibility of the first stage decision variables.

D. Solution Method and Algorithm

The CCG method is used to decompose CID into a master problem and a sub-problem. For clarity, the master problem is written compactly as

$$\max_{\mathbf{x}=[\mathbf{x}_1^T\ \mathbf{x}_2^T]^T} \mathbf{c}^T\mathbf{x}_1$$

$$s.t\ \mathbf{A}\mathbf{x}_1\leq\mathbf{b}\ ,\ \mathbf{B}\mathbf{x}_2=\mathbf{d},\ \mathbf{x}_2\in\{0,1\},\&\text{CCG cuts} \quad (12)$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are continuous and binary variables of the first stage. The sub-problem (11) is written compactly as

$$\max_{\mathbf{u}\in U(\boldsymbol{x})} \min_{\mathbf{y},\mathbf{s}\geq 0} \mathbf{1}^T\mathbf{s}\,,$$

$$s.t\ \mathbf{C}\mathbf{y}+\mathbf{s}\leq\mathbf{f}\ ,\ \mathbf{D}\mathbf{y}+\mathbf{E}\mathbf{u}+\mathbf{F}\mathbf{x}\leq\mathbf{g} \quad (13)$$

where $\mathbf{s}$ is slack vector; $\mathbf{1}$ is a $N-1$ dimensional vector with each element being 1; N is the number of buses and $\mathbf{u}$ is uncertainty.

Replacing the inner-level problem with its dual, problem (13) can be reformulated as

$$\max_{\mathbf{u}\in U(\mathbf{x}),\boldsymbol{\lambda},\boldsymbol{\omega}} -\mathbf{f}^T\boldsymbol{\lambda}+\mathbf{u}^T\mathbf{E}^T\boldsymbol{\omega}+\mathbf{x}^T\mathbf{F}^T\boldsymbol{\omega}-\mathbf{g}^T\boldsymbol{\omega}$$

$$s.t\ \mathbf{C}^T\boldsymbol{\lambda}+\mathbf{D}^T\boldsymbol{\omega}=\mathbf{0},\ \mathbf{1}+\boldsymbol{\lambda}\geq\mathbf{0},\ \boldsymbol{\lambda}\geq\mathbf{0},\ \boldsymbol{\omega}\geq\mathbf{0} \quad (14)$$

Note that (14) is a bilinear problem that can be linearized with big-M method. Suppose $\mathbf{u}^*$ is an optimal solution of (14), then the CCG constraints are generated as

$$\mathbf{C}\mathbf{y}^l\leq\mathbf{f}\ ,\ \mathbf{D}\mathbf{y}^l+\mathbf{E}\mathbf{u}^*+\mathbf{F}\mathbf{x}\leq\mathbf{g} \quad (15)$$

where $\mathbf{y}^l$ is newly created variables at l-th iteration.

The Algorithm of the proposed model is given as

Step 1: Set iteration index k=0

Step 2: Solve problem (12) and obtain the optimal solution $\mathbf{x}^*$

Step 3: Fix $\mathbf{x}$ in (14) at $\mathbf{x}^*$ and solve problem (14). Obtain the optimal value $F(\mathbf{x}^*)$ and optimal solution $\mathbf{u}^*$.

Step 4: Check the Convergence. If $F(\mathbf{x}^*)$=0, return $\mathbf{x}^*$ and stop; otherwise add constraints (15) to (12), update k=k+1 and return to Step 2.

III. Case Study

The CID is tested on IEEE 33-bus distribution network [4], where 8 PV arrays are connected at bus 14, 15, 16, 17, 21, 24, 31 and 32, respectively. The capacity of each PV array is 800 kW. The PV solar power is calculated using the actual solar irradiance data provided by [5]. The lower bound of PV output is set as 20% of forecasted value. The algorithm is implemented on MATLAB with CPLEX 12.6 solver.

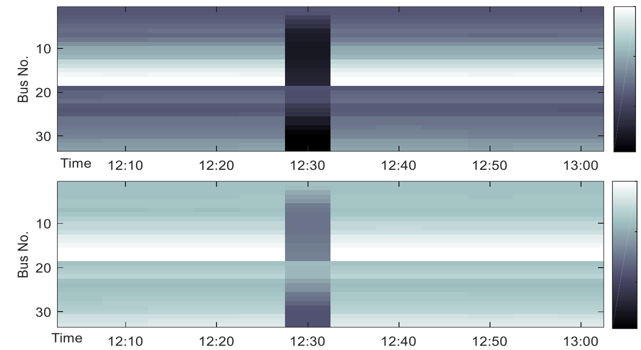

Fig. 1. (a) Bus voltage under DM(top). (b) Bus voltage under CID(bottom).

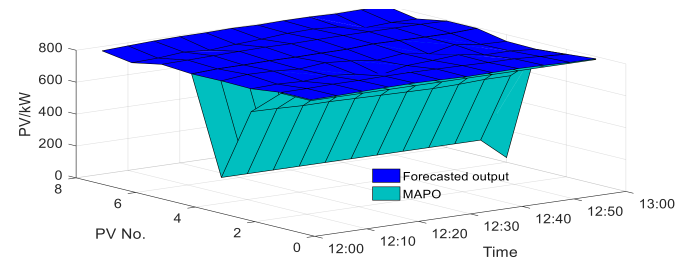

Fig. 2. Maximum PV output and forecasted PV output

Deterministic model (DM) is used as a benchmark, whose objective is to minimize PV curtailment without considering the PREs. The voltages at each bus using DM and CID are compared in Fig. 1. Severe undervoltage is observed when a PRE takes place during 12:30 to 12:35 with DM. In contrast, the bus voltage magnitudes under CID are always within the acceptable ranges. It is because MAPO is coordinated with OLTC to ensure the distribution network security. Therefore, CID outperforms DM in preventing PRE-induced voltage violations. Particularly, MAPO as well as the forecasted PV output are plotted in Fig.2. It is observed that MAPO remains the same with the forecast value for most PV inverters and the PV curtailment is relatively small (6.3% of total PV generation), which validates the effectiveness of the proposed model in coping with the PREs.

IV. Conclusion

This letter proposes a novel two-stage coordinated intra-hour dispatch model for DN considering PV ramp events. Case study verifies the effectiveness of the proposed model in addressing the PRE-induced voltage violations.

References

[1] E. Dall'Anese, S. V. Dhople, and G. B. Giannakis, "Optimal dispatch of photovoltaic inverters in residential distribution systems," *IEEE Transactions on Sustainable Energy,* vol. 5, pp. 487-497, 2014.

[2] T. Ding, C. Li, Y. Yang, J. Jiang, Z. Bie, and F. Blaabjerg, "A two-stage robust optimization for centralized-optimal dispatch of photovoltaic inverters in active distribution networks," *IEEE Transactions on Sustainable Energy,* vol. PP, pp. 1-1, 2016.

[3] H. G. Yeh, D. F. Gayme, and S. H. Low, "Adaptive VAR control for distribution circuits with photovoltaic generators," *IEEE Transactions on Power Systems,* vol. 27, pp. 1656-1663, 2012.

[4] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Transactions on Power Delivery,* vol. 4, pp. 1401-1407, 1989.

[5] "Hong Kong Observatory," [online]. Available: http://www.hko.gov.hk/.